Frank Jenko

Capitalizing on a better understanding of plasma turbulence

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Acknowledgements

Turbulence in Laboratory
& Astrophysical Plasmas

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University of Valencia
Université Libre de Bruxelles
University of Wisconsin-Madison

Some introductory words

Due to the huge range of spatio-temporal scales involved in the dynamics of fusion plasmas, *ab initio* simulations will remain very challenging (although invaluable)

There are great incentives to develop reduced models with a reasonable balance between accuracy and efficiency

This requires **a solid understanding of the fundamental physical processes** (here: plasma turbulence)

Topics addressed in this talk

A few remarks concerning the GENE code

On the applicability of quasilinear theory (QLT)

A limit of QLT: Turbulence at finite β

Damped modes, cascades, and Large Eddy Simulations

Smith & Hammett, PoP 1997

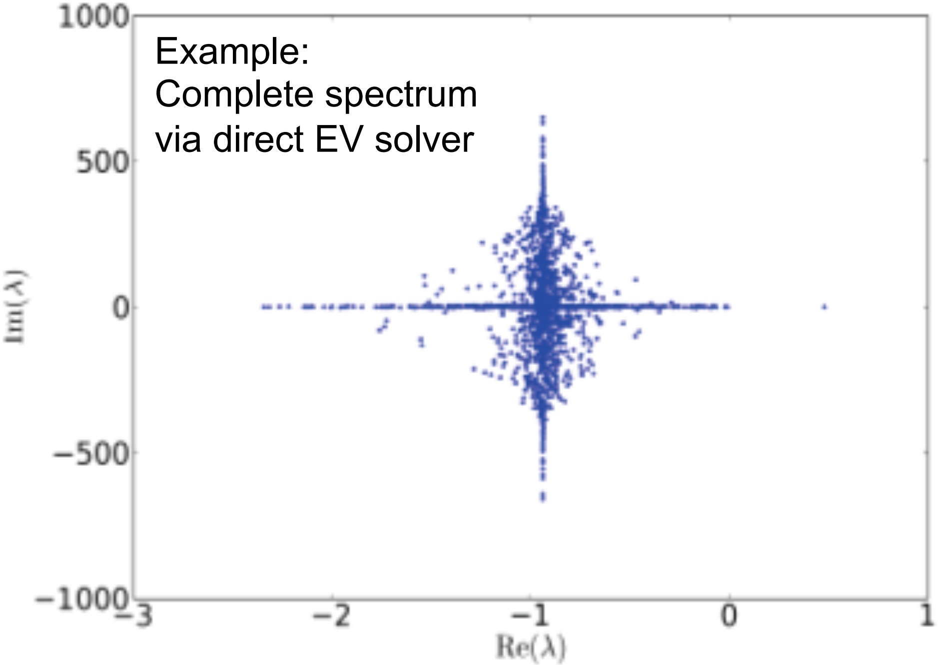
Turbulent transport of energetic particles (...cosmic rays)

A few remarks concerning GENE

The gyrokinetic code GENE http://gene.rzg.mpg.de

GENE is a physically comprehensive Vlasov code:

- allows for kinetic electrons & electromagnetic fluctuations, collisions, and external ExB shear flows – and for local and global simulations
- can be used as **initial value** or **eigenvalue** solver



Close collaborations with experts in applied mathematics and computer science (e.g., J.E. Roman, Valencia)

Due to tailored new developments of the "**S**calable **L**ibrary for **E**igenvalue **P**roblem **c**omputations" (SLEPc), the speed of GENE as a linear eigenvalue solver has been increased by **>10^2** (now only a few seconds per run)

Iterative EV solvers via SLEPc

An extension of PETSc…
J.E. Roman, Valencia

Available methods:

- Power Iteration (also Inverse Power Iteration, Rayleigh Quotient Iteration)
- Subspace Iteration with Rayleigh-Ritz projection
- Arnoldi method
- Lanczos method
- Krylov-Schur (with and without Harmonic Extraction)
- Davidson methods (as of recently)

Recent result:

Using the preconditioned Jacobi-Davidson method is about twelve times faster as Krylov-Schur method with harmonic extraction and twice as fast as the preconditioned Krylov-Schur method.

C. Kowitz, F. Merz

Exceptional points

Kammerer, Merz, and Jenko, Phys. Plasmas 15, 052102 (2008)

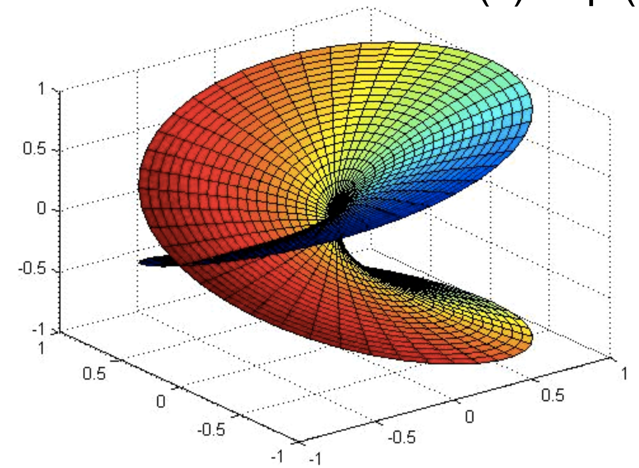
Different microinstabilites (usually considered as separated) can be transformed into each other via continuous parameter changes.

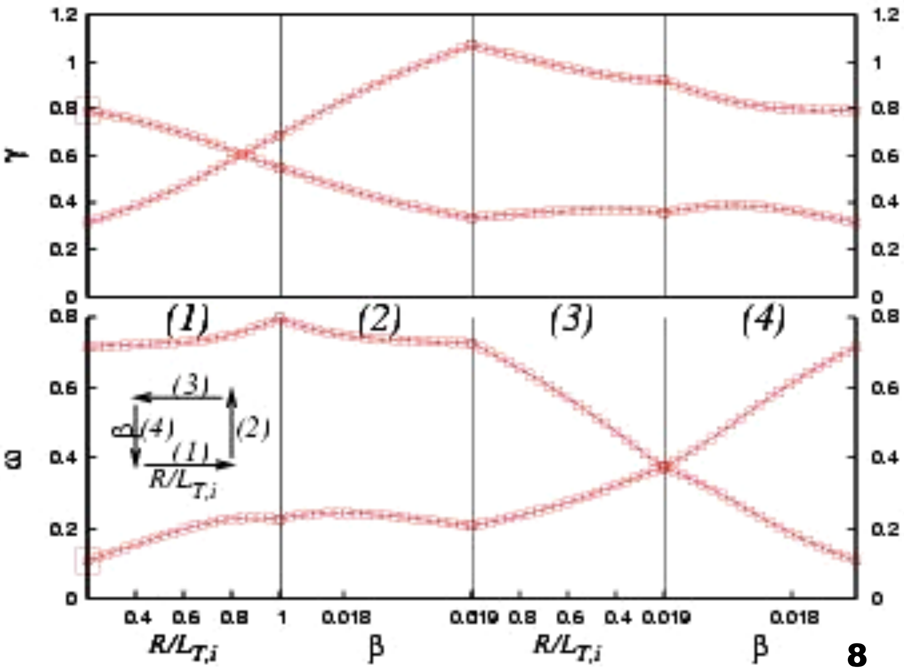
The non-Hermiticity of the linear gyrokinetic operator leads to *Exceptional Points*.

Here, both eigenvalues *and* eigenvectors are identical.

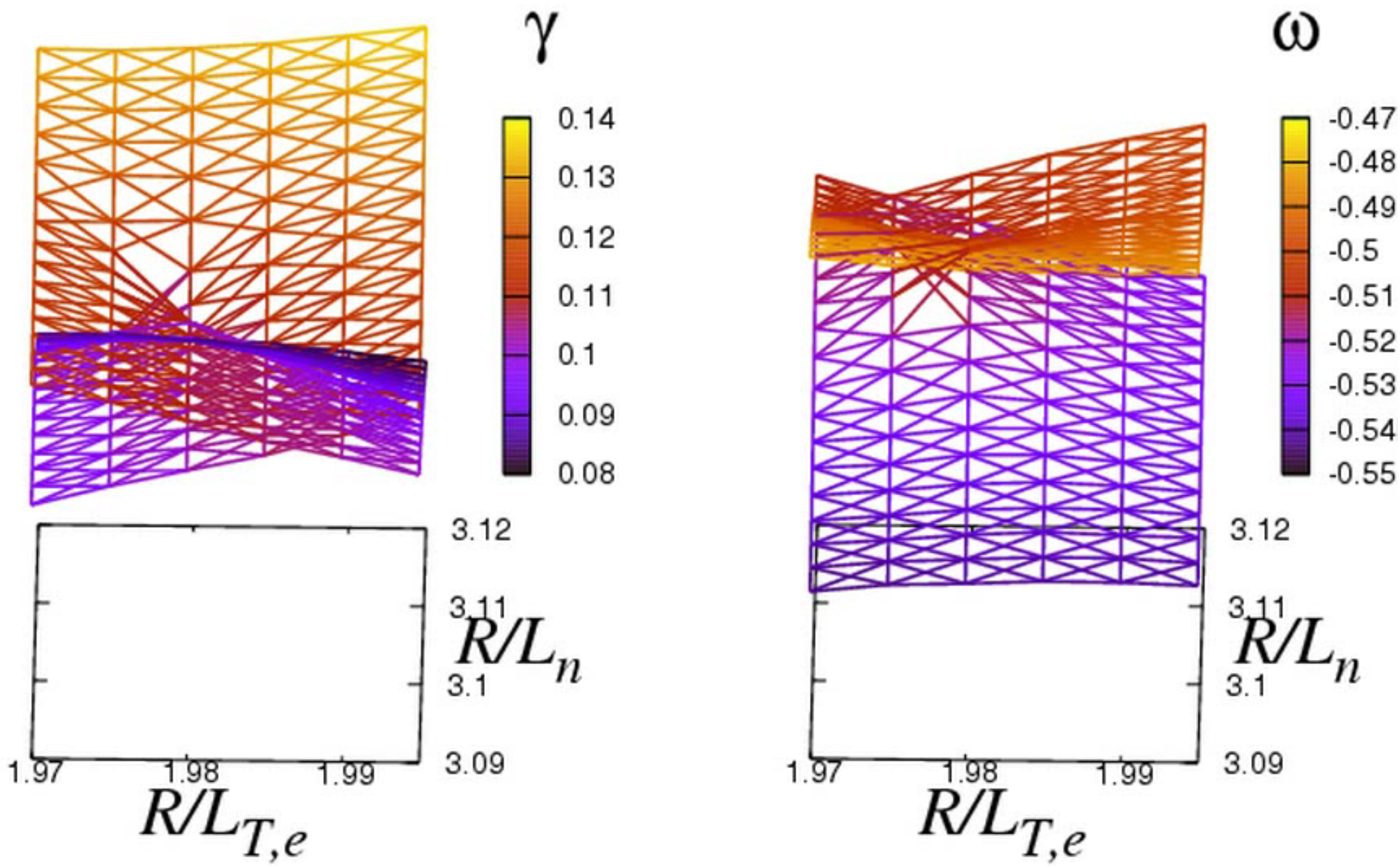
Similar: quantum physics etc.

