Turbulent Heating and Fluctuation Characteristics in Alfvenic Turbulence

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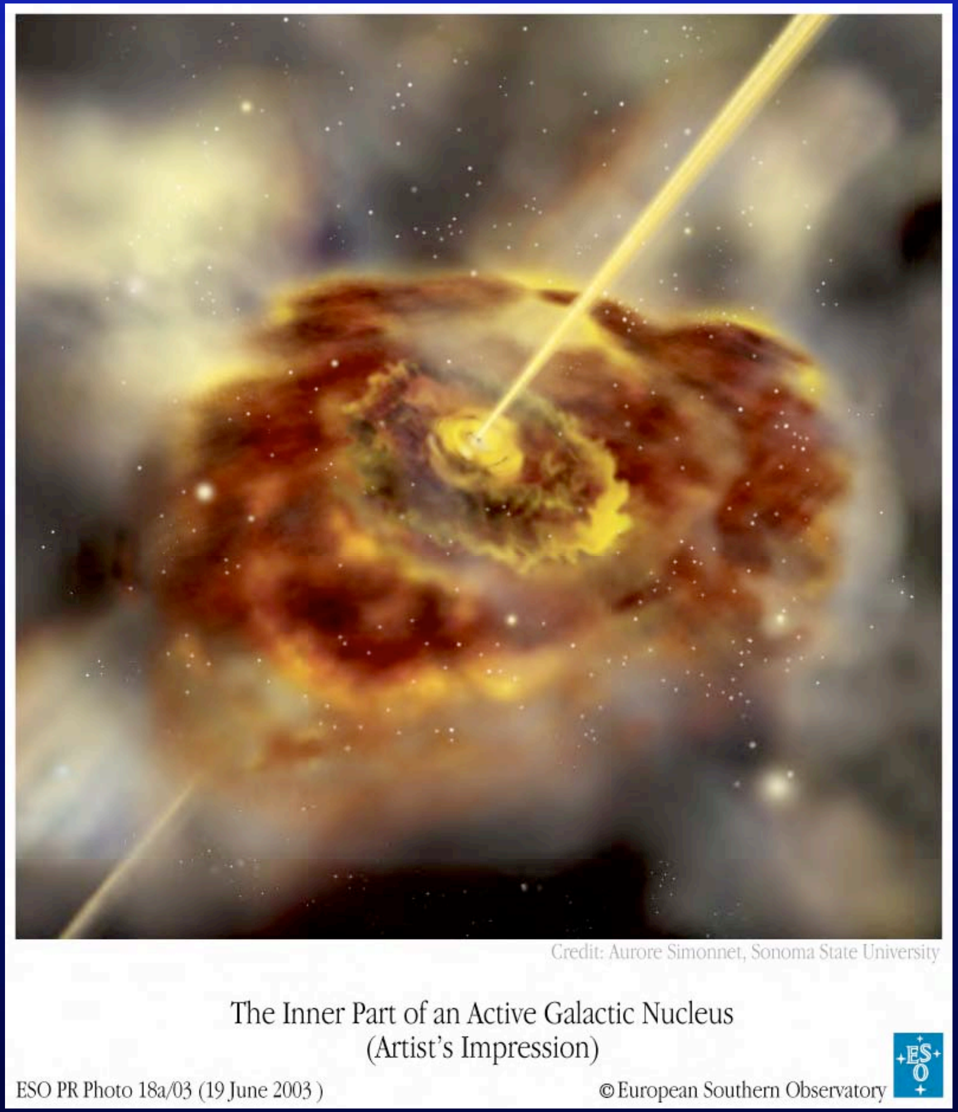
Center for Multiscale Plasma Dynamics

Extending established first-principles, microscopic, kinetic simulation techniques to problems that intrinsically involve the slow evolution of the macroscopic system, and validating simulations against experimental observations.

http://cmpd.umd.edu

Astrophysical Plasmas are Turbulent



- **Magnetohydrodynamic turbulence driven at very large scales: transport is MHD process**
- **Turbulent energy absorbed at microscopic scales: heating is gyrokinetic process**
- **Electrons radiate; ions swallowed: Emitted radiation is strong function of gyrokinetic physics!**

Project Overview

- Many examples of MHD turbulence in nature:

  Interstellar medium, solar wind, accretion flows, *etc.*
- Alfvenic component does not directly heat plasma $(E_{\|} = 0)$
- Fundamental question:

  How is MHD turbulence dissipated in a collisionless plasma?

$$\omega = \pm k_{\|} v_A$$

Alfven

$$\omega = \pm k_{\|} v_A \left(\frac{\gamma\beta}{\gamma\beta + 1} \right)^{1/2}$$

Slow magnetosonic

$$\omega = \pm k_{\perp} v_A \left(\gamma\beta + 1 \right)^{1/2}$$

Fast magnetosonic

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Slow magnetosonic

$$\cancel{\omega = \pm k_{\perp} v_A \left(\gamma\beta + 1\right)^{1/2}}$$

~~Fast magnetosonic~~

Ignore fast wave for this talk

So no fast shocks

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Slow magnetosonic

$$4\pi \delta p_\perp + B\,\delta B = 0$$

Pressure balance

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Alfven

$$\cancel{\omega = \pm k_{\|} v_A \left(\frac{\gamma\beta}{\gamma\beta + 1} \right)^{1/2}}$$

Slow magnetosonic
is damped when
$k_{\|} \lambda_{mfp} \sim 1$

Barnes, 1966

Project Overview

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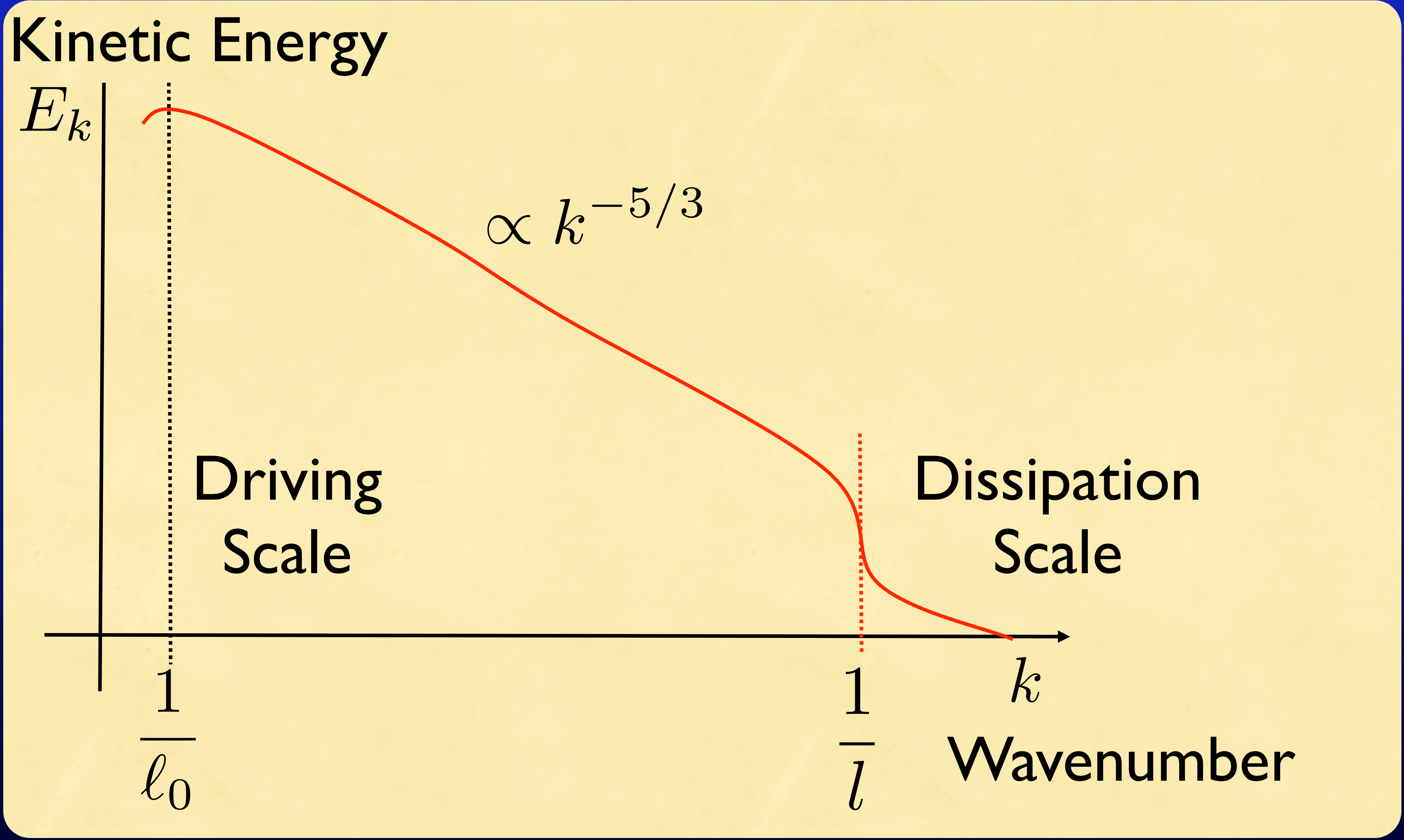
Alfven

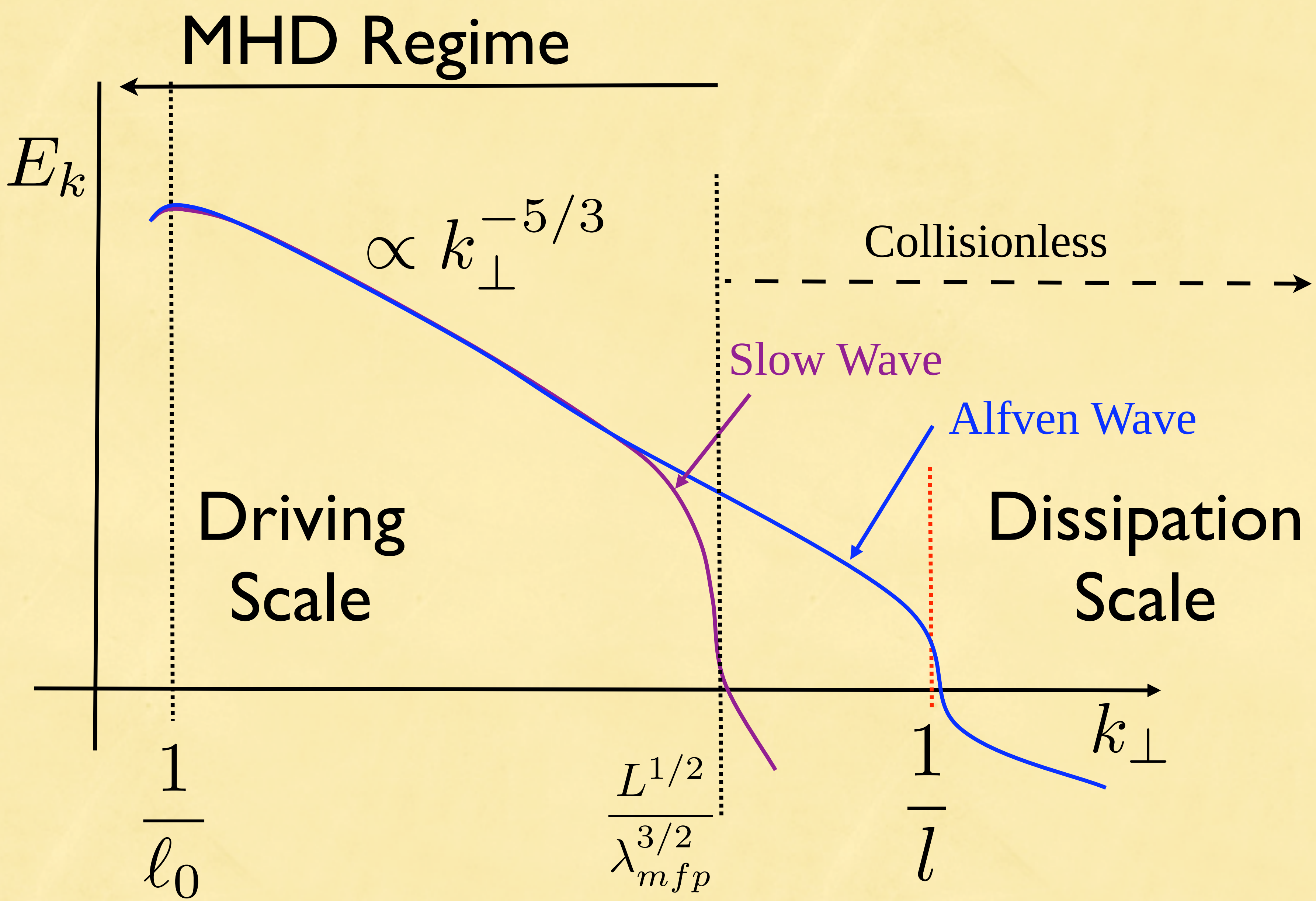
Alfven waves cascade to small scales without damping.

