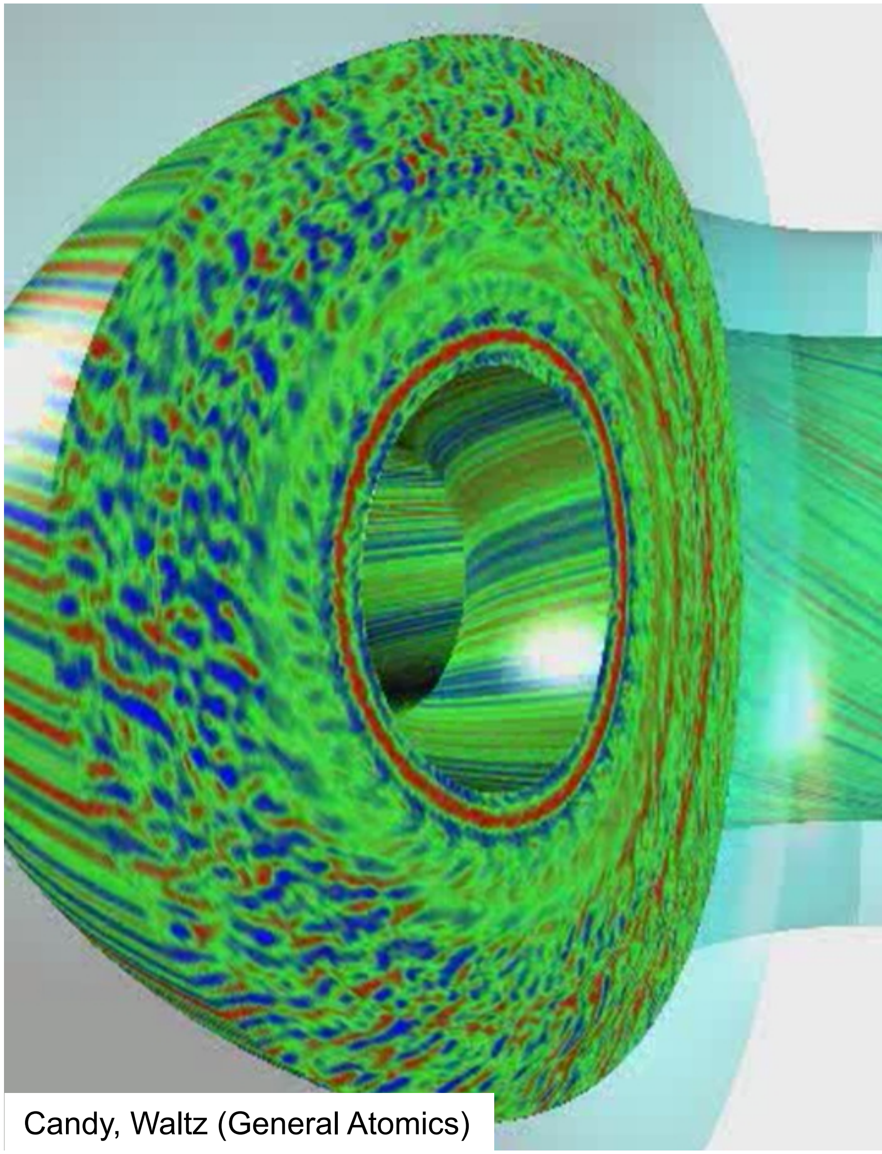
Intro to Tokamak Turbulence

NUF Fusion & Plasmas Summer School, Princeton, June 10, 2014



Candy, Waltz (General Atomics)

Greg Hammett
Princeton Plasma Physics Lab (PPPL)
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I. Brief Perspective on Fusion

II. Billiard Balls & Chaos Theory

III. Physical picture of instabilities in toroidal magnetic fields driven by effective-gravity / bad-curvature, based on inverted-pendulum and Rayleigh-Taylor analogies.

(some of these slides will be skipped)

Need to aggressively pursue a portfolio of alternative energy in the near term (10-30 years)

Needed to deal with global warming, energy independence, & economic issues

- improved building & transportation efficiency
- plug-in hybrid, CNG vehicles
- wind power
- concentrated solar
- photovoltaic
- storage (hourly, daily, monthly, seasonal)
- clean coal with CO2 sequestration
- synfuels+biomass with CO2 sequestration
- fission nuclear power plants
- …

However, there are uncertainties about all of these energy sources: cost, quantity, intermittency, storage, side-effects. How much CO2 can be stored underground long term, and at what cost? Energy demand expected to > triple throughout this century as poorer countries continue to develop.

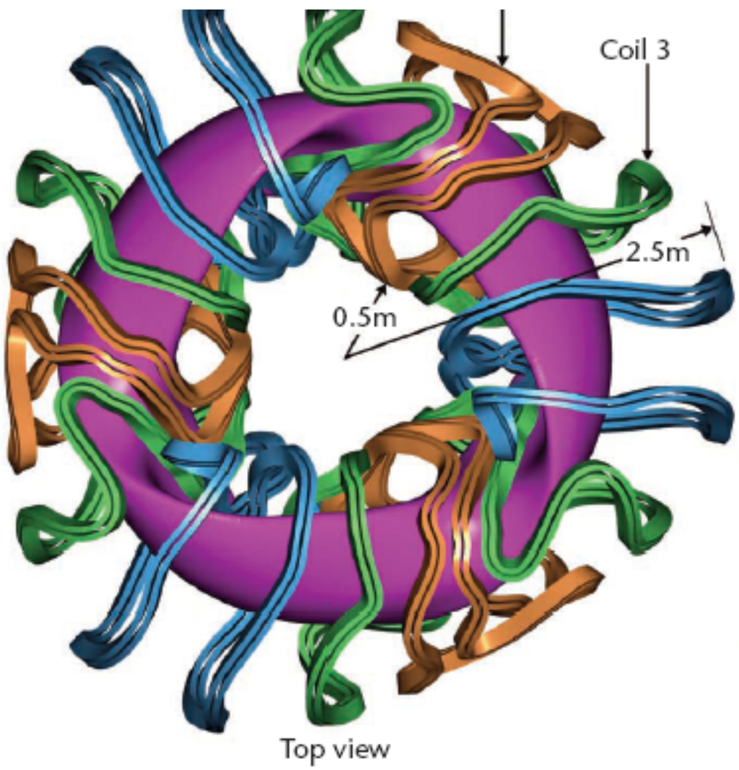
Because of uncertainties, particularly on the longer time scale (>50 years), still need to explore fusion.

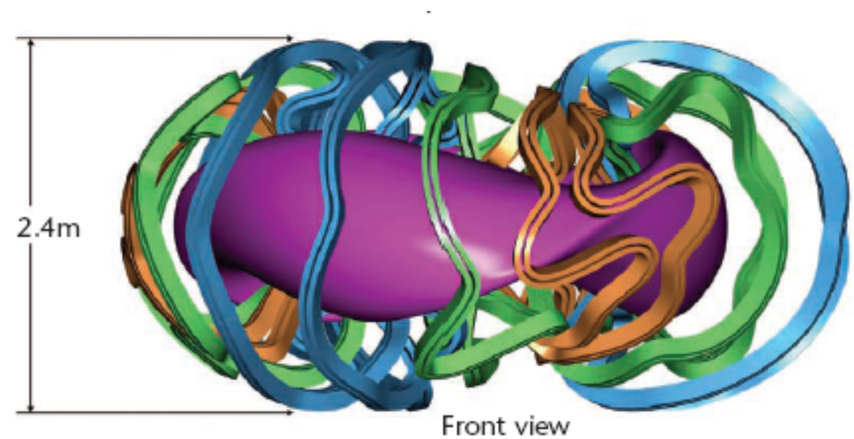
My perspective on fusion questions:

- Fusion energy is hard and it will take a lot of time, but it's an important problem, we've been making progress, and there are interesting ideas to pursue that could make it more practical:
  - "advanced tokamak" regimes, spherical torus
  - spontaneous spinning reduces turbulence?
  - Liquid metal walls: handle power loads better, "black hole" wall reduces cold neutral recycling & improves performance. LTX, NSTX, …
  - Stellarators: After 40+ years of research, a hidden symmetry discovered that improves performance
  - High-temperature superconductors can run at higher magnetic field (recent advances in last 5 years for practical coils) → smaller fusion devices
  - FRCs, RFPs,
  - …

Improved Stellarators Being Studied

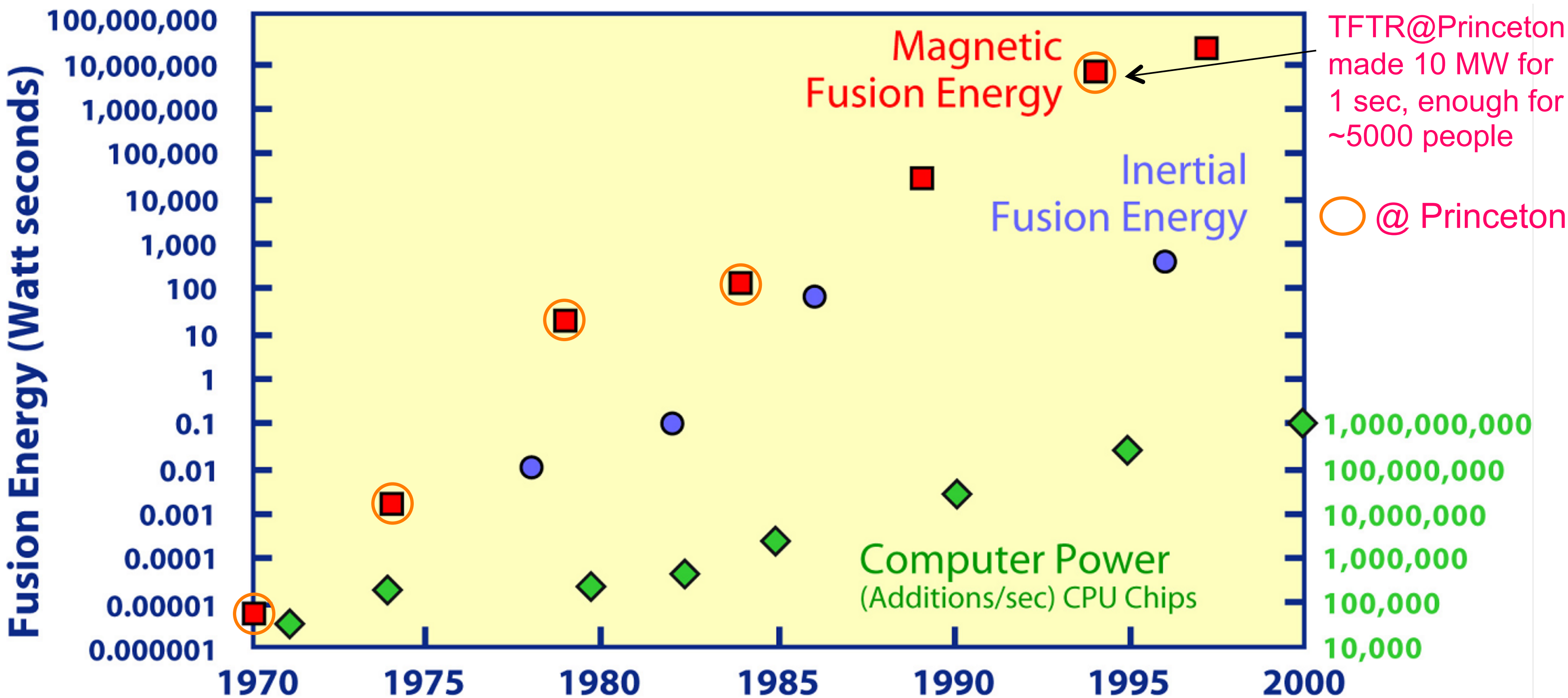
- Originally invented by Spitzer ('51), the unique idea when fusion declassified ('57)
- Mostly abandoned for tokamaks in '69. But computer optimized designs now much better than slide rules. Now studying cost reductions.
- Quasi-symmetry discovered in late 90's: don't need vector ***B*** exactly symmetric toroidally, |***B***| symmetric in field-aligned coordinates sufficient to be as good as tokamak.
- Magnetic field twist & shear provided by external coils, not plasma currents, inherently steady-state. Stellarator expts. find they don't have Greenwald density limit or hard beta limit & don't disrupt. Quasi-symmetry allows plasma spin to reduce turbulence?





~$1B W7-X stellarator starting up in Germany, grad student opportunities

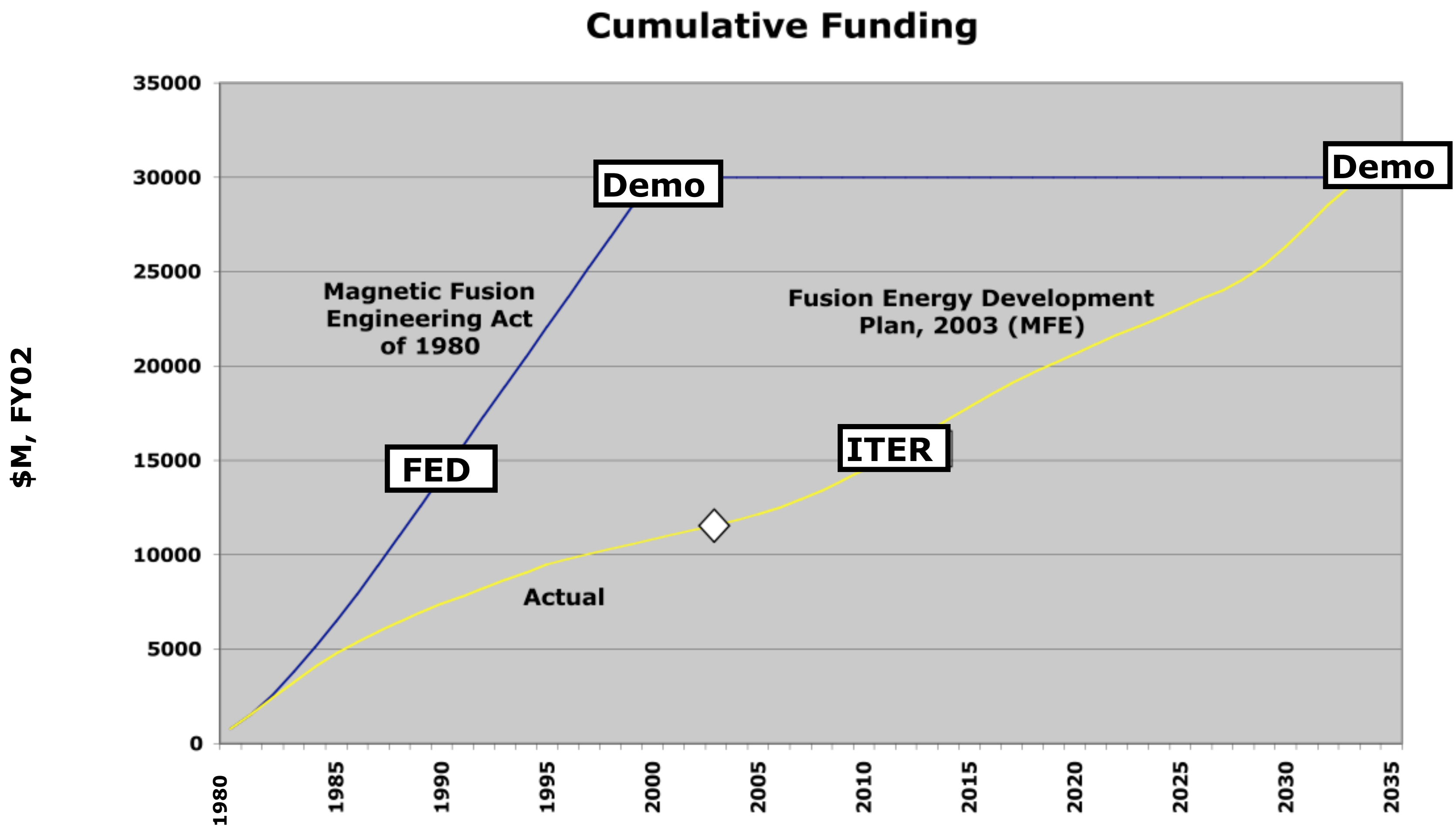
Progress in Fusion Energy has Outpaced Computer Speed



ITER goal to produce 200,000 MJ/pulse (~300 MW), 10^7 MJ/day of fusion heat).
NIF goal to produce 20 MJ/pulse (and /day) of fusion heat.

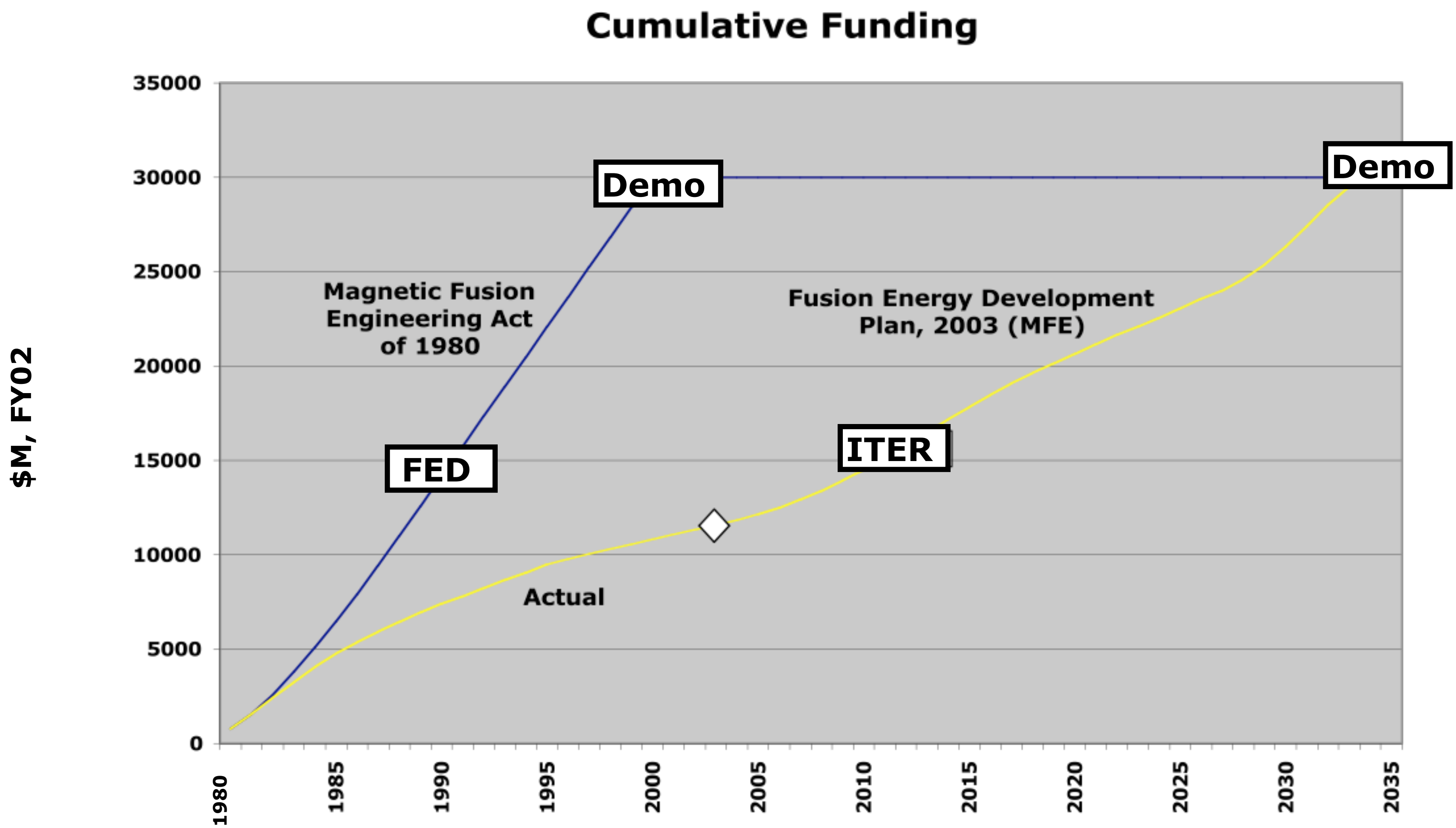
Some of the progress in computer speed can be attributed to plasma science.

