THEORY-BASED MODELS OF TURBULENCE AND ANOMALOUS TRANSPORT IN FUSION PLASMAS

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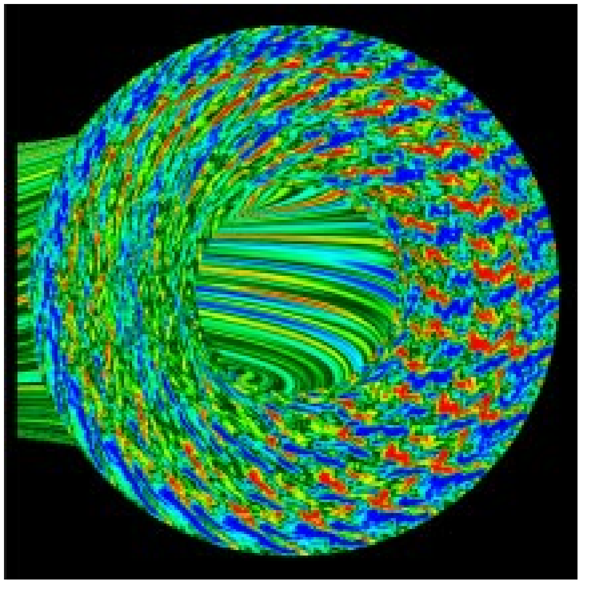
APS Centennial, Atlanta, March, 1999



Close collaboration with M.A. Beer (PPPL), W. Dorland (Univ. of Maryland), M. Kotschenreuther (Univ. of Texas), R.E. Waltz (General Atomics).

Acknowledgments: A. Dimits, G.D. Kerbel (LLNL), T.S. Hahm, Z. Lin, P.B. Snyder (PPPL), S.E. Parker (U. Colorado), and many others.

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THEORY-BASED MODELS OF TURBULENCE AND ANOMALOUS TRANSPORT IN FUSION PLASMAS

I. Simple picture of plasma microinstabilities

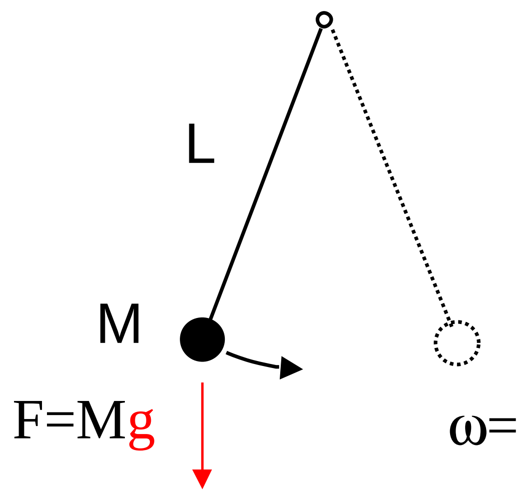
- **Inverted pendulum $\rightarrow$ Rayleigh-Taylor $\rightarrow$ Magnetic curvature instability.**
- **Difference between MHD and micro-instabilities/drift-waves.**

II. Complexity and challenge of plasma turbulence

- **nonlinear, chaotic, wide-range of space and time scales**
- **theoretical and computational advances made in tackling these problems.**

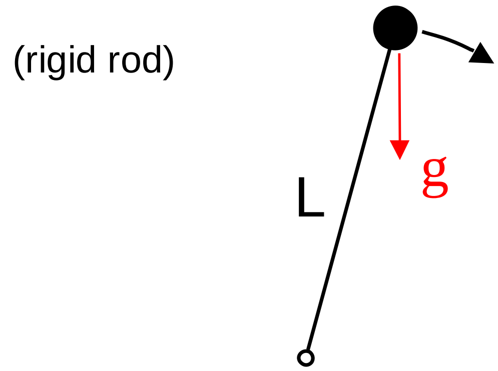
III. Comparisons with experiments, remaining challenges.

Stable Pendulum



$\omega=(g/L)^{1/2}$

Unstable Inverted Pendulum

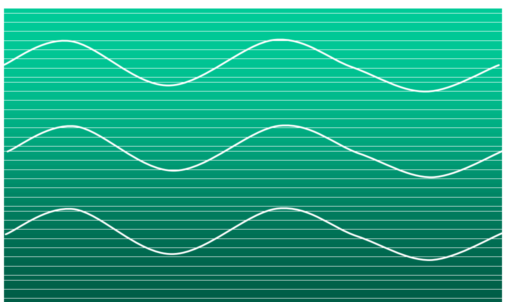


$\omega= (-g/|L|)^{1/2} = i(g/|L|)^{1/2} = i\gamma$

Instability

Density-stratified Fluid

$\rho=\exp(-y/L)$

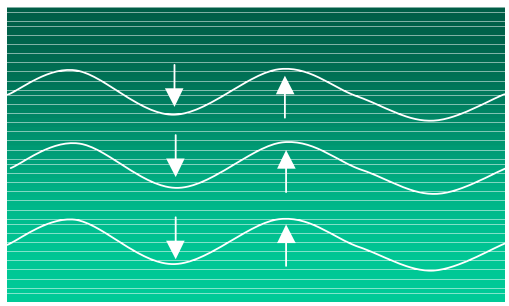


stable $\omega=(g/L)^{1/2}$

Inverted-density fluid

$\Rightarrow$Rayleigh-Taylor Instability

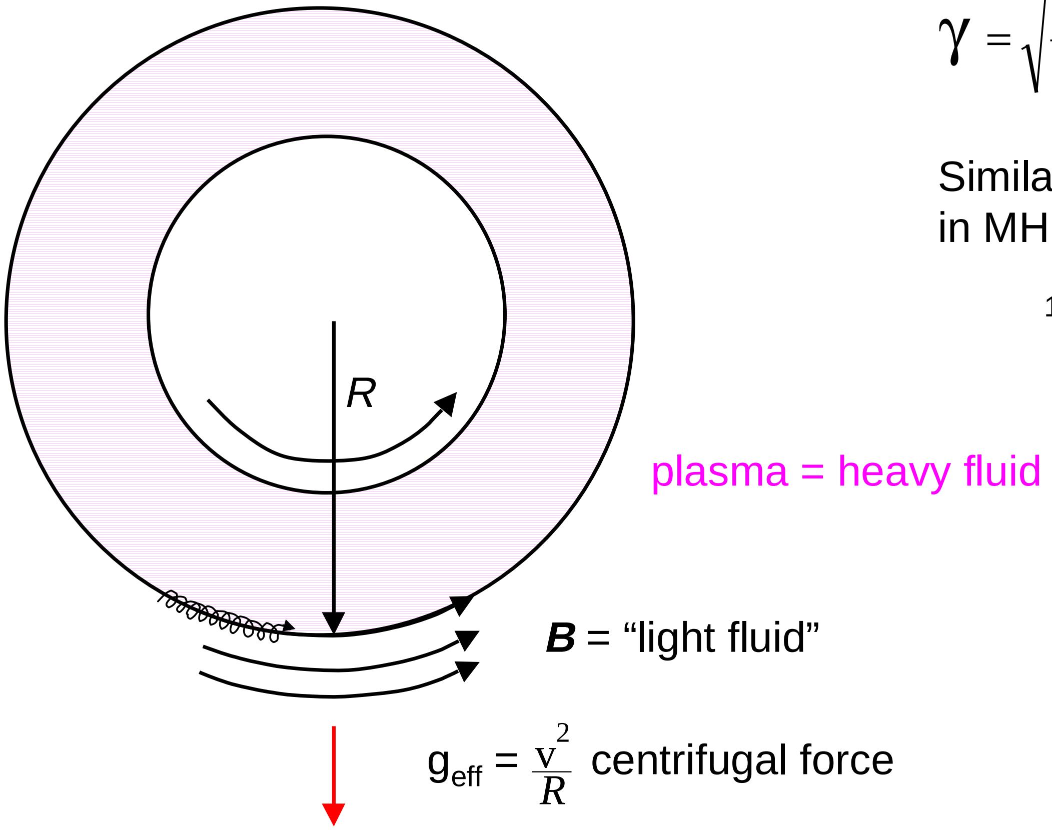
$\rho=\exp(y/L)$



Max growth rate $\gamma=(g/L)^{1/2}$

“Bad Curvature” instability in plasmas
≈ Inverted Pendulum / Rayleigh-Taylor Instability

Top view of toroidal plasma:



