# Gyrokinetics & GS2,
# a physical picture of "bad-curvature", flux-tube/global, geometry...

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GS2 Users Workshop

PPPL Jan 8-10, 2002

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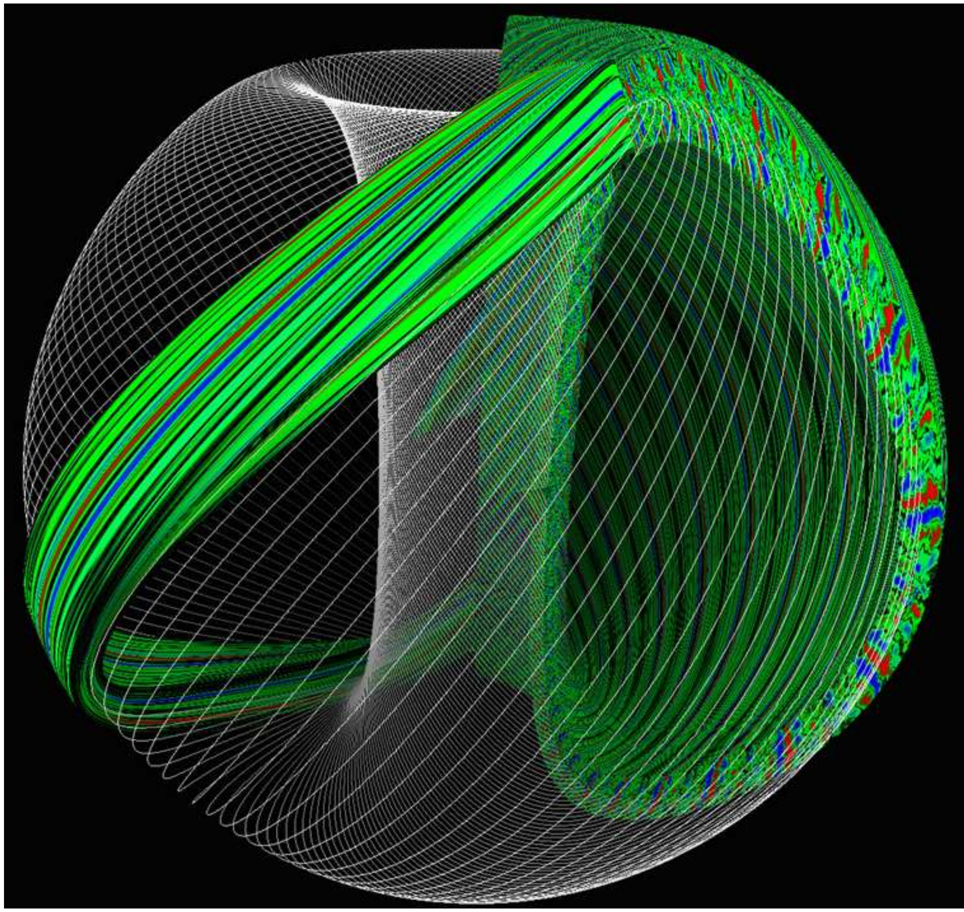
GS2 nonlinear gyrokinetic code

http://gk.umd.edu

http://gs2.sourceforge.net

Plasma Microturbulence Project

http://fusion.gat.com/theory/pmp



## Definition of Poisson bracket

**Bill wrote convective time derivative in the gyrokinetic equation including the nonlinearity as:**

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \{\chi, f\}$$

**Simplifying to $\chi = \Phi$ (the electrostatic potential), the "Poisson bracket" can be written as**

$$\begin{aligned} \{\Phi, f\} &= \frac{\partial \Phi}{\partial x}\frac{\partial f}{\partial y} - \frac{\partial \Phi}{\partial y}\frac{\partial f}{\partial x} \\ &= \hat{z} \times \nabla\Phi \cdot \nabla f \\ &= c\frac{\mathbf{E} \times \mathbf{B}}{B^2} \cdot \nabla f \\ &= \mathbf{v}_{E\times B} \cdot \nabla f \end{aligned} \tag{1}$$

**(once normalizations put back in).**

**Borrowing from previous slides:**

http://w3.pppl.gov/∼hammett/fsp-gk.pdf

**p.1-9: simple physical picture of ”bad-curvature” drive**
**p. 15 small scale fluctuations**
**p. 17 geometry start**

**Aligning coordinate system with magnetic field reduces resolution requirements by a factor of $\rho_* = \rho/L$. Very efficient.**

**http://w3.pppl.gov/$\sim$mbeer/ictp1.ps**

**p.17/18 flux tube coordinates and annular toroidal wedge equivalent**

**If flux tube has a toroidal width of $2\pi/n_0$, and a radial extent from the $q = q_1$ surface to the $q = q_1 + m_0/n_0$, then the flux-tube domain is mathematically exactly equivalent to simulating a toroidal wedge of a toroidal annulus (a slice of a hollow Bundt cake).**

