

Physical mechanism of enhanced stability from negative shear in tokamaks: Implications for edge transport and the L-H transition

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The enhancement of stability to ballooning modes from negative shear in tokamaks is shown to be a simple consequence of the orientation of the convective cell with respect to the toroidally outward effective gravity, \vec{g} . For modest positive shear, convective cells remain oriented along \vec{g} as they map along field lines. In contrast, for negative shear or very positive shear convective cells twist strongly away from \vec{g} and are less strongly driven. The twist of convection cells is controlled by the profile of the vertical magnetic field along the outer midplane, B_z . Twist is a minimum in regions where B_z is independent of the major radius. Transport should be highest in such locations. Resistive ballooning modes in the tokamak edge are strongly stabilized by modest values of negative shear. Tokamak discharges with finite values of β_p develop regions of local negative shear on the outside midplane of the plasma torus. This local negative shear should self-stabilize resistive ballooning modes at finite values of the poloidal beta. This effect may impact the transition to high confinement operation (H-mode). © 1996 American Institute of Physics. [S1070-664X(96)02705-4]

Recent results from experiments on the Tokamak Fusion Test Reactor¹ (TFTR) tokamak suggest that negative magnetic shear has a strong stabilizing influence on tokamak instabilities.² The DIII-D¹ experiments have also suggested similar behavior.³ That negative shear could exert stabilizing influences on ideal ballooning modes was known but not appreciated in early work by Greene and Chance.⁴ There were indications in the early work by Kadomtsev and Pogutse⁵ that negative shear has a stabilizing impact on non-ideal microinstabilities. More recently Kessel *et al.*⁶ demonstrated that reversed shear in the core of a tokamak resulting from bootstrap-driven currents has a surprisingly strong stabilizing impact on these nonideal instabilities. Recent experiments have borne out this prediction and sparked further theoretical and experimental investigations into this phenomenon.

In this paper, we show that there is a simple physical explanation for why negative shear is stabilizing which should apply to all curvature driven instabilities in tokamaks. This picture is supported by analytical calculations and three dimensional (3-D) numerical simulations of edge turbulence. The surprising conclusion is that modest negative shear has a strongly stabilizing influence on curvature driven resistive ballooning modes in the edge.

The basic physics of the negative shear stabilization of curvature driven instabilities can be understood by first recognizing that convective cells oriented in the direction of the major radius R are driven by the curvature in a tokamak with modest β and, second, noting that positive magnetic shear tends to maintain the ∇R orientation of flute disturbances while negative shear twists disturbances toward the vertical (z). We first review the first point by showing the classic Rosenbluth-Longmire physical picture of the interchange. Shown in Fig. 1 is the poloidal cross-section with the centerline to the left and the toroidal magnetic field B_ϕ inwards. In a tokamak with modest β the magnetic drifts of the elec-

trons and ions are dominated by the curvature and ∇B drifts which are in the z direction as shown. A local region of high density, shown by the shaded cell in Fig. 1, causes a charge separation of the electrons and ions as a result of these drifts. The resulting poloidal electric field causes the high density region to drift outward to the right which further enhances the local density in comparison to the ambient density since ∇n points to the left. Vertically oriented disturbances, whether on the outside of the torus or on the top or bottom, are only weakly affected by the particle drifts and are therefore not driven.

In high temperature tokamak discharges disturbances in the temperature, density or potential tend to map along the field lines because of the high parallel transport. The 3-D structure of a disturbance on the outside midplane of the torus can therefore, to lowest order, be obtained by mapping the field lines. The simplest case, when the magnetic shear $\hat{s} = q' r / q = 0$, is illustrated in Fig. 2(a). In this case, the field lines at the two ends of the shaded disturbance have the same pitch so that the horizontal orientation of the disturbance on the outer midplane becomes vertical when the disturbance maps to the top of the torus. It should be clear from this figure that even when $\hat{s} = 0$, the spiraling of the magnetic field is stabilizing since the unfavorable horizontal orientation of the disturbance on the outside midplane has become vertical at the top of the torus and is no longer driven by the particle drifts. The case of $\hat{s} > 0$ is illustrated in Fig. 2(b). In this case the pitch of the magnetic field on the small r side of the disturbance is greater than on the large r side so that the outside of the disturbances lags the inside as the disturbance maps down the field line. The curious result is that the orientation of the disturbance retains some horizontal component so the magnetic drifts continue to drive the disturbance at the top of the torus. Positive magnetic shear is therefore destabilizing. The case $\hat{s} = 1$ causes the disturbance to remain exactly horizontal for poloidal angles corresponding to the