Growth rate:

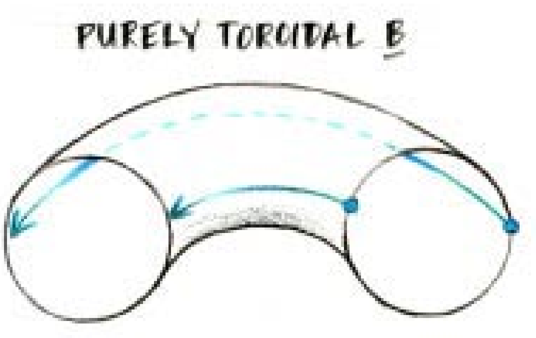
$$\gamma = \sqrt{\frac{g_{eff}}{L}} = \sqrt{\frac{\mathbf{V}_t^2}{RL}} = \frac{\mathbf{V}_t}{\sqrt{RL}}$$

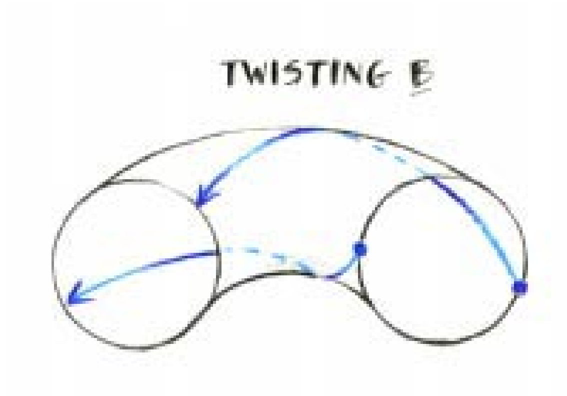
Similar instability mechanism in MHD & drift/microinstabilities

$1/L = \nabla p/p$ in MHD,
$\propto$ combination of ∇n & ∇T in microinstabilities.

The Secret for Stabilizing Bad-Curvature Instabilities

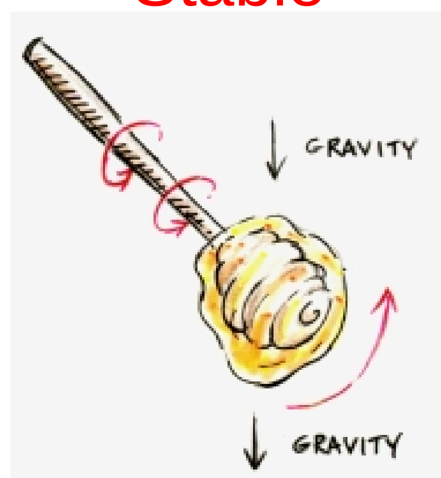
Twist in ***B*** carries plasma from bad curvature region to good curvature region:





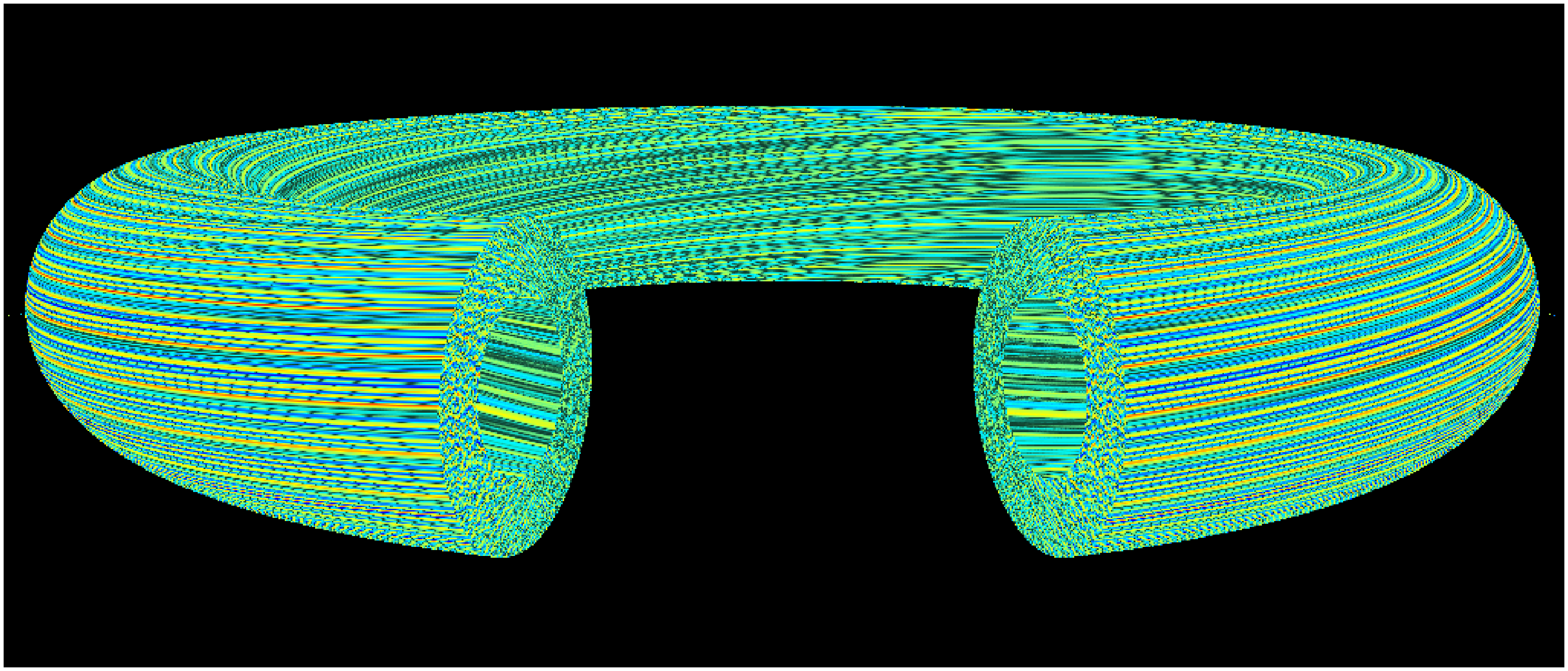
Unstable

Stable



Similar to how twirling a honey dipper can prevent honey from dripping.

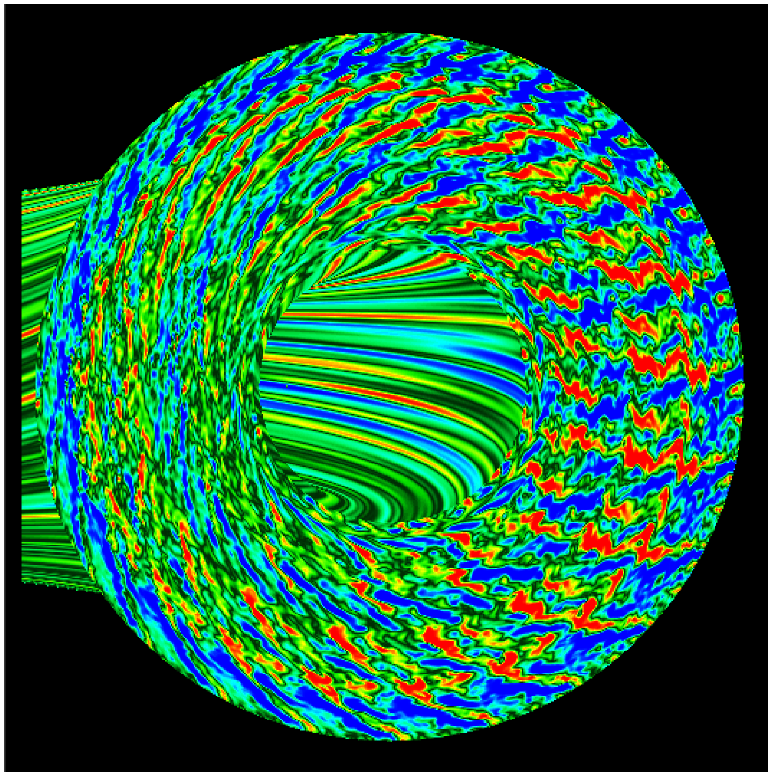
Cut-away view of tokamak turbulence simulation



Waltz (General Atomics), Kerbel (LLNL), et.al., gyrofluid simulations. Similar pictures from gyrokinetc particle simulations.

Lots more pictures at www.acl.lanl.gov/GrandChal/Tok/gallery.html.