Fusion Research Has Never Received Budget Needed To Fully Developed It



Plot from R.J. Goldston in 2003

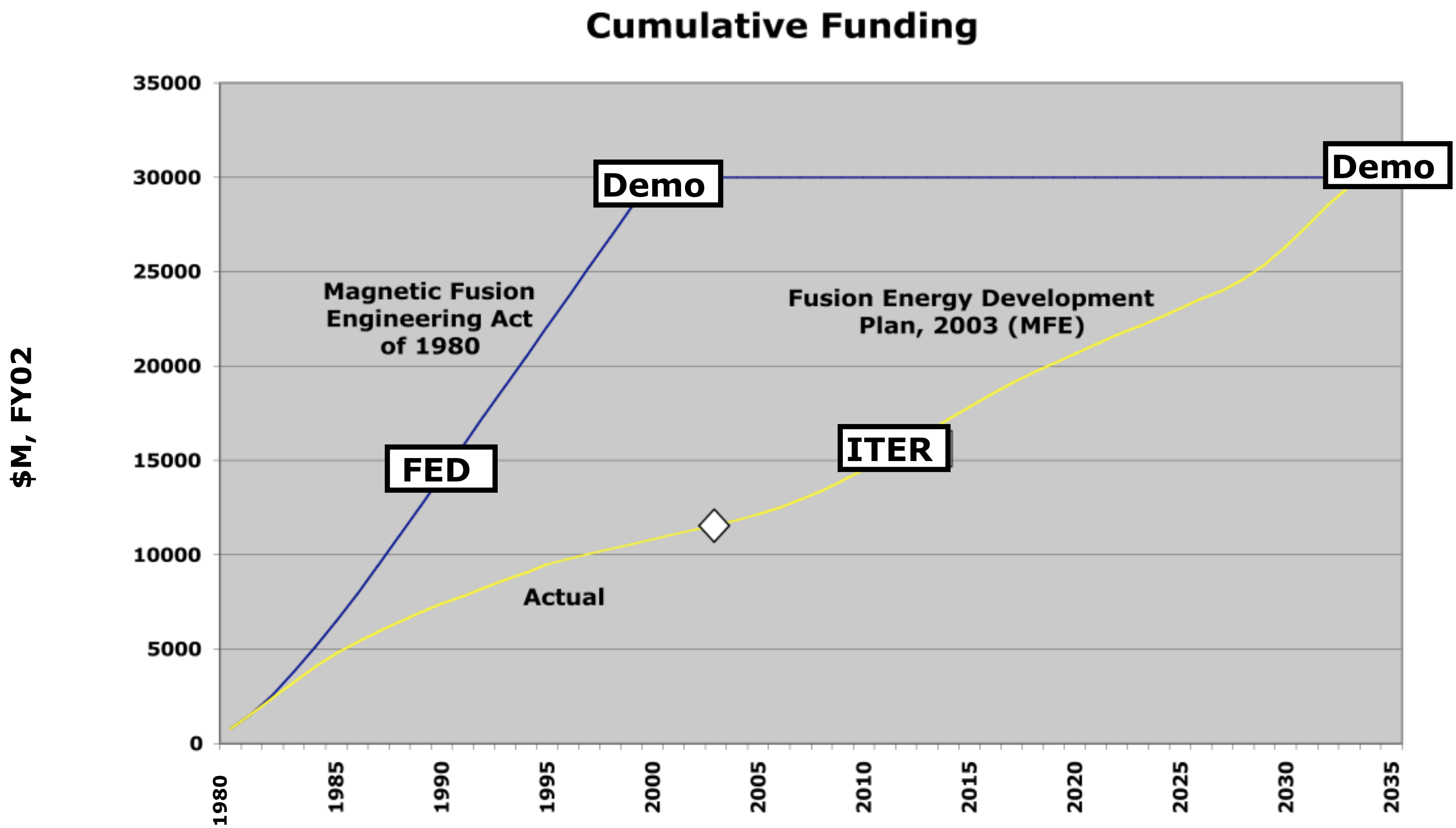
Fusion Research Has Never Received Budget Needed To Fully Developed It



Einstein: Time is relative,

Plot from R.J. Goldston in 2003

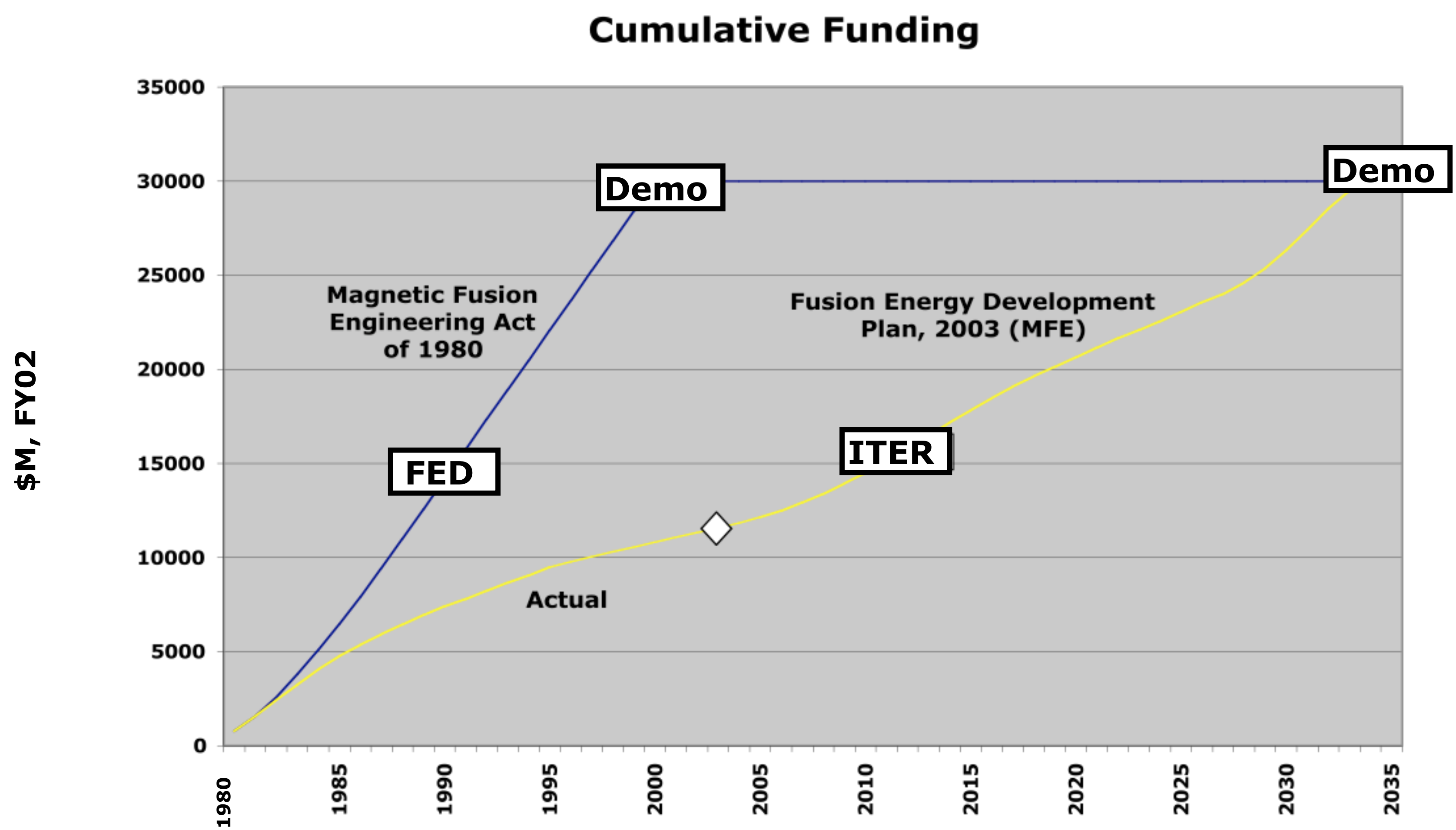
Fusion Research Has Never Received Budget Needed To Fully Developed It



Einstein: Time is relative, Measure time in $$

Plot from R.J. Goldston in 2003

Fusion Research Has Never Received Budget Needed To Fully Developed It



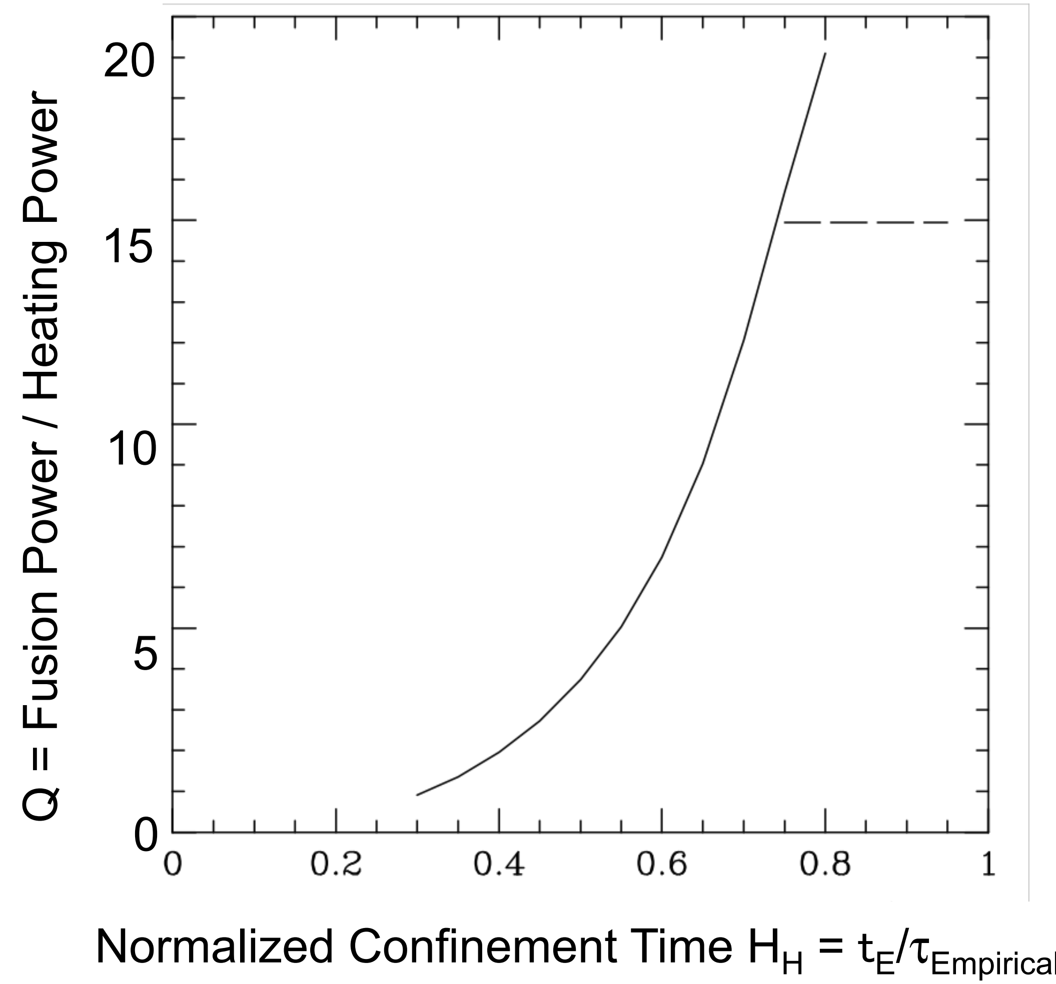
The fusion program should do the best it can with the funding available to learn about fusion, find ways to improve it and bring down its cost. Aim to provide the scientific basis for a larger funding initiative someday to fully develop it.

~$80B total development cost is tiny compared to >$100 Trillion energy needs of 21st century & potential costs of global warming. Still 67:1 payoff after discounting 50+ years if fusion is just 10% cheaper than best environmentally acceptable alternative. Goldston IAEA 2006 http://www-naweb.iaea.org/napc/physics/fec/fec2006/html/node132.htm

Plot from R.J. Goldston in 2003

Fusion performance depends sensitively on confinement

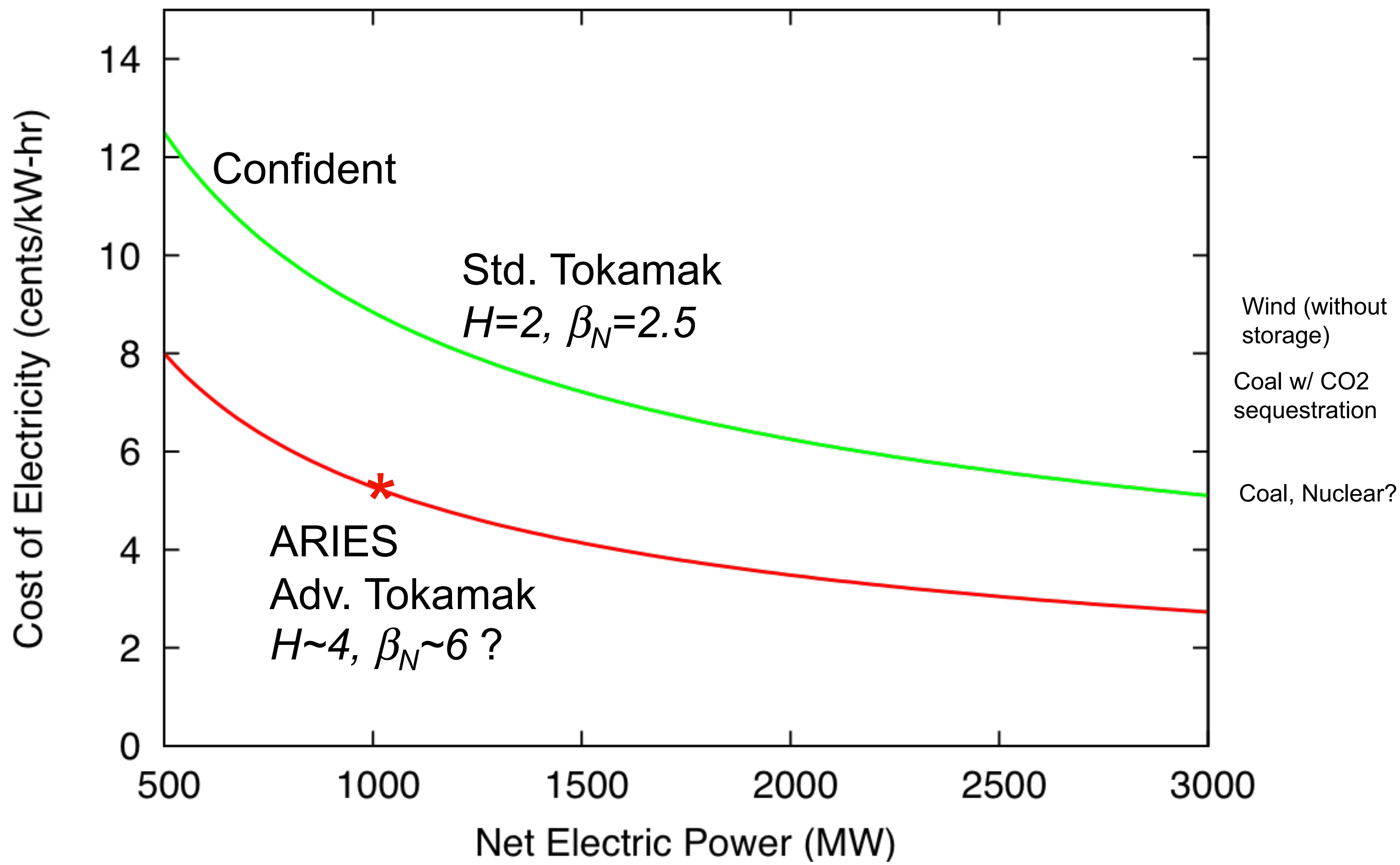
Sensitive dependence on turbulent confinement causes some uncertainties, but also gives opportunities for significant improvements, if methods of reducing turbulence extrapolate to larger reactor scales.



$$\frac{dW}{dt} = P_{ext} + P_{fusion} - \frac{W}{\tau_E}$$

Caveats: best if MHD pressure limits also improve with improved confinement.
Other limits also: power load on divertor & wall, …

↓ turbulence & ↑ β could significantly improve fusion



(Relative cost estimates in Galambos et al. study, see ARIES studies for more detailed & lower costs estimates, including potential engineering advances)

From Galambos, Perkins, Haney, & Mandrekas 1995 Nucl.Fus. (very good), scaled to match ARIES-AT reactor design study (Fus. Eng. & Des. 2006), http://aries.ucsd.edu/ARIES/

↓ turbulence & ↑ β could significantly improve fusion

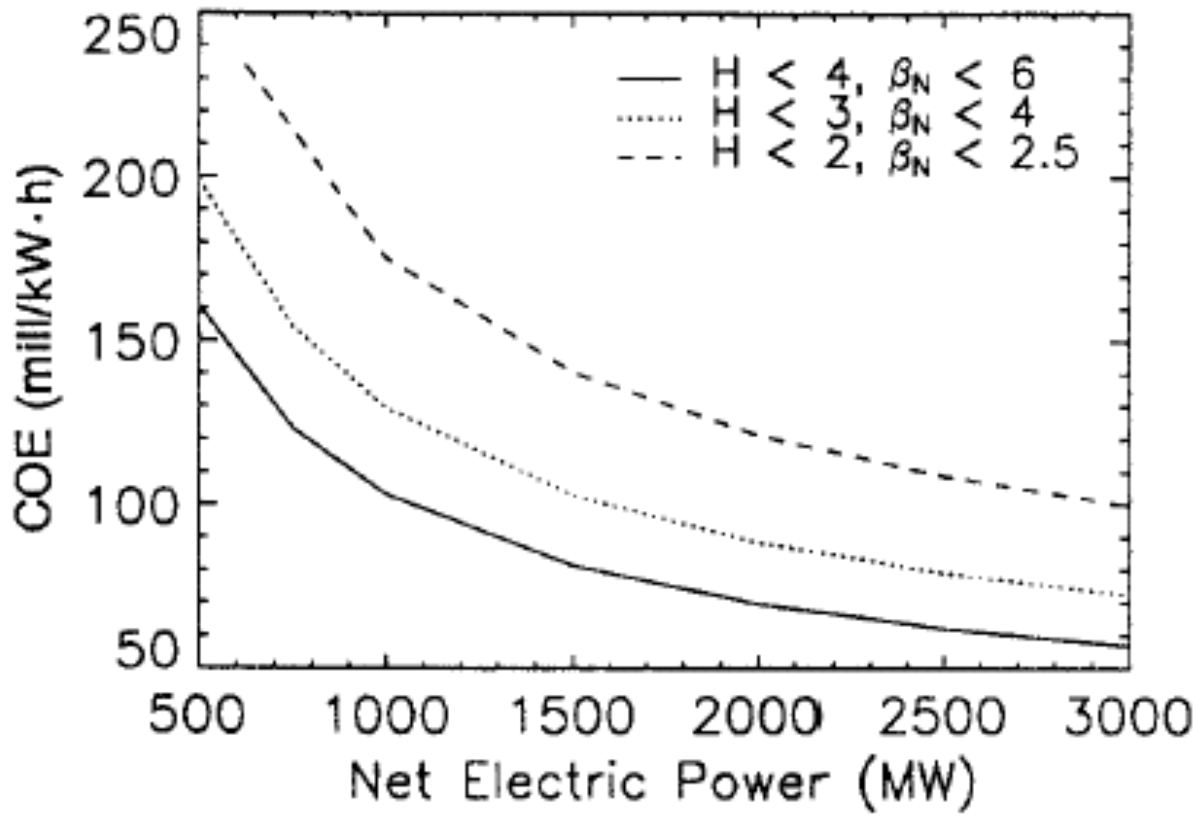


FIG. 4. Minimum COE steady state reactor parameters versus the net electric output. Cases are shown for three physics levels: (a) present day levels that would be sustainable in a non-transient manner in a conservatively designed system ($H \leq 2, \beta_N \leq 2.5$), (b) moderately improved physics ($H \leq 3, \beta_N \leq 4$) and (c) advanced physics ($H \leq 4, \beta_N \leq 6$).