In this talk, we consider the fate of the Alfven wave cascade at small scales.

Turbulence Spectrum: Fluid Theory



Turbulence Spectrum: MHD Theory



Strongly Turbulent Alfven Cascade is Anisotropic

Goldreich & Sridhar, 1995

- Alfven waves stirred at scale ℓ_0 with velocity v_0

Energy flux from scale ℓ to scale $\ell/2$

$$\epsilon = \frac{\rho v_\ell^2}{2\tau_\ell}$$

Constant from scale to scale

Cascade time $\tau_\ell = \frac{\ell}{v_\ell}$ strong interaction $\rightarrow v_\ell = v_0 \left(\frac{\ell}{\ell_0}\right)^{1/3}$

GS argue that the scale ℓ is the perpendicular scale and that the parallel scale is set by "critical balance."
Nonlinear rate = Linear rate

$$\frac{v_\ell}{\ell} = k_\| v_A$$

$$k_\| = \frac{v_0}{v_A} k_\perp^{2/3} \ell_0^{-1/3} \ll k_\perp$$

Smaller scales are progressively more anisotropic

$$\frac{k_\parallel}{k_\perp} \simeq \left(\frac{v_0}{v_A}\right) (k_\perp \rho_i)^{-1/3} \left(\frac{\rho_i}{\ell_0}\right)^{1/3}$$

- Anisotropy goes like $(\rho_i/\ell_0)^{1/3}$
- Small parameter for astrophysical systems:

* ISM: $(\rho_i/\ell_0)^{1/3} \sim 10^{-4}$

* SMBH Accretion Disk: $(\rho_i/\ell_0)^{1/3} \sim 10^{-3}$

* Solar Wind: $(\rho_i/\ell_0)^{1/3} \sim 0.02$

- Frequency also scales with $(\rho_i/\ell_0)^{1/3}$:

$$\frac{\omega}{\Omega_i} \simeq \left(\frac{v_0}{v_{th}}\right) (k_\perp \rho_i)^{2/3} \left(\frac{\rho_i}{\ell_0}\right)^{1/3}$$

Gyrokinetics 101

Taylor and Hastie; Catto; Antonsen and Lane; Frieman and Chen;; Dubin

- Vlasov-Boltzmann-Maxwell eqns in low frequency, anisotropic limit

$$\frac{Df}{Dt} = C(f,f) \qquad \frac{\omega}{\Omega_c} \ll 1 \qquad \frac{k_\parallel}{k_\perp} \ll 1$$

- Expansion parameter is $\epsilon = \frac{\rho}{L} \sim \frac{\omega}{\Omega} \sim \frac{k_\parallel}{k_\perp}$ $\qquad \frac{\rho}{L} \ll 1$
- Perturbations ordered small:

$$f = F_0 + \delta f \qquad \frac{\delta f}{F_0} \sim \frac{v_E}{v_{th}} \sim \frac{\delta B}{B} \sim \epsilon$$

- Rigorously correct, fully nonlinear kinetic physics in this limit
- No particular ordering of $\beta, k_\perp \rho, T_i/T_e, m_e/m_i, \nu/\omega$, etc.
- Time evolution of equilibrium allowed at order ϵ^3

but not correctly calculated, until now (multiscale physics!)

Gyrokinetic Equation, Ordering

Small perturbations $$f = F_0 + \delta f$$

Evolve only non-adiabatic part $$\delta f = h + \frac{q}{T}\langle \Phi \rangle F_0$$

$$\frac{\partial h}{\partial t} + \frac{c}{B}\left[\langle \chi \rangle, h\right] + v_{||}\hat{b}\cdot\nabla h = \frac{q}{T}\frac{\partial \langle \chi \rangle}{\partial t}F_0 + \mathcal{C}(h)$$

Nonlinear gyrokinetic equation

Gyrokinetic Equation, Ordering

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$$\frac{dn_0}{dt} = 0$$

$$\frac{3}{2}n_0\frac{dT_0}{dt} = q\int d^3x\int d^3v\, h\frac{\partial\chi}{\partial t}$$

Slow evolution of background

Gyrokinetic Equation, Ordering

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$$\frac{dn_0}{dt} = 0$$

$$\frac{3}{2}n_0\frac{dT_0}{dt} = q\int d^3x \int d^3v\, h\frac{\partial\chi}{\partial t}$$

$$\frac{3}{2}n_0\frac{dT_0}{dt} = -T_0\int d^3x \int d^3v\, \frac{h\mathcal{C}(h)}{F_0}$$

Thermodynamics: heating properly related to entropy change

Gyrokinetic Equation, Ordering

Small perturbations $$f = F_0 + \delta f$$

Evolve only
non-adiabatic part $$\delta f = h + \frac{q}{T}\langle\Phi\rangle F_0$$

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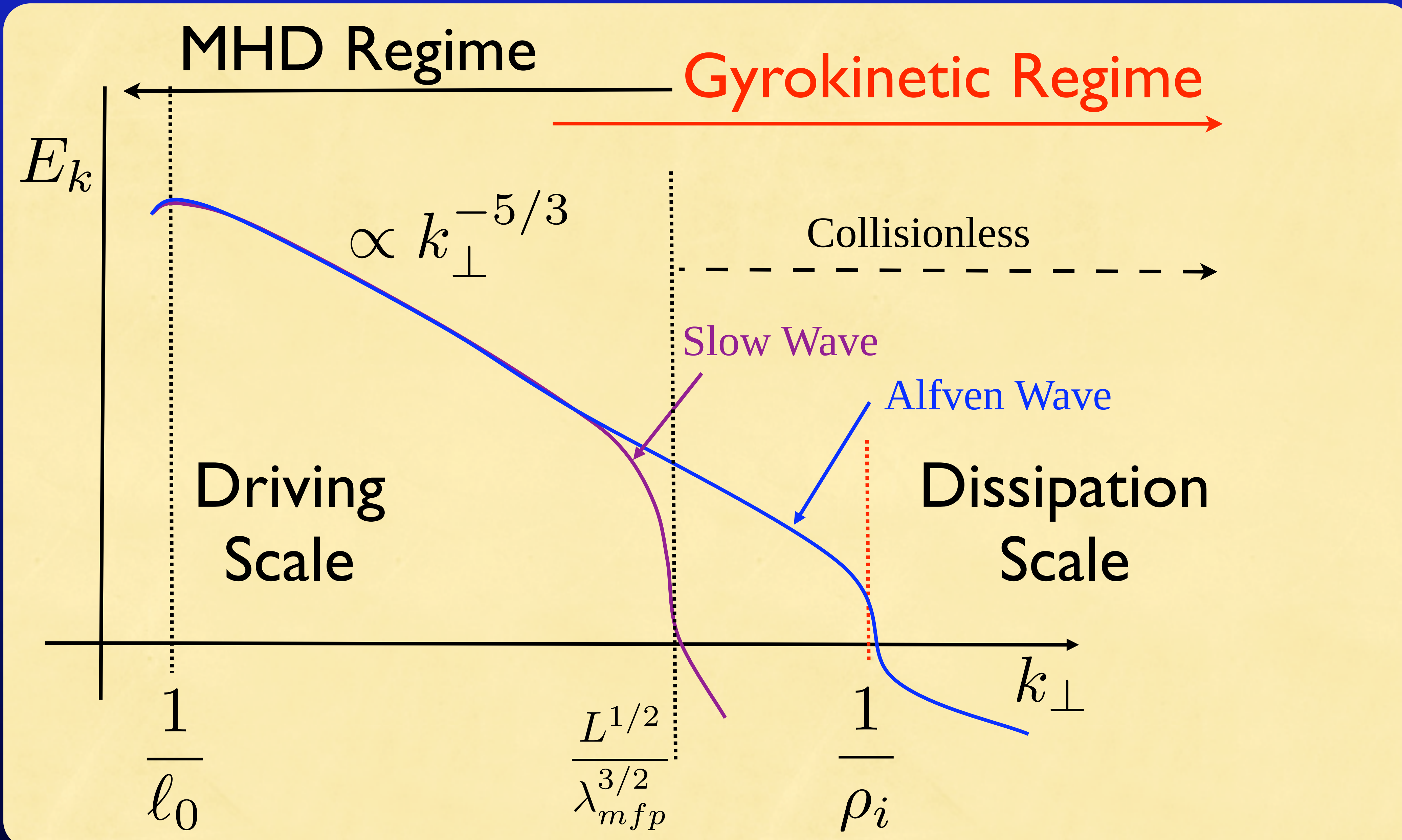
$$\frac{3}{2}n_0\frac{dT_0}{dt} = -T_0\int d^3x\int d^3v\,\frac{h\mathcal{C}(h)}{F_0}$$

$$\mathcal{E} = \int\frac{d^3x}{V}\left\{\int d^3v\sum_s\left[\frac{T_0\delta f^2}{2F_0}\right] + \frac{(\delta B)^2}{8\pi}\right\}$$

Nonlinearly conserved
gyrokinetic energy identified

Turbulence Spectrum: MHD Theory



Collisionless absorption at gyroradius scales in turbulent plasma

- Anistropic cascade brings in gyrokinetics
- Quataert & Gruzinov calculated linear damping of shear Alfven waves using hot plasma dispersion relation; predicted conditions for strong absorption by ions (or electrons)
- Problem has appealing universal aspect: absorption is unlikely to be a function of large-scale physics. Absorption is a function of only a few plasma parameters
- Excellent opportunity to solve a basic problem in plasma physics; hard b/c high accuracy required, 5-D