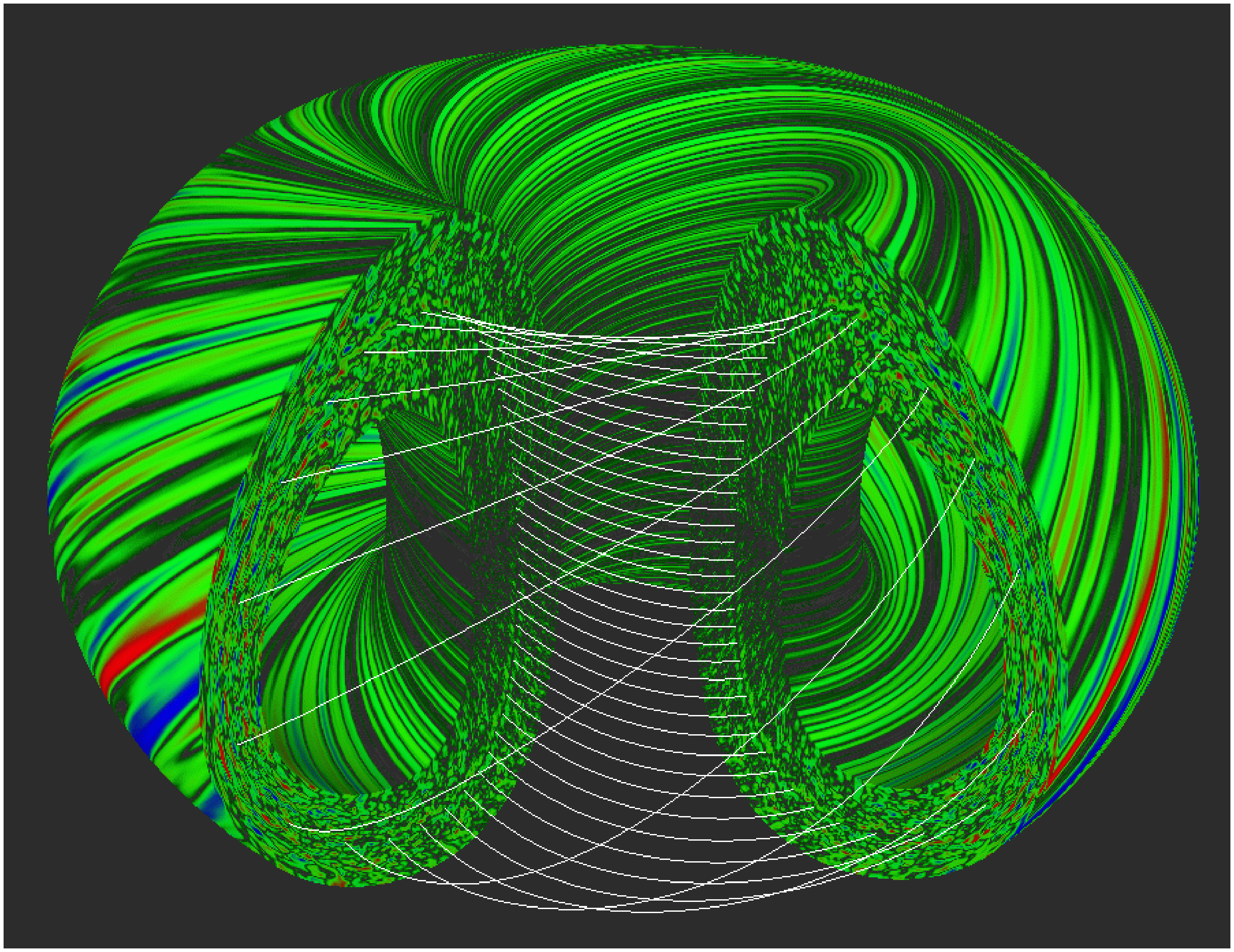
Simulations of Tokamak Plasma Turbulence



- **Realistic simulations made possible by advances in plasma theory, experimental insights, and parallel supercomputers.**
- **Fundamental science: fascinating physics of plasma turbulence.**
- **Applications: studying ways to reduce turbulence and the cost of a fusion energy power plant.**

General Atomics (San Diego), NERSC (Livermore/Berkeley), PPPL (Princeton), IFS (U.Texas, Austin), ACL (Los Alamos), part of the Numerical Tokamak Project, a DoE/HPCC Computational Grand Challenge.

Simulations can handle realistic non-circular geometry



Turbulence can be reduced by strong plasma shaping in advanced tokamaks, spherical tori, etc.

General Atomics (San Diego), NERSC (Livermore/Berkeley), PPPL (Princeton), IFS (U.Texas, Austin), ACL (Los Alamos), part of the Numerical Tokamak Project, a DoE/HPCC Computational Grand Challenge.

"Defrosting Magnetic Field Lines"*

MHD (macrostability) vs. Drift waves (microstability)

Electron fluid force Eq:

Inertia — Pressure Force — Lorentz E&B Forces — e-i collisional drag

$$\rho_e \frac{d\vec{u}}{dt} = \underbrace{-\nabla p_e - \rho_e \frac{e}{m}\left(\vec{E} + \frac{\vec{u}_e \times \vec{B}}{c}\right)}_{\text{Drift-waves}} \underbrace{- \nu_{ei}\rho_e(\vec{u}_e - \vec{u}_i)}_{\text{+ resistive MHD}}$$

Ideal MHD

Compare terms 2 & 4

$$\frac{\nabla p_e}{enu_e B/c} \sim k\rho_i \ll 1 \text{ in MHD}$$

$$\omega \approx \omega_* \sim \frac{v_t}{L} k_y \rho_i$$

Keeping term 2 ⇒ new small-scale modes
not in regular MHD theory. "Drift waves",
"Ion Temperature Gradient modes",... "microinstabilities"

Ideal MHD

$$\underset{\sim}{E} + \frac{\underset{\sim}{u}_{fluid} \times \underset{\sim}{B}}{c} = \underset{\sim}{E}' = 0$$

Plasma = perfect conductor

$\underset{\sim}{E}' = 0$ in frame moving with fluid.

Magnetic field is "frozen" in to plasma: Plasma & $\underset{\sim}{B}$ move together.

*Horton and Berk

Component ∥ to $\underset{\sim}{B}$:

$$\nabla_\parallel p_e = -en E_\parallel$$

$$= en\left(\nabla_\parallel \Phi + \frac{1}{c}\frac{\partial A_\parallel}{\partial t}\right)$$

Drift-modes allow electrostatic modes. $\underset{\sim}{E} \times \underset{\sim}{B}$ move plasma without bending $\underset{\sim}{B}$.

Ideal MHD $E_\parallel = 0$

Φ must induce $\tilde{\underset{\sim}{B}}$

(As $\beta \uparrow$, must include magnetic component.)

Small (except for $k_y \rho_i \sim 1$)

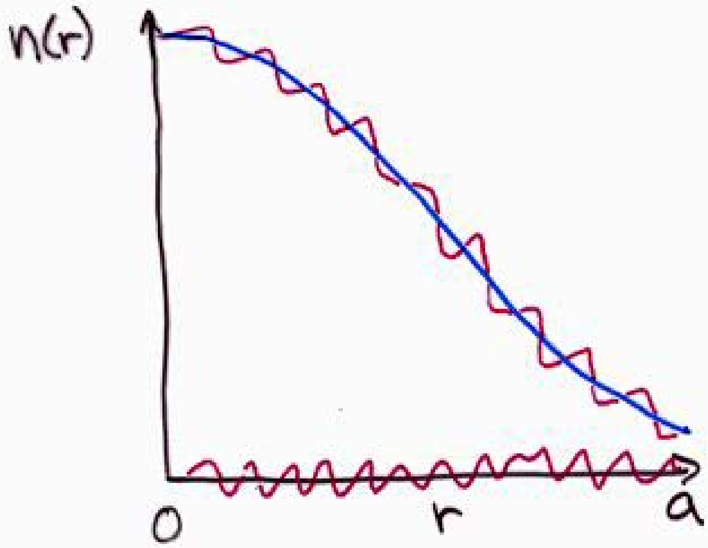
Drift frequency $\omega \sim \omega_* = \frac{V_{ti}}{L_n}\left(k_y \rho_i\right)$

Drift waves lower ω than MHD : $\frac{\omega}{k_\parallel} \sim V_{ti}$, $\frac{\omega}{k_\perp} \sim$ drift speeds of ions + trapped electrons

Wave-particle resonances important

Landau-damping (& inverse)

Microinstabilities are small-amplitude but still nonlinear



$$n = n_0(r) + \tilde{n}(\underset{\sim}{x}, t)$$

$$n_0 \gg \tilde{n}$$

but $\nabla n_0 \sim \nabla \tilde{n}$

Can locally flatten or reverse total gradient that was driving instability.