Riemann surface of f(z)=sqrt(z)





Exceptional points (cont'd)

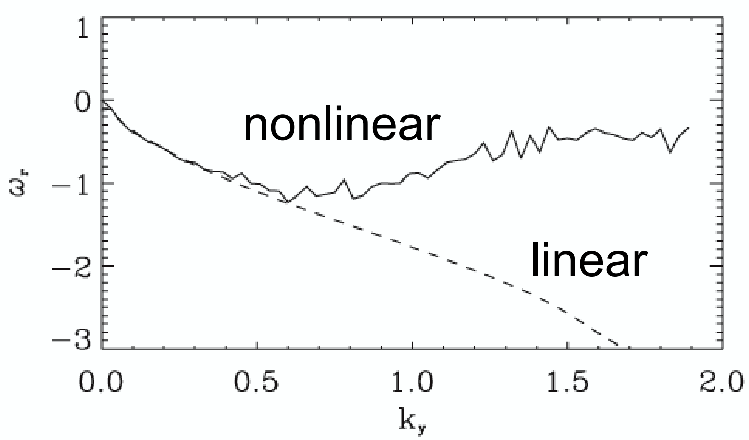


On the applicability of quasilinear theory: TEM turbulence

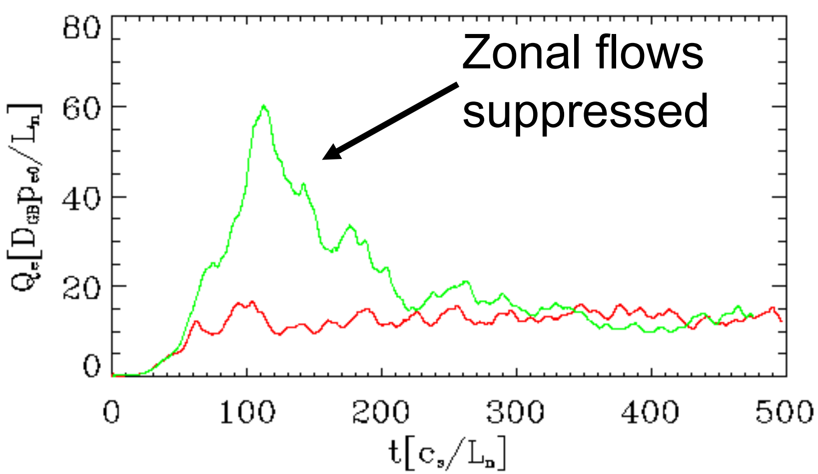
Features of TEM turbulence

Saturated phase of TEM turbulence simulations:

- In the drive range, nonlinear and linear frequencies are identical



- In the drive range, there is no significant shift of cross phases w.r.t. linear ones



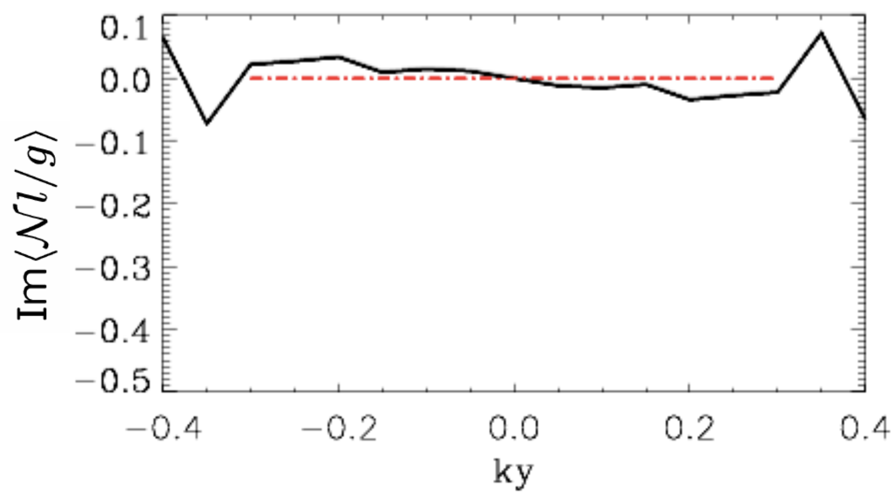
– No dependence of transport level on zonal flows [Dannert & Jenko 2005, Ernst et al. 2009]

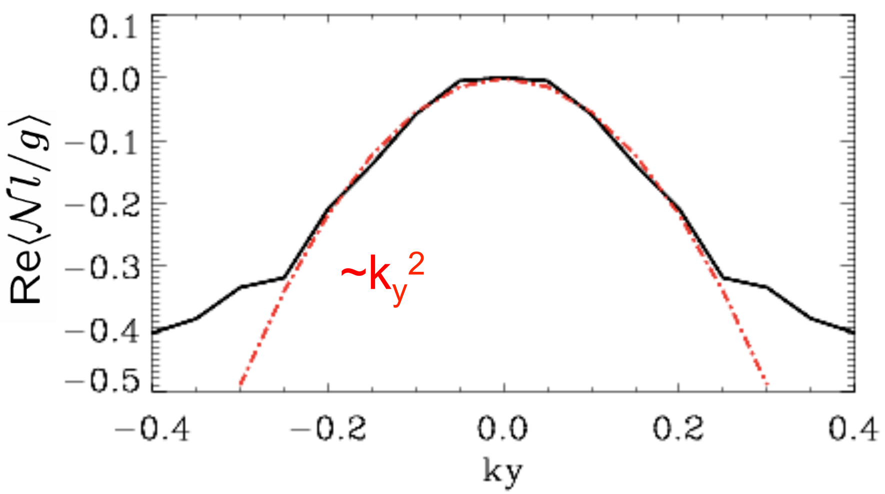
Statistical analysis of the ExB nonlinearity

Merz & Jenko, PRL 2008

ExB nonlinearity in the low-k_y range: large transport contributions; small random noise, while the coherent part can be written as:

$$\boxed{\mathcal{N}l[g] \simeq D(-k_\perp^2)g = D\nabla_\perp^2 g}$$

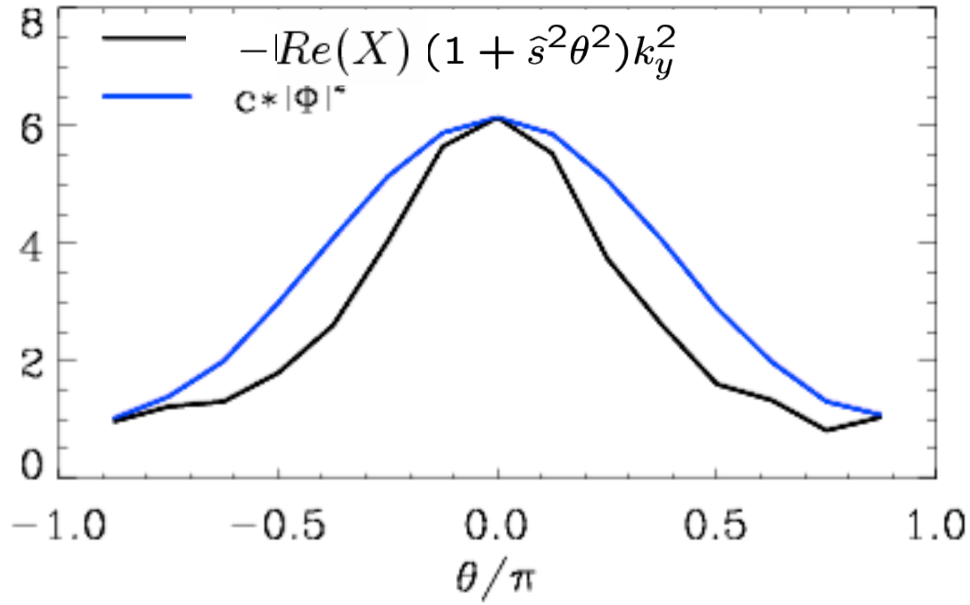




This is in line with various theories, including Resonance Broadening Theory (Dupree), MSR formalism (Krommes), Dressed Test Mode Approach (Itoh)

Parallel structure of diffusivity

- Dependence on parallel coordinate:
  $\approx |\Phi|^2$



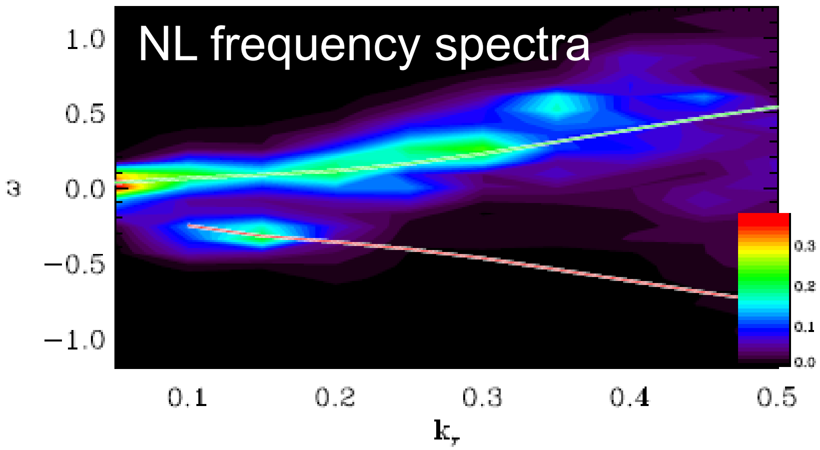
- Integration with parallel weighting yields effective wave number $\langle k_\perp^2 \rangle := \int d\theta D(\theta) k_\perp^2 \simeq c \int d\theta \left| \Phi^2(\theta) \right| k_\perp^2$
- Quasilinear equation: $\dfrac{\partial g}{\partial t} = \mathcal{L}g + \mathcal{N}l[g] \simeq (i\omega_r + \gamma - D_0 \langle k_\perp^2 \rangle) g$
- Stationarity implies $D_0 \sim \dfrac{\gamma}{\langle k_\perp^2 \rangle}$

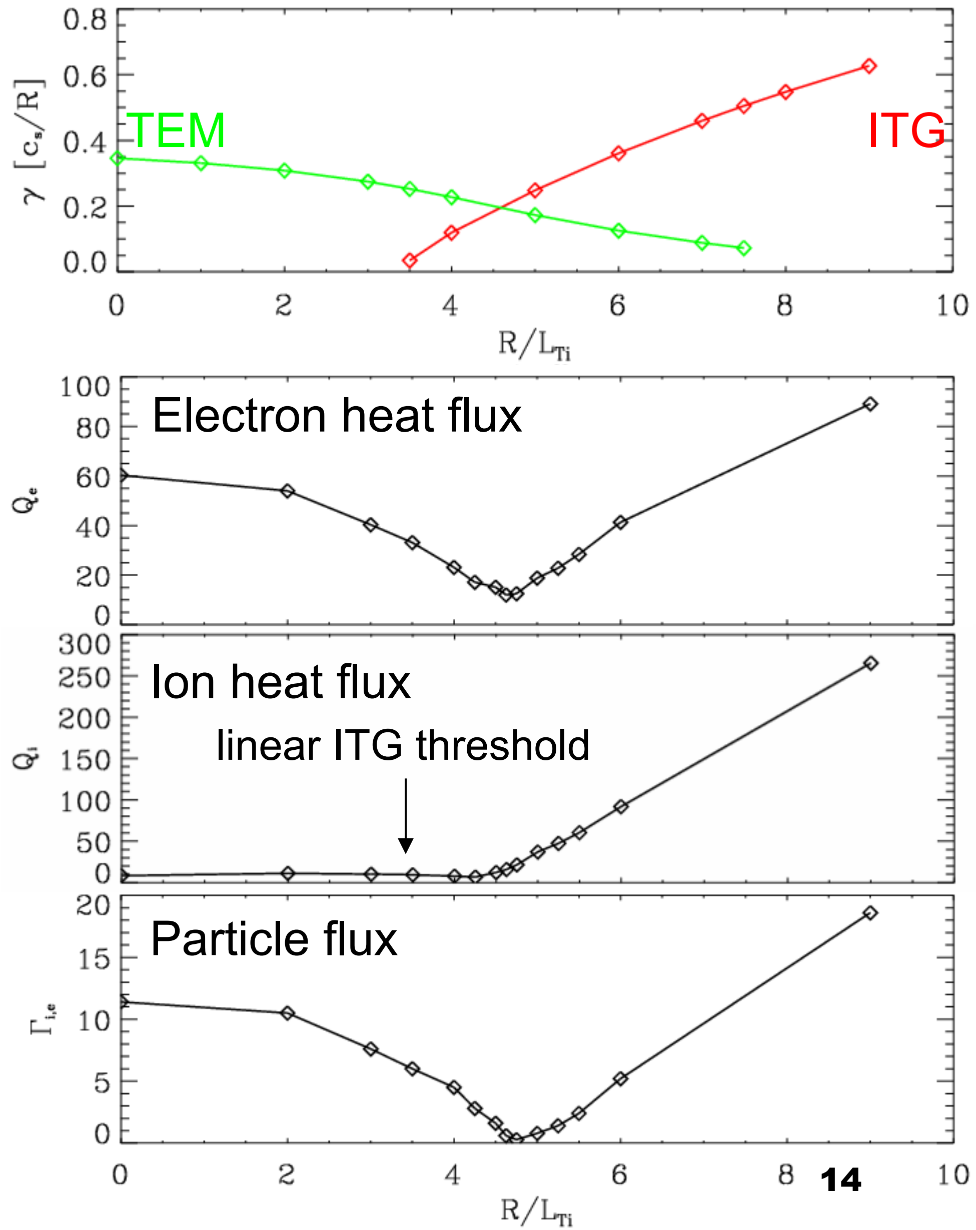
Tested successfully in many cases, but...