Remainder of this talk

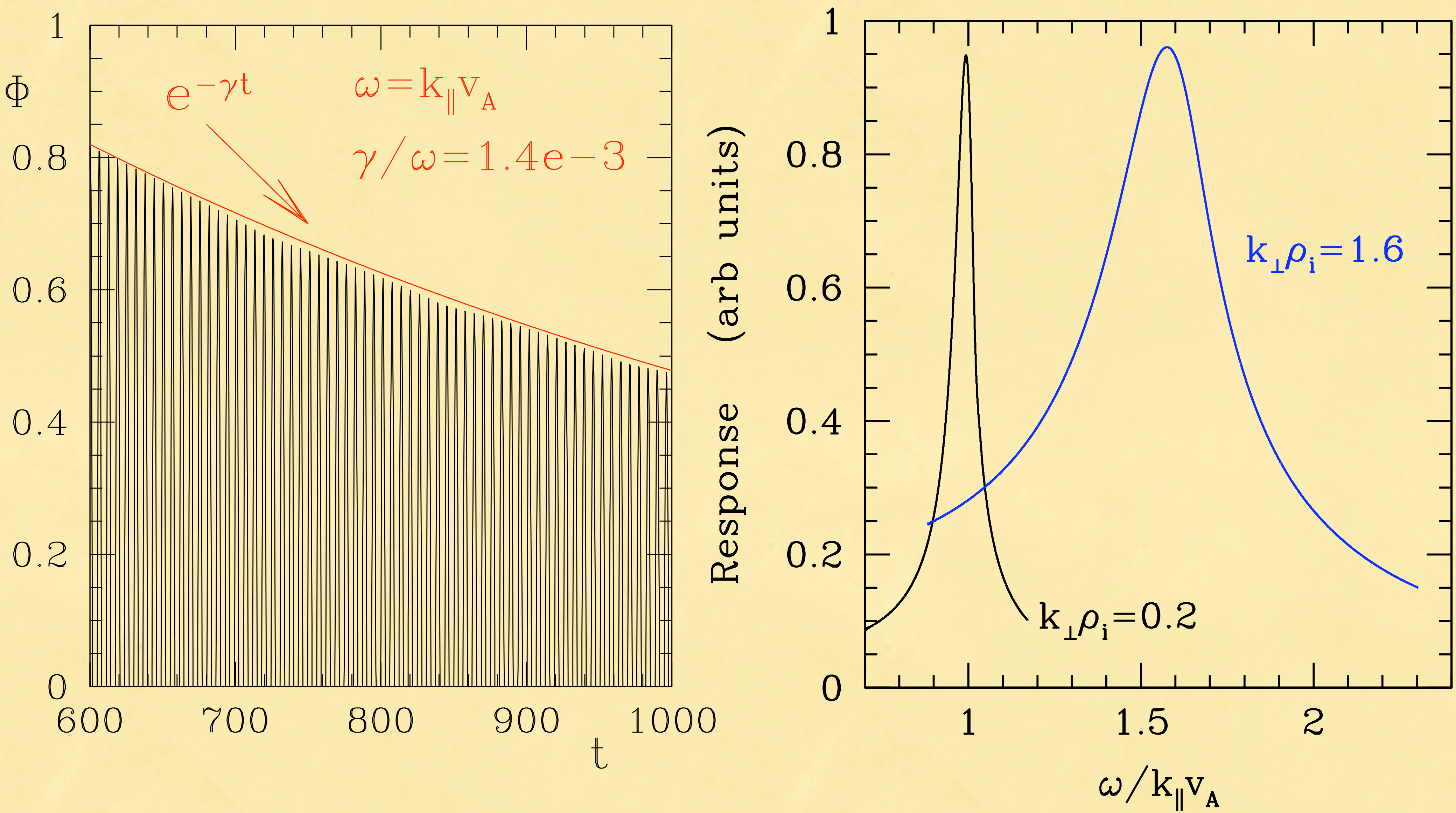
- Linear physics: Demonstration that GS2 properly calculates damping, heating
- Nonlinear physics
  - ➡ Alfven cascade

    (scales large compared to ion gyroradius)
  - ➡ Kinetic Alfven cascade

    (scales between ion gyroradius and electron gyroradius)
  - ➡ Transition region: absorption, spectra

    (ion gyroradius scale)
  - Conclusions

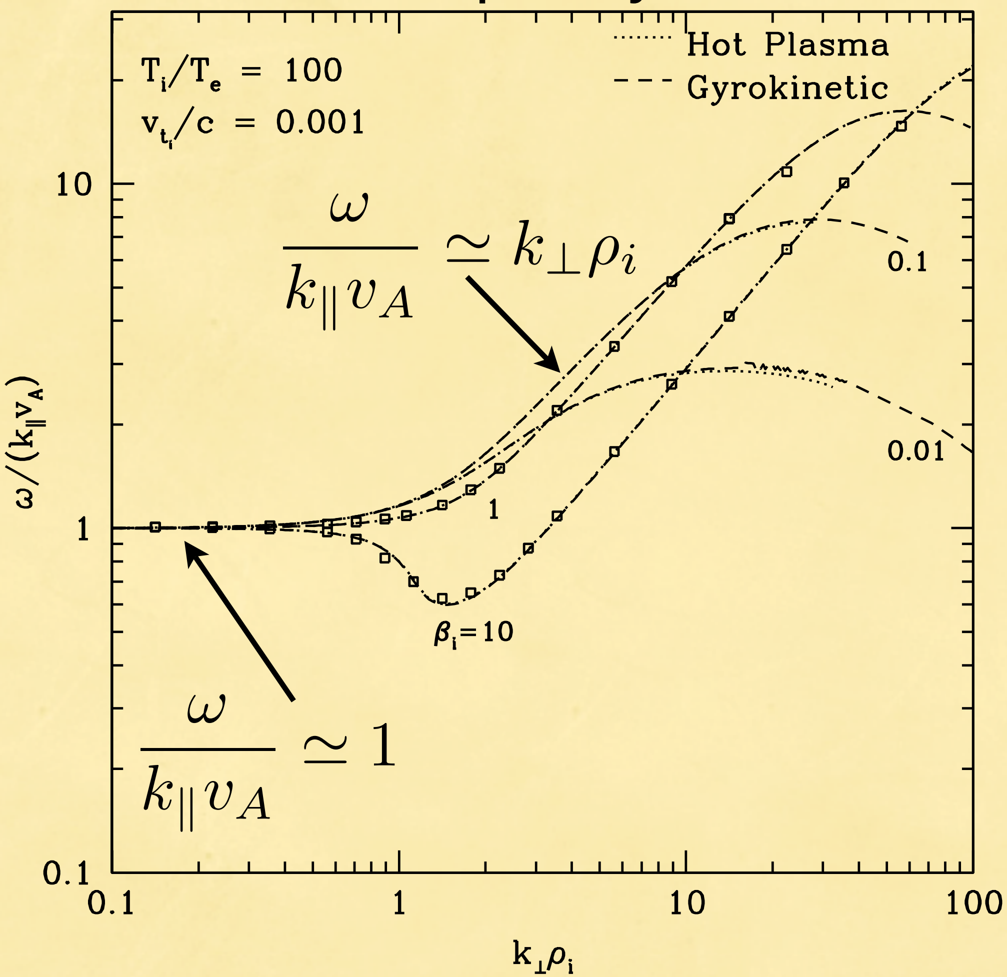
Landau, Barnes Damping in GS2



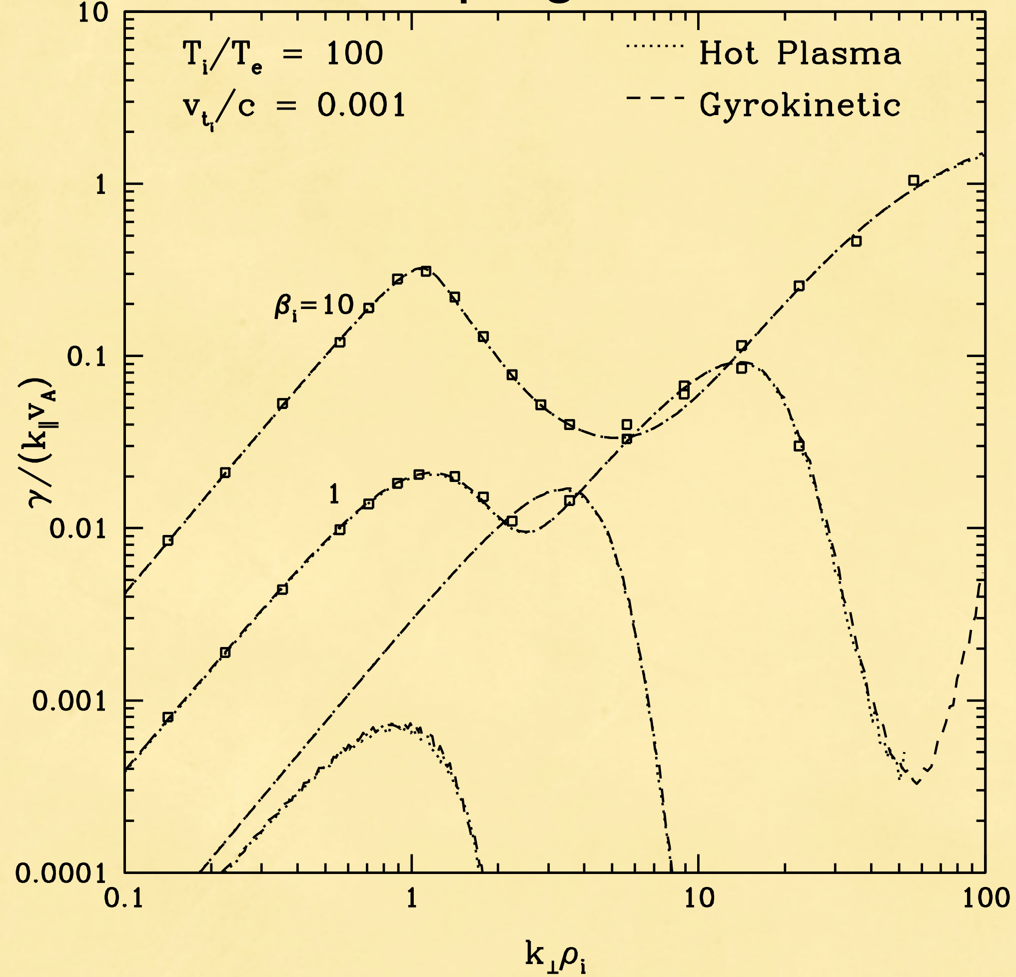
- Damping measured directly, or by fitting Lorentzian (driven system)

Gyrokinetics, Hot Plasma Dispersion Relations Agree; GS2 Accurate

Frequency



Damping rate



- Alfven wave couples to slow mode at $k_\perp \rho_i \sim 1$
- Damping is a function of $\beta, T_i/T_e, etc.$