* Turbulence causes loss of plasma to the wall, but confinement still $\times 10^5$ better than without $\underset{\sim}{B}$.

If no $\underset{\sim}{B}$, loss time $\sim \frac{a}{V_t} \sim 1\ \mu\text{sec}$

with $\underset{\sim}{B}$, expts. measure $\sim 0.1 - 1.0$ sec.

Extreme Range of Time (& Space) Scales To Cope With in Plasmas

PPPL#91X0674

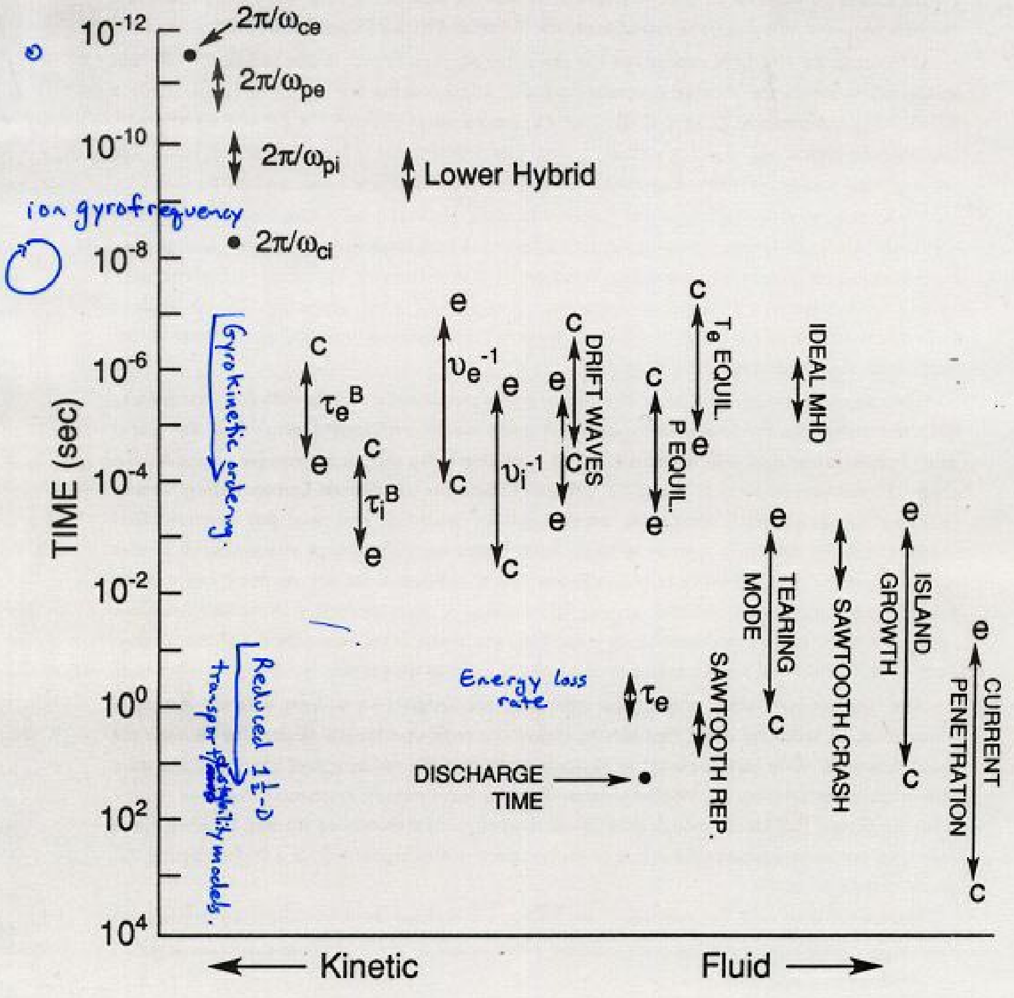


FIGURE III-1: TIME SCALES FOR TPX.
e DENOTES EDGE AND c DENOTES CENTER

Vlasov, Boltzmann, Liouville Eq:

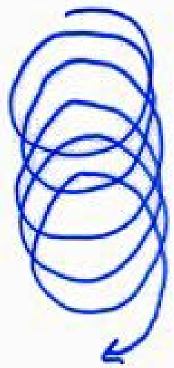
Particle Distribution

$$\frac{\partial f(\mathbf{x},\mathbf{v},t)}{\partial t} + \mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{x}} + \frac{q}{m}\underbrace{\left(\mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c}\right)}_{\text{Nonlinear, } \mathbf{E} \,\&\, \mathbf{B} \text{ depend on } f \text{ through Maxwell's Eqs.}}\cdot\frac{\partial f}{\partial \mathbf{v}} = C(f)$$

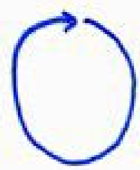
Nonlinear Gyrokinetic Eq. 1982-88

(Frieman & Chen, W.W. Lee, Dubin, Krommes, Hahm, Brizard...)

linear gyrokinetics 1960's & 70's.



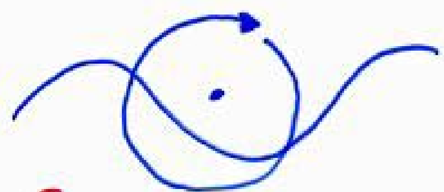
$=$



$+$ $\downarrow$ $\mathbf{V}_{drift}$

$$\frac{c}{B^2}\langle \mathbf{E}\rangle\times\mathbf{B} + \cdots$$

Φ



Possible to eliminate fast gyrofrequency time scales & retain nonlinear dynamics & $k_\perp \rho_i \sim 1$

Major Theoretical & Algorithmic Speedups

For moderate size tokamak $\frac{L}{\rho} \sim 400$

Speedup

Nonlinear Gyrokinetic Eq. Avg. Fast Gyromotion $\frac{\Omega_{ci}}{\omega_*} \sim \frac{L}{\rho}$ — $\times 400$

δf Method
(Dimits, Kotschenreuther, Parker, Lee, ...)

Reduce $\frac{1}{\sqrt{N}}$ Noise $\left(\frac{\delta f}{f}\right) \sim \frac{\rho}{L}$ — $\times 10^3 - 10^5$

$$f(\underset{\sim}{x}, \underset{\sim}{v}, t) = \underset{\text{Smooth}}{f_0} + \underset{\text{Discrete particle w/ weights}}{\delta f}$$

Also useful for accelerators, ICF, ...

Efficient Coordinates — $\times 40$
(Cowley, Beer, Hammett, Dimits)
Nonlinear extension of ballooning transform
related to Bloch transform in solid state
Coordinates aligned with twisting sheared $\underset{\sim}{B}$ $\frac{\Delta_\parallel}{\Delta_\perp} \sim \frac{L}{\rho}\frac{a}{qR}$

Reduced simulation size flux-tube $\left(\frac{a}{L_{\perp sim}}\right)^2$ — $\times \sim 16$

Gyro-Landau Fluid Closure Approx. — $\times 20+$

Bounce-avg electrons // implicit methods $k_\parallel v_{te}/\omega$ — $\times 10 - 50$

Massively Parallel Supercomputers past 10 years — $\times 100$

Caution compute time $\propto \left(\frac{L}{\rho}\right)^3$

Fluid closure approximations for collisionless limit

(Hammett & Perkins, Chang & Callen, Dorland, Beer, Waltz, ...)