Limits of QLT: ITG/TEM interactions

Merz & Jenko, NF 2010

- Linear growth rates (k_y=0.25), using GENE as an EV solver
- TEM regime: Electron heat flux is suppressed, not increased
- ITG regime: Nonlinear upshift of critical R/L_{Ti}
- Nonlinear ITG/TEM coexistence

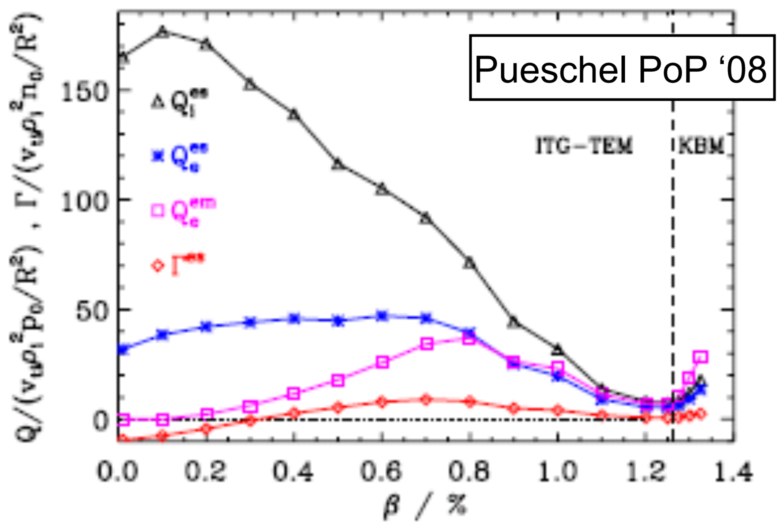


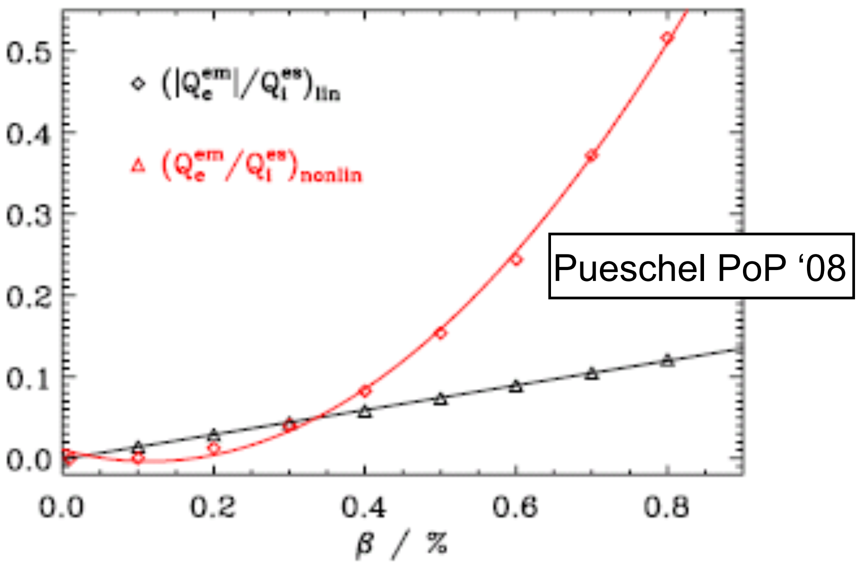


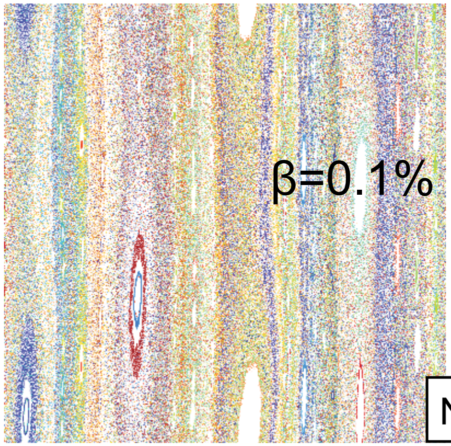
Another limit of QLT: Turbulence at finite β

Recent results on finite β turbulence

- **Magnetic electron heat transport** can approach (or even surpass) electrostatic transport as β **increases** [Candy PoP '05, Pueschel PoP `08]
- Magnetic transport violates quasilinear theory – **β^2-scaling**







Nevins PRL`11

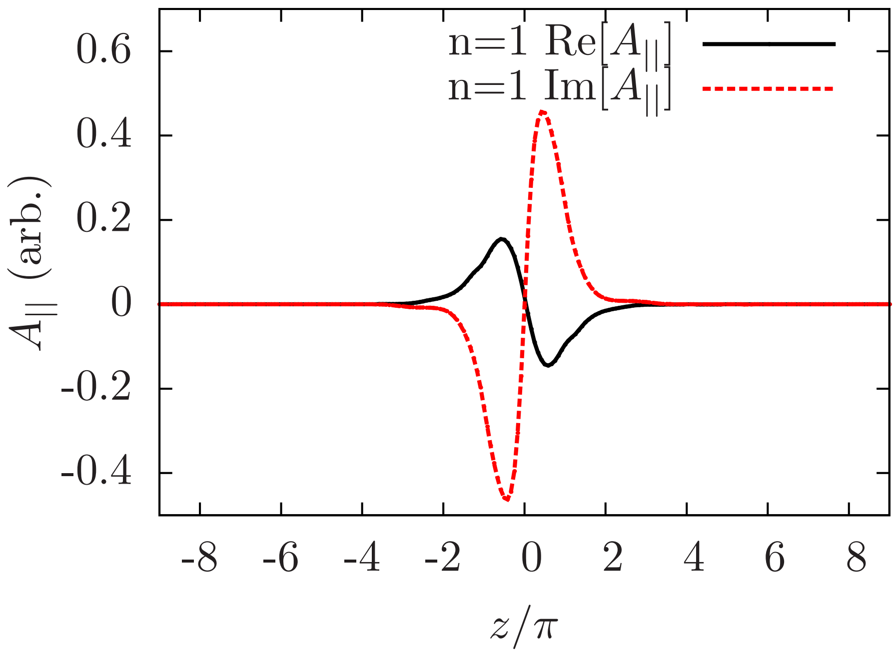
- Observation of near-ubiquitous **magnetic stochasticity** – even at low values of β (Nevins PRL '11, Wang PoP '11)

...needs an explanation!

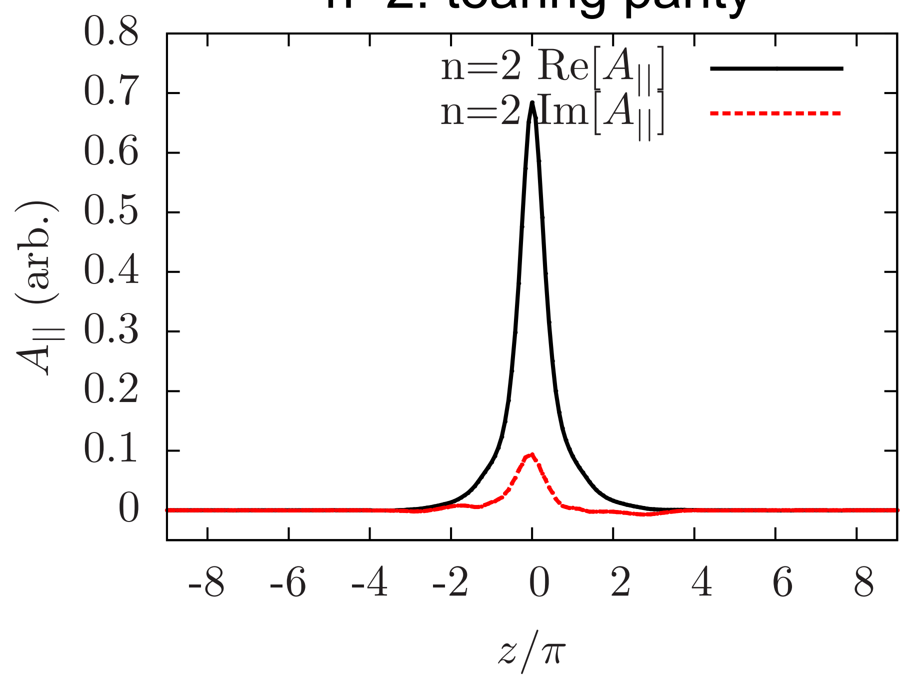
Proper orthogonal decomposition

$$A_{\|k}(z,t) = \sum_n A_{\|k}^{(n)}(z) h_k^{(n)}(t) \qquad k_y\rho_s = 0.2, k_x\rho_s = 0, \text{ and } \beta = 0.003$$

n=1: ballooning parity



n=2: tearing parity



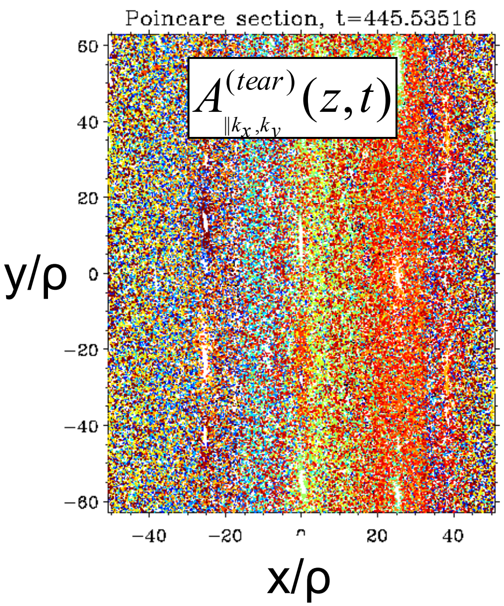
„n=1“ matches very closely the mode structure of the unstable ITG mode (in the drive range)