Lost Wave Energy Goes into Heat

- Poynting's theorem can be derived from Maxwell's equations:

$$\frac{\partial u}{\partial t} + \nabla \cdot \mathbf{S} = -\mathbf{J} \cdot \mathbf{E}$$

- u is the electromagnetic energy density. $\mathbf{S}$ is the Poynting flux.
- Integrate over spatial domain to find

$$\frac{d\mathcal{E}_{\mathrm{em}}}{dt} = -\int d^3x \, \mathbf{J} \cdot \mathbf{E}$$

Lost Wave Energy Goes into Heat

- Electromagnetic energy:

$$\frac{d\mathcal{E}_{\rm em}}{dt} = -\int d^3x\, \mathbf{J}\cdot\mathbf{E}$$

- Total plasma heating can be derived from kinetic equation:

$$\left(\frac{d}{dt} + \mathbf{v}\cdot\nabla + \mathbf{a}\cdot\nabla_v\right) f = -\mathcal{C}(f,f)$$

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- Electromagnetic energy:

$$\frac{d\mathcal{E}_{\rm em}}{dt} = -\int d^3x\, \mathbf{J}\cdot\mathbf{E}$$

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$$\sum_s \int d^3x\, d^3v\, \frac{1}{2} m v^2 \left[\left(\frac{d}{dt} + \mathbf{v}\cdot\nabla + \mathbf{a}\cdot\nabla_v\right) f = -\mathcal{C}(f,f)\right]$$

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$$\sum_s \int d^3x\, d^3v\, \frac{mv^2}{2}\left[\left(\frac{d}{dt} + \mathbf{v}\cdot\nabla + \frac{q}{m}\left(\mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c}\right)\cdot\nabla_v\right) f = -\mathcal{C}(f,f)\right]$$

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- Result of integration is

$$\frac{d\mathcal{E}_k}{dt} = \int d^3x\, \mathbf{J}\cdot\mathbf{E}$$

Lost Wave Energy Goes into Heat

- Electromagnetic energy:

$$\frac{d\mathcal{E}_{\rm em}}{dt} = -\int d^3x\, \mathbf{J}\cdot\mathbf{E}$$

- Total plasma heating can be derived from kinetic equation:

$$\frac{d\mathcal{E}_K}{dt} = \int d^3x\, \mathbf{J}\cdot\mathbf{E}$$

- These relations are true on the heating timescale; must average over dynamical timescale. Signal can be noisy!
- Helpful to use gyrokinetic thermodynamics; entropy does not decrease, can be measured directly, and used to calculate thermalized turbulent energy, species by species.

Entropy and Heating in Gyrokinetics

- Start with gyrokinetic equation:

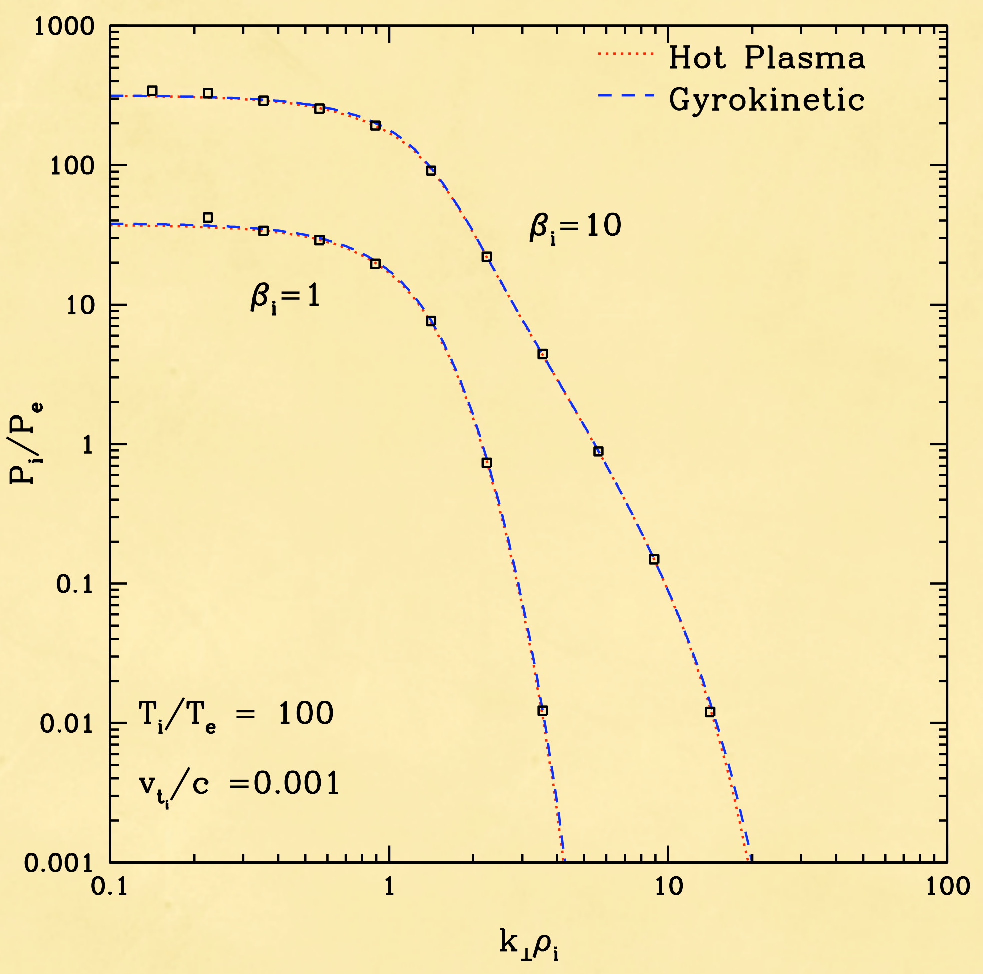
$$\frac{\partial h}{\partial t} + \frac{c}{B}\left[\langle\chi\rangle, h\right] + v_{\|}\hat{b}\cdot\nabla h = \frac{q}{T}\frac{\partial\langle\chi\rangle}{\partial t}F_0 + \mathcal{C}(h)$$

- Multiply by *h* and integrate over space, velocities

$$\frac{d\mathcal{E}_k}{dt} = T_0\frac{dS}{dt} = -T_0\int d^3x\, d^3v\, h\mathcal{C}(h)$$

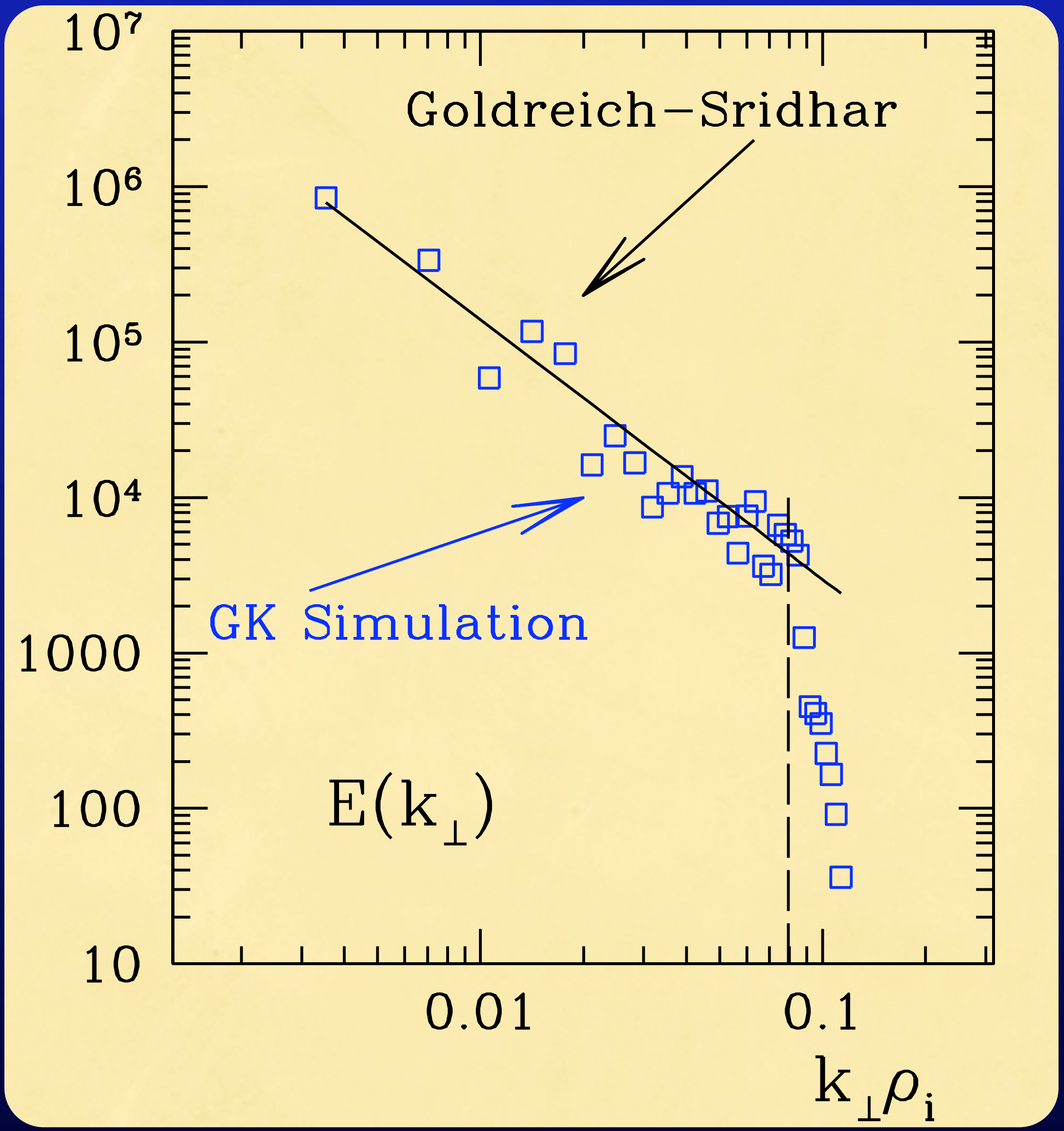
- Collision operator (or other dissipative terms) is negative definite; relations good species by species
- Essential technique for extracting heating from nonlinear simulations -- requires accurate collision operator (which GS2 has)

GS2 Resolves Extreme Ratios of Ion/Electron Heating



Three datasets here: direct evaluation of the hot plasma dispersion relation; direct evaluation of the gyrokinetic dispersion relation; and points calculated with GS2