$$f(\vec{x}, v_\parallel, v_\perp, t) \quad \to \quad \int d^3v\, f\, v_\parallel^j v_\perp^k$$

$$5D + t \quad \to \quad 3D + t \ \times$$ **6 moments**

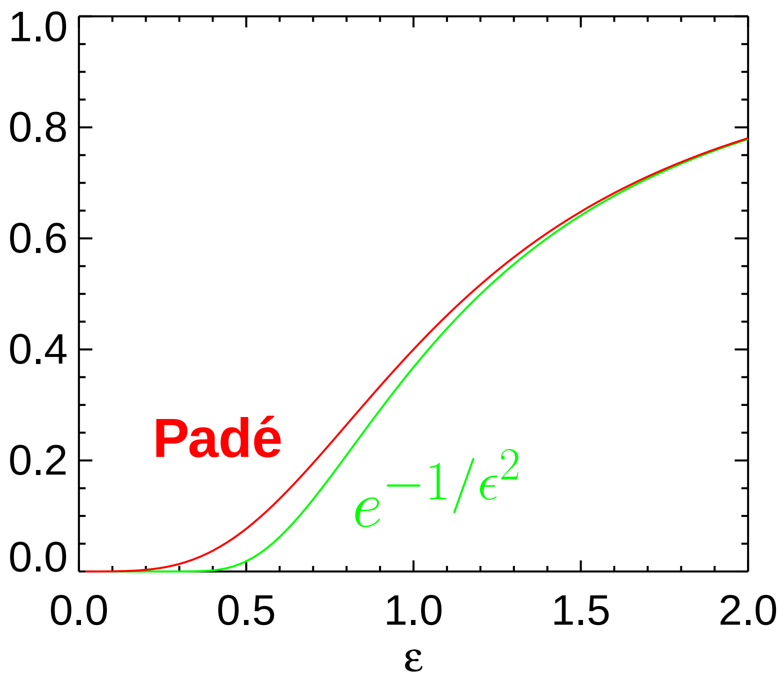
(Density, avg. flow, parallel and perp pressures and heat fluxes)

Example closure: Heat conductivity:

$$\chi_\parallel = \frac{v_t^2}{\underbrace{\nu_{collisions}}_{\text{Chapman-Enskog Collisional Transport}} + \underbrace{|k_\parallel| v_t}_{\text{Collisionless limit}}}$$

Chapman-Enskog Collisional Transport

Collisionless limit n-pole approx. to Landau damping (and inverse) $1/|k_\parallel| \to$ non-local integral operator



$$e^{-\omega^2/k^2 v_t^2} = e^{-1/\epsilon^2}$$

Taylor expansion vanishes to all orders, but Padé approx. useful:

$$\approx \frac{\epsilon^4}{0.5 + \epsilon^2 + \epsilon^4}$$

IFS-PPPL Transport Model

Kotschenreuther, Dorland, Beer, Hammett '94

• **Based on nonlinear gyrofluid simulations of ITG turbulence to map out structure of ion thermal conductivity χ_i, & on linear gyrokinetic calc of growth rates and critical gradients.**

$$\chi_i = \rho_i^2 \frac{v_{ti}}{R} \left(\frac{R}{L_T} - \frac{R}{L_{T,crit}(p_j)} \right)^{1,1/2} F(p_j) \underbrace{\left(1 - \frac{\overbrace{\gamma_{shear}}^{\text{Hahm-Burrell ExB shear}}}{\gamma_{lin}} \right)}_{\text{Waltz gyrofluid fit}}$$

$$p_j = \left(\frac{R}{L_T}, \frac{R}{L_n}, \frac{T_i}{T_e}, q, \hat{s}, Z_{eff}, \nu_*, \frac{r}{R}, \ldots \right)$$

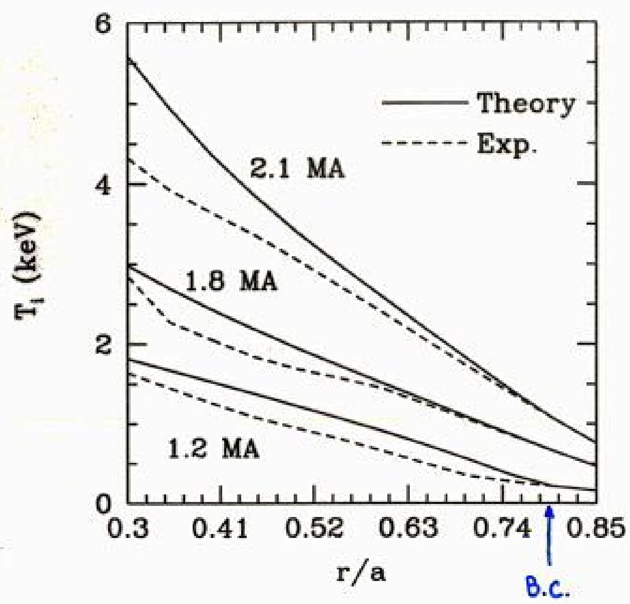
$$\chi_e/\chi_i = \text{quasilinear}$$

• **Brought together scalings from many analytic theories into a single formula. Comprehensive enough to explain many observed trends in standard tokamak operating regimes, including some improved confinement regimes (given edge B.C.'s)**

IFS-PPPL transport model represented a significant advance. But a more complete model is needed:

- **advanced tokamak regimes (negative shear, high β, strong shaping)**
- **internal transport barriers: suppress χ_i & D_e, but large χ_e ??!!**
- **particle and momentum transport (presently just heat transport)**
- **edge turbulence**
- **better shear in equilibrium $E \times B$, $\omega_*(r)$, $\eta_i(r)$**
- **better zonal flows, gyrofluid/gyrokinetic diffs**

Comparison of measured & predicted ion temperature

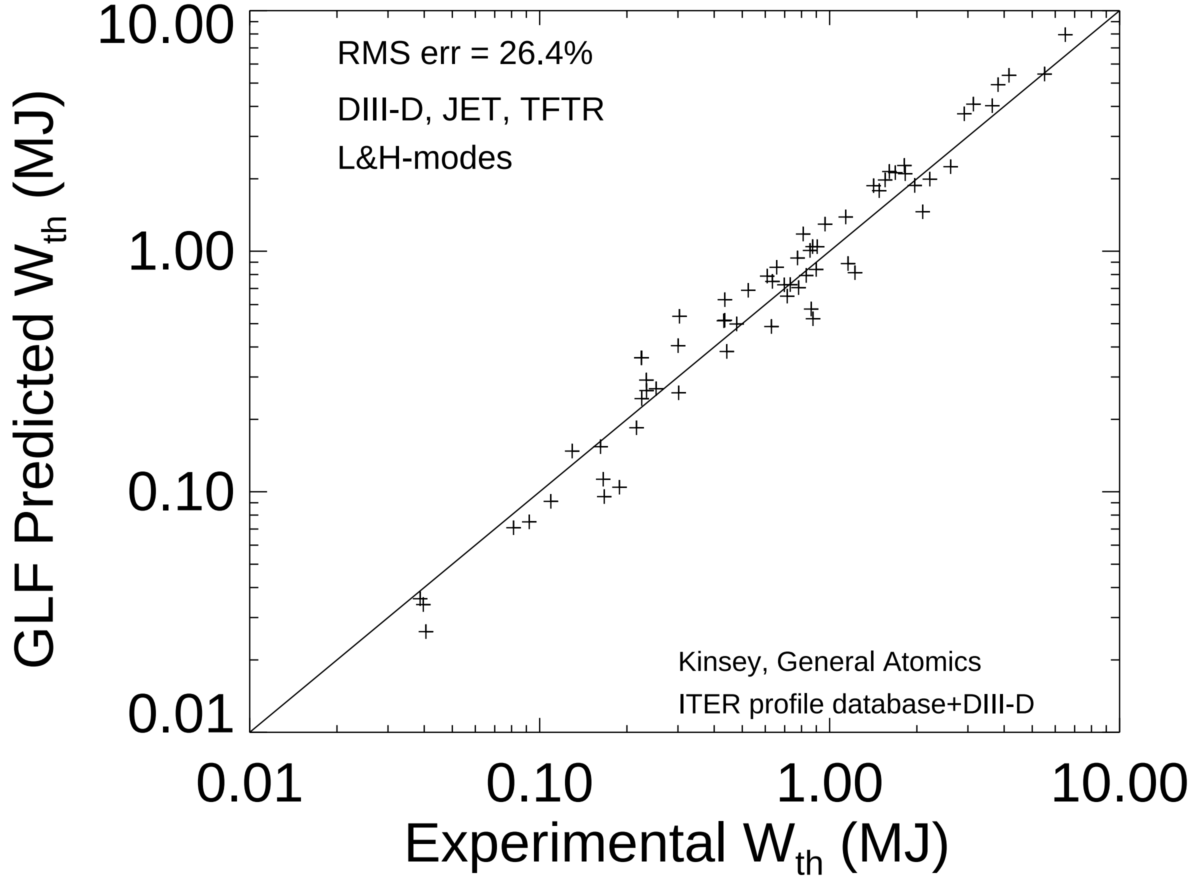


Heat flux $= -n\chi_i \nabla T_i$

Example of T_i predicted by IFS-PPPL model of ion thermal diffusivity χ due to ITG-turbulence. Follows improvement with plasma current ($\approx 30\%$ error, but sawteeth neglected..., better current scaling than most early models).