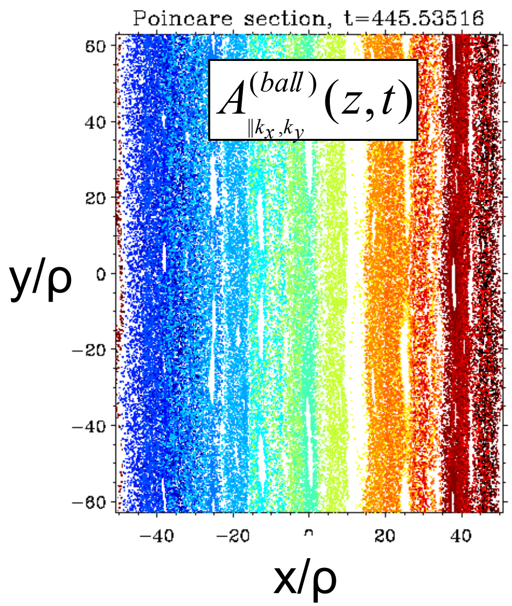
„n=1“ and „n=2“ capture almost all of the transport and field-line stochasticity

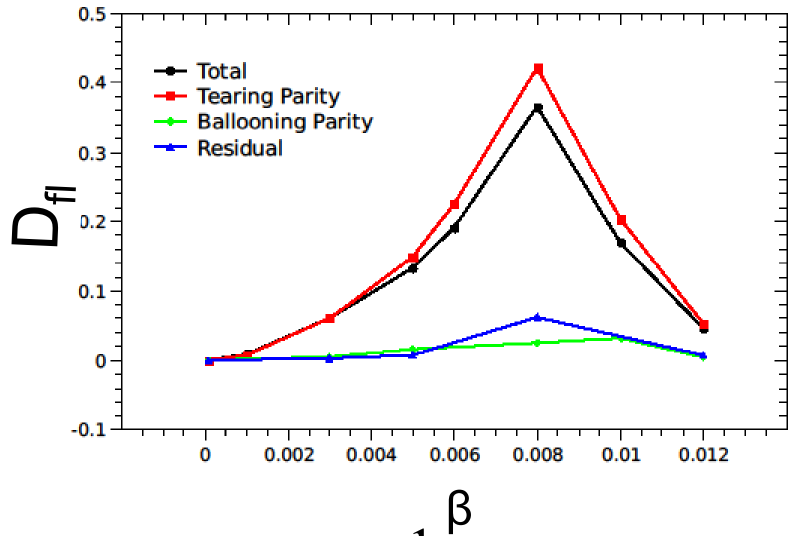
Origin of magnetic stochasticity

- **Ballooning parity** modes: no reconnection / stochastic fields
- **Tearing parity** modes: reconnection
- First two modes (ballooning and tearing) plus residual modes

$$A_{\|k_x,k_y}(z,t) = A^{(ball)}_{\|k_x,k_y}(z,t) + A^{(tear)}_{\|k_x,k_y}(z,t) + A^{(res)}_{\|k_x,k_y}(z,t)$$





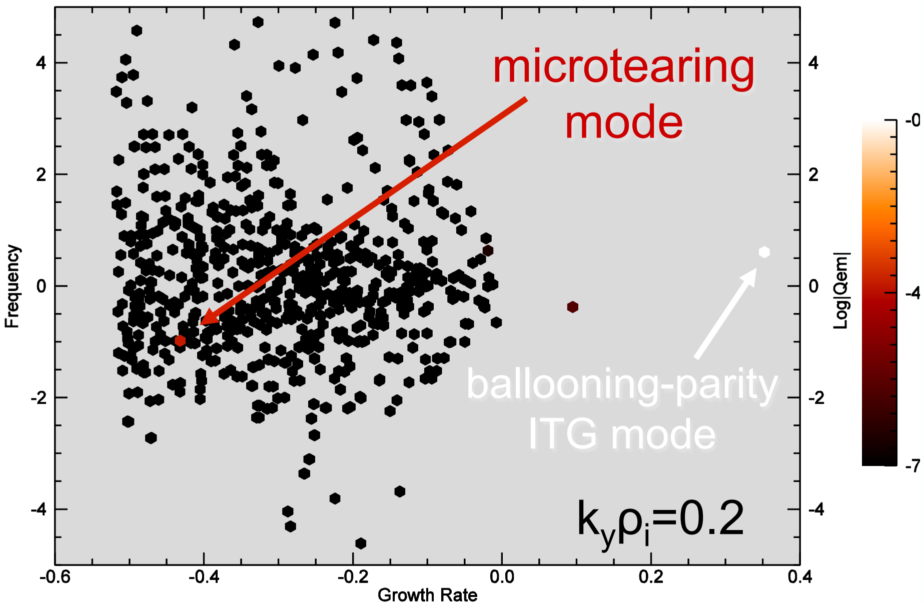


$$D_{fl} = \lim_{l\to\infty} \frac{1}{l}\left\langle \left| r_i(l) - r_i(0) \right|^2 \right\rangle$$

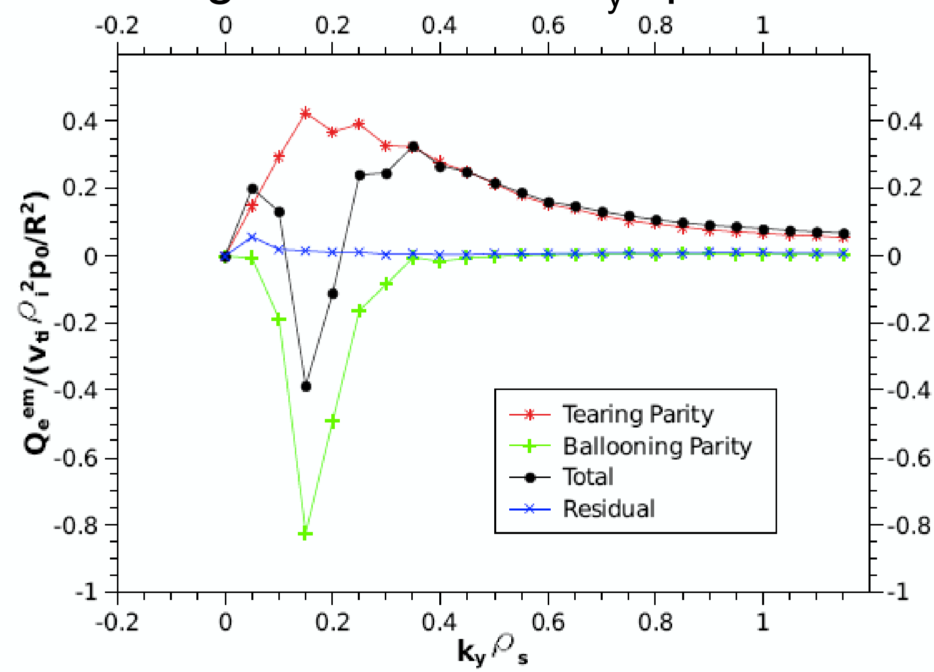
Tearing component causes stochasticity

NL excitation of tearing-parity mode

GENE eigenvalue spectrum



Magnetic heat flux k_y spectrum



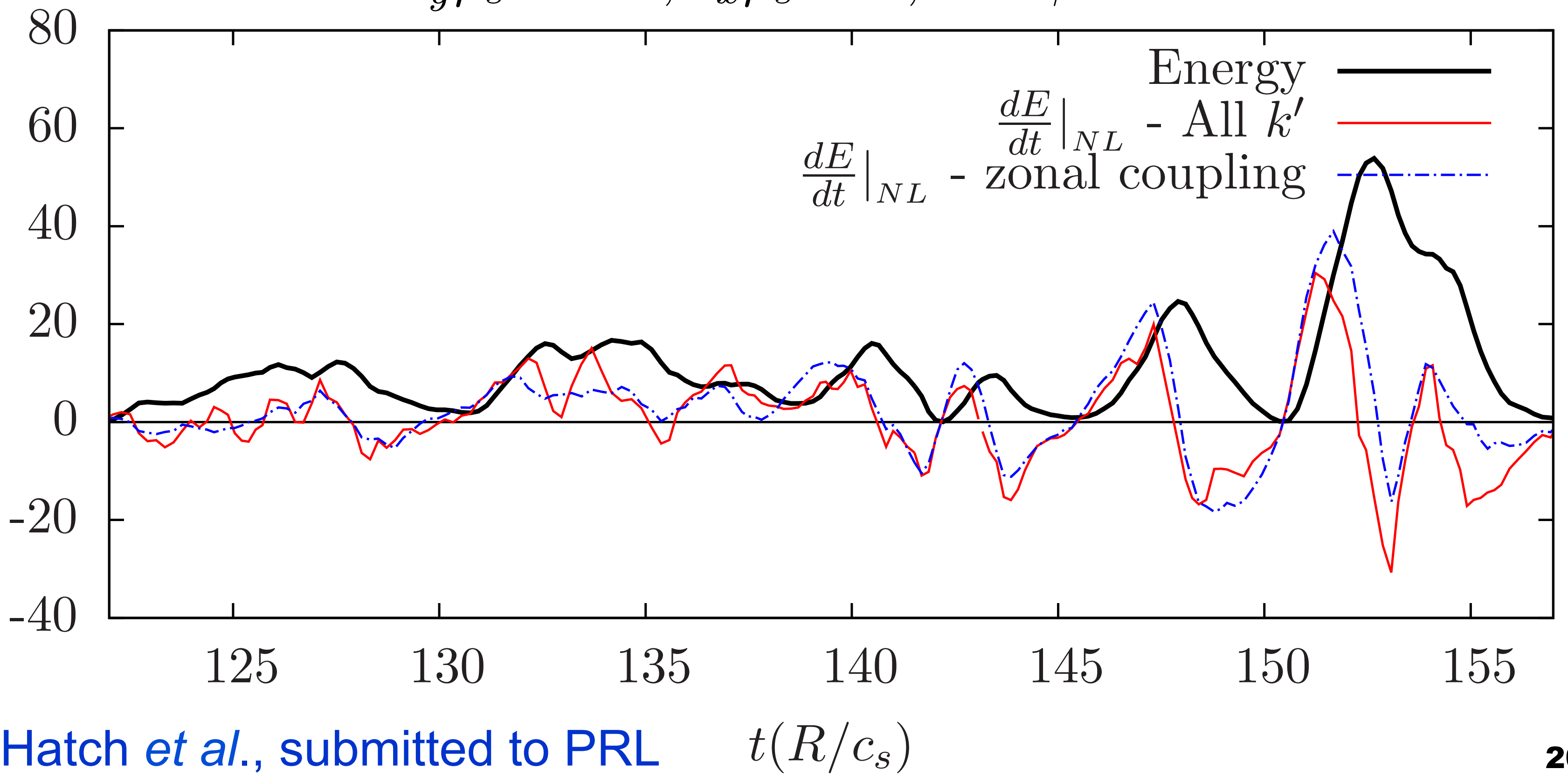
A linearly stable microtearing mode is nonlinearly excited – at higher β values, it dominates the total electron heat flux

Hatch *et al.*, submitted to PRL

Excitation via zonal modes (!!)

The free energy in the „n=2“ mode (black) along with the total nonlinear drive (red) and the nonlinear drive via coupling to zonal modes only (blue), in the saturated turbulent state.