Nonlinear Studies: Alfven Cascade

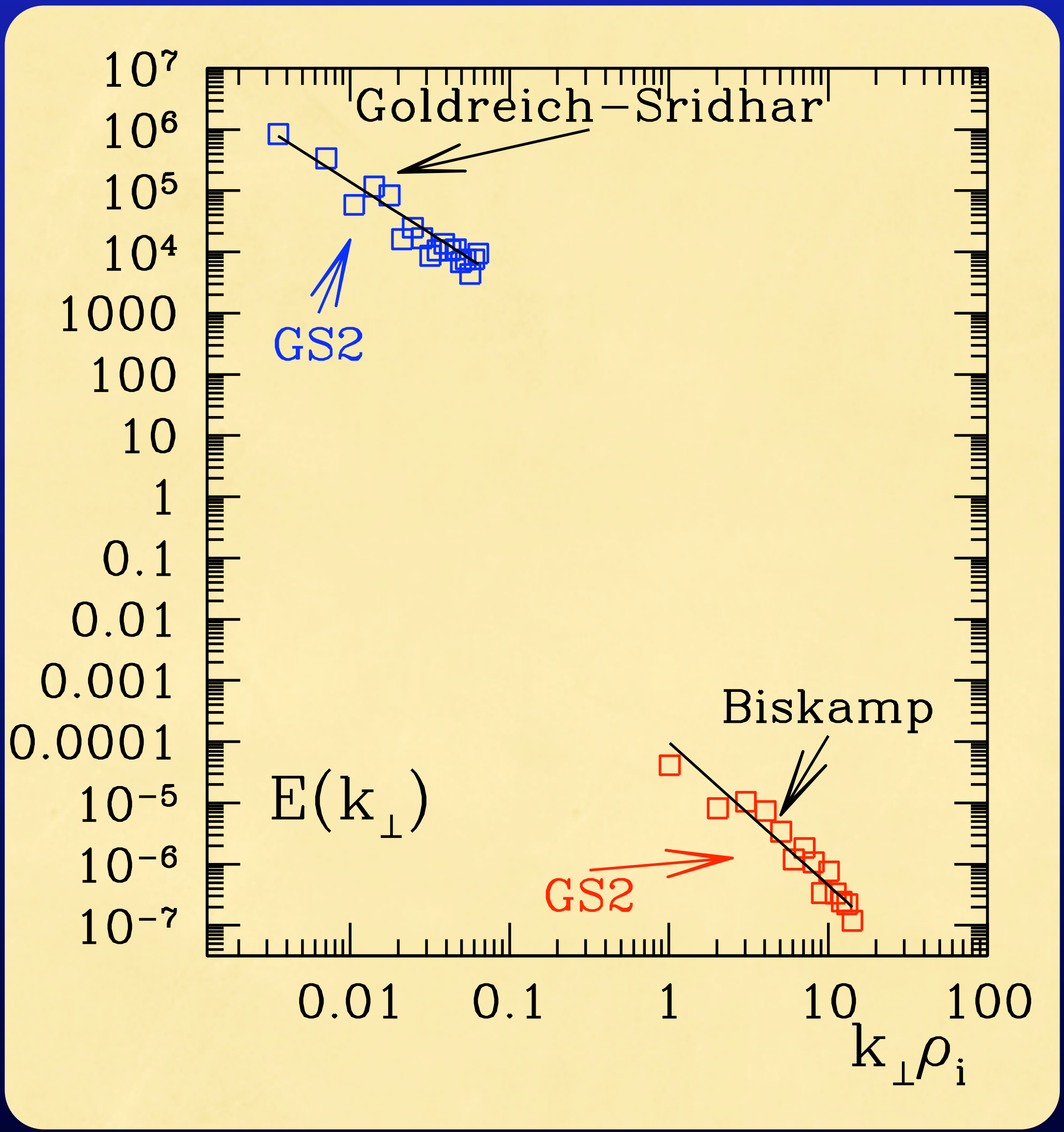


- Run GS2 in a large domain
- Stir with Langevin antenna at box scale
- Drain energy at small scales with hyper-viscosity and hyper-resistivity (as is normal in MHD)
- Find good agreement between expected spectrum of turbulent energy fluctuations and theory:

$$E(k_\perp) \sim k_\perp^{-5/3}$$

$$(\beta = 4 = 400\%)$$

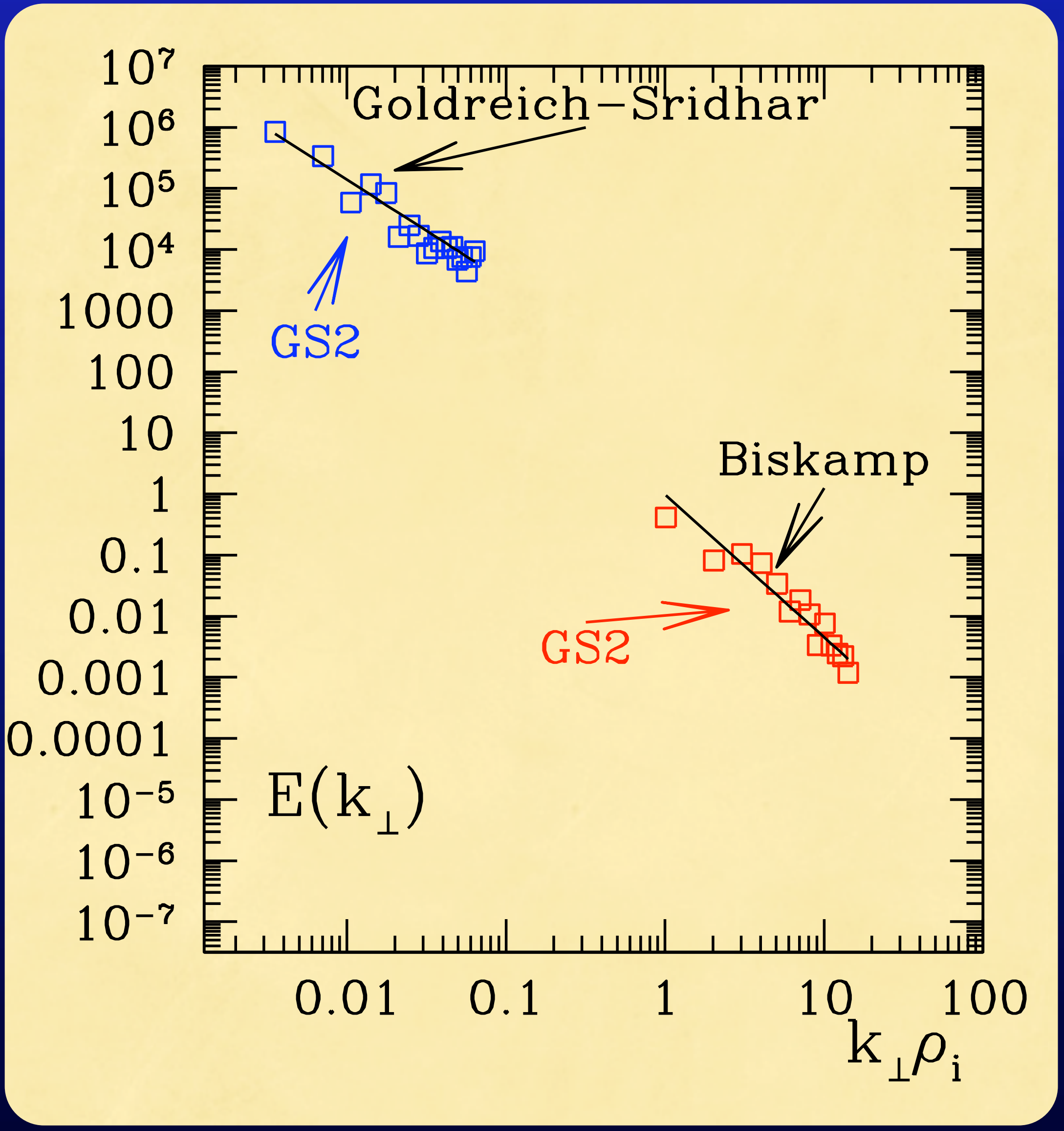
Nonlinear Studies: KAW Cascade



- Run GS2 in a small domain
- Stir with Langevin antenna at box scale
- Drain energy at small scales with hyper-viscosity and hyper-resistivity (as is normal in EMHD)
- Find good agreement between expected spectrum of turbulent energy fluctuations and theory:

$$E(k_\perp) \sim k_\perp^{-7/3}$$

Free Parameter to be Fixed by Physics in the Absorption Region



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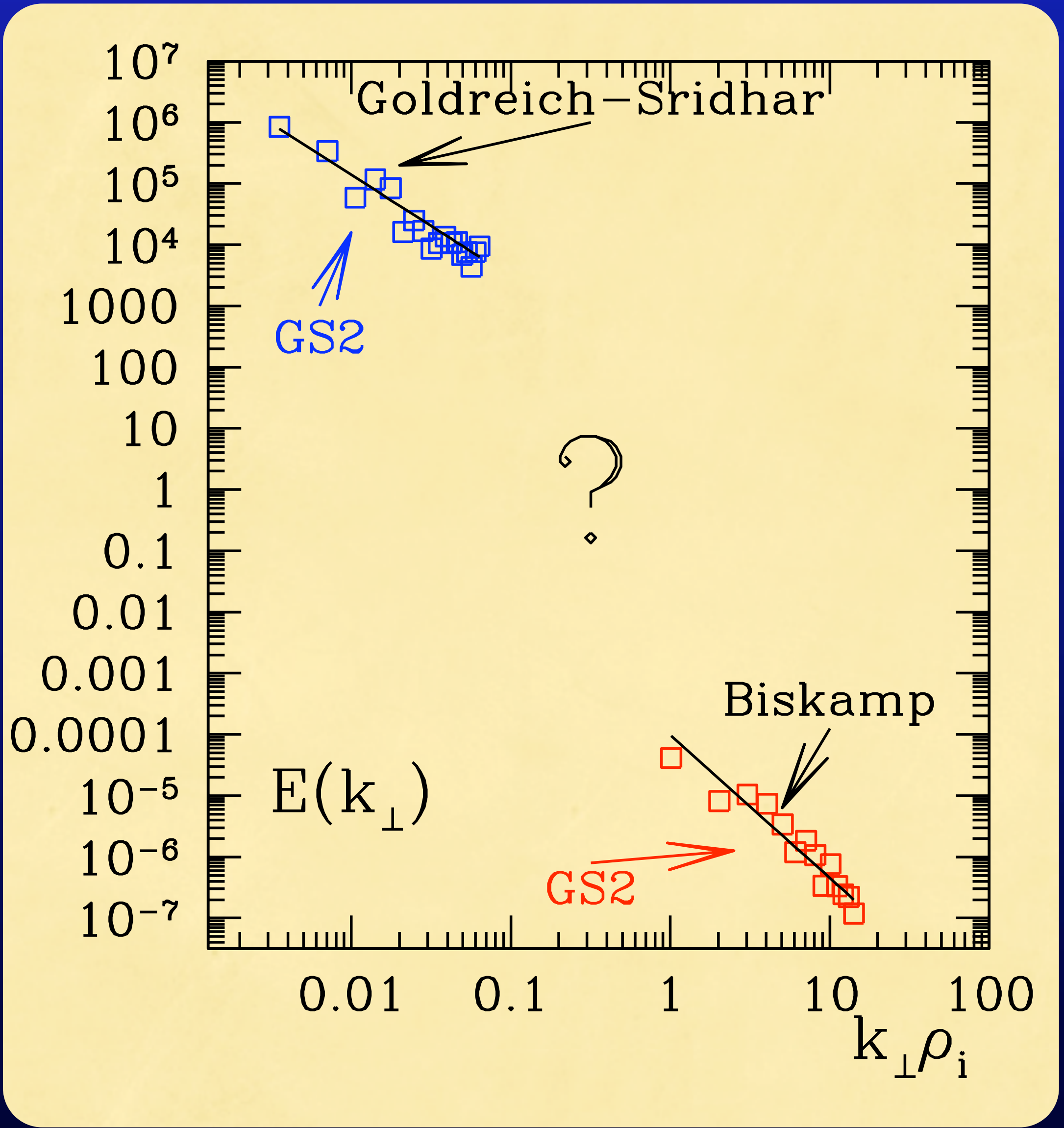