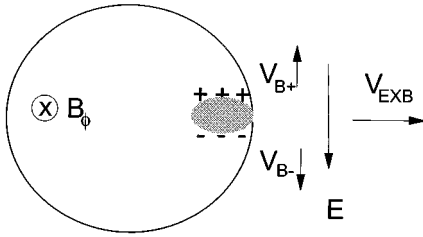


FIG. 1. The Rosenbluth-Longmire picture of the interchange in a torus.

outside of the torus and is therefore generally the worst possible value of shear for stability. The physical basis is discussed further below. Finally, in Fig. 2(c) we show the mapping for $\hat{s} < 0$. In this case the pitch of the magnetic field on the large r side of the disturbance is larger than that on the small r side. The result is that the disturbance flips completely over by the time it reaches the top of the torus. In this orientation the drifts of the particles work against the drive on the outside of the torus since the positive charge on the top of the disturbance (see Fig. 1) is now on the bottom while the electron and ion drifts are unchanged. Thus, negative \hat{s} is strongly stabilizing. Instability can only occur if the disturbance is very strongly ballooning. In the case of ideal MHD this requires a higher β for instability while in the case of resistive ballooning modes in the edge the most unstable modes shift to shorter wavelengths.⁷

While the above physical picture is presented for fluid-like ballooning or resistive ballooning modes, we expect this picture to apply to any flute-like mode driven by toroidal curvature. The mathematical underpinning for what we have

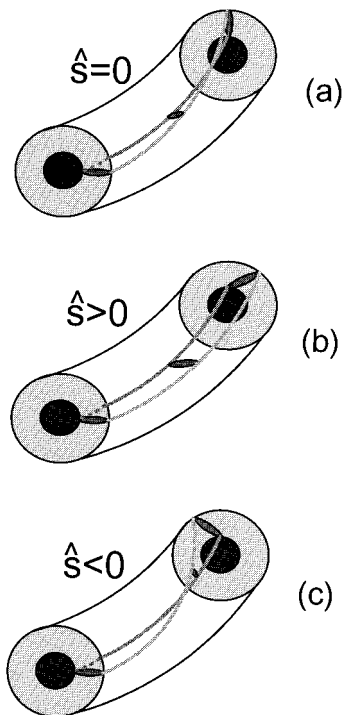


FIG. 2. Mapping of a disturbance for (a) $\hat{s} = 0$, (b) $\hat{s} > 0$, and (c) $\hat{s} < 0$.

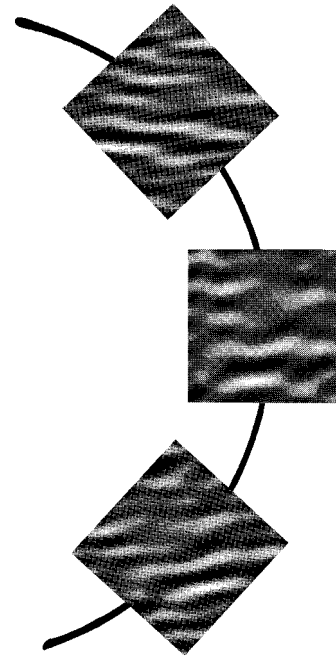


FIG. 3. Grey-scale plots of \tilde{n} for $\hat{s} = 1$.

described is the curvature term in the vorticity equation. This term represents the drive for many curvature-driven instabilities and is of the form $\vec{v}_{dm} \cdot \nabla \tilde{p}$ where in fluid theory \vec{v}_{dm} is the magnetic drift velocity of a thermal particle and \tilde{p} is the perturbed pressure. In a kinetic description of instabilities driven by circulating particles $\vec{v}_{dm} = (c/qB)\hat{b} \times (mv_{\parallel}^2 \vec{\kappa} + \mu \nabla B)$ is the velocity of an individual particle and \tilde{p} is replaced by the perturbed distribution function. The perturbed pressure is always strongly controlled by parallel transport process which causes \tilde{p} to vary slowly along a field line. As a result, the physical picture presented in Fig. 2 appears to be generic.

The heuristic pictures shown in Figs. 1–2 are supported by numerical simulation of drift resistive ballooning modes in the tokamak edge plasma. The basic equations and numerical techniques used to solve the nonlinear 3-D equations have been described elsewhere^{8,9} so we do not repeat the details here. Our goal is to simply demonstrate that the basic ideas presented here are consistent with the numerical simulations. In Figs. 3 and 4 we show evidence for the twisting of the disturbances as described earlier. The simulations are carried out in a flux tube coordinate system which wraps around the torus following field lines as shown in Fig. 2.⁹ We start the simulations with random initial perturbations and allow the system to evolve in time until the spatial structure of the growing disturbances becomes well developed. In Fig. 3 we show grey-scale plots of \tilde{n} in the plane transverse to the local magnetic field for a case with $\hat{s} = 1.0$ at three locations along the flux tube, corresponding to $\theta = -\pi/4$, $\theta = 0$ and $\theta = \pi/4$. The cuts have been projected onto the poloidal plane of the tokamak and oriented in the appropriate directions so that the results can be easily visualized. At the outside midplane the disturbances are radially extended. For this positive \hat{s} case the disturbances retain their horizontal orientation