$k_y \rho_s = 0.2, k_x \rho_s = 0,$ and $\beta = 0.003$

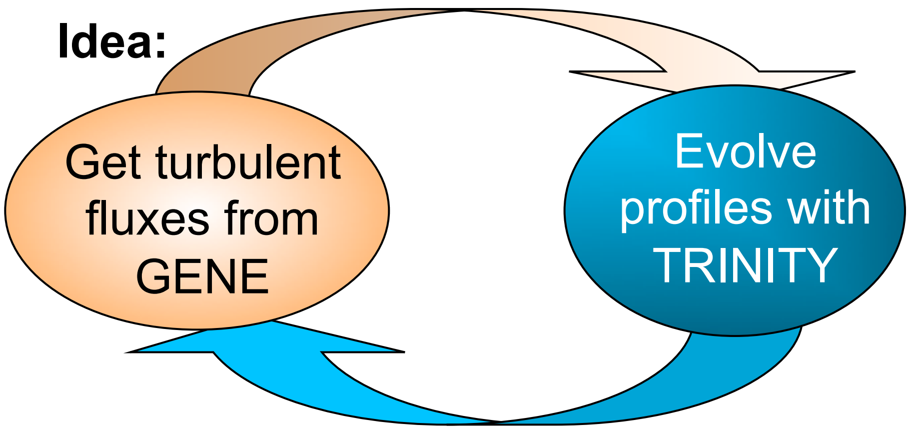


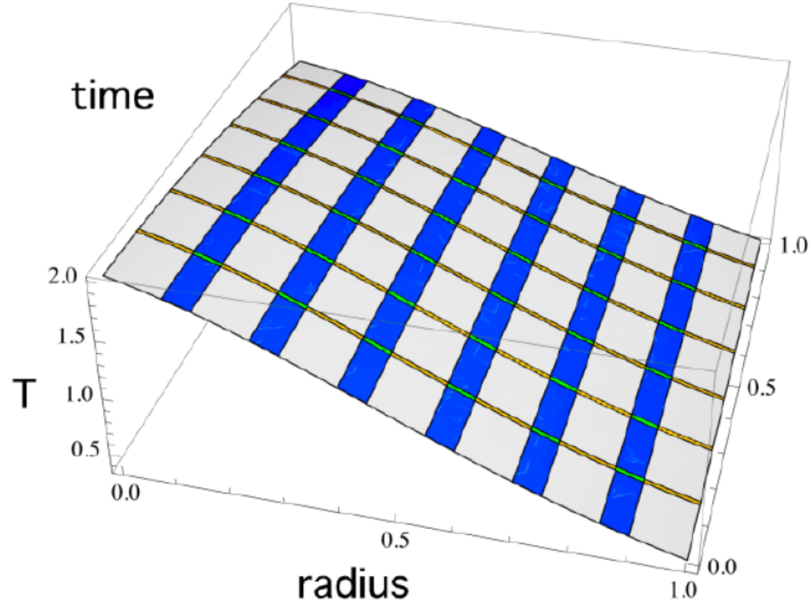
Hatch *et al.*, submitted to PRL

Damped modes, turbulent cascades & Large Eddy Simulations

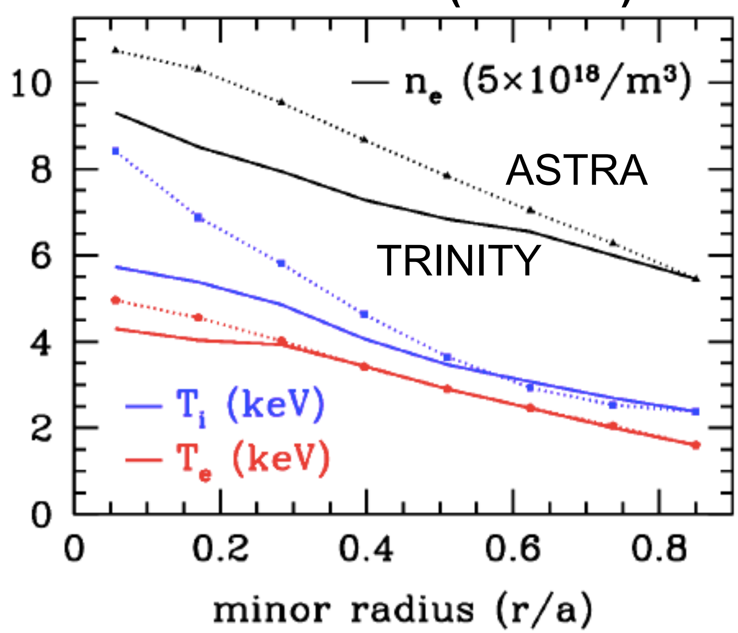
Coupling GENE and TRINITY

Idea:





AUG #13151 (H-mode)



Barnes *et al.*, PoP 2010

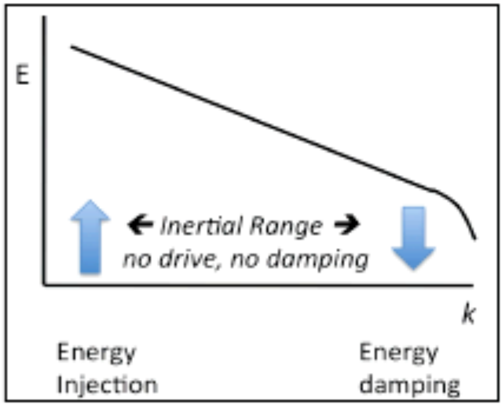
Observed deviations possibly due to:

- *shear flow effects*
- *uncertainties in q profile*

Computational cost much lower than for flux-driven global simulations, but still too high for frequent usage

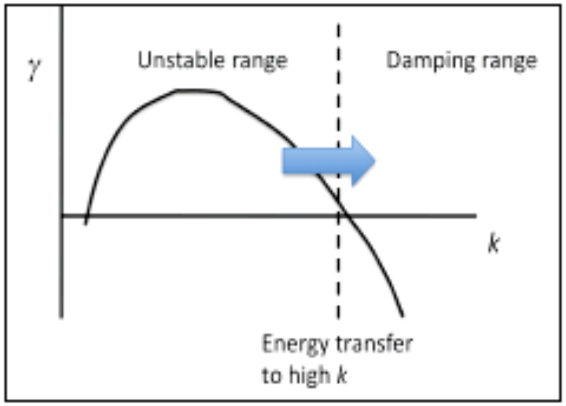
Turbulence in fluids and plasmas – Three basic scenarios

1.Hydrodynamic cascade



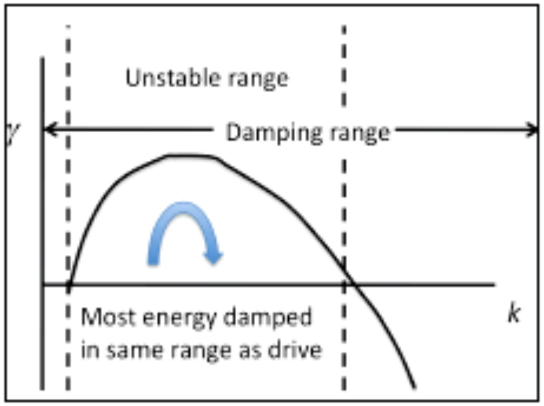
Inertial range
→ no dissipation
→scale invariant dynamics
→power law spectrum

2. Conventional μ-turbulence



Energy transfer to high *k*
like hydro – no inertial range
adjacent unstable, damping ranges

3. Saturation by damped eigenmode

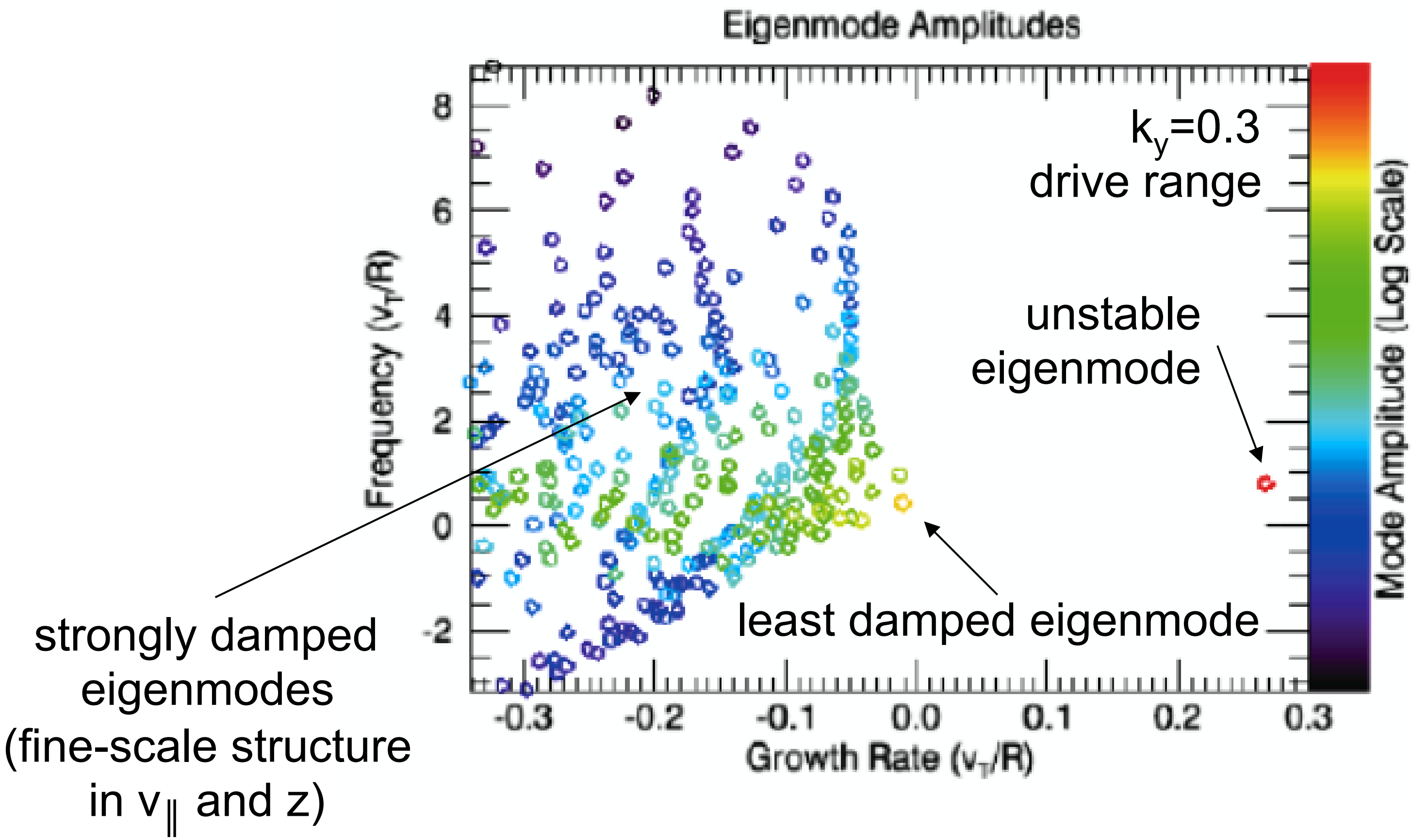


Energy can go to high *k*
but most of it is lost at low *k* in driving range

…in collaboration with P. W. Terry

Excitation of damped eigenmodes

Using GENE as a linear eigenvalue solver to analyze nonlinear ITG runs via projection methods, one finds…



Energetics

Turbulent free energy consists of two parts:

$$\mathcal{E}_f = \sum_j \int d\Lambda \frac{T_{0j}}{F_{0j}} \frac{f_j^2}{2}, \qquad \mathcal{E}_\phi = \sum_j \int d\Lambda q_j \frac{\bar{\phi}_1 f_j}{2}.$$

Drive and damping terms:

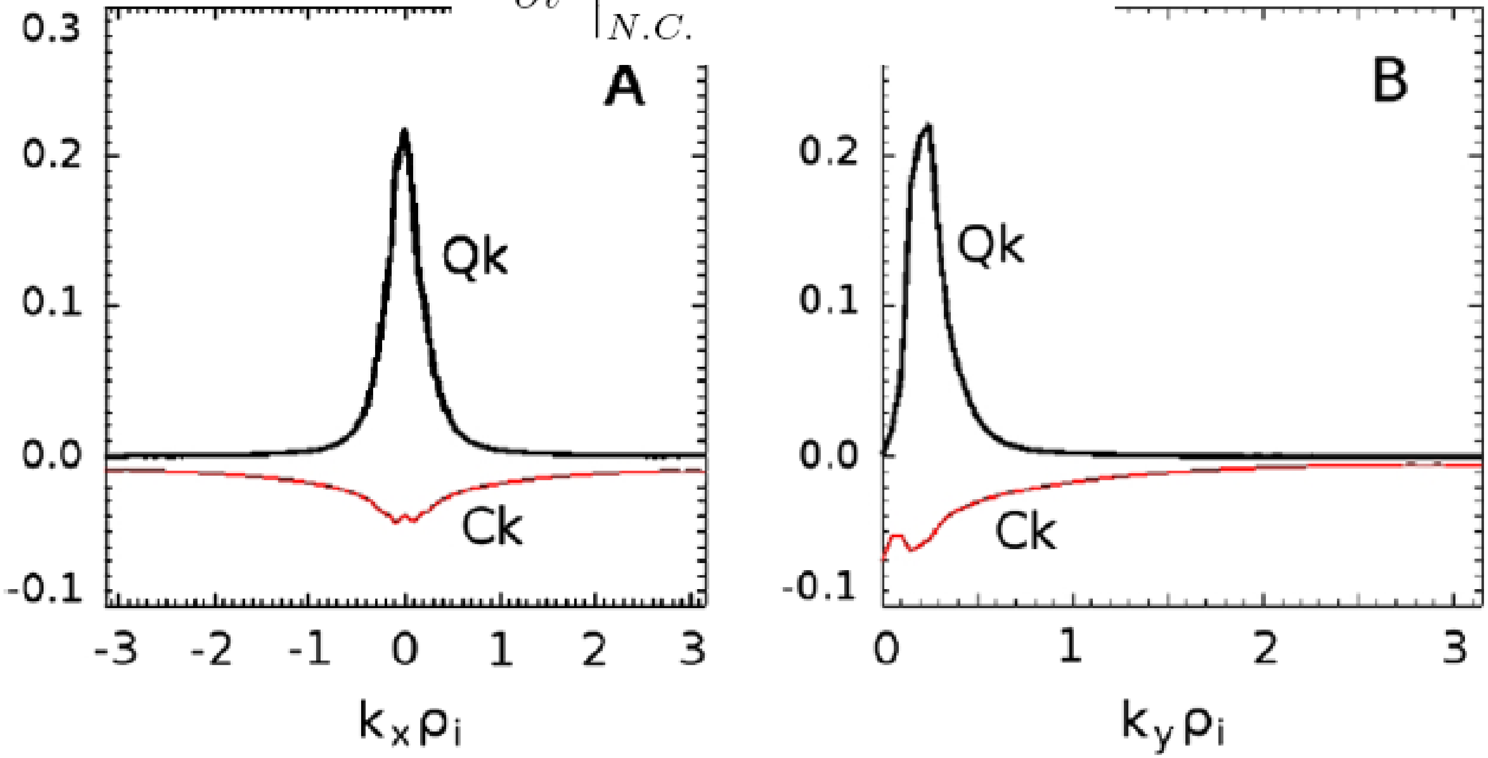
$$\frac{\partial \mathcal{E}}{\partial t} = \sum_j \int d\Lambda \frac{T_{0j}}{F_{0j}} h_j \frac{\partial f_j}{\partial t} = \mathcal{G} - \mathcal{D} \qquad h_j = f_j + (q_j \bar{\phi}_1 / T_{0j}) F_{0j}$$

$$\mathcal{G} = -\sum_j \int d\Lambda \frac{T_{0j}}{F_{0j}} h_j \cdot \left[\omega_n + \left(v_\parallel^2 + \mu B_0 - \frac{3}{2}\right)\omega_{Tj}\right] \times F_{0j} \frac{\partial \bar{\phi}_1}{\partial y}$$

$$\mathcal{D} = -\sum_j \int d\Lambda \frac{T_{0j}}{F_{0j}} h_j (\mathcal{D}_z f_j + \mathcal{D}_{v_\parallel} f_j).$$

Energetics in wavenumber space

$$\left.\frac{\partial E_k}{\partial t}\right|_{N.C.} = Q_k + C_k + N_k$$



Damped eigenmodes are responsible for significant dissipation in the drive range (!)

Hatch, Terry, Jenko, Merz & Nevins, PRL 2011

Resulting spectrum decays exponentially @lo k, asymptotes to power law @hi k

Spectrum from k space attenuation of $T(k)$ by dissipation $\alpha E(k)$:

$$\frac{dT(k)}{dk} = \frac{d(v_k^3 k)}{dk} = aE(k)$$

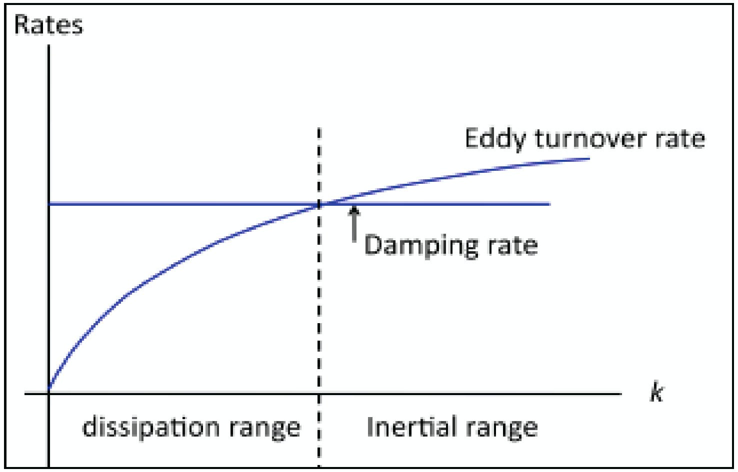
nonlinear energy transfer rate

Corrsin closure procedure: $v_k^3 k = v_k^2 \cdot v_k k = E(k)k \cdot \varepsilon^{1/3} k^{-1/3} k$

Solving attenuation ODE:

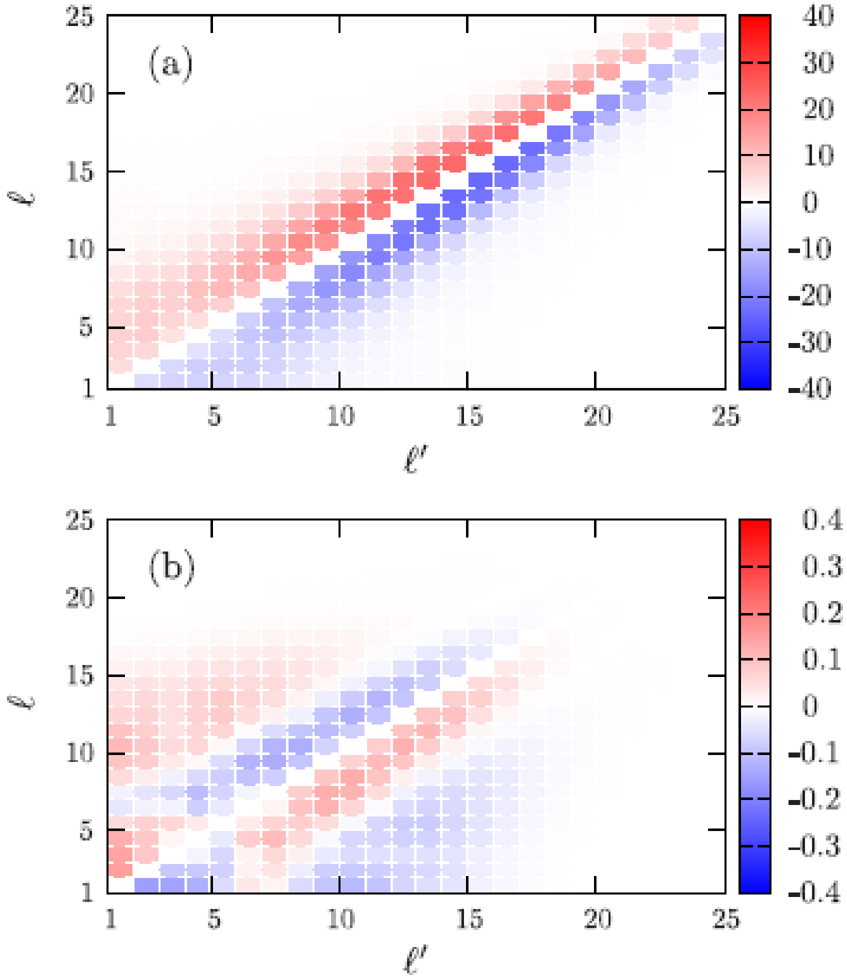
$$E(k) = \beta \varepsilon^{2/3} k^{-5/3} \exp\left[\frac{3}{2}\alpha \varepsilon^{-1/3} k^{-2/3}\right]$$

Spectrum becomes power law in range where eddy turnover rate exceeds constant dissipation rate



Hatch *et al.*, PRL 2011
Terry et al., submitted to PoP

Shell-to-shell transfer of free energy



$$\mathcal{E}_f = \sum_i \int d\Lambda \frac{T_{0j}}{F_{0j}} \frac{f_j^2}{2},$$

ITG turbulence (adiabatic electrons); logarithmically spaced shells

Entropy contribution dominates; exhibits very local, forward cascade

$$\mathcal{E}_\phi = \sum_j \int d\Lambda q_j \frac{\bar{\phi}_1 f_j}{2}.$$

Banon Navarro *et al.*, PRL 2011

Application: Gyrokinetic LES models

LES filter in DNS domain: $\partial_t f_{ki} = L[f_{ki}] + N[\phi_k, f_{ki}] - D[f_{ki}]$

$$\partial_t \overline{f_k} = L[\overline{f_k}] + N[\overline{\phi_k}, \overline{f_k}] + T_{\overline{\Delta},\Delta^{\mathrm{DNS}}} - D[\overline{f_k}]$$

Sub-grid term: $T_{\overline{\Delta},\Delta^{\mathrm{DNS}}} = \overline{N}[\phi_k, f_k] - N[\overline{\phi_k}, \overline{f_k}] \approx c_\perp k_\perp^4 h_{ki}$

Free energy spectra vs $c_\perp$:

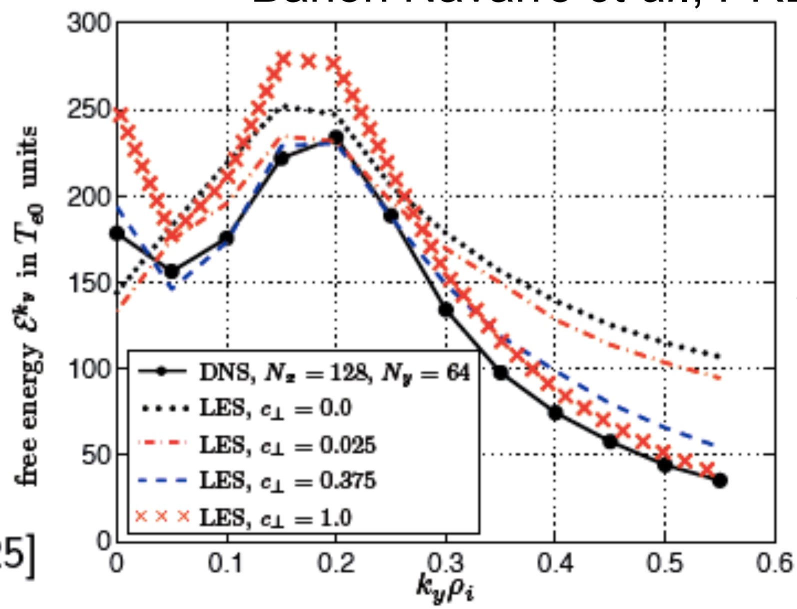
Cyclone Base Case (ITG)