Kotschenreuther, Dorland, Beer, Hammett, Phys.Plas.95

Transport Model Based on Turbulence Simulations Follows Many Experimental Trends



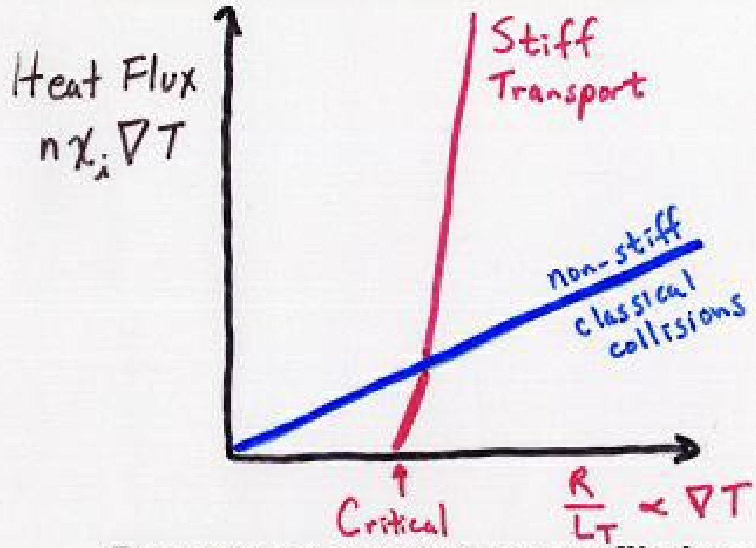
- **GLF23 transport model by Waltz et.al fit to Beer et.al. nonlinear 3-D gyrofluid simulations of ITG/trapped-electron turbulence.**

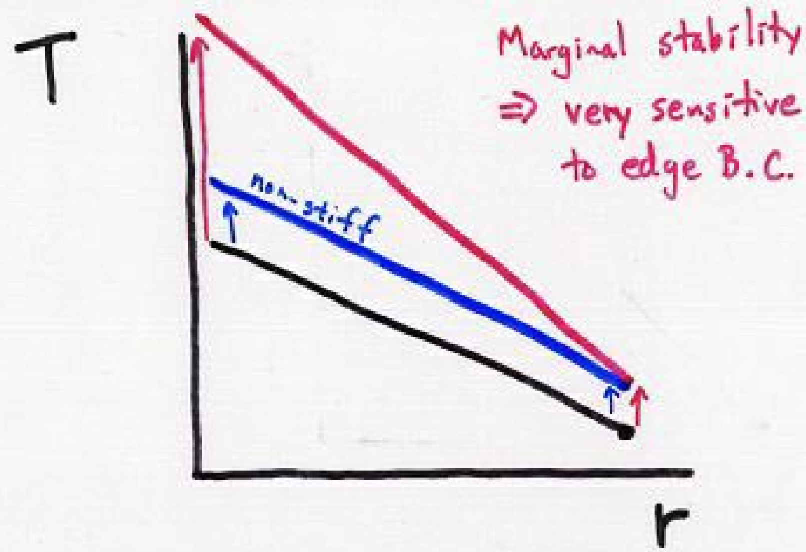
- Encouraging results so far, but many caveats: uses measured density and rotation profiles, uses measured temperatures at $r/a = 0.9$, electrostatic turbulence simulations need extension to magnetic fluctuations, gyrofluid/gyrokinetic discrepancy, etc... Much future work needed to be more accurate over a wider range of plasma parameters.

- **Rescaled GLF23, $\downarrow \chi$ and $E \times B$ shear, improves to RMS error $\approx 19\%$.**

Large χ's predicted by many 1980's analytic ITG theories lead to the proposal that temperature gradients would be forced to near marginal stability

(for example Biglari, Diamond, Rosenbluth, Phys. Fluids **B1**, 109 (1989), Horton et.al. Phys. Fluids **B4**, 953 (1992), Bateman PF B4, 634 (1992) and refs therein).





Resulting temperature profile is more sensitive to critical gradient than to magnitude of χ. Core temperature becomes very sensitive to boundary condition if there is perfect marginal stability:

$$T = T_0 e^{-r/L_T crit} \qquad T(r) = T_0 e^{-r/L_{Tcrit}}$$

Helps explain experimental sensitivity to edge boundary conditions (neutral recycling, wall conditions, supershots, edge transport barriers). Similar to the largest fusion reactor in the solar system....

Solar Convection Zone Near Marginal Stabili

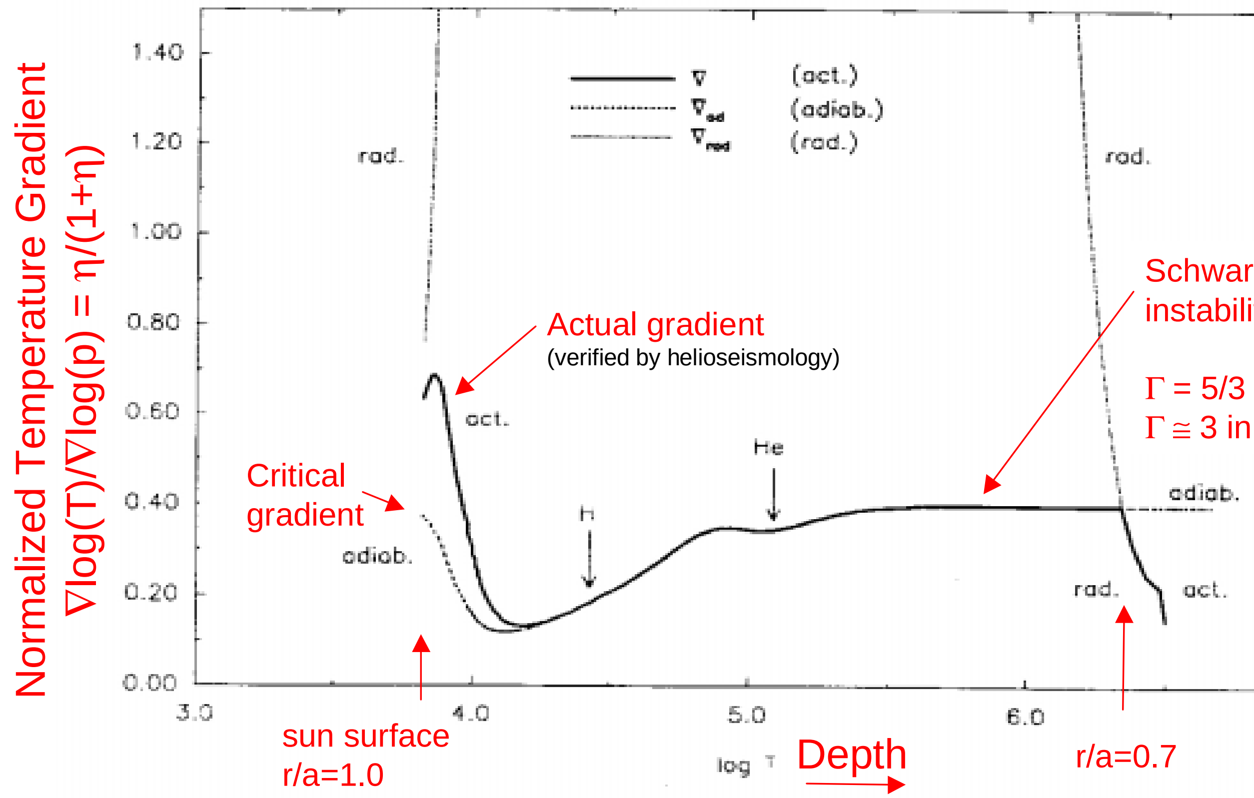
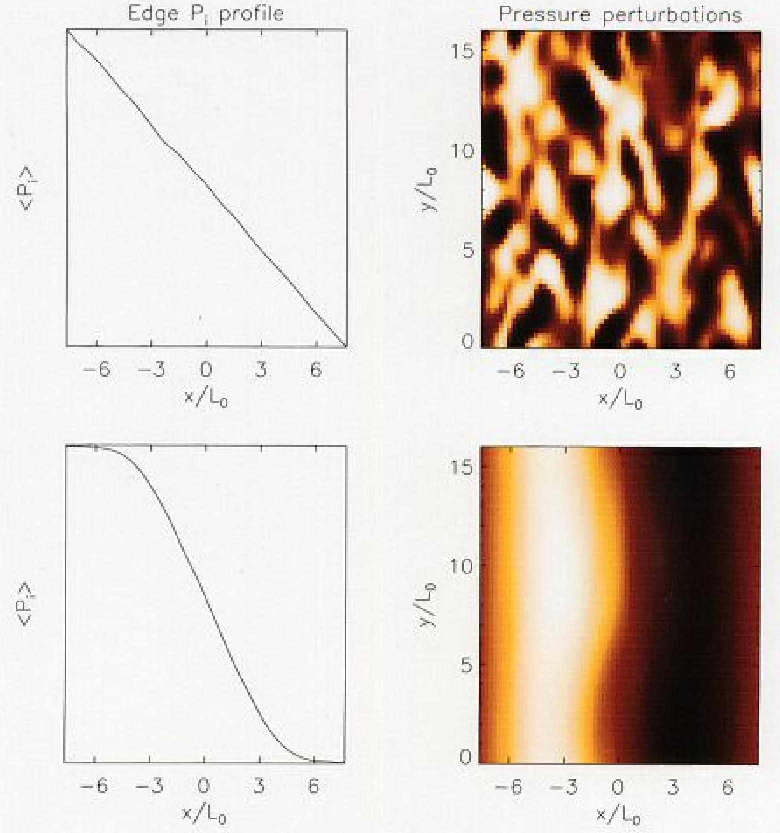