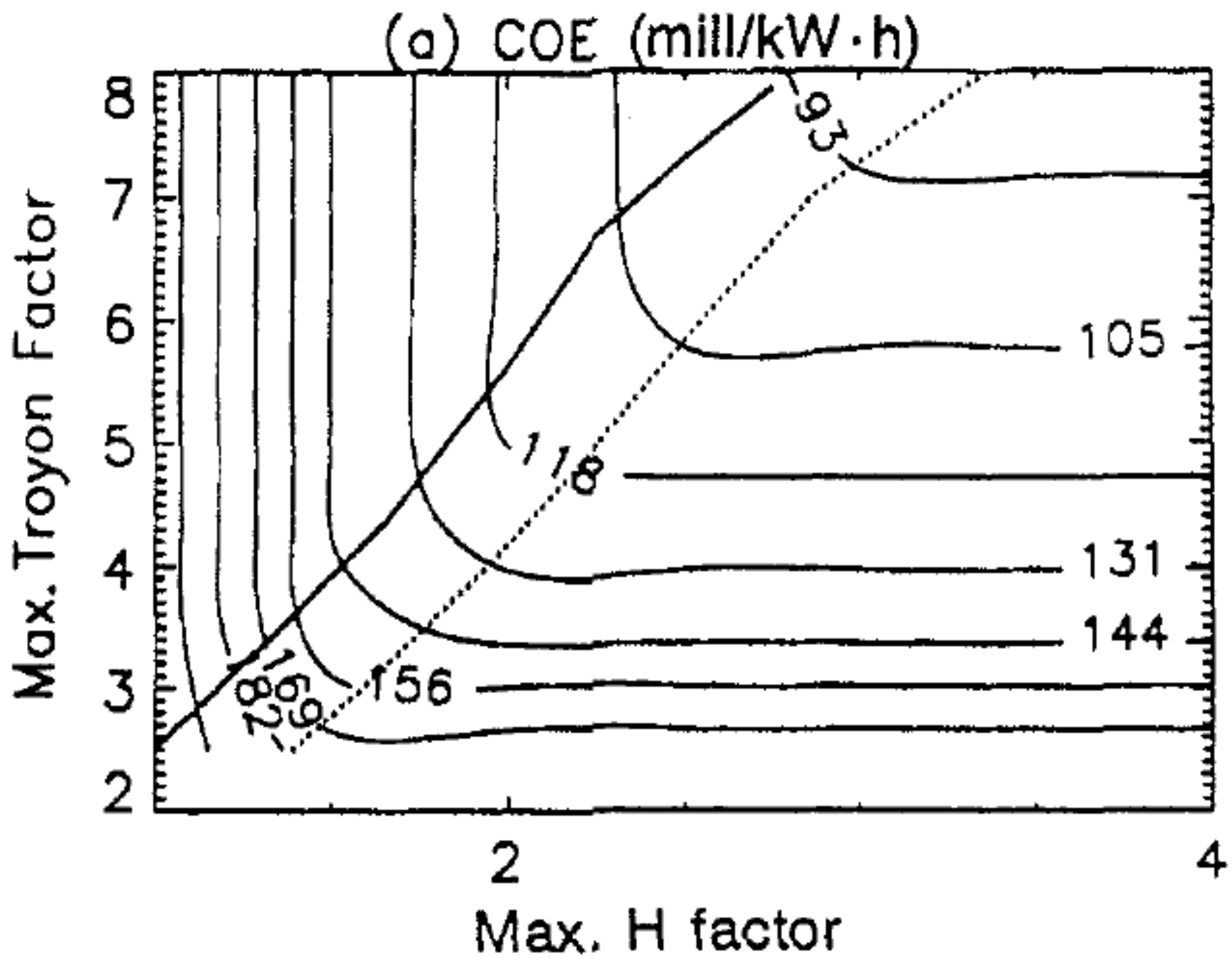
Improved confinement factor H helps even in very large reactor-scale devices. (Have to increase H & β together.)

↑H → ↑ P_{fusion} and/or
↓I_p ↓& ↓current drive

(Relative cost estimates in Galambos et al. study, see ARIES studies for more detailed & lower costs estimates, including potential engineering advances)

Galambos, Perkins, Haney, & Mandrekas 1995 Nucl. Fus.

Fusion Reactors benefit from improving Confinement Time and Beta limits simultaneously

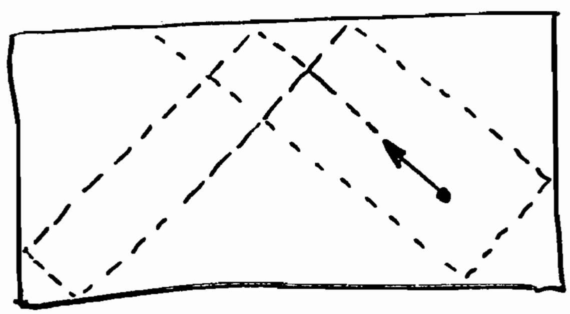


Galambos, Perkins, Haney, & Mandrekas 1995 Nucl. Fus.

A brief intro to Chaos theory

Many simple systems are chaotic

Frictionless billiard balls on a billiard table. [Macintosh program — John Cary Univ. of Colorado.]



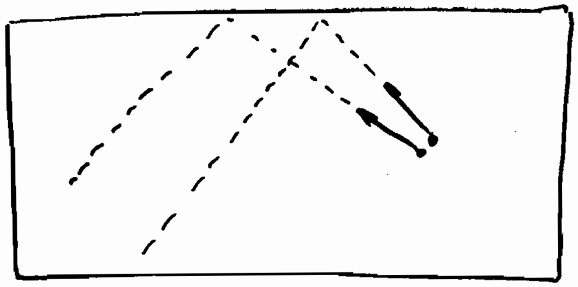
Rectangular table — regular motion

ball is restricted to traveling in one of 4 directions

Billiard chaos slides from
Charles Karney, PPPL (~1993)

Start two balls close to each other

(ignore collisions between balls)



separation of balls grows <u>slowly</u>

Two characteristics of regular motion

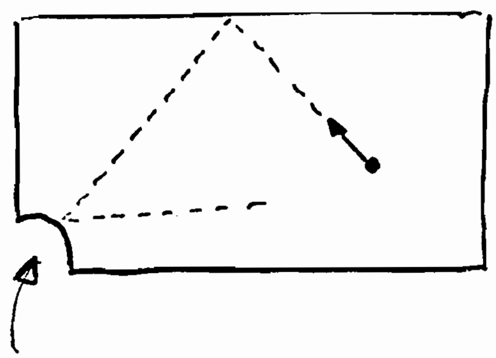
- motion is restricted

- errors grow slowly

  — prediction is possible

  (for a fairly long time)

Take a "bite" out of table



makes motion irregular

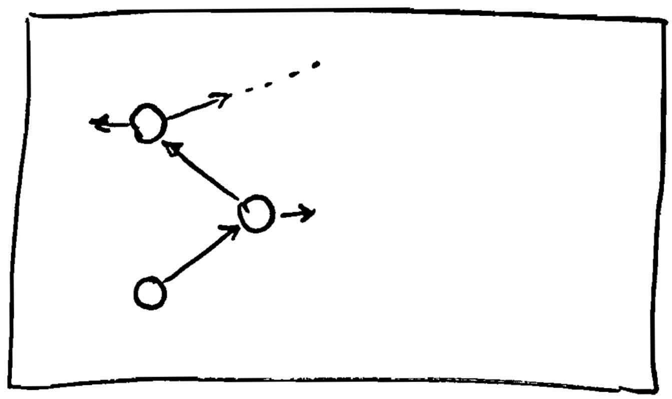
Two characteristics of irregular motion

- no restriction on motion —
  every point is visited at every angle

- errors grow quickly (exponentially)
  - errors (in initial placement or unevenness of surface) eventually swamp the motion
  - (long term) prediction is impossible

The growth of errors is very rapid

frictionless billiard table with several colliding balls



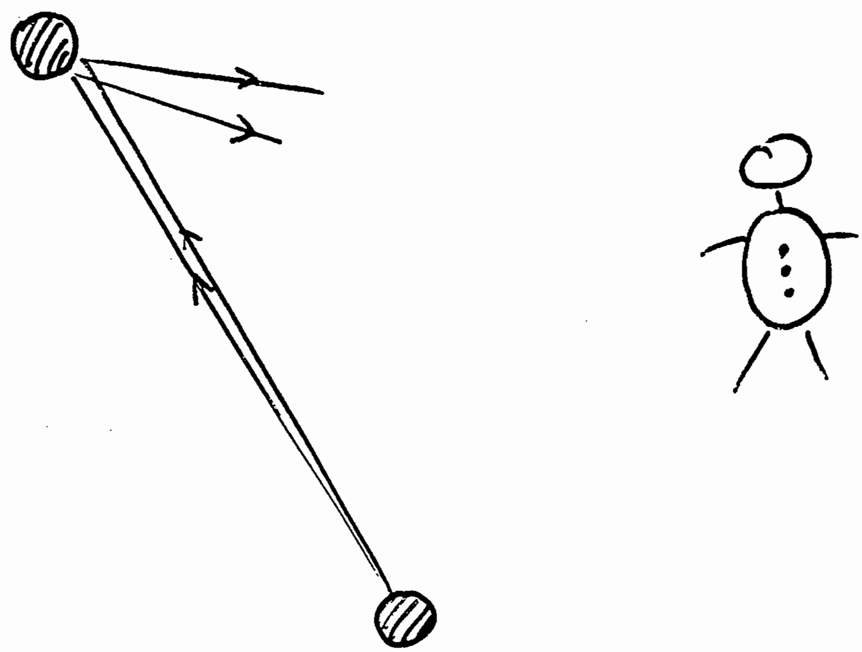
Knowing initial positions and speeds of balls, try to predict their positions after several collisions.

THIS IS IMPOSSIBLE!

(see M.V. Berry, in *Topics in Nonlinear Dynamics*, S. Jorna, Ed., (AIP, 1978)).

Suppose your friend enters the room to observe your experiment.

He will gravitationally attract the billiard balls



* [This will completely alter their motion after about 10 collisions.

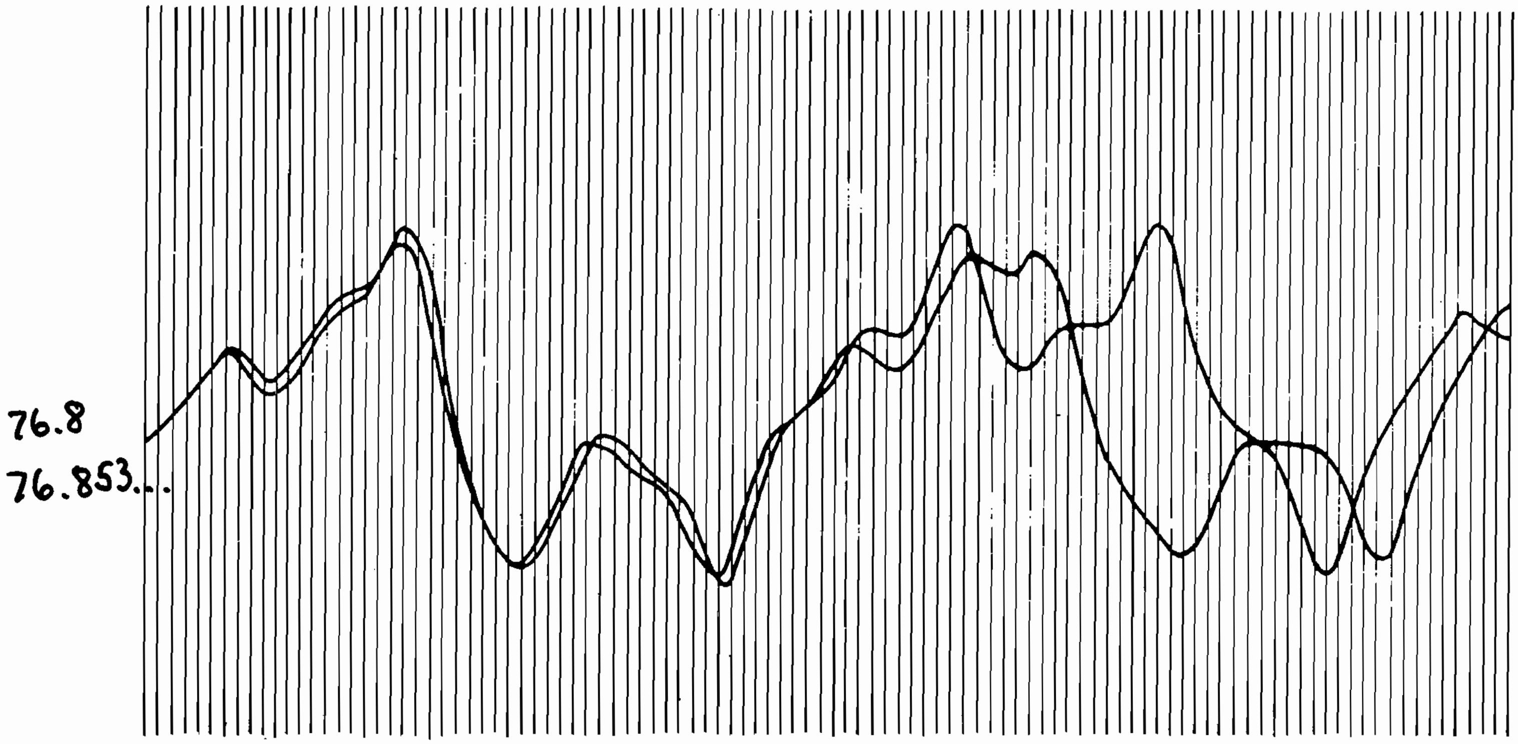
The same effect prevents weather forecasts beyond about 1 week.

* Homework: Prove this! Hints: ① Gravitational Constant $G = 6.67 * 10^{-11}$ Newton-m^2/Kg^2, ② Consider all balls fixed except for one, ③ Don't try to be precise, just get order-of-magnitude effects + scaling...

Consider a large collection of air molecules at room temperature and pressure. Can show that the gravitational force due to an electron located at the edge of the universe is enough to make the trajectories completely different after about ~60 collisions.

Exercise 5.1 in Statistical Mechanics summary chapter of

T. Padmanabhan, Theoretical Astrophysics, Vol. I: Astrophysical Processes.



HOW TWO WEATHER PATTERNS DIVERGE. From nearly the same starting point, Edward Lorenz saw his computer weather produce patterns that grew farther and farther apart until all resemblance disappeared. (From Lorenz's 1961 printouts.)

From Gleick's Chaos (1987)

practical purposes, the cycles would be predictable—and eventually uninteresting. To produce the rich repertoire of real earthly weather, the beautiful multiplicity of it, you could hardly wish for anything better than a Butterfly Effect.

The Butterfly Effect acquired a technical name: sensitive dependence on initial conditions. And sensitive dependence on initial conditions was not an altogether new notion. It had a place in folklore:

"For want of a nail, the shoe was lost;
For want of a shoe, the horse was lost;
For want of a horse, the rider was lost;
For want of a rider, the battle was lost;
For want of a battle, the kingdom was lost!" *

In science as in life, it is well known that a chain of events can have a point of crisis that could magnify small changes. But chaos meant that such points were everywhere. They were pervasive. In systems like the weather, sensitive dependence on initial conditions was an inescapable consequence of the way small scales intertwined with large.

* Quote from George Herbert (1593–1633)

Quoted in J. Gleick, Chaos, Making a New Science, (Viking, New York, 1987)

J.C. Maxwell, Matters and dynamics (1877):
"There is a maxim in studying the physical phenomena, that is, 'Same phenomena result in same.' This is true in all cases.
And there is another maxim, 'Similar phenomena result in similar.' This is appropriate for many phenomena too, but not all."

James Clerk Maxwell (1831–1879)

Maxwell's Eqs – Unified electrical & magnetic forces

Maxwellian distribution – "bell-shaped curve" important in thermodynamics & theory of gases.

Maxwell was the first scientist to understand chaos *

"No one, I suppose, would assign to free will a more than infinitesimal range. No leopard can change his spots, nor can any one by merely wishing it, or, as some say, *willing* it, introduce discontinuity into his course of existence... In the course of this our mortal life we more or less frequently find ourselves in a physical or moral watershed, where an imperceptible deviation is sufficient to determine into which of two valleys we shall descend."

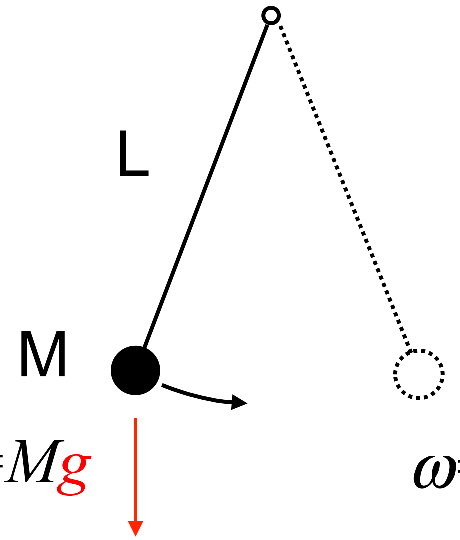
From an essay concerning the debate between determinism & free will, delivered at Cambridge Univ, 1873. Quoted by Hunt & Yorke, Nonlinear Science Today 3, p. 1 (1993).

"the existance of unstable conditions renders impossible the prediction of future events, if our knowledge of the present state is only approximate, & not accurate..."