- ⋆ $c_\perp$ too small
  - ⇒ not enough dissipation
- ⋆ $c_\perp$ too strong
  - ⇒ overestimates injection
- ⋆ $c_\perp = 0.375$ good agreement

→ "plateau" for $c_\perp \in [0.25, 0.625]$

→ holds for k_x

Bañon Navarro *et al.*, PRL 2011



Morel *et al.*, PoP 2011

Substantial savings in computational cost: Here, a factor of 20

Self-adjustment of model parameters

Test filter in DNS domain: $\partial_t \widehat{f}_k = L[\widehat{f}_k] + N[\widehat{\phi_k}, \widehat{f}_k] - D[\widehat{f}_k] + T_{\widehat{\Delta},\Delta^{\mathrm{DNS}}}$

Test filter in LES domain: $\partial_t \widehat{f}_k = L[\widehat{f}_k] + \widehat{N}[\overline{\phi_k}, \overline{f_k}] - D[\widehat{f}_k] + \widehat{T}_{\overline{\Delta},\Delta^{\mathrm{DNS}}}$

$\widehat{\overline{\cdots}} = \widehat{\cdots}$ ...for the Fourier cut-off filters used here

One thus obtains the (Germano) identity:

$$T_{\widehat{\Delta},\Delta^{\mathrm{DNS}}} = \widehat{T}_{\overline{\Delta},\Delta^{\mathrm{DNS}}} + \widehat{N}[\overline{\phi_k}, \overline{f_k}] - N[\widehat{\phi_k}, \widehat{f}_k] = \widehat{T}_{\overline{\Delta},\Delta^{\mathrm{DNS}}} + T_{\widehat{\Delta},\overline{\Delta}}$$

Approximate sub-grid terms and minimize error:

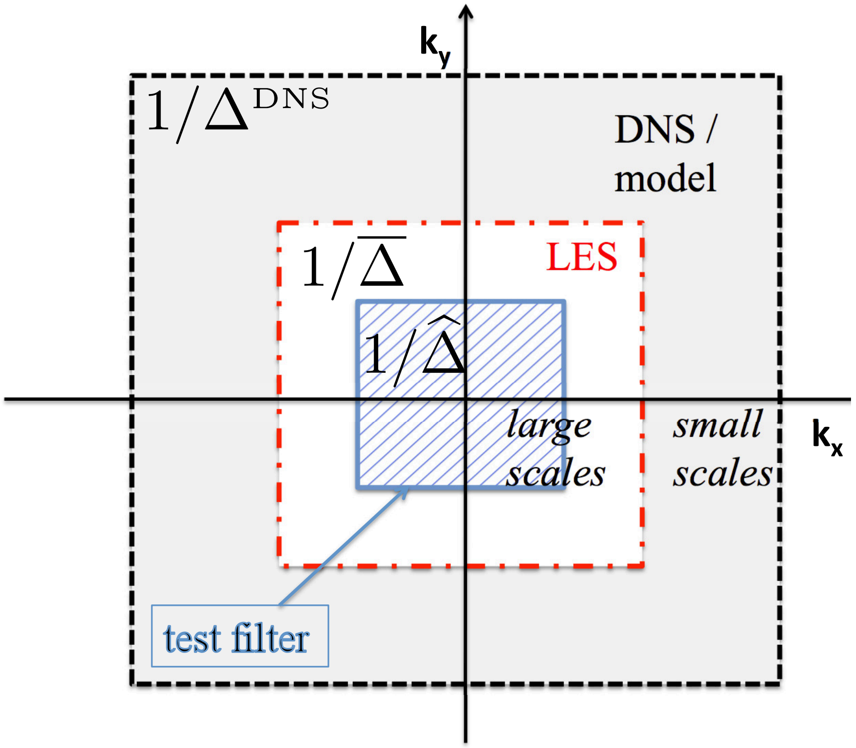
$$T_{\widehat{\Delta},\Delta^{\mathrm{DNS}}} \approx M_{\widehat{\Delta}} \quad ; \quad T_{\overline{\Delta},\Delta^{\mathrm{DNS}}} \approx M_{\overline{\Delta}} \qquad M_{\widehat{\Delta}} \approx \widehat{M}_{\overline{\Delta}} + T_{\widehat{\Delta},\overline{\Delta}}$$

$$d^2 = \left\langle \left(T_{\widehat{\Delta},\overline{\Delta}} + \widehat{M}_{\overline{\Delta}} - M_{\widehat{\Delta}} \right)^2 \right\rangle_\Lambda$$

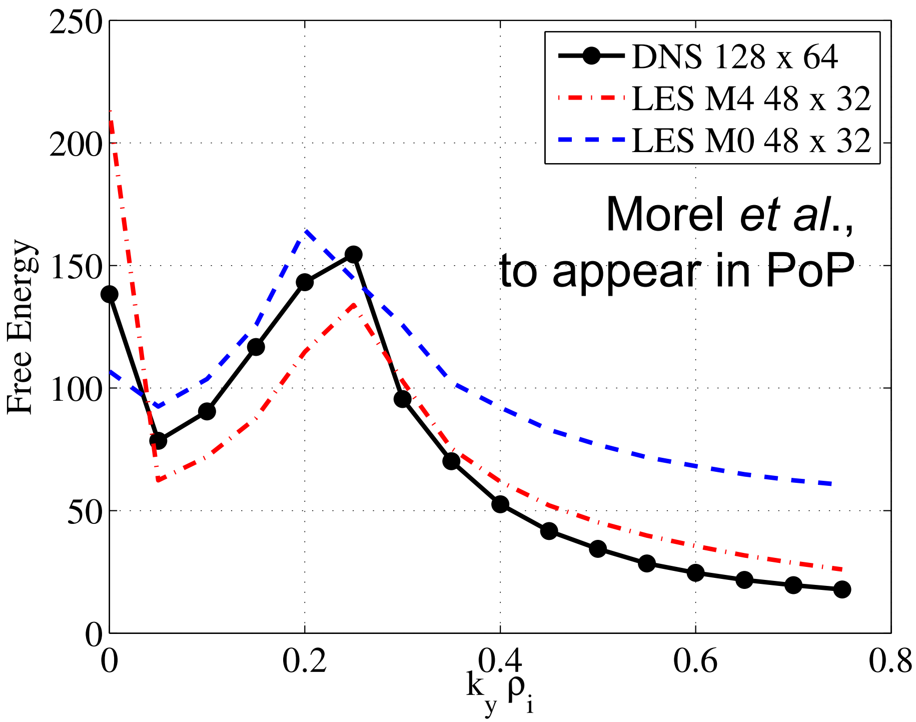
...this procedure yields explicit expressions for the model parameter(s)

The "dynamic procedure" in practice

Schematic of „dynamic procedure"



Free energy spectra
(w/ and w/o model)



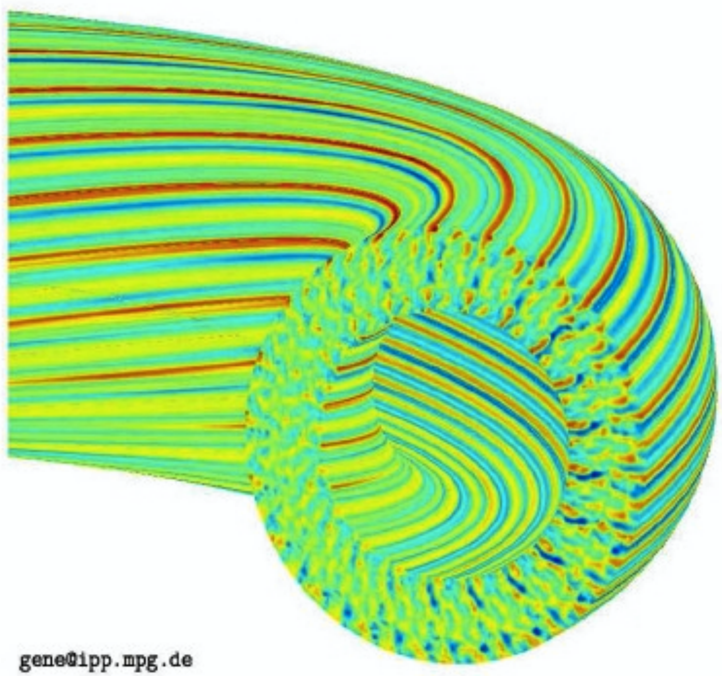
LES techniques are likely to reduce the simulation effort substantially without introducing many free parameters. This offers an interesting perspective…

Turbulent transport of energetic particles

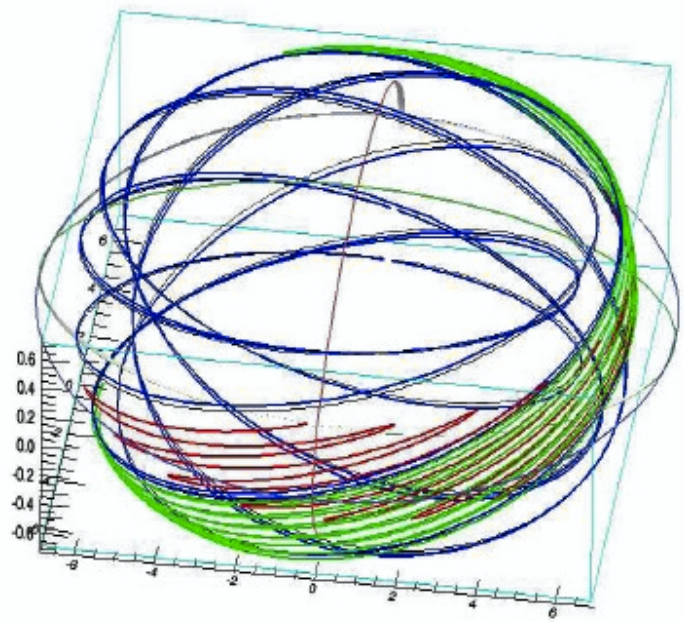
Diffusion and decorrelation

Diffusivity and Lagrangian velocity autocorrelation function (Taylor, 1922):

$$D_x(t) = \frac{1}{2}\frac{d}{dt}\langle \delta x^2(t)\rangle = \int_0^t d\xi \, \langle v_x(0) v_x(\xi) \rangle \equiv \int_0^t d\xi \, L_{v_x}(\xi)$$

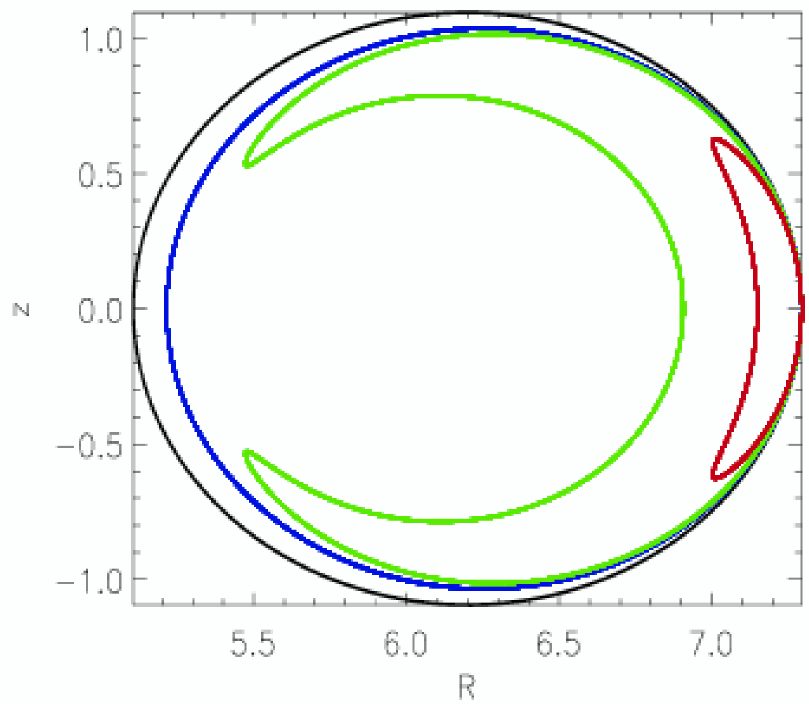


Turbulence in a torus (GENE)