**Toroidal periodicity on wedge means effectively simulating $n = 0, n_0, 2n_0, 3n_0, \ldots$.**

**I.e., $n = 87$ mode similar to $n = 86$ mode, can use coarser grid in $k_\theta = m/r = nq/r$.**

**Example: $n_0 = 10$, radius from $q = 2.2$ to $q = 2.3$, $k_\theta\rho \approx (m/r)\rho = 0.1, 0.2, 0.3, \ldots$ for $a/\rho = 400$, or**

$$n_0 \approx \frac{a}{\rho_{ref}} \frac{r}{aq} \min(k_\theta \rho_{ref})$$

**Because turbulent eddies are so extended along field lines, while having a short correlation length perpendicular to the field line, it is more natural to think in terms of elongated flux-tube simulation domain. In this case these quantization/periodicity constraints aren't really relevant in most cases of turbulent interest (see papers by Cowley, Kulsrud, and Sudan, and Beer, Cowley, Hammett, etc.).**

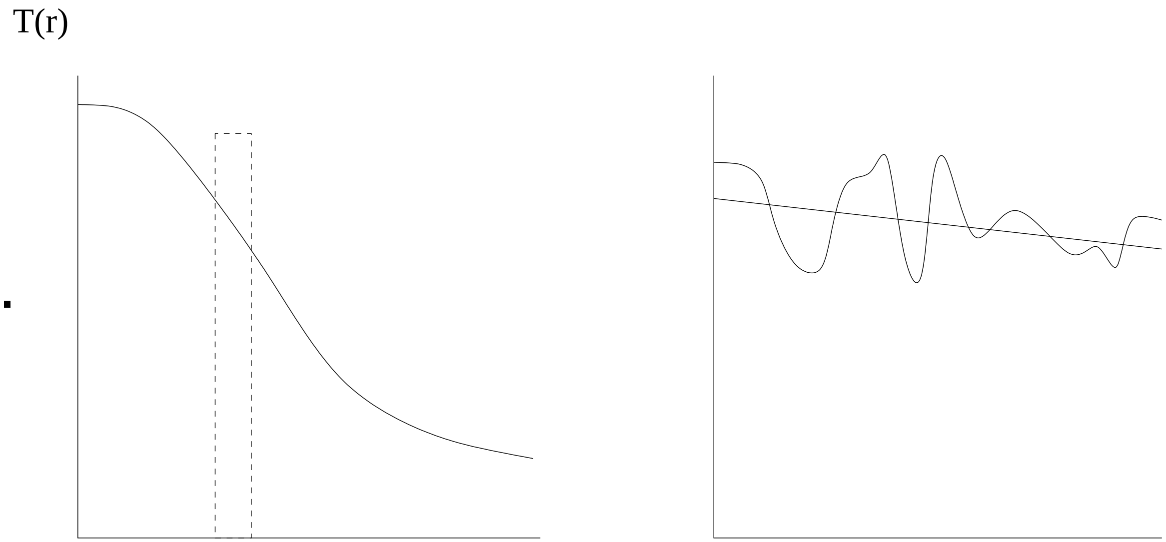
**p. 16: Key is to make the box more than a few decorrelation lengths in all directions so opposite sides of box are random with respect to each other. Then can assume statistical homogeneity: the statistics of the range of sizes of eddies that go in and out one side of the box is the same as at the other side of the box.**

**Use this "statistical similarity" to assume exact periodicity. Provides natural boundary conditions for small scale homogeneous turbulence.**

**Standard flux-tube simulations make further assumptions for simplicity: geometrical quantities vary along field line but are assumed to not vary much across the width of a thin flux tube. For example assume $|B|$ is constant in directions perpendicular to the flux tube when calculating $J_0(k_\perp v_\perp/\Omega)$, etc. [Paper by Adil Hassam et.al. discuss extension of usual flux-tube geometry to include terms needed to get Stringer spin-up.]**

**Though constant gradients kept in terms like $\omega_{*T} \propto \nabla(T)$, variations of equilibrium $T_0(r)$ across box neglected if it appears without gradient (like in gyroradius for Bessel functions)**

**Though background temperature gradient is constant on scale of box, local $\nabla T(r,t)$ allowed to self-consistently fluctuate.**




**These are standard, widely-used, two-scale/periodic approximations, going back to ballooning mode theory, Hasegawa-Mima, homogeneous turbulence simulations in Navier-Stokes fluids, Balbus-Hawley shearing-box simulations of Magneto-Rotational-Instability in astrophysical accretion disks...**