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Consider Two Cases: Low and High Beta

- "Low" beta case has $\beta = 4 = 400\%$ and $T_i/T_e = 100$

  Expect ions to dominate absorption, but absorption weak

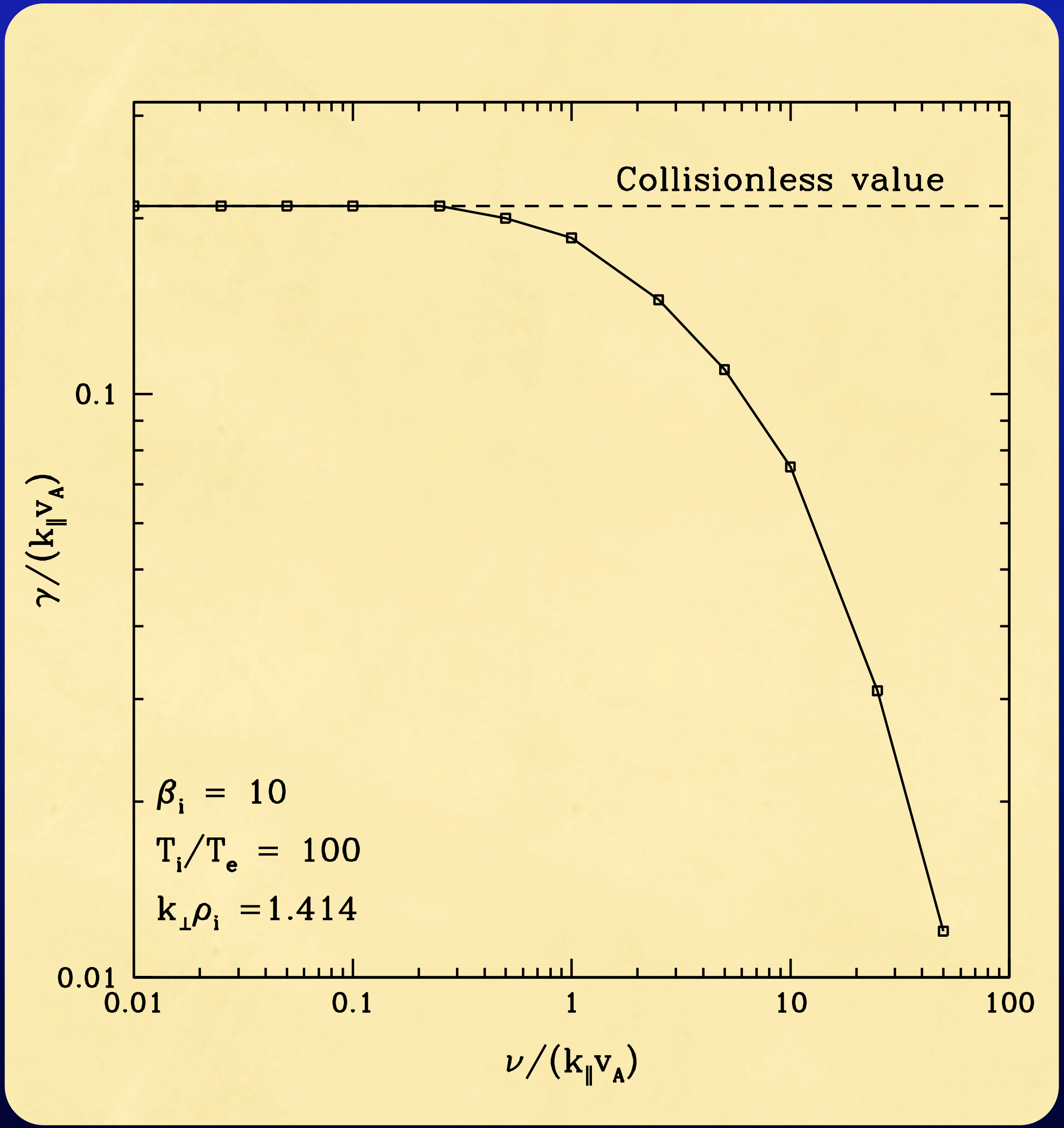
  Use hyper-(visc, res) to mimic transfer to smaller scales

- High beta case has $\beta = 40 = 4000\%$ and $T_i/T_e = 100$

  Expect ion to dominate absorption and absorption to be strong

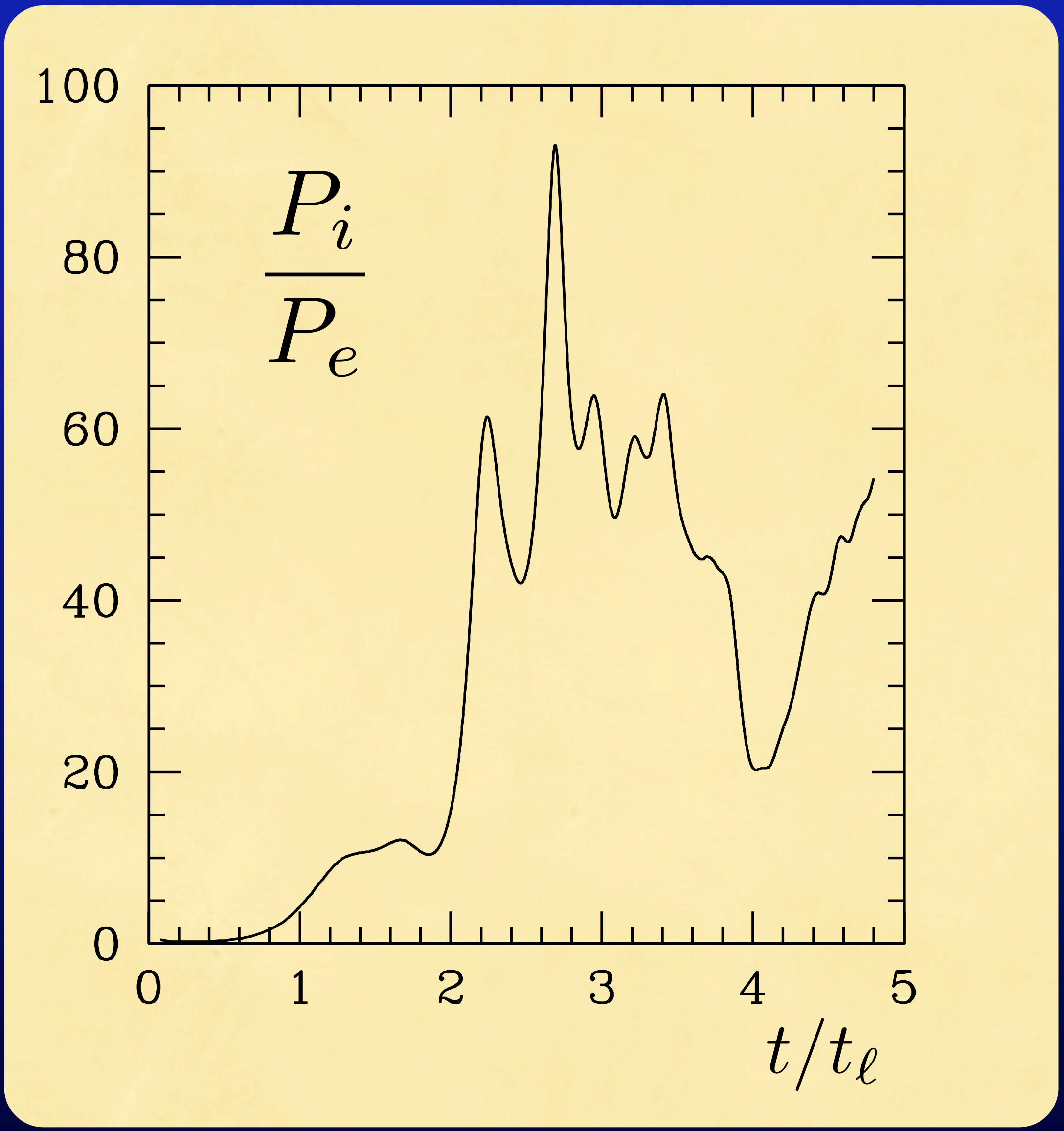
- In both cases, require collisions to make system irreversible, but use values small enough that waves are not affected.

Aside: Collisions Can Reduce Damping



- For large values of collision frequency, Alfven waves and slow modes are undamped
- (You already knew that!)
- Effect is shown in series of runs at left

Low Beta Case: Ions Heated, but…

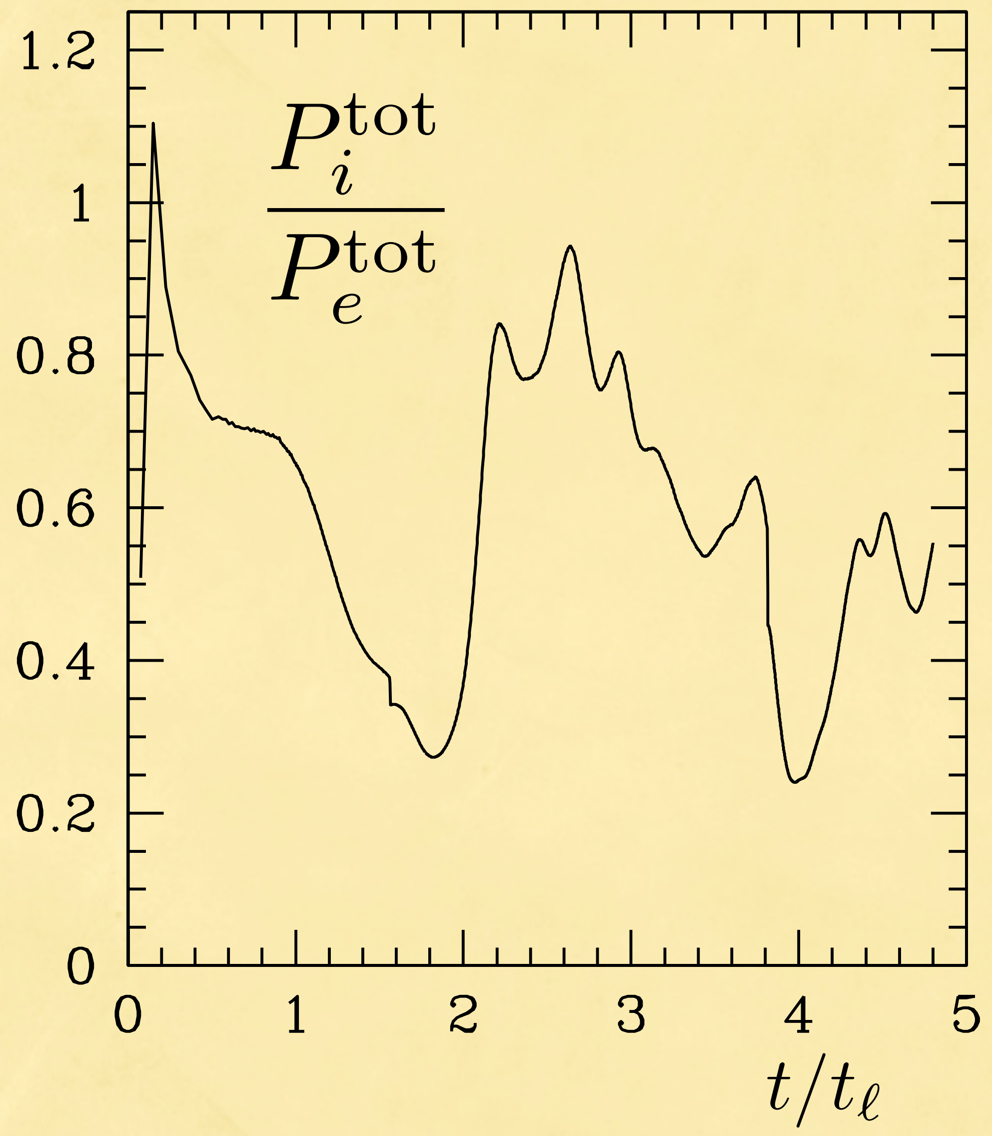


- Thermal energy deposited in ions is much larger than thermal energy in electrons

$$P_s = -T_s \int h\,\mathcal{C}(h)$$

- But heating is dominated by hyper-diffusive terms
- Energy mostly cascading through