Figure 2: Temperature gradients ∇_{rad}, ∇_{ad}, and ∇ as functions of $\log T$ for a mixing leng
of the solar convection zone (Spruit 1977). At the bottom of the convection zone (log
depth $\approx 2 \cdot 10^5$ km) the actual temperature gradient changes from $\nabla \approx \nabla_{rad}$ to $\nabla \approx \nabla$
superadiabaticity becomes significant only in the surface layers ($\log T < 4.$) where the w
is driven. The ionization regions of hydrogen and helium are indicated; the adiabatic tem
gradient decreases there due to the effect of latent heat. (from Spruit, 1977 Ph.D., in Schüssler '92

Example of Edge Turbulence Simulations

EM effects play fundamental role in 3D simulations of L/H transition

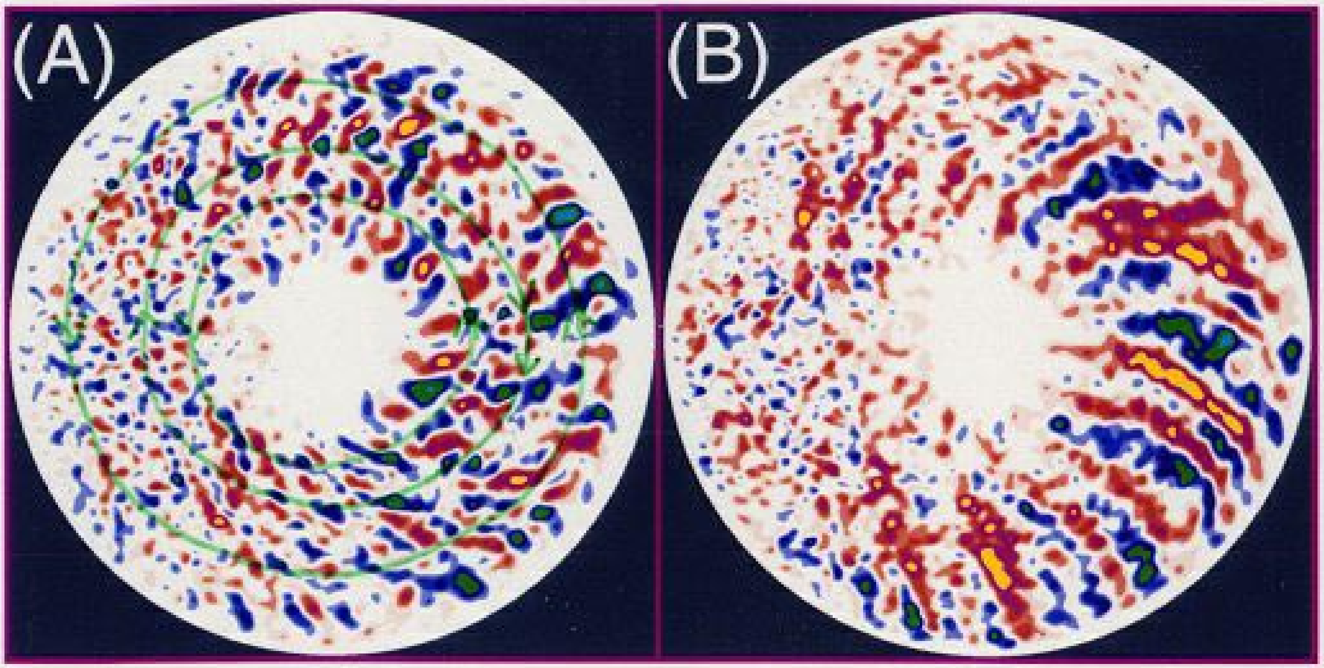


- **Increasing edge pressure gradient in simulations leads to stronger magnetic fluctuations. Above critical threshold, these cause transport barrier to spontaneously form.**

3D Drift-Braginskii
Rogers & Drake

Gyrokinetic Simulations of Plasma Microinstabilities: turbulence decorrelation by zonal flows

Sheared zonal flows (generated by turbulence) causes eddies to break up radially ⇒ ↓ transport.



With Flow **Without Flow**

Turbulence reduction via sheared plasma flow (A), compared to case with flow suppressed (B). [Z. Lin *et al.*, **Science** 281, 1835 (1998)]

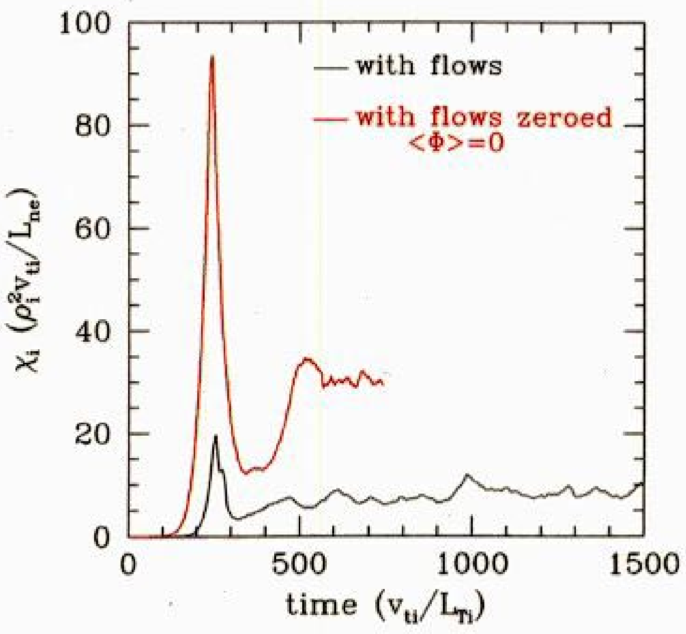
Higher quality image available at w3.pppl.gov/∼zlin/gyrokinetic.html

Turbulence-generated Zonal Flows

Hasegawa-Wakatani '87 3D resistive drift interchange

Carreras et.al. '91
Diamond + Kim '91 } suggests this happens only very close to the edge

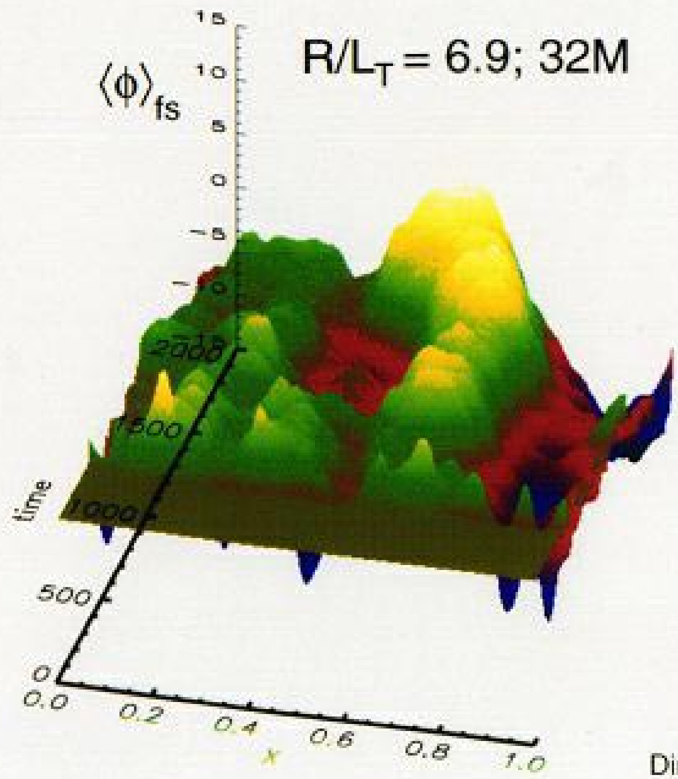
Dorland et.al. '93
Beer et.al. '93 } Show zonal flows very important for core ITG turbulence also.