Next: An Intuitive picture of plasma instabilities driven by “bad curvature”

-- based on analogy with Inverted pendulum / Rayleigh-Taylor instability

Stable Pendulum

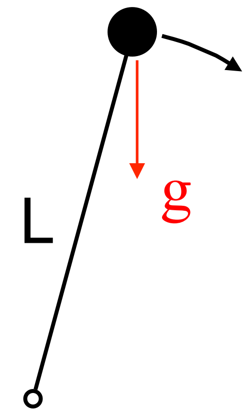


$F=Mg$

$\omega=(g/L)^{1/2}$

Unstable Inverted Pendulum

(rigid rod)

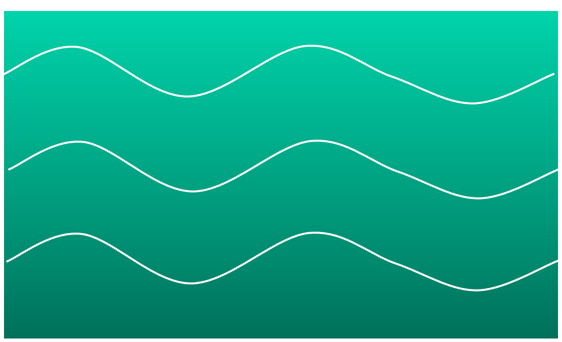


$\omega= (-g/|L|)^{1/2} = i(g/|L|)^{1/2} = i\gamma$

└ Instability

Density-stratified Fluid

$\rho=exp(-y/L)$

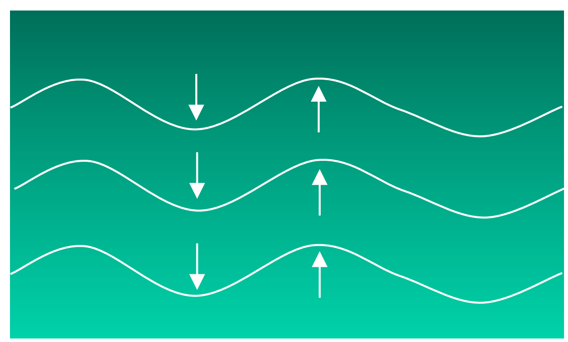


stable $\omega=(g/L)^{1/2}$

Inverted-density fluid

⇒Rayleigh-Taylor Instability

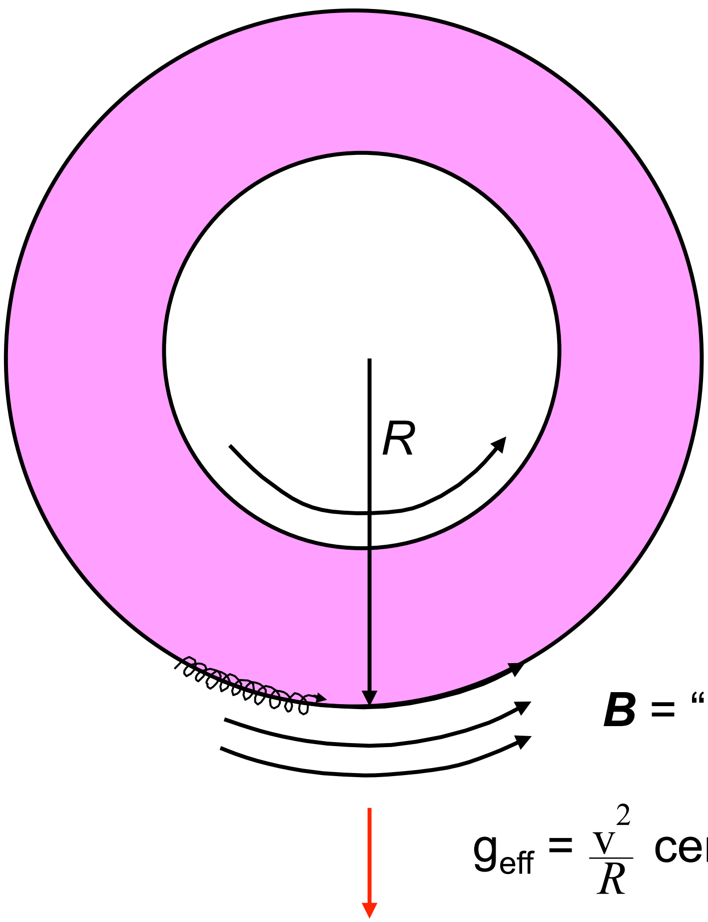
$\rho=exp(y/L)$



Max growth rate $\gamma=(g/L)^{1/2}$

"Bad Curvature" instability in plasmas
≈ Inverted Pendulum / Rayleigh-Taylor Instability

Top view of toroidal plasma:



$g_{eff} = \frac{v^2}{R}$ centrifugal force

Growth rate:

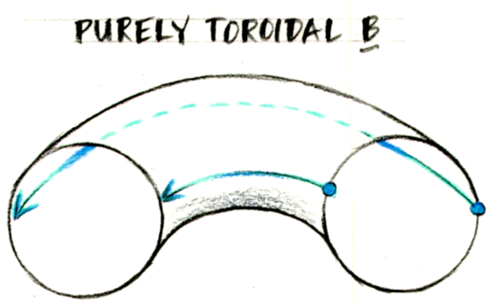
$$\gamma = \sqrt{\frac{g_{eff}}{L}} = \sqrt{\frac{v_t^2}{RL}} = \frac{v_t}{\sqrt{RL}}$$

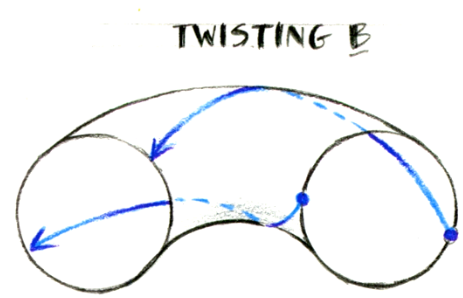
Similar instability mechanism
in MHD & drift/microinstabilities

1/L = |∇p|/p in MHD,
∝ combination of ∇n & ∇T
in microinstabilities.

The Secret for Stabilizing Bad-Curvature Instabilities

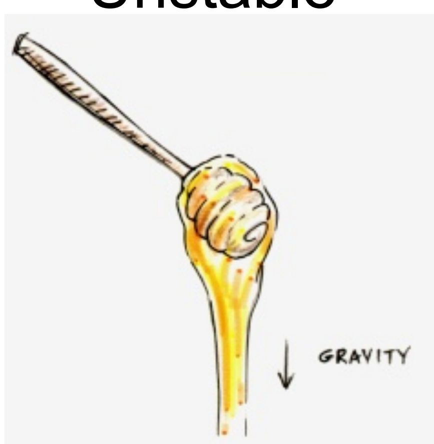
Twist in ***B*** carries plasma from bad curvature region to good curvature region:

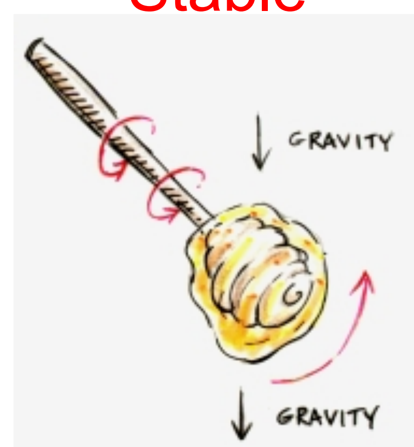




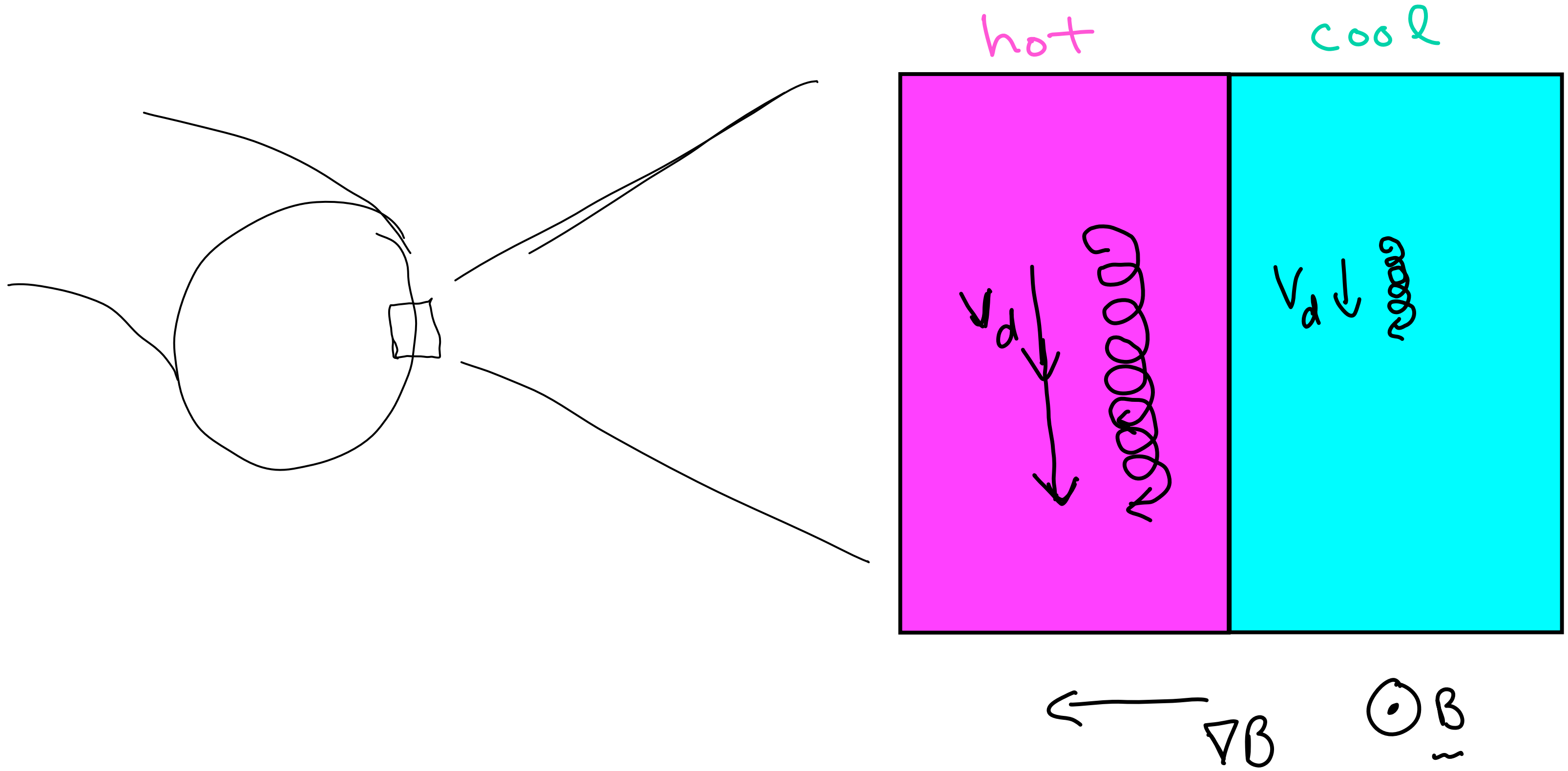
Unstable

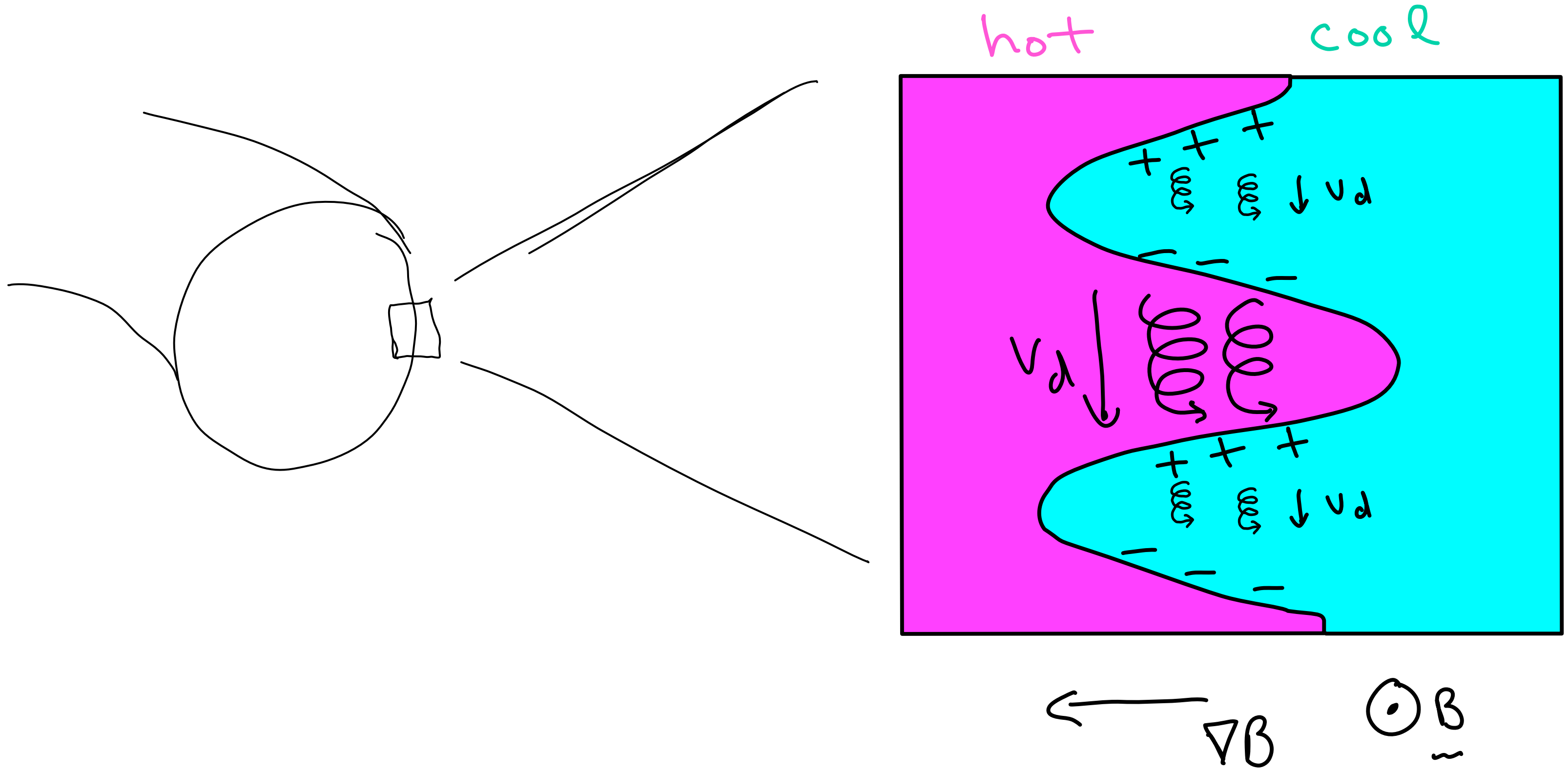
Stable



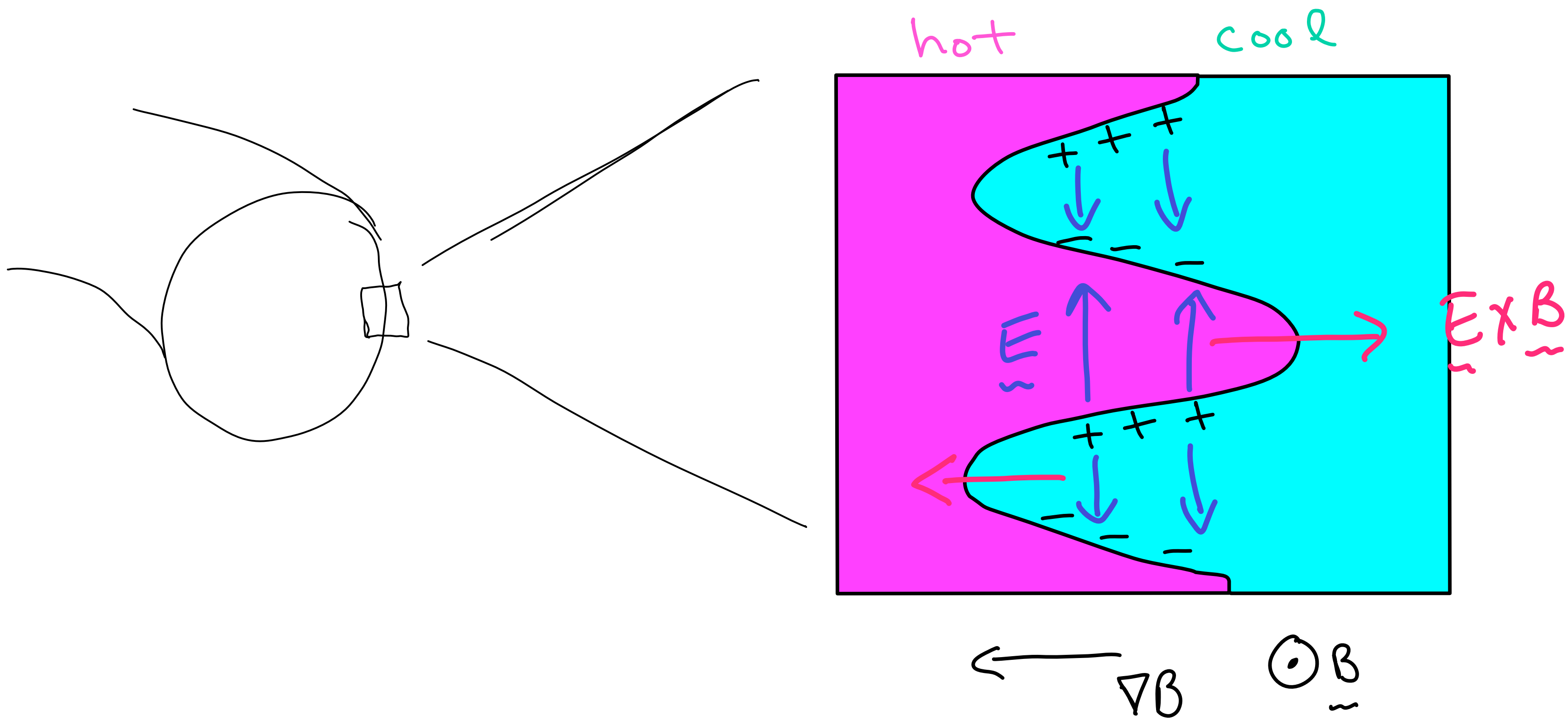


Similar to how twirling a honey dipper can prevent honey from dripping.


hot
cool
V_d
V_d
∇B
⊙ B

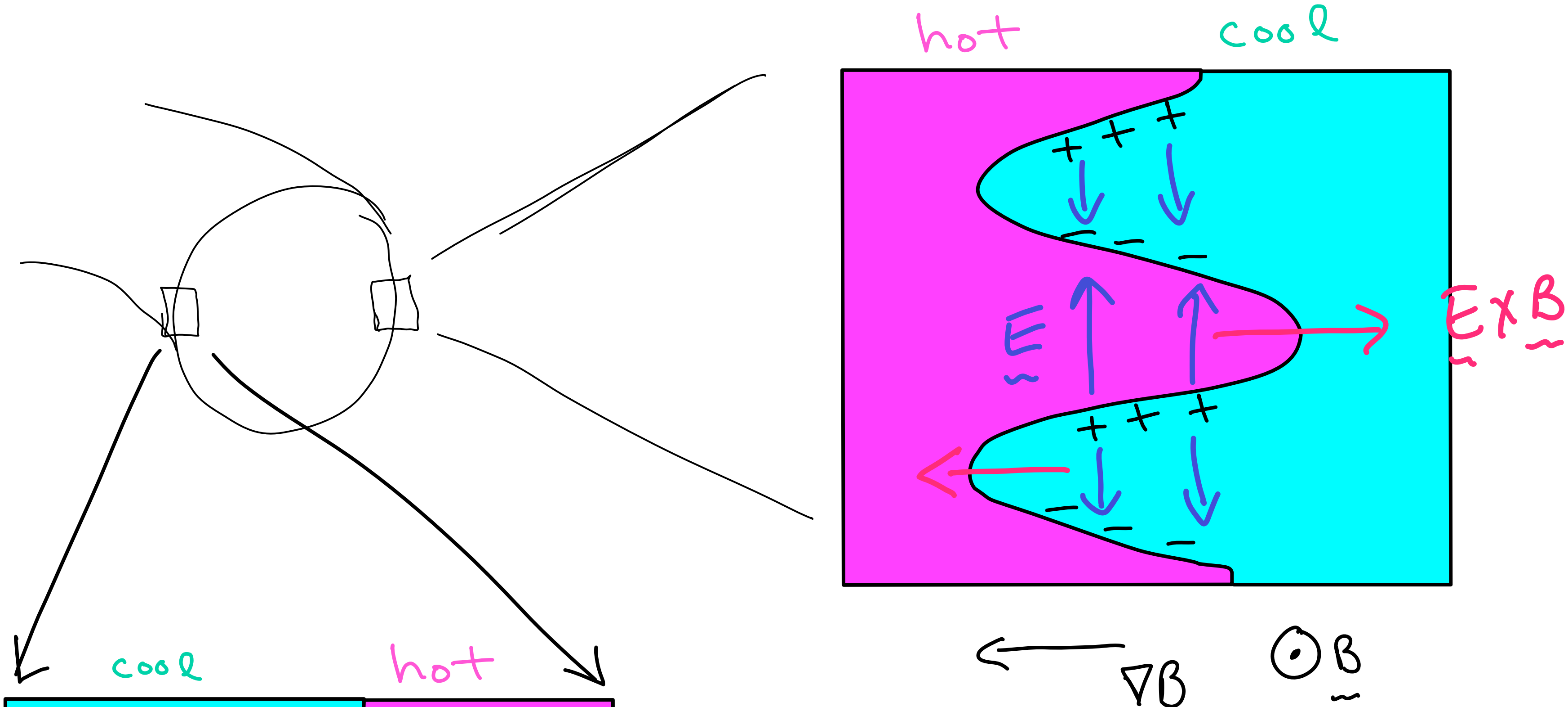


hot
cool
v_d
v_d
v_d
∇B
$\odot B$



Higher energy particles ∇B drift faster, creates charge separation & thus E field, causes E x B flow that further accentuates perturbation. Positive feedback ⇒ instability.