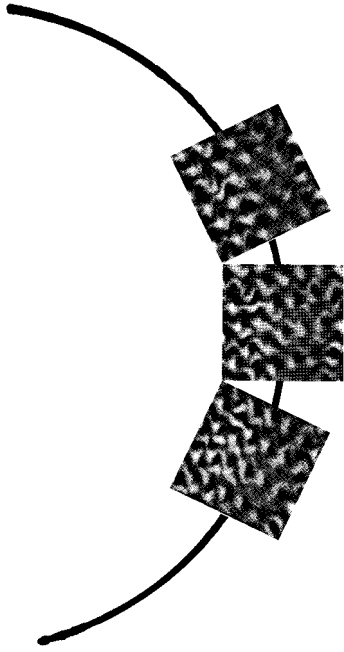


FIG. 4. Same as Fig. 3 but $\hat{s} = -1$.

tation at $\theta = \pm \pi/4$ as expected from Fig. 2(b) and are strongly unstable. In Fig. 4 we show grey-scale plots from a simulation with $\hat{s} = -1$ but for angles $\theta = -\pi/8$, $\theta = 0$ and $\theta = \pi/8$. The ballooning was so strong for this value of \hat{s} that the disturbances are hardly visible at $\theta = \pm \pi/4$. Note that for negative \hat{s} the disturbances at $\theta = \pm \pi/8$ have already twisted into the vertical direction. The growing fluctuations also have much smaller scale lengths than in the case with positive \hat{s} and grow only weakly. As expected from the structure of the disturbances in Figs. 3 and 4, the particle flux is also much more spatially localized on the outside of the torus when \hat{s} is negative compared with \hat{s} positive.

The discussion of the orientation of disturbances as they map along field lines in Fig. 2 was presented in the context of the magnetic shear since it was the recent TFTR and DIII-D negative shear experiments which have been the driver of these ideas. However, conventional magnetic shear described by \hat{s} really is not the fundamental parameter which controls the twist of curvature driven disturbances. The twist of the disturbances is completely controlled by the radial profile of B_z , the vertical component of \vec{B} on the outside midplane. This is because the vertical translation dZ of a disturbance as it projects along \vec{B} is given by $Rd\phi B_z/B_\phi$. The shear in its vertical rate of translation controls the twist of the disturbance. The dimensionless parameter which controls twist is therefore $s_b = rd \ln(B_z)/dr = 1 - \hat{s}$. If B_z is independent of r , corresponding to $s_b = 0$ or $\hat{s} = 1$, the disturbances remain horizontal as they map along \vec{B} . We expect

this case to cause the largest transport. The irony is that $\hat{s} = 1$ is often chosen as a typical parameter for tokamak stability analysis. If B_z increases (decreases) with r , corresponding to s_b positive (negative), disturbances twist counterclockwise (clockwise). Either situation should produce less transport from curvature-driven modes than the case of B_z constant.

Although negative shear is usually not associated with the edge of a tokamak, the outward shift of the flux surfaces in a finite β_p torus can be sufficient to cause B_z to sharply increase with radius. This effect causes resistive ballooning modes to twist counterclockwise as the disturbance projects along the magnetic field and should be stabilizing. This is the same effect which causes ideal ballooning modes to enter the region of second stability.^{10,11}

In summary, a simple explanation has been given for the enhanced stability resulting from negative shear in tokamaks. What controls the stability of curvature-driven modes is the twist of the disturbances as they project along the magnetic field in high temperature plasma. Horizontally oriented disturbances which do not twist should produce the largest transport. The parameter which controls the twist is the radial gradient of B_z rather than the usual magnetic shear parameterized by \hat{s} . In dimensionless form this twist parameter is $s_b = rd \ln(B_z)/dr = 1 - \hat{s}$. For s_b positive (negative) disturbances twist in the counter-clockwise (clockwise) direction as they project along \vec{B} . Large values of the twist parameter strongly stabilize tokamak edge turbulence. Plasmas with high values of $(a^2/R)d\beta_p/dr$ naturally form regions of positive twist (negative shear) on the outside midplane of the plasma torus.^{11,12} This region of high local twist may play a role in the stabilization of edge turbulence at the L-H transition.

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