Low Beta Case: Ions Heated, but…

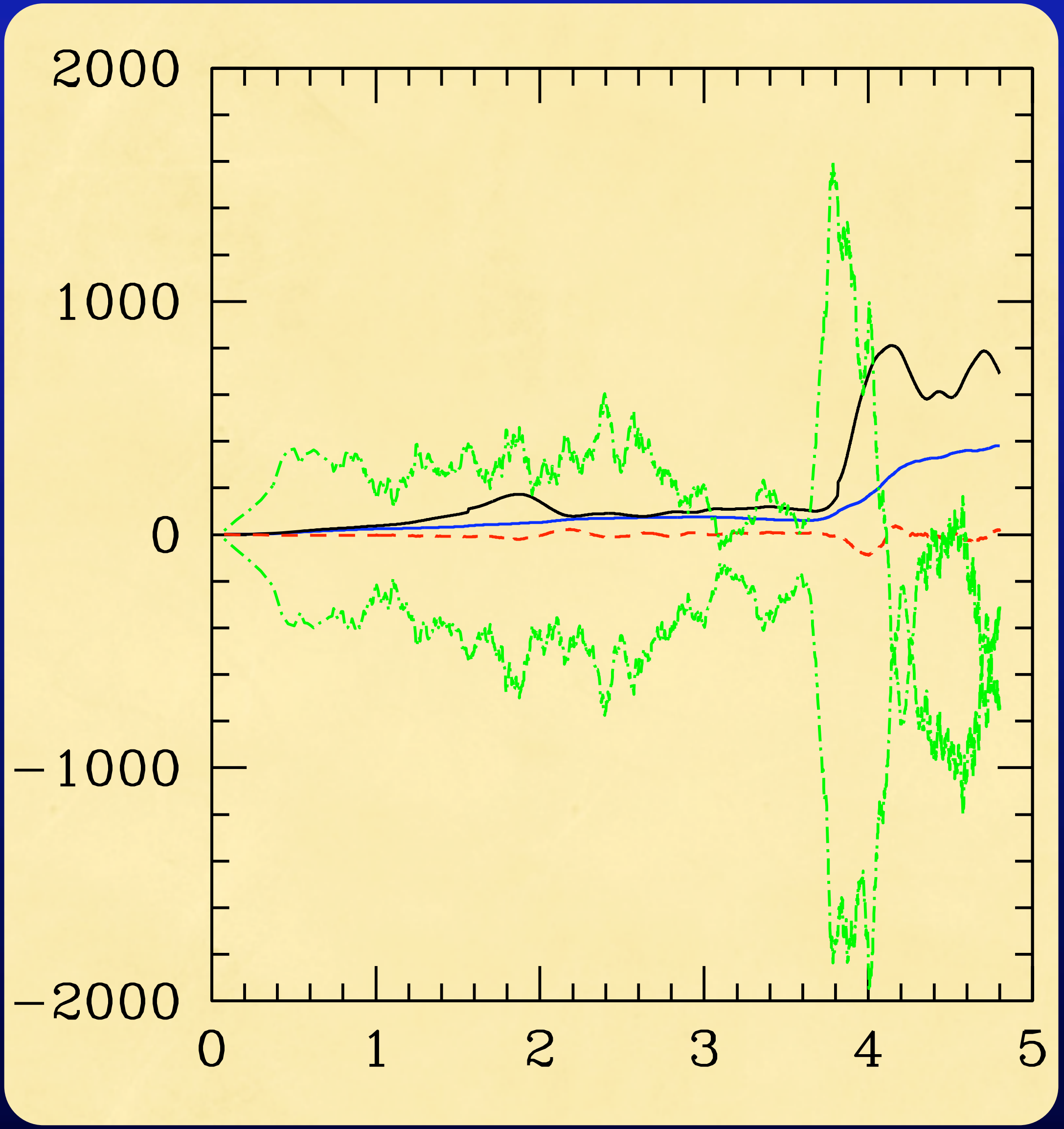


- Thermal energy deposited in ions is much larger than thermal energy in electrons
- But heating is dominated by hyper-diffusive terms

$$P_s^{\rm tot} = q\int h\,\frac{\partial\chi}{\partial t}$$

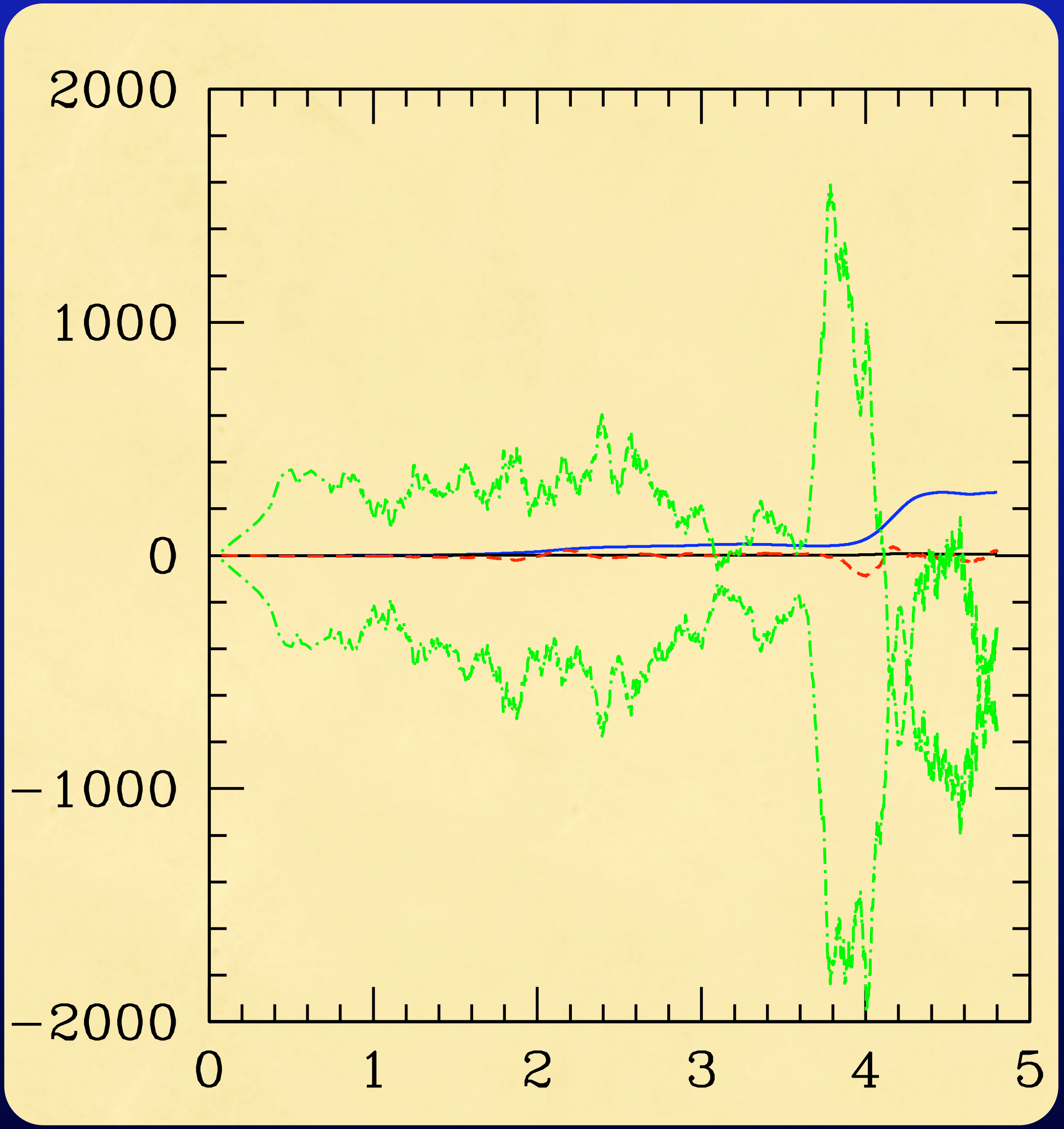
- Energy mostly cascading through; diagnostic sees equal heating