Rosenbluth-Longmire picture



Can repeat this analysis on the good curvature side & find it is stable. (Leave as exercise.)

Rosenbluth-Longmire picture

Twist in B stabilizes unless

growth rate in bad-curvature region	>	propagation from bad-curvature to good curvature regions

MHD works well to lowest order in plasmas, so RHS ⇒

$$\frac{v_t}{\sqrt{RL}} > k_{\parallel} v_A \sim \frac{v_A}{qR}$$

Square:

$$\frac{v_t^2 \, q^2 R^2}{v_A^2 \, RL} > 1$$

$$LHS = \frac{\beta}{2} \frac{q^2 R}{L} = \frac{1}{2} q^2 R \left| \frac{\partial \beta}{\partial r} \right| = \frac{1}{2} \alpha_{MHD}$$

An aside to define some tokamak terminology (ι used in stellarator literature):

$\iota =$ "rotational transform" (or "twisting rate")

$q = \frac{1}{\iota} =$ "safety factor" or "inverse rotational transform"
(or "inverse twisting rate")

$q =$ # of times a field line goes around toroidally
in order to go once around poloidally

$$\mathrm{q} \approx \frac{rB_{tor}}{RB_{pol}}$$

Note: older stellarator literature (< ~ late 1990s) defined "iota bar":

$\iota\!\!\!-\, = \iota / (2\pi) = 1/q$

$q \approx 1.6$ in the upper right figure 2 slides back.

While MHD works well to lowest order in plasmas, there are next-order FLR corrections that defrost the magnetic field & allow $E_{\parallel} \neq 0$ & allow the plasma to move separately from $\underset{\sim}{B}$.

Still have sound waves that can connect good & bad curvature region. Unstable if:

$$\gamma > \text{connection rate}$$

$$\frac{v_t}{\sqrt{RL}} > \frac{v_t}{qR}$$

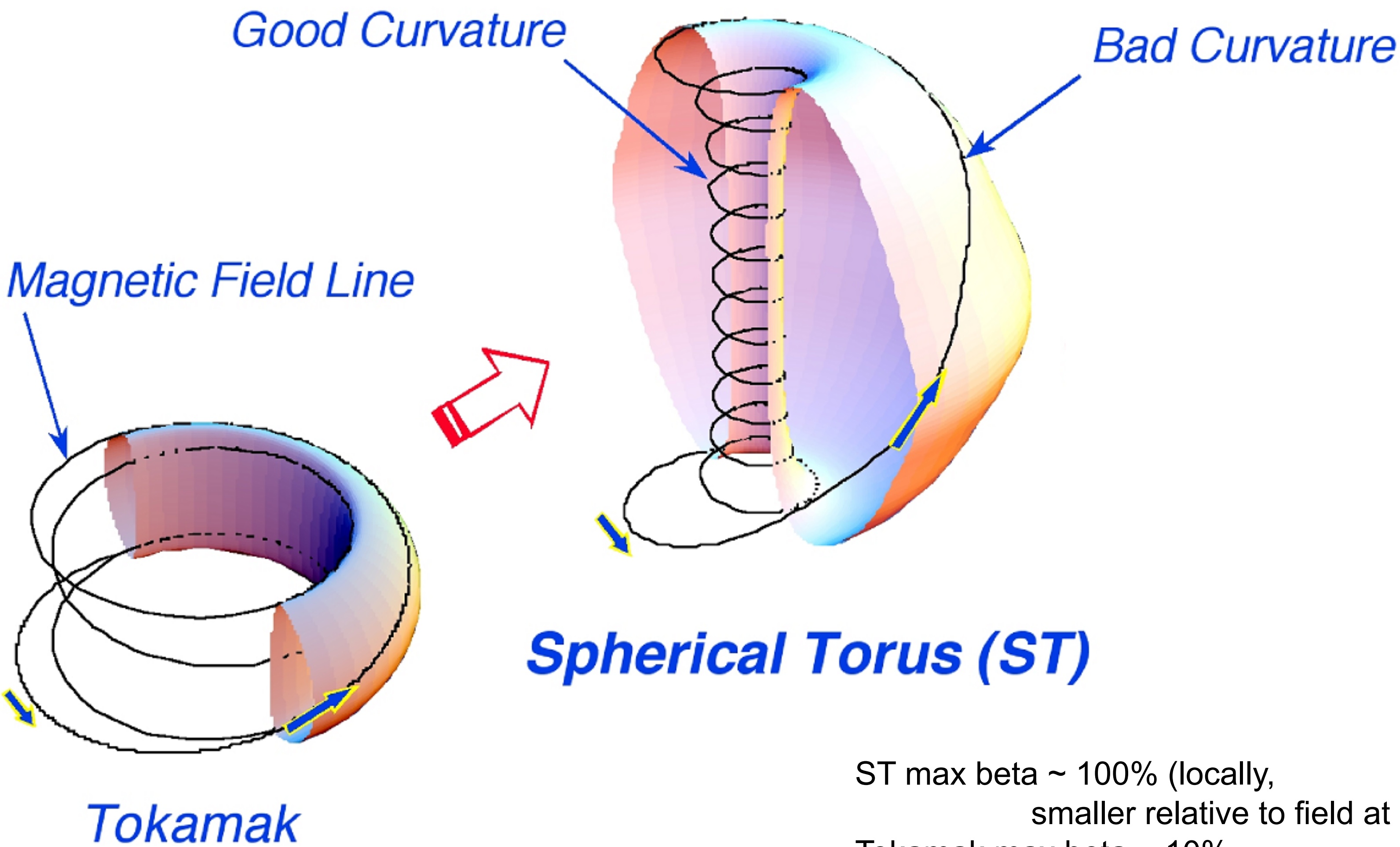
Rough, but tells us $\frac{R}{L}$ is important...

$$\boxed{\frac{R}{L} > \frac{1}{q^2}}$$

$\frac{1}{L} \sim \frac{\nabla T}{T}$

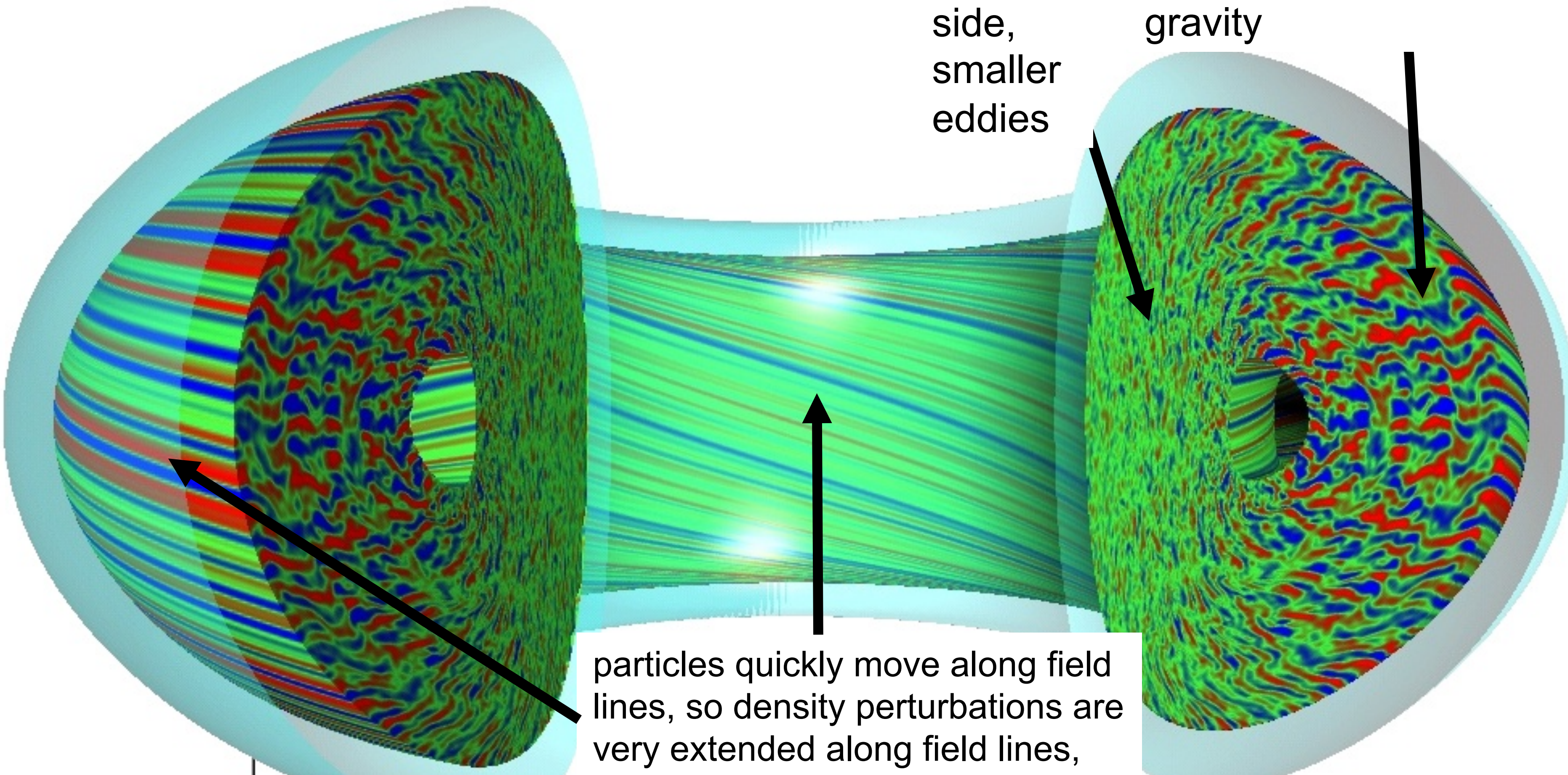
or $\sim \nabla p / p$

Spherical Torus has improved confinement and pressure limits (but less room in center for coils)



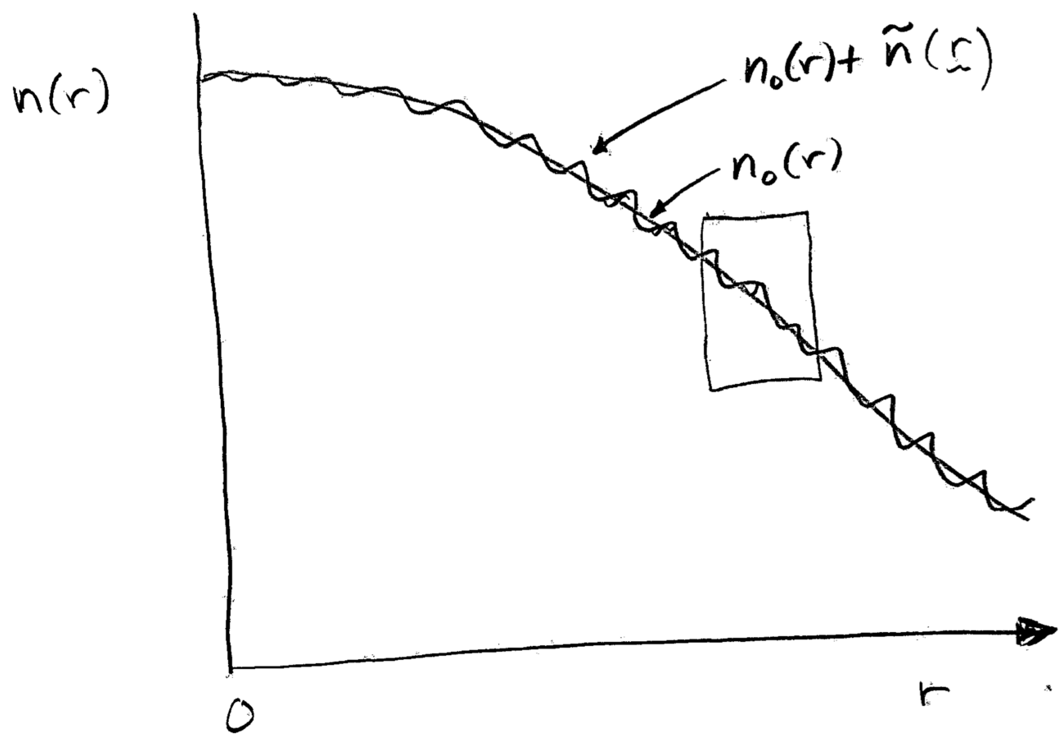
ST max beta ~ 100% (locally,
smaller relative to field at coil)
Tokamak max beta ~ 10%

These physical mechanisms can be seen in gyrokinetic simulations and movies



Note: plots such as on the last page make it look like there is extremely large turbulence in a tokamak. In fact, the relative density fluctuations are quite small, $\tilde{n}/n_0 \sim 0.1\% - 1\%$ in the core region. What is being plotted on the last page are just the density fluctuations $\tilde{n}(\boldsymbol{x})$, because if we tried to plot contours of the total density $n(\boldsymbol{x}) = n_0(r) + \tilde{n}(\boldsymbol{x})$, you couldn't see the small amplitude fluctuations (see below).

Even with the turbulence, particles are confined a factor of $\sim 10^5$ longer than if there was no magnetic field, so the tokamak is confining the particles quite well, but we could improve fusion a lot if we could improve the confinement another factor of 2.



For low-frequency fluctuations, $\omega << k_{||} v_{te}$, electrons have a Boltzmann response to lowest order along a field line:

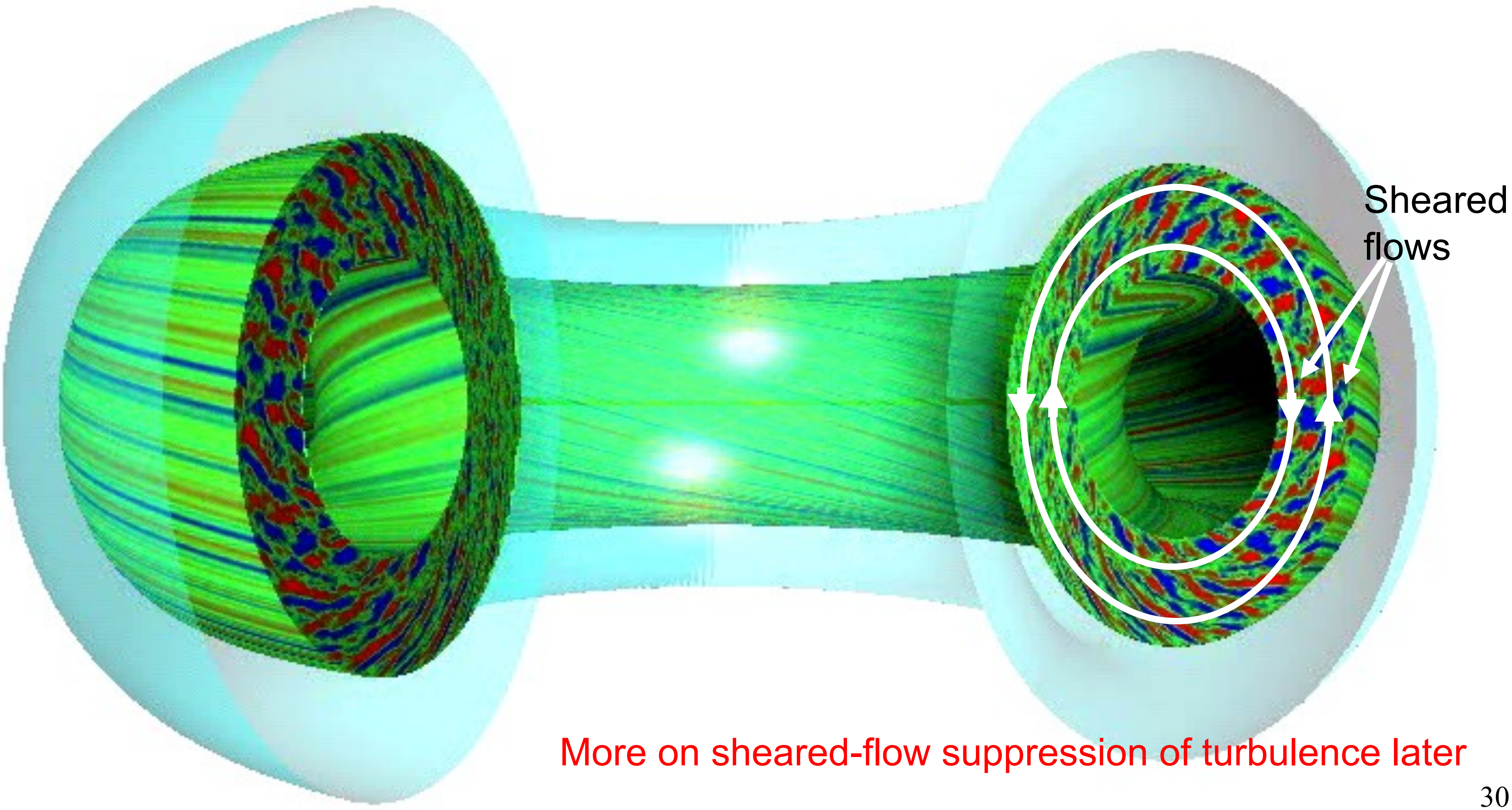
$$n_e(\vec{x},t) = C(r)e^{|e|\phi/T_{e0}}$$

$$\approx n_{e0}\left(1+\frac{|e|\phi}{T_{e0}}\right)$$

$$\delta n \sim n_{e0}\frac{|e|\phi}{T_{e0}}$$

So contours of density fluctuations are also contours of constant potential, and so represent stream lines for the ExB drift. (Like stream lines in 2D fluid flow.) Can illustrate this with a sketch…

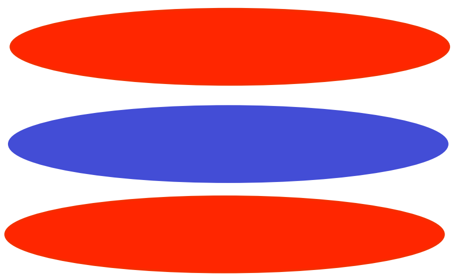
Movie https://fusion.gat.com/theory-wiki/images/3/35/D3d.n16.2x_0.6_fly.mpg from http://fusion.gat.com/theory/Gyromovies shows contour plots of density fluctuations in a cut-away view of a GYRO simulation (Candy & Waltz, GA). This movie illustrates the physical mechanisms described in the last few slides. It also illustrates the important effect of sheared flows in breaking up and limiting the turbulent eddies. Long-wavelength equilibrium sheared flows in this case are driven primarily by external toroidal beam injection. (The movie is made in the frame of reference rotating with the plasma in the middle of the simulation. Barber pole effect makes the dominantly-toroidal rotation appear poloidal..) Short-wavelength, turbulent-driven flows also play important role in nonlinear saturation.