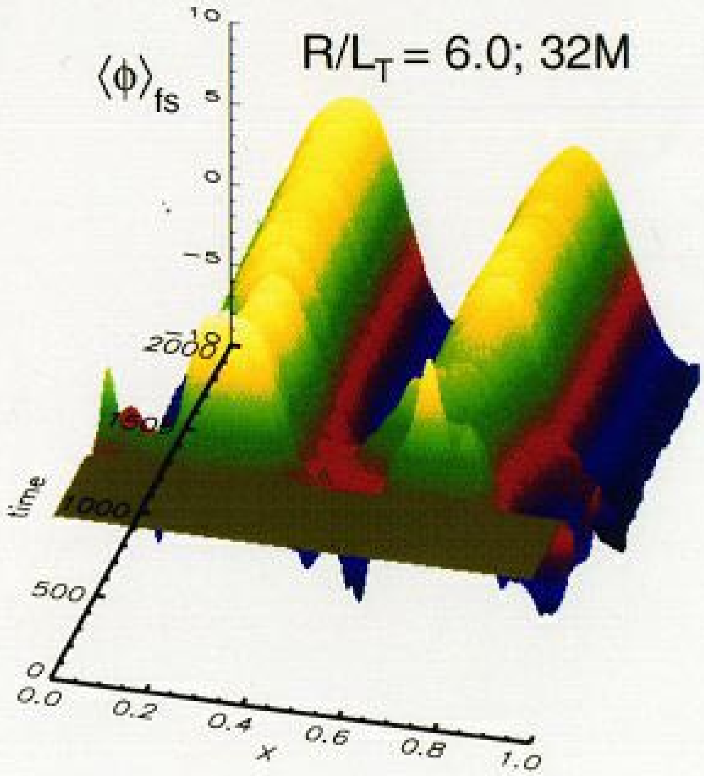


Rosenbluth-Hinton show there should be a component of the zonal flows that are linearly undamped (missed by original gyrofluid closure) + $\chi \downarrow$ even more, as shown by Dimits particle simulations.

FLUX-SURFACE-AVERAGED POTENTIALS

- Zero-χ_i states show stationary $\langle\phi\rangle_{fs}$ structures, unlike nonzero-χ_i states.
- Max. ExB shear $\approx 3\times\gamma_{max}$; E$\times$B stabilization and radial transport barriers are playing a significant role.





Dimits et al., LLNL

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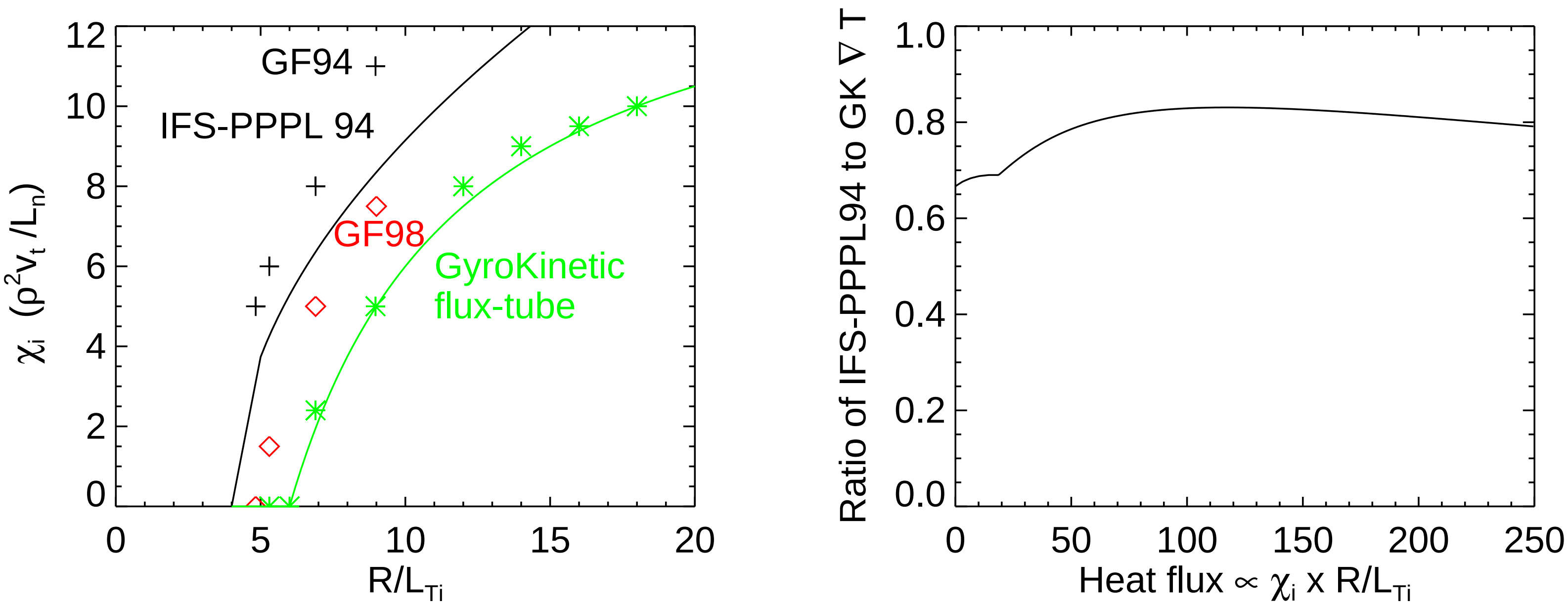
Gyrofluid/gyrokinetic (GF/GK) simulation differences $\rightarrow$ 20-33% change in predicted temperature gradient



- **Dimits (LLNL): good convergence in his gyrokinetic particle simulations**

- **New neoclassical gyrofluid closure significantly improves GF/GK comparison.**

- **Turning this plot around, for a fixed amount of heat flux $\propto \chi \nabla T$, the temperature gradient predicted by the original gyrofluid-based IFS-PPPL model is 20-33% low. But $P_{fusion} \propto T^2$, and so may increase by $\times 2$ or more.**

- **Nonlinear upshift in critical gradient may depend on: Rosenbluth-Hinton undamped zonal flows $\uparrow$ with elongation (W. Dorland), $\downarrow$ with weak collisions (Z. Lin), $\downarrow$?? with non-adiabiatic electrons [may limit inverse cascade that drives zonal flows (Diamond, Liang, Terry-Horton, Waltz, ...) and $\uparrow$ turbulent viscosity].**

CONCLUSIONS

Major progress has been made during the past 10 years in direct 3D simulations of plasma turbulence and in reduced transport models.

Reasonable agreement with core temperature profiles (∼30%) in many cases, but more work needed to resolve significant uncertainties (edge turbulence, zonal flows, electron dynamics, ...).

Relatively complete simulations should be achievable soon...[†]

Also: many ways to reduce turbulence and improve performance (sheared flows, IBW, edge beams, density peaking, high beta advanced tokamak designs with strong Shafranov shift and shaping, ...)

[†]**But needs a lot of hard work, more complete physics in codes, and new generation of computers.**

P.S.: The content of the above slides is the same as I used in my talk at the Atlanta APS meeting (March, 1999), though I converted of few of them to be typest instead of just scanning in handwritten slides.