**Flux-tube also natural for full implementation of "standard gyrokinetics". Global codes are starting to investigate what are thought to be some of the most important finite $\rho_*$ corrections, but finite $\rho_*$ corrections in the gyrokinetic equation or in the assumed equilibrium have not been completely worked out.**

**?? Describe Waltz picture of ExB shear $\rho_*$ effects, showing gyro-Bohm, Bohm, etc. regimes...**

$E \times B$ **shear suppression of turbulence model based on BDT theory, Waltz approximation:**

$$\chi_i = \chi_{GB}\left(1 - \frac{\gamma_{ExB}}{\gamma_{lin}}\right)$$

$$\chi_i \approx \frac{cT}{eB}\frac{\rho}{L}\left(1 - \frac{\gamma_{ExB}}{\gamma_{lin}}\right)$$

**where $\gamma_{lin} \sim v_t/L$, and**

$$\gamma_{ExB} \sim \frac{\partial v_{ExB,\theta}}{\partial r} \sim \frac{v_t \rho}{L^2}$$

**(using $enE_r \sim \partial p/\partial r$), leads to**

$$\chi_i \sim \frac{cT}{eB}\rho_*(1 - C\rho_*)$$

**Has gyro-Bohm, Bohm, worse-than-Bohm (Goldston ;-) ) regimes**

**Similar stabilizing effects if radial derivative of poloidal group velocity $\omega/k_\theta \sim \omega_*/k_\theta$ included. Global codes are studying this.**

**Waltz-Candy source term seems reasoanble to me (though I haven't checked all the details of it). Given time-averaged profiles $T_0(r), n_0(r)$, etc. Microinstabilities would cause these profiles to relax to be uniform. If one is very close to marginal stability, only a little bit of relaxation would shut off the turbulence. There must be some source/sink terms (neutral beams, RF, radiation, charge-exchange losses), that maintains the profiles against turbulent diffusion. Waltz adds a time-averaged source to do this (beam heating, etc. is constant in time), whatever is required to match the time-averaged profiles $T_0(r)$ etc. given by the experiments..**

**Profiles are allowed to fluctuate in time self-consistently.**

**Check: Waltz-Candy compared bounded simulation using compensating sources/sinks with periodic simulation using no sources/sinks and got good agreement.**

**p.15: characteristics of:**
**ITG (Ion Temperature Gradient driven instability)**
**TEM (Trapped electron mode)**

**Antonsen et.al. picture of why negative magnetic shear is stabilizing:**

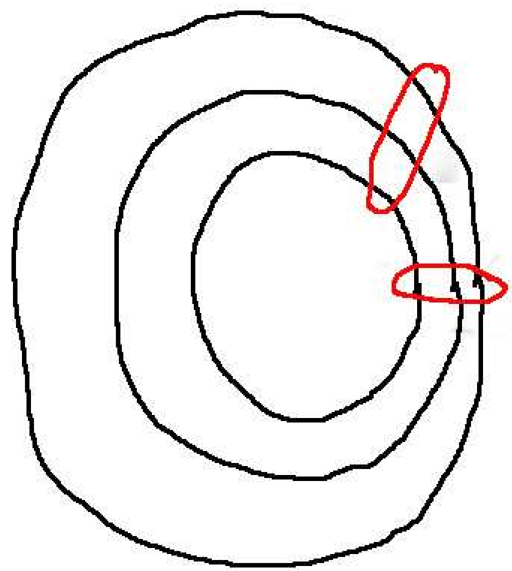
**Antonsen, Drake, Guzdar, Hassam, Lau, Liu, Novakovskii, PoP 96**

$$B_{pol} \approx \frac{B_p(r)}{(1+\frac{r}{R}\cos\theta)(1+\Delta'\cos\theta)}$$

**Squeezing of flux surfaces by Shafranov shift** $\Delta' = dR_0/dr \approx -(r/R)(\beta_{pol} + \ell_i/2)$ **can increase** $B_{pol}$ **a lot.**

**Local magnetic shear** $\propto (d/dr)B_{pol} \propto (d/dr)\beta_{pol}$ **leads to important parameter** $\alpha = -Rq^2 d\beta/dr$ **of the** $s-\alpha$ **model.**

**High** $\alpha$ **gives rise to a local negative magnetic shear.**



**Probably dominant stabilizing term in ST's.**
**See recent paper by Clarisse Bourdelle using GS2 to study various effects:**

**http://w3.pppl.gov/ hammett/gyrofluid/papers/2002/bp_clarisse.pdf**

**Emily Belli has studied other shaping effects, finds radial derivative of elongation is more important that elongation itself, etc.:**

**Belli, Hammett, Budny, Dorland, APS 02**