More on sheared-flow suppression of turbulence later

Sheared flows can suppress or reduce turbulence

Most Dangerous Eddies:
Transport long distances
In bad curvature direction



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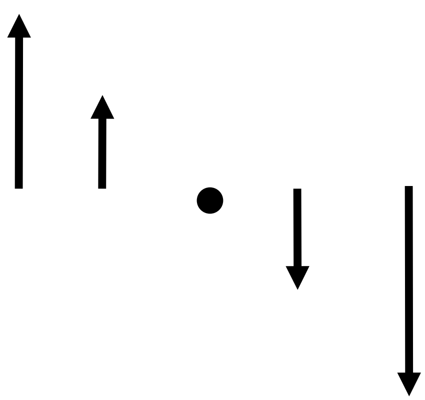
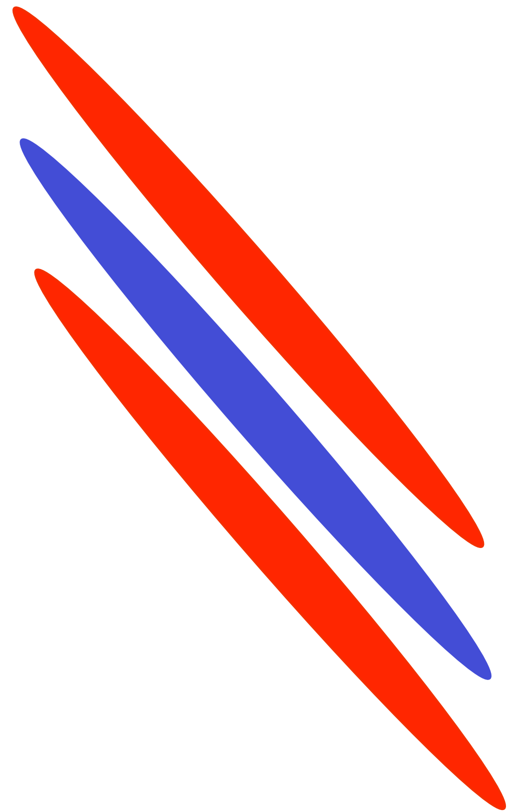
Sheared Flows



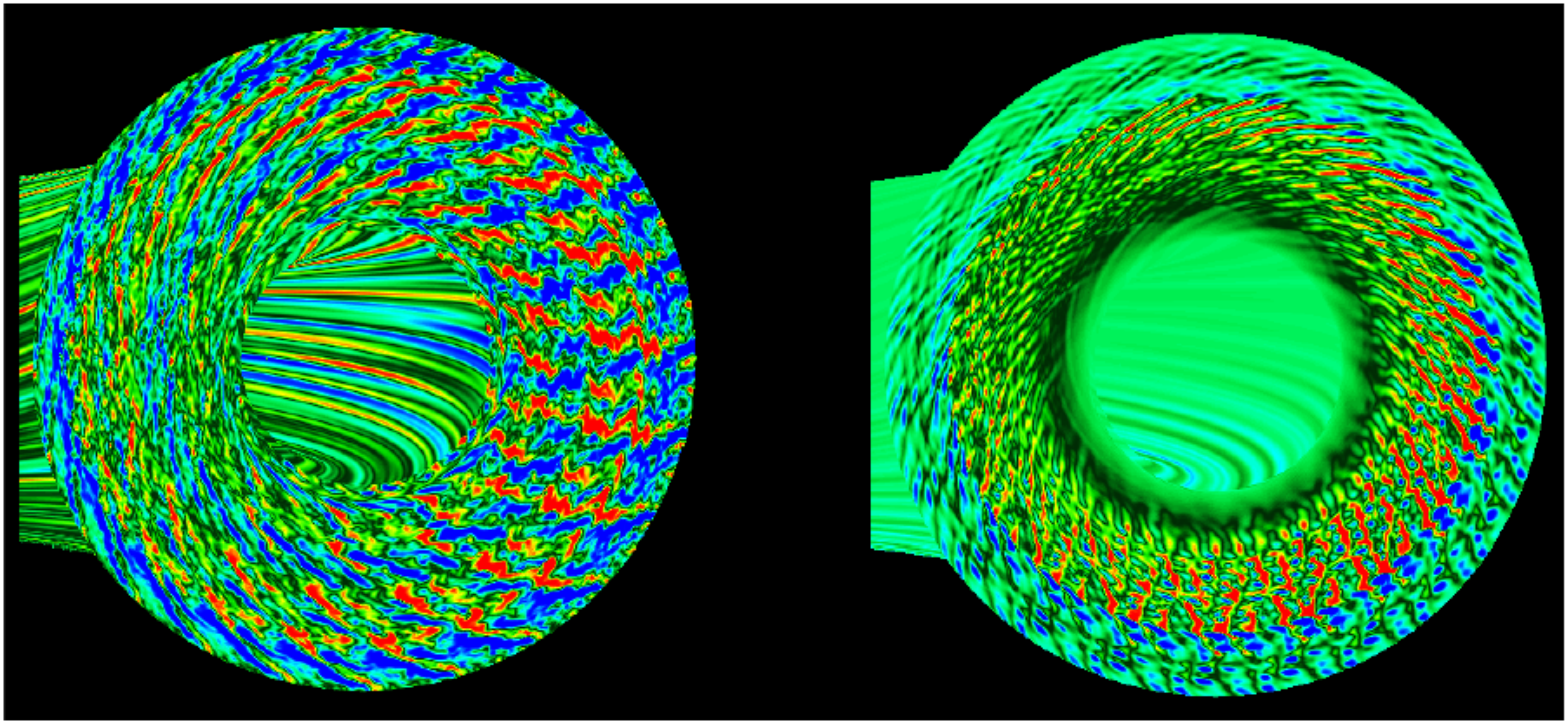
Sheared Eddies
Less effective



Eventually break up

Biglari, Diamond, Terry (Phys. Fluids1990),
Carreras, Waltz, Hahm, Kolmogorov, et al.

Sheared ExB Flows can regulate or completely suppress turbulence (analogous to twisting honey on a fork)



Dominant nonlinear interaction between turbulent eddies and ±θ-directed zonal flows.

Additional large scale sheared zonal flow (driven by beams, neoclassical) can completely suppress turbulence

Waltz, Kerbel, Phys. Plasmas 1994 w/ Hammett, Beer, Dorland, Waltz Gyrofluid Eqs., Numerical Tokamak Project, DoE Computational Grand Challenge

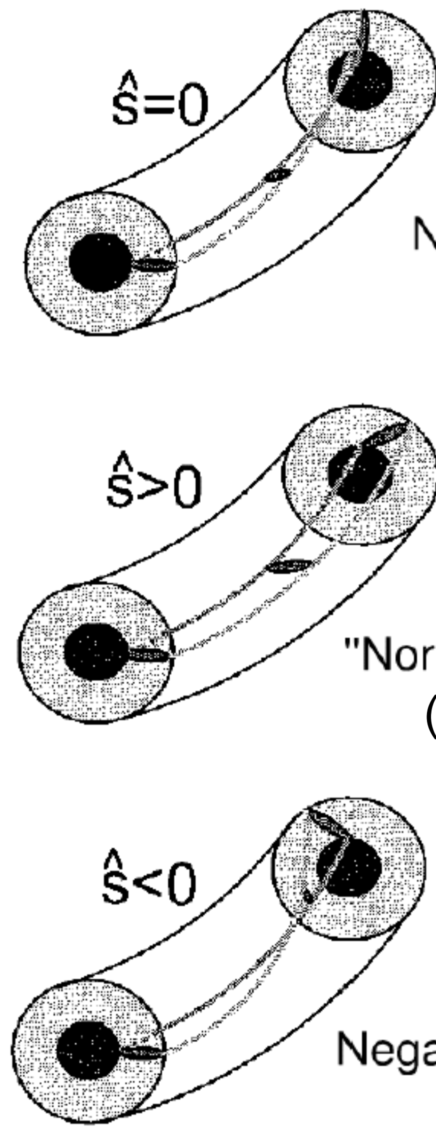
Simple picture of reducing turbulence by negative magnetic shear

Particles that produce an eddy tend to follow field lines.

Reversed magnetic shear twists eddy in a short distance to point in the ``good curvature direction''.

Locally reversed magnetic shear naturally produced by squeezing magnetic fields at high plasma pressure: ``Second stability'' Advanced Tokamak or Spherical Torus.

Shaping the plasma (elongation and triangularity) can also change local shear



No Magnetic Shear

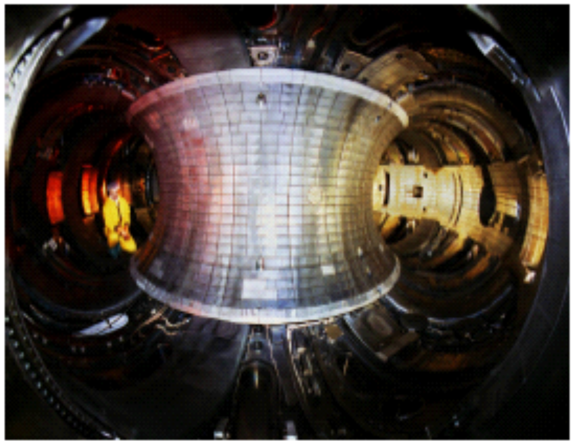
"Normal" Magnetic Shear
(in std tokamaks)

Advanced Tokamaks

Negative Magnetic Shear
“Normal” in stellarators

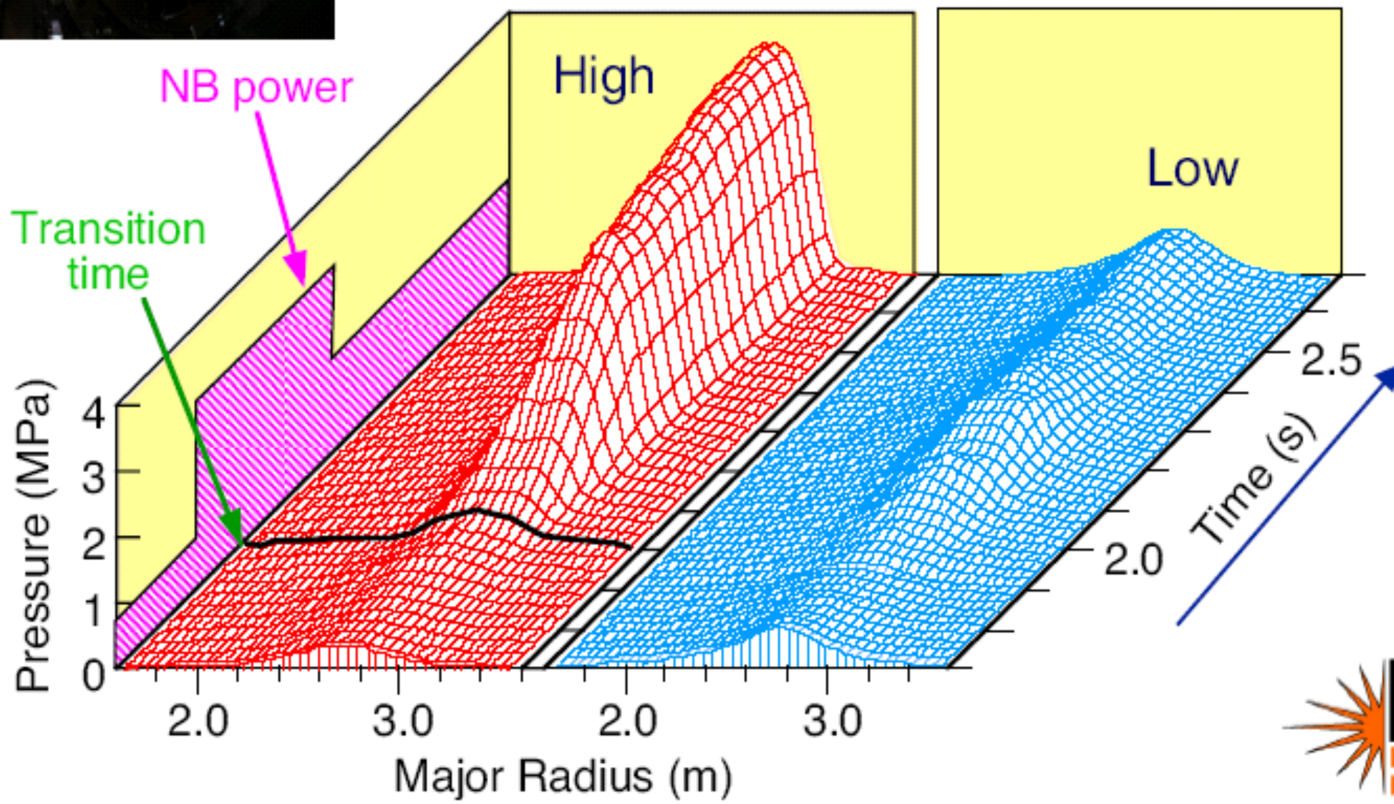
Fig. from Antonsen, Drake, Guzdar et al. Phys. Plasmas 96
Kessel, Manickam, Rewoldt, Tang Phys. Rev. Lett. 94

Fascinating Diversity of Regimes in Fusion Plasmas. What Triggers Change? What Regulates Confinement?



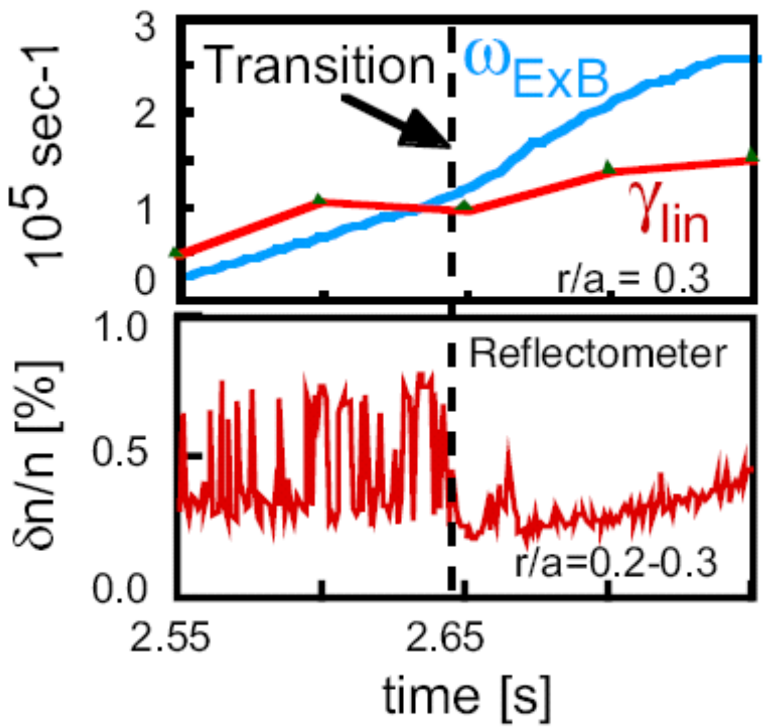
TFTR

- Two regimes with very different confinement for similar initial conditions and neutral beam heating
- Access depends on plasma heating and reducing current density on axis
- Can we attribute a difference in turbulence to these two different confinement regimes?

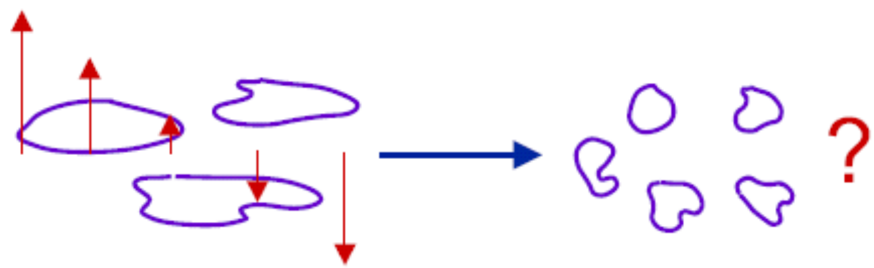


R. Nazikian et al.

Transition to Enhanced Confinement Regime is Correlated with Suppression of Core Fluctuations in TFTR



- Theory predicts fluctuation suppression when rate of shearing (ω_{ExB}) exceeds rate of growth (γ_{lin})
- Outstanding issue: Is suppression accompanied by radial decorrelation?



- Similar suppression observed on JET (X-mode reflectometer) and DIII-D (FIR Scattering)

Hahm, Burrell, Phys. Plas. 1995, E. Mazzucato et al., PRL 1996.