Heating is Not Dominant Effect



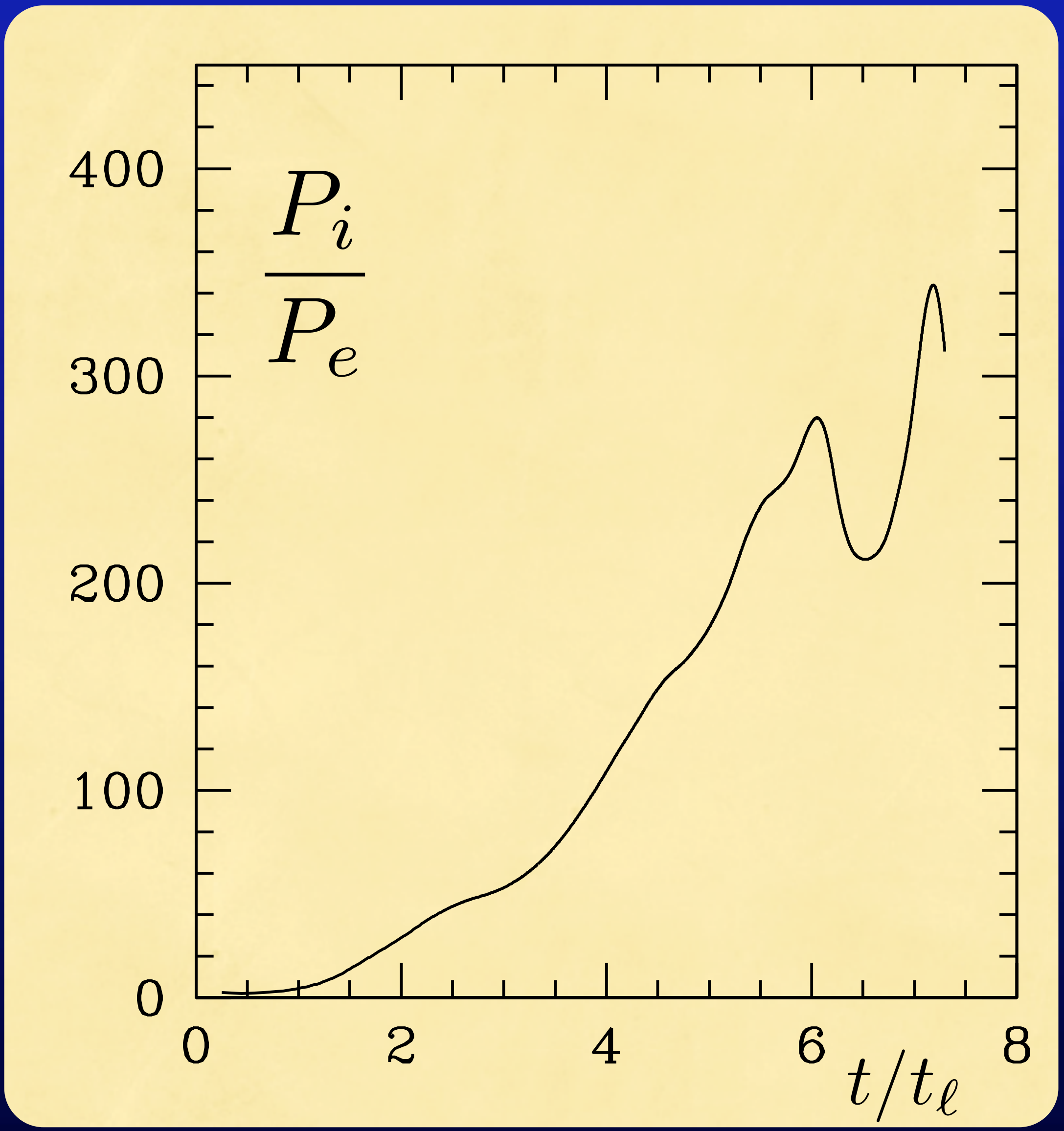
- Various terms in total power balance shown
- Sum to the red curve, which should be zero
- Large (green) signals are instantaneous power input and instantaneous change in total energy

Thermalization is Small Effect



- Blue curve is ion thermalization
- Black curve is electron thermalization
- Need full power of pseudo-spectral algorithms in GS2 to see this signal in the turbulence!

High Beta Case: Ion Heating Dominant



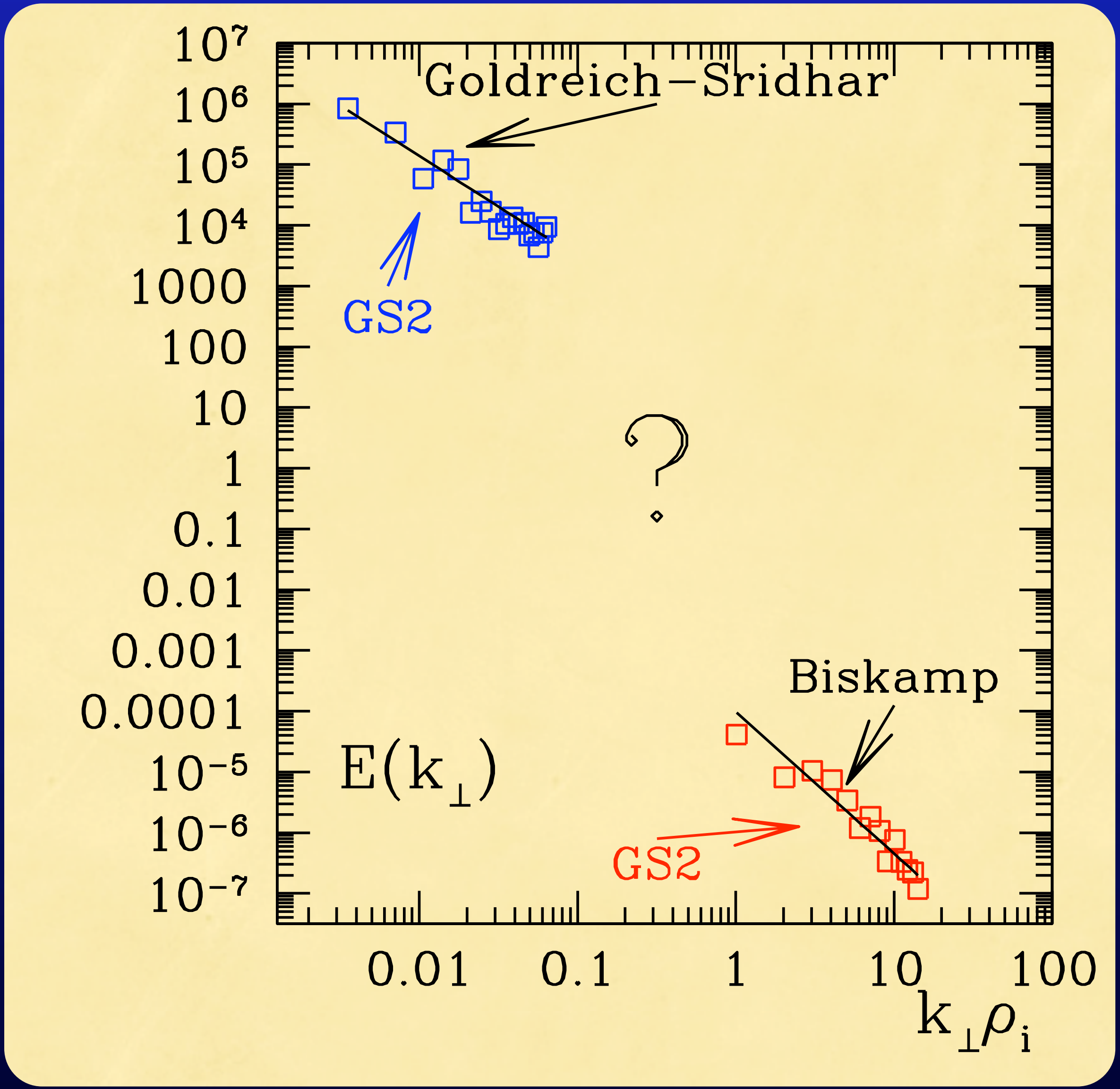
- Ion heating is much larger than electron heating (as expected for these parameters)

$$P_s = -T_s \int h\, \mathcal{C}(h)$$

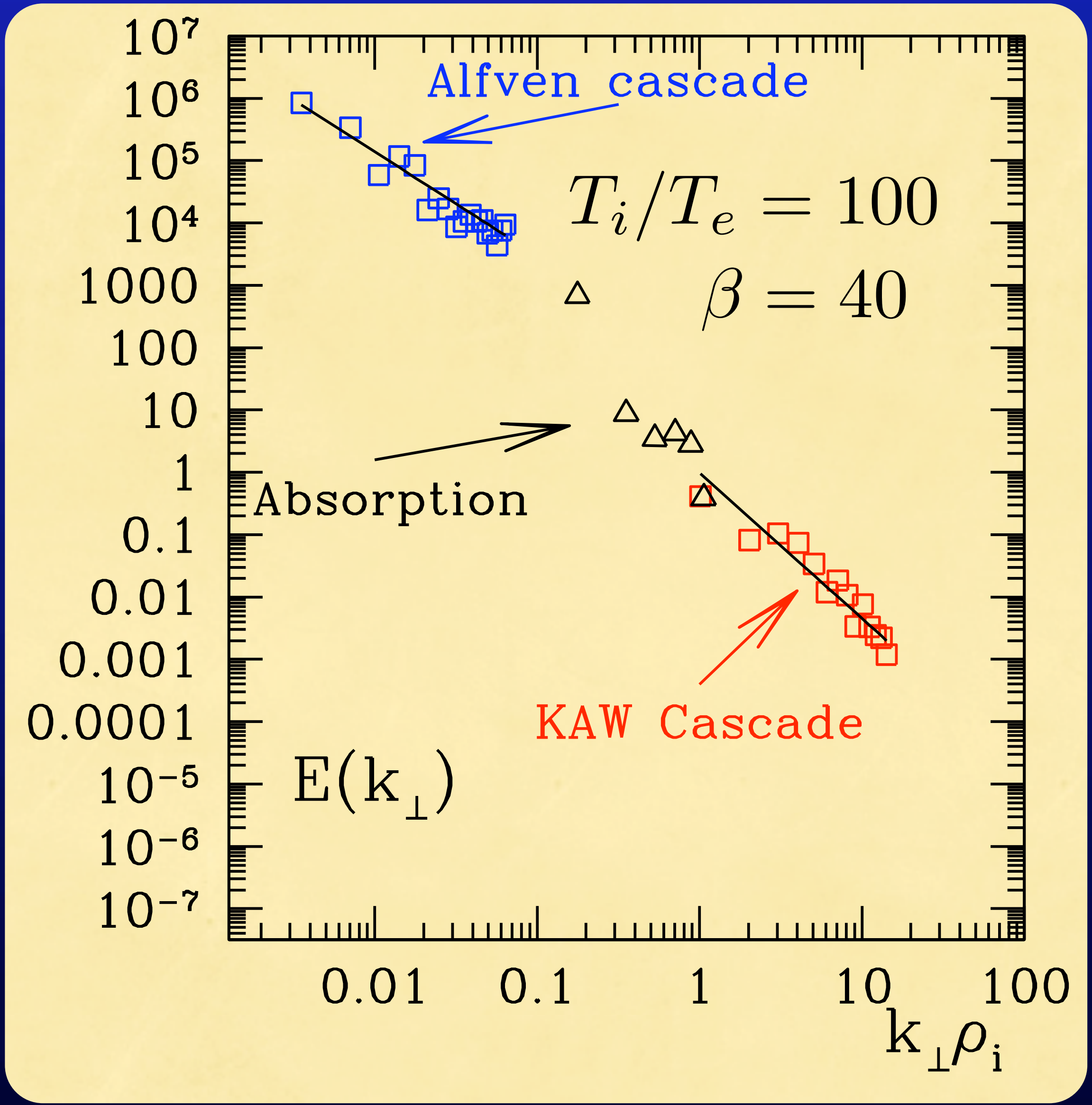
- Ion heating dominant because Barnes damping sees large ion magnetic moment

$$(T_i/T_e \gg 1)$$

What Does Spectrum Look Like?



Significant Absorption in High Beta Case



- First calculation of absorption and spectra from Alfven cascade
- High beta case is easiest: will be more challenging to extend this to the rest of the interesting parameter space

Summary and Conclusions

- Gyrokinetics has application to astrophysical plasmas
- Energy absorption can be calculated for realistic physical parameters
- Transport time-scale equations in the gyrokinetic hierarchy derived
- Gyrokinetic energy, entropy relations derived and tested
- Parametric studies of the heating and spectral characteristics are plausible: two main parameters: $\beta, T_i/T_e$