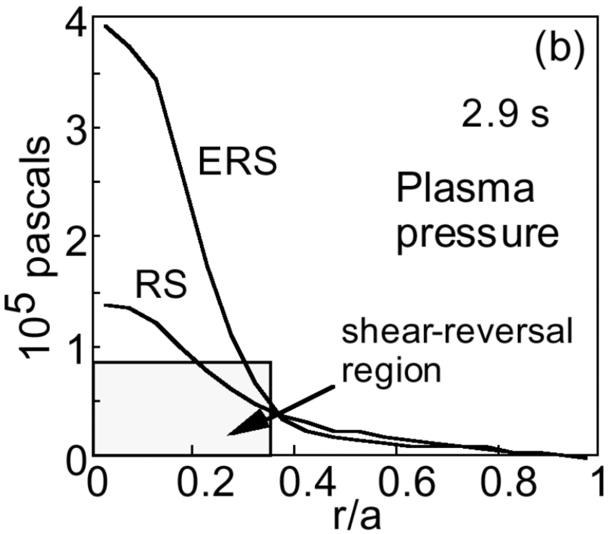
PPPL PRINCETON PLASMA PHYSICS LABORATORY

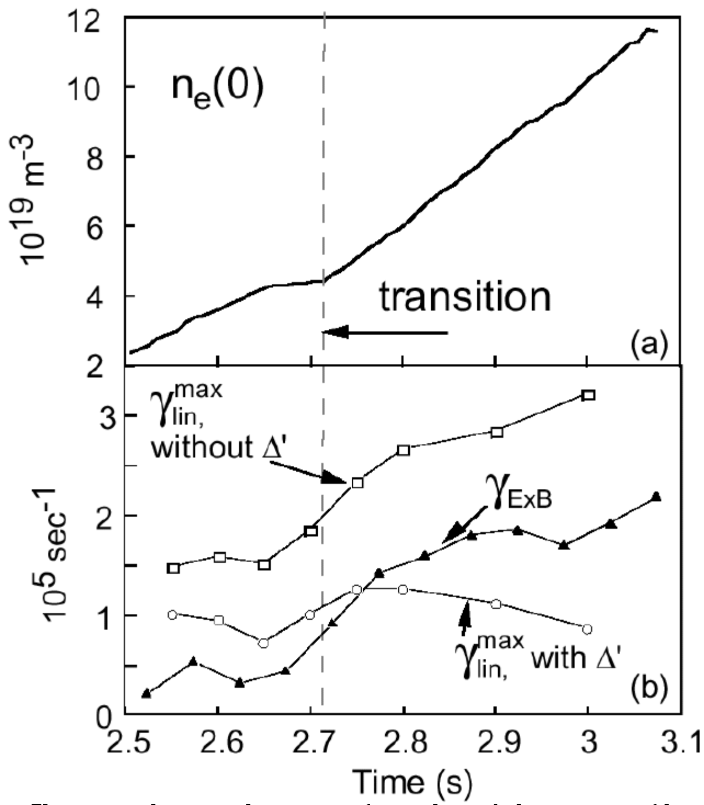
R. Nazikian et al.

I usually denote the shearing rate as γ_{s} or γ_{ExB} instead of ω_{ExB} because it is a dissipative process and isn't like a real frequency. The shearing rate (in a simple limit of concentric circular flux surfaces) is

$$\gamma_s \approx \frac{d\text{v}_{ExB,\theta}}{dr}$$

All major tokamaks show turbulence can be suppressed w/ sheared flows & negative magnetic shear / Shafranov shift





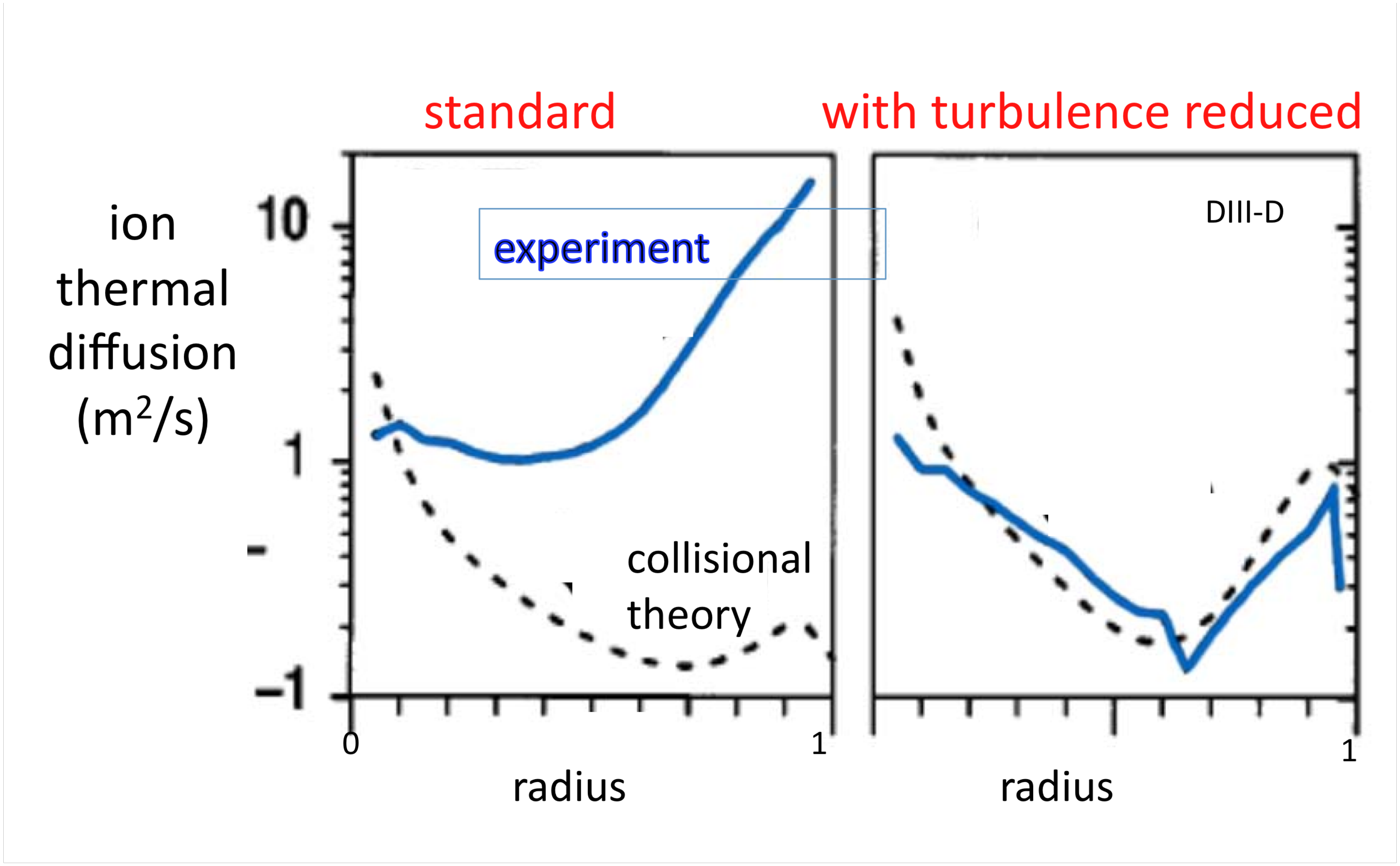
Synakowski, Batha, Beer, et.al. Phys. Plasmas 1997

Internal transport barrier forms when the flow shearing rate dv_θ/dr > ~ the max linear growth rate γ_{lin}^{max} of the instabilities that usually drive the turbulence.

Shafranov shift Δ' effects (self-induced negative magnetic shear at high plasma pressure) also help reduce the linear growth rate.

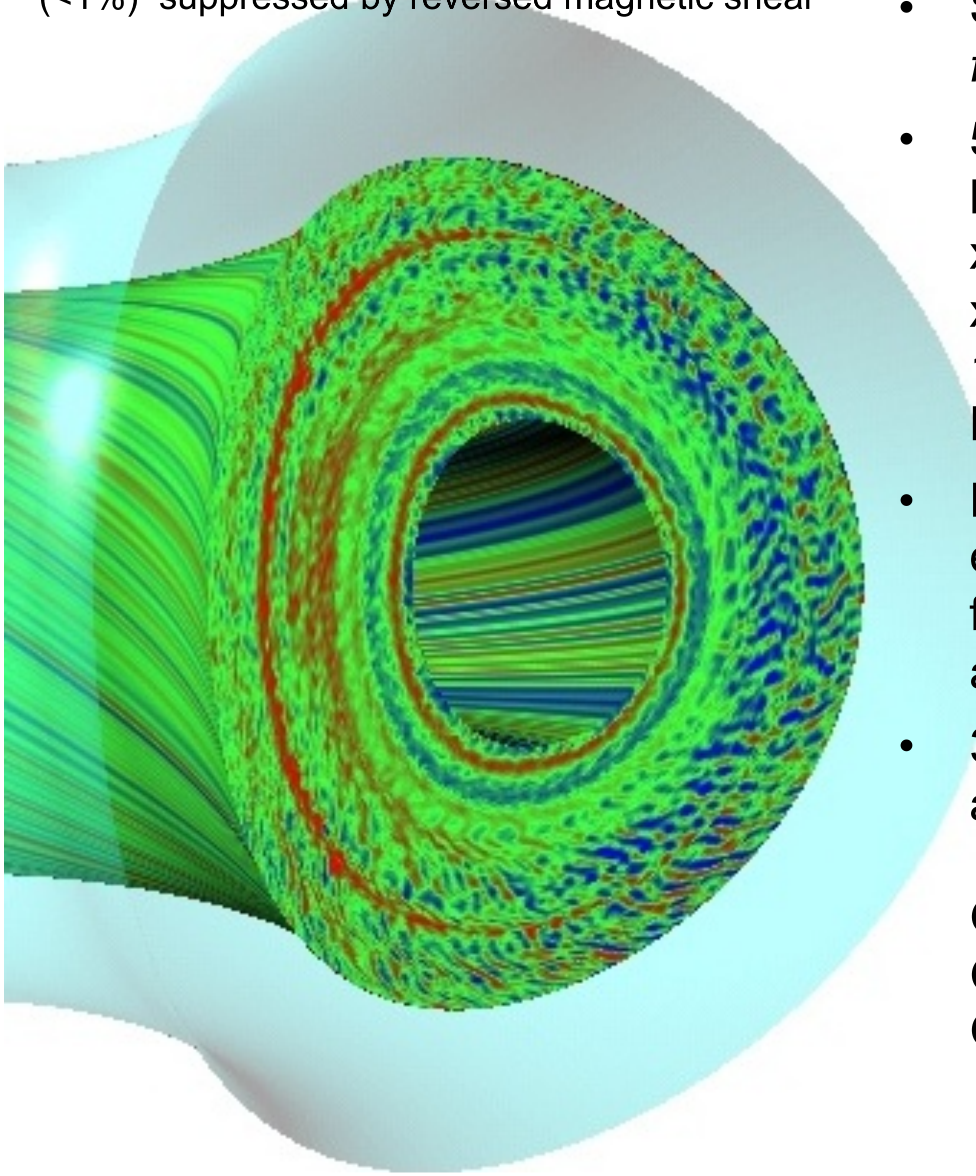
Advanced Tokamak goal: Plasma pressure ~ x 2, $P_{fusion} \propto$ pressure^2 ~ x 4

Turbulence suppression mechanisms really work:
Ion Transport level can be reduced to minimal collisional level in some cases.



Fairly Comprehensive 5-D Gyrokinetic Turbulence Codes Have Been Developed

small scale, small amplitude density fluctuations (<1%) suppressed by reversed magnetic shear



- Solve for the particle distribution function $f(r,\theta,\alpha,E,\mu,t)$ (avg. over gyration: 6D → 5D)
- 500 radii x 32 complex toroidal modes (96 binormal grid points)
  x 10 parallel points along half-orbits
  x 8 energies x 16 $v_{||}/v$
  12 hours on ORNL Cray X1E with 256 MSPs
- Realistic toroidal geometry, kinetic ions & electrons, finite-β electro-magnetic fluctuations, collisions. Sophisticated algorithms.
- 3 most widely used comprehensive codes all use “continuum”/Eulerian algorithms:

  GS2 (Dorland et al.)
  GYRO (Candy et al.)
  GENE (Jenko et al.)

Center for the Study of Plasma Microturbulence

- A DOE, Office of Fusion Energy Sciences, SciDAC (Scientific Discovery Through Advanced Computing) Project

- devoted to studying plasma microturbulence through direct numerical sumulation

- National Team (& 2 main codes):
  - GA (Waltz, Candy)
  - U. MD (Dorland)
  - MIT (D. Ernst)
  - LLNL (Nevins, Cohen, Dimits)
  - PPPL (Hammett, …)

- They've done all the hard work …

Continuum/Eulerian approach: Solve Vlasov/Boltzmann PDE (like the Liouville Eq.) to find the density of particles in phase space, $f(\vec{x}, \vec{v}, t)$:

$$\frac{\partial f}{\partial t} = -\vec{v} \cdot \nabla f - \frac{q}{m} \left(\vec{E} + \vec{v} \times \vec{B} \right) \cdot \nabla_v f$$

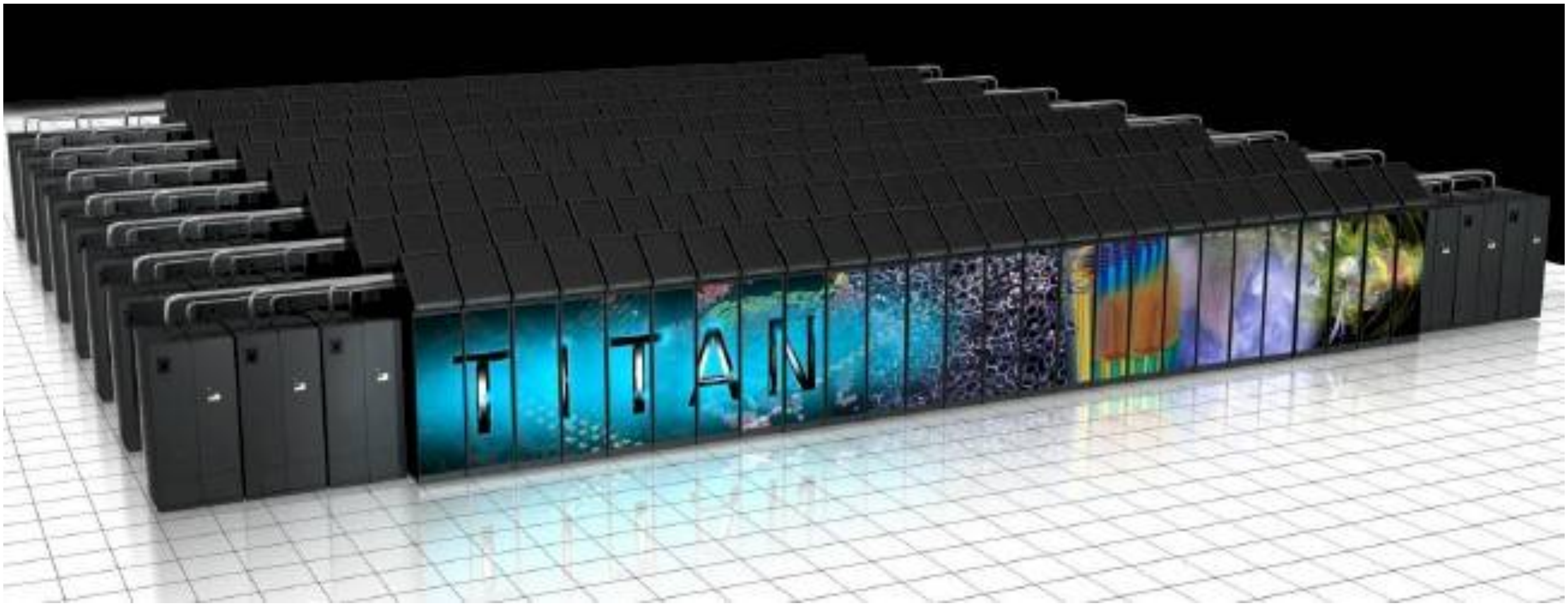
Particle/Lagrangian approach (a Monte Carlo sampling of phase-space), for a large number of particles:

$$\frac{d\vec{x}_i}{dt} = \vec{v}_i, \qquad \frac{d\vec{v}_i}{dt} = \frac{q_i}{m_i}(\vec{E}(\vec{x}_i, t) + \vec{v}_i \times \vec{B}(\vec{x}_i, t))$$

Use "gyrokinetic" equations to average over the fast gyration of particles and eliminate irrelevant high-frequency dynamics (using sophisticated asymptotic theory), so we only need to solve for $f(\vec{x}, v_{||}, v_\perp)$. Average these equations over all velocity to derive fluid equations that describe the evolution of the density, momentum, and pressure of the plasma. Leads to MHD (MagnetoHydroDynamics).

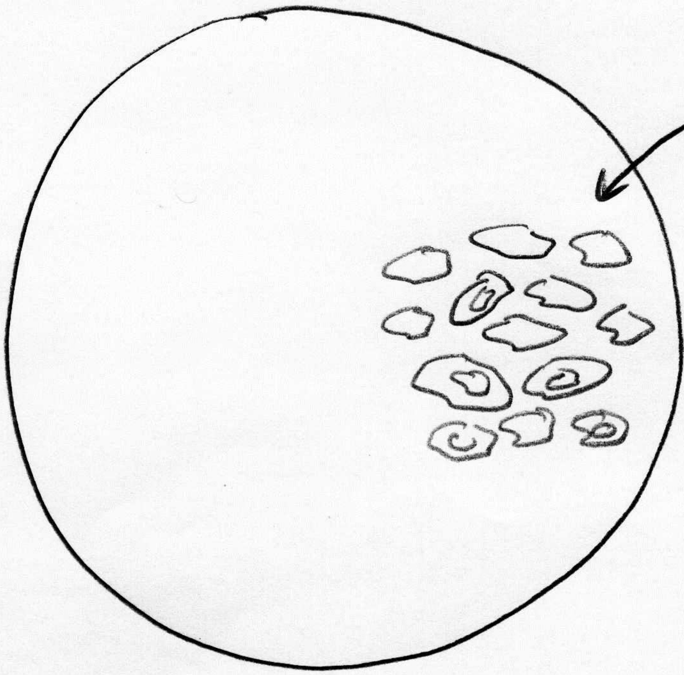
Several DOE "Scientific Discovery Through Advanced Computing" (SciDAC) projects for fusion energy.

Plasma physics advanced computing also in: astrophysics (Stone: MHD turbulence and shocks, Spitkovsky: PIC sims of supernova shocks), space physics, solar storms (Bhattacharjee, Johnson), & Max-Planck/Princeton Center for Plasma Physics.



Cray Titan Supercomputer @ Oak Ridge: World's fastest (Fall, 2012):
300,000 AMD Opteron cores, 19,000 GPUs
20 Petaflops (2×10^{16} flop/s ~1/10 human)
$100M, $9M/y electricity

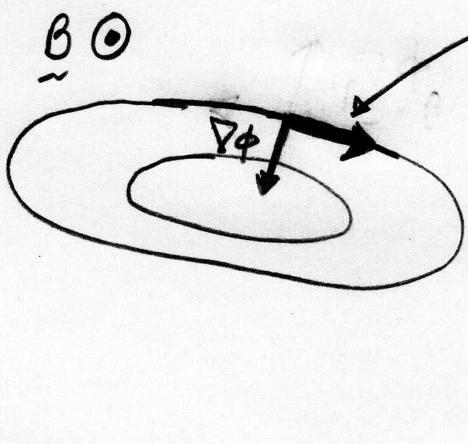
Mixing Length Estimates of Turbulent Diffusion in Tokamaks



Contours of electrostatic potential $\tilde{\Phi}(\underset{\sim}{x})$ in a tokamak.

$\underset{\sim}{E} \times \underset{\sim}{B}$ drift causes particles to drift along lines of constant Φ:

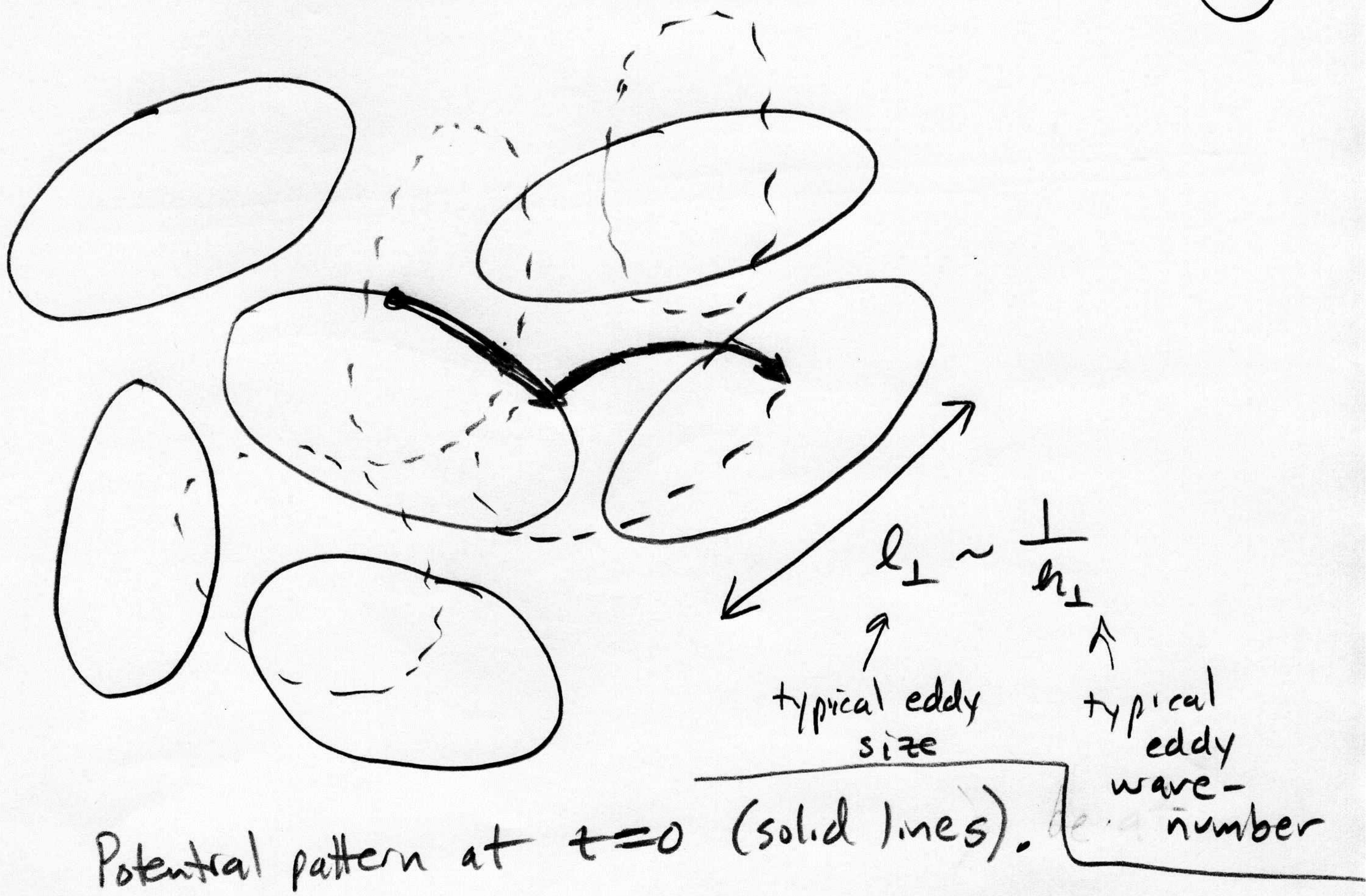
I.e., there is no flow in the direction of



$$\underset{\sim}{V}_{E\times B} = \frac{\underset{\sim}{E} \times \hat{b}}{B} = \frac{\hat{b} \times \nabla\tilde{\Phi}}{B}$$

so $V_{E\times B} \cdot \nabla\tilde{\Phi} = 0$

$\tilde{\Phi}(\underset{\sim}{x})$ a.k.a. "the stream function"



Potential pattern at $t=0$ (solid lines).

After a decorrelation time Δt, there are new eddy patterns (dotted lines).

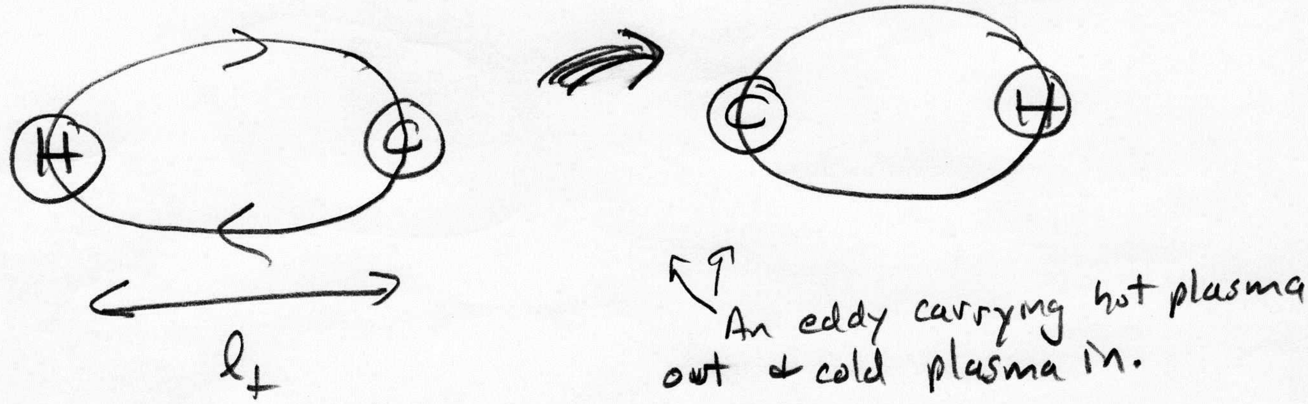
Can treat this as a random walk diffusion process:

$$D = \frac{1}{2}\frac{\langle (\Delta x)^2 \rangle}{\Delta t} \sim \frac{(V_{E\times B}\,\Delta t)^2}{\Delta t}$$

$$\sim V_{E\times B}^2\,\Delta t$$

Typical assumption in "strong turbulence" regimes is that the decorrelation time Δt is such that a typical eddy advects about half-way around it's circumference

$$V_{E\times B}\,\Delta t \sim l_\perp$$



After this time, the temperature gradient (or density gradient, or whatever was driving the eddy) has locally reversed sign & is no longer driving this eddy. Then new eddies are driven. So

$$D \sim V_{E\times B}\, l_\perp$$

Assume $V_{E\times B} = \dfrac{\hat{b}\times\nabla\tilde{\phi}}{B}$ has a "typical"

magnitude (i.e., an RMS, root-mean-square)

of $V_{E\times B} \sim \dfrac{\phi_{rms}}{\ell_\perp}\dfrac{1}{B}$ where ϕ_{rms} is an RMS

potential amplitude & $\ell_\perp$ a typical eddy size.

Now

$$D \sim V_{E\times B}\,\ell_\perp \sim \frac{\phi_{rms}}{B}$$

But how big is ϕ_{rms}?

Electrons move along field lines very quickly

(because they are light compared to ions):

$$V_{te} = \sqrt{\frac{T_e}{m_e}} \gg V_{ti}$$

Electrons are in a Boltzmann thermal equilibrium along a magnetic field line, so

$$n_e = G\, e^{e\tilde{\phi}(\underline{x})/T_{eo}}$$

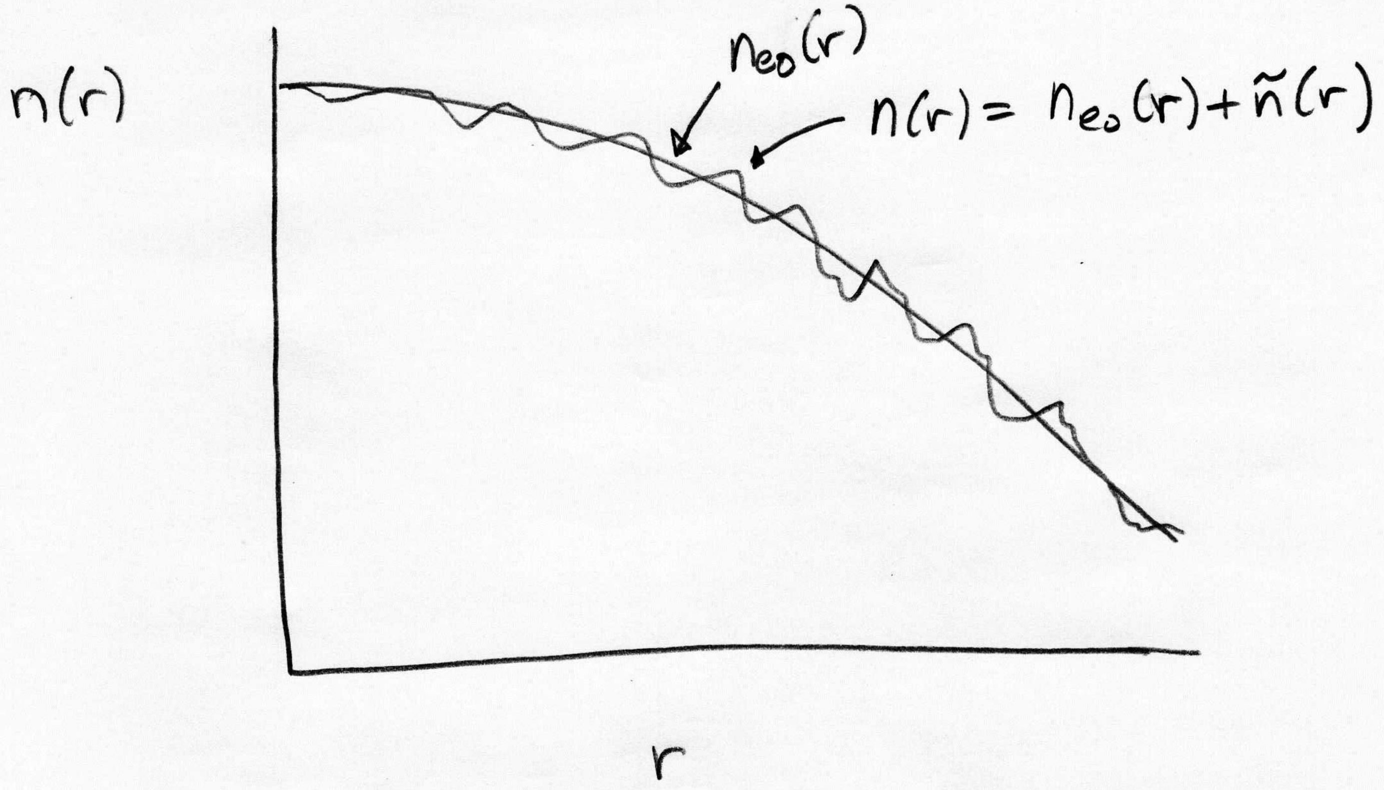
(caution: the symbol "e" is used 3 different ways here.)

$$n_e \approx G\left(1 + \frac{e\tilde{\phi}}{T_{eo}}\right)$$

$$= n_{eo} + \underbrace{n_{eo}\frac{e\tilde{\phi}(\underline{x})}{T_{eo}}}$$

$\tilde{n}_e$ = electron density fluctuations ∝ potential fluctuations.

or $$\tilde{\phi} = \frac{\tilde{n}_e}{n_{eo}}\frac{T_{eo}}{e}$$



Instabilities driven by density (or temperature) gradients tend to saturate when $\nabla \tilde{n}$ is large enough to offset ∇n_0 so the total density gradient vanishes at some places:

$$\nabla \tilde{n}_e \sim \nabla n_{eo}$$

$$\frac{\tilde{n}_{e,rms}}{l_\perp} \sim \frac{n_{eo}}{L_n}$$

Thus $D \sim \frac{\phi_{rms}}{B} \sim \frac{\tilde{n}_e}{n_{eo}} \frac{T_{eo}}{eB}$

$$\sim \frac{l_\perp}{L_n} \frac{T_{eo}}{eB}$$

Define sound speed $c_s = \sqrt{\frac{T_{eo}}{m_i}}$

typical gyroradius $\rho_s = \frac{c_s}{\Omega_c}$, $\Omega_c = \frac{eB}{m_i}$

& use the result that typical turbulent eddy sizes in tokamaks are of order $l_\perp \sim (3-10)\rho_s$, gives:

$$D \sim (3-10) \frac{\rho_s}{L_n} \underbrace{c_s \rho_s}_{\doteq 16 D_{Bohm}}$$

This is called "Gyro-Bohm Diffusion", because it is reduced from Bohm diffusion by a large factor, $\frac{\rho_s}{L_n} \sim \frac{\rho_s}{a} \sim 10^{-3}$

This is a rough estimate of turbulent diffusion in a tokamak. It gives roughly the proper scalings, but more detailed calculations (involving $10^4 - 10^6$ CPU hours of heavy-duty supercomputing) are needed to find numerical coefficients (& effects of magnetic fluctuations, & many other things).

A key process left out of this formula is a "critical gradient", that there is no turbulence unless the gradients are sufficiently steep ($1/L_n$ sufficiently large).

Basic scaling: $\tau_E \sim \frac{a^2}{D} \sim \frac{a^3}{c_s \rho_s^2} \sim \frac{a^3 B^2}{T^{3/2}}$

So to get higher τ_E for fusion, need a larger machine (a) &/or stronger magnetic field. (& the confinement gets worse at higher temperature).

Further Reading for Newcomers to Plasmas

- The textbook by Goldston and Rutherford, “Introduction to Plasma Physics”, is aimed at an advanced undergraduate level, and is a good place to start for those looking for a systematic treatment of plasma physics. In the back are several chapters that deal with the types of instabilities that drive small-scale turbulence in tokamaks (including the ITG instability and drift wave instabilities in simple slab geometry).
- Wesson’s text book, “Tokamaks”, is a nice compendium, and has sections on simple models of plasma turbulence and transport.
- Someday I should write up a more systematic description of the ideas I discuss here about simple pictures of ITG turbulence mechanisms, subtle effects of critical gradients, and a survey of ways to reduce turbulence.
- John Krommes, “The Gyrokinetic Description of Microturbulence in Magnetized Plasmas”, Ann. Rev. of Fluid Mechanics 44, 175 (2012), http://dx.doi.org/10.1146/annurev-fluid-120710-101223 This is a survey of very interesting new results in tokamak turbulence. It discusses some cutting-edge research that is quite complicated, but tries to do so in way that gets some of the main ideas across to a broad audience of scientists outside of fusion research.

- Ph.D. Dissertations are a good place to look for beginners in a field, because they often contain useful tutorials or pointers to good references in the beginning sections. On the topic of tokamak turbulence, I would suggest dissertations by my recent students Luc Peterson and Jessica Baumgaertel, which are linked to at http://w3.pppl.gov/~hammett/papers/. (Granstedt’s thesis is also very good, but has less intro material on turbulence.)
- My second Ph.D. student’s thesis (Mike Beer 1995) has a good tutorial on the toroidal ITG mode:
  http://w3.pppl.gov/~hammett/collaborators/mbeer/afs/thesis.html
  Presents a tutorial on fundamentals and physical pictures of ITG mode, and the first comprehensive 3D gyrofluid simulations (gyrofluid equations include models of FLR & kinetic effects like Landau damping) of ITG and TEM turbulence in realistic toroidal geometry. Documents the important role of turbulence-generated zonal flows in saturating toroidal ITG turbulence, and the major reduction of ITG turbulence by using a proper adiabatic electron response that does not respond to zonal electric fields with $E_{||}=0$ (also shown in slab limit in Dorland’s earlier thesis).

ITG Turbulence References

- Early history:
  - slab eta_i mode: Rudakov and Sagdeev, 1961
  - Sheared-slab eta_i mode: Coppi, Rosenbluth, and Sagdeev, Phys. Fluids 1967
  - Toroidal ITG mode: Coppi and Pegoraro 1977, Horton, Choi, Tang 1981, Terry et al. 1982, Guzdar et al. 1983… (See Beer's thesis)

- Romanelli & Briguglio, Phys. Fluids B 1990
- Biglari, Diamond, Rosenbluth, Phys. Fluids B 1989
  These two are detailed analytic papers on ITG dispersion relations and mixing-length estimates of turbulent transport. The Biglari et al. paper shows some interesting tricks for manipulating the plasma dispersion function Z (used also in Beer's thesis).

More ITG References (2)

- Online links to some of these papers are at http://w3.pppl.gov/~hammett/papers/

- Kotschenreuther, Dorland, Beer, Hammett, PoP 1995,
  Presents the "IFS-PPPL" transport model, based on nonlinear gyrofluid ITG simulations and linear gyrokinetic simulations for a more accurate critical gradient. The first transport model comprehensive enough to successfully predict the temperature profiles in the core region of tokamaks over a wide range of parameters, including explaining the improved confinement of "supershots" and H-modes relative to L-modes. Also emphasized the importance of marginal stability effects that make core temperature profiles sensitive to edge temperature boundary conditions.

- Jenko, Dorland, Hammett, PoP 2001
  improved, fairly accurate critical gradient for ETG/ITG instabilities, fit to a large number of linear numerical gyrokinetic simulations (and recovers previous analytic results in various limits)

- "Comparisons and Physics Basis of Tokamak Transport Models and Turbulence Simulations", Dimits et al, PoP 2000
  Detailed cross-code comparisons of gyrofluid and full gyrokinetic codes for ITG turbulence (the "cyclone" case here is an oft-used benchmark test). Demonstrated that gyrofluid codes had too much damping of zonal flows and missed the "Dimits" nonlinear shift in the effective critical gradient. (These errors were not large enough to significantly affect previous predictions using gyrofluid-based models about the performance of the 1996 ITER design.) Later improvements to gyrofluid closures reduce the discrepancies.

More ITG References (3)

- Jenko & Dorland et al, PoP 2000, Dorland & Jenko et al. PRL 2000
  discovery that ETG turbulence is much stronger than expected from simple scaling from ITG turbulence, because of the important difference between the adiabatic species response to zonal flows.

- Jenko & Dorland, PRL 2002 http://prl.aps.org/abstract/PRL/v89/i22/e225001
  interesting explanation of the differences between ITG & ETG nonlinear saturation levels in various regimes based on secondary instability analysis, relative importance of Rogers (perpendicular/zonal flow) vs. Cowley (parallel flow) secondary instabilities.

- "Anomalous Transport Scaling in the DIII-D Tokamak Matched by Supercomputer Simulation", Candy & Waltz, PRL 2003, https://fusion.gat.com/THEORY/images/e/e7/Candy-PRL03.pdf
  One of the first comprehensive simulations by the GYRO code, similar to the Kotschenreuther-Dorland continuum gyrokinetic turbulence code, but extended from the local limit to consider non-local/global effects that can break gyro-Bohm scaling.

Gyrokinetic Turbulence Code References

- Below are 3 widely-used gyrokinetic codes for comprehensive 5-D plasma turbulence simulations. These 3 codes use "continuum" methods with a grid in phase-space, instead of the random sampling of Particle-in-Cell (PIC) algorithms. These 3 codes are relatively comprehensive, handling fully electromagnetic fluctuations with a kinetic treatment of electrons and multiple ion species, collision operators, and general non-circular tokamak geometries. They are actively being used to compare with experiments and to understand the underlying physics of the turbulence.

  - GS2 (Kotschenreuther & Dorland, IFS/Texas & Maryland) the first fully electromagnetic nonlinear gyrokinetic code, optimized for the small ρ_* thin-annulus / flux-tube local limit, and can also handle stellarators: http://gyrokinetics.sourceforge.net/

  - GENE (Jenko et al., Garching) similar to GS2 originally, extended to non-local/global effects like GYRO, and for stellarators: http://www.ipp.mpg.de/~fsj/gene

  - GYRO (Candy and Waltz et al., General Atomics), inspired by GS2, but extended to non-local global effects that can break gyro-Bohm scaling: http://fusion.gat.com/theory/Gyro

  - There are several PIC codes that have also been used to study aspects of tokamak turbulence with various levels of approximation, including GEM, GTS, GTC, XGC, …