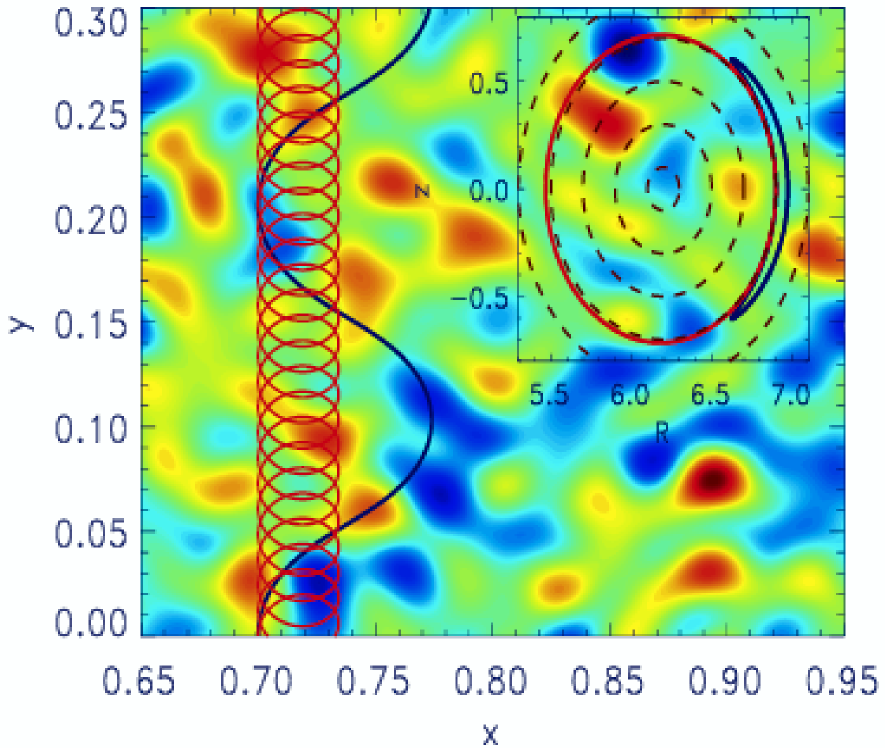


Drift orbits in the torus

Energetic particle trajectories



Drift orbits in the R-z plane



Drift orbits in field aligned coordinates

Key question: How do the particles decorrelate?

Validity of orbit averaging (OA)?

Distinguish different regimes:

- For deeply trapped, thermal particles, OA holds (for low frequencies)
- For passing, thermal particles, OA applies only marginally (one turn)
- For energetic particles, OA does not apply

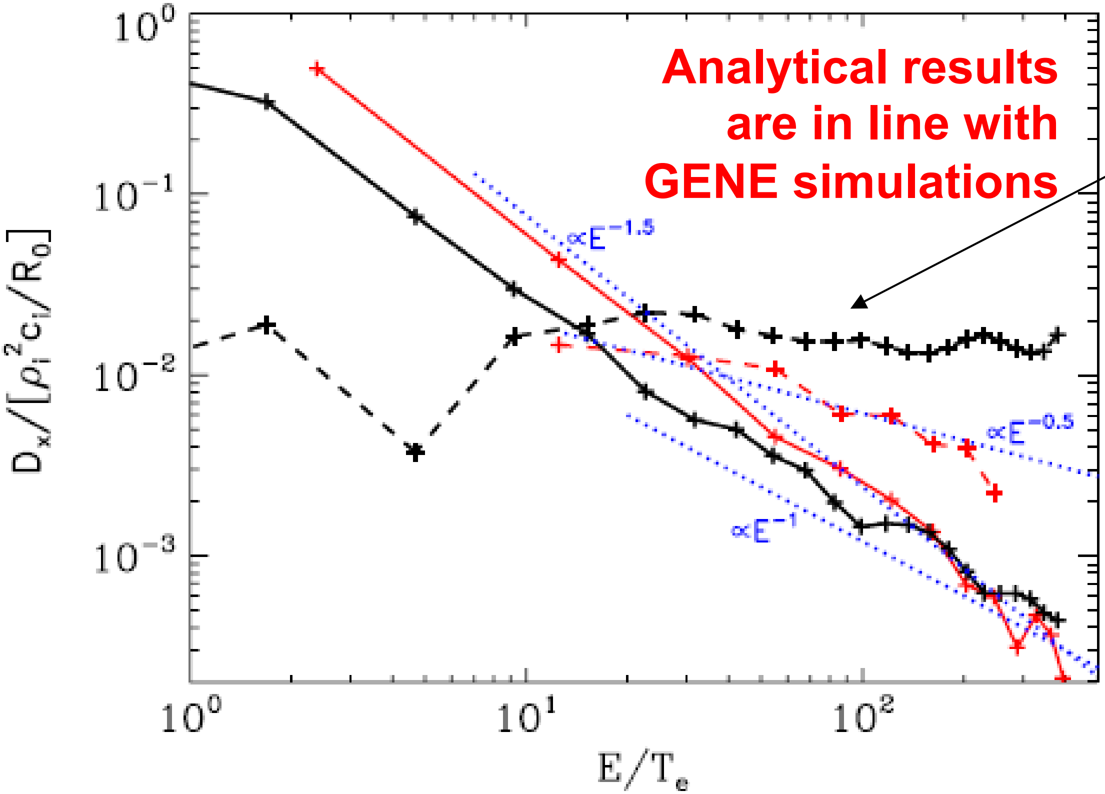
precession frequency ~ E, transit frequency ~ $E^{1/2}$

quasi-periodicity (requirement of OA) is violated

OA is generally not applicable to energetic particles; perpendicular decorrelation dominates

Jenko *et al*., PRL 2011

Turbulent transport of fast ions



Magnetic transport at large $v_\parallel/v$ is (almost) independent of particle energy:

$$D \sim v_\parallel^2 \left(\frac{\widetilde{B}_r}{B_0} \right)^2 \tau$$

Hauff *et al.*, PRL 2009

May explain fast radial current redistribution observed experimentally

Similar: Transport of runaway electrons

Hauff & Jenko, PoP 2009

Experimental evidence for anomalous transport of beam ions

DIII-D plasmas with NBI

Anomalies relative to the classical prediction are correlated with low E/T_e

Theoretical estimates of (electrostatic) fast-ion diffusivities comparable to experimental levels

Heidbrink *et al.*, PRL 2009

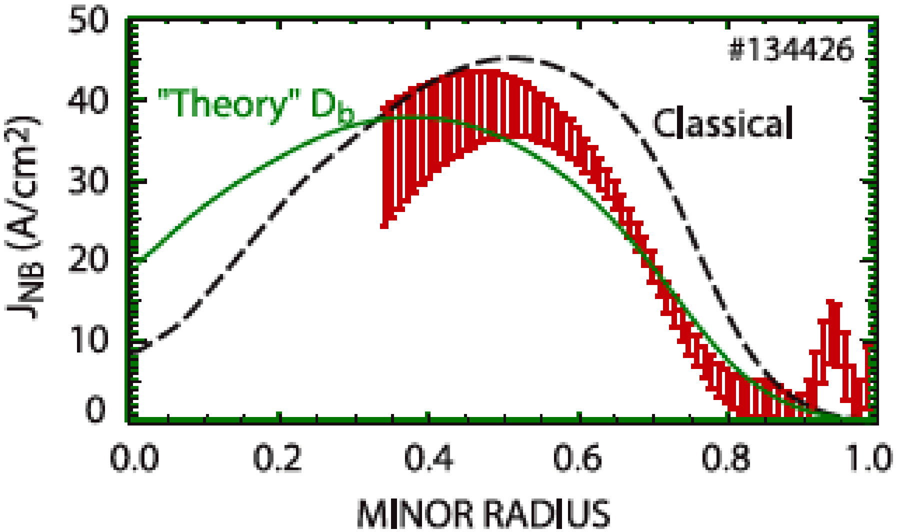


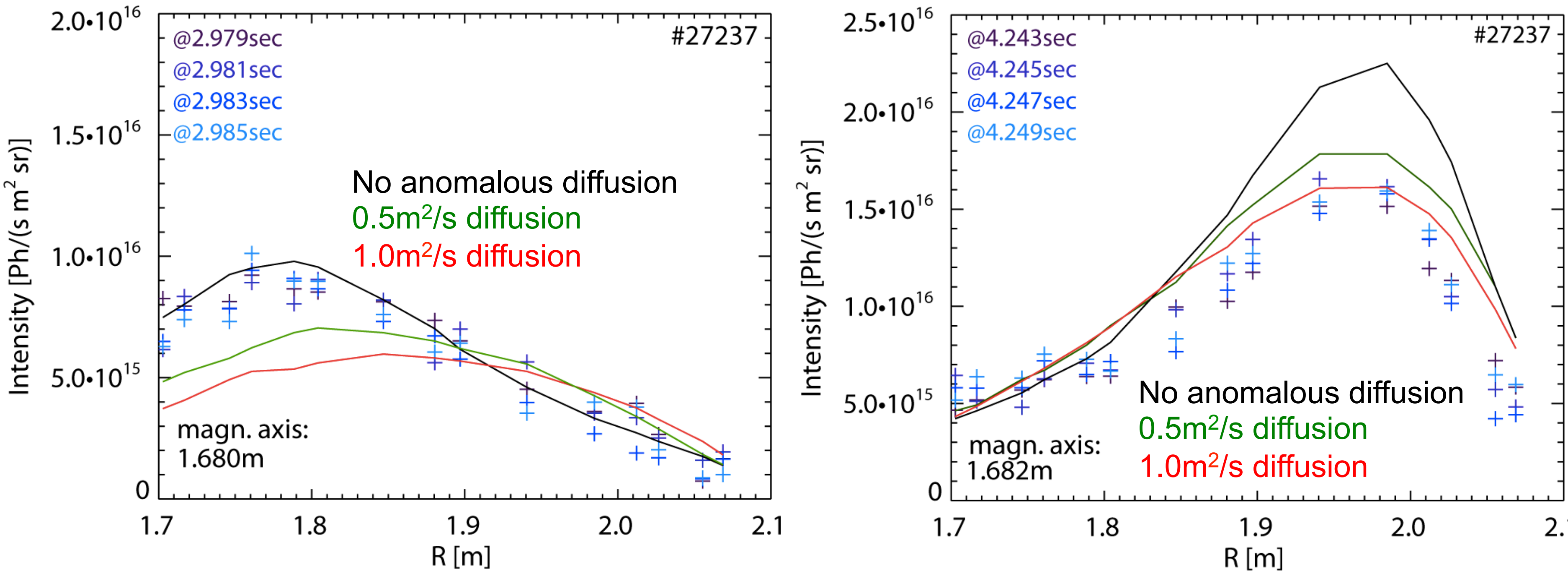
FIG. 2 (color online). Measured beam-driven current (symbols), classical prediction (dashed line), and theory-based prediction (solid line) versus ρ for the 7.2 MW discharge.

New direct sim-exp comparisons are currently underway

Recent FIDA measurements @ AUG

5MW of on-/off-axis NBI heating

Geiger *et al.*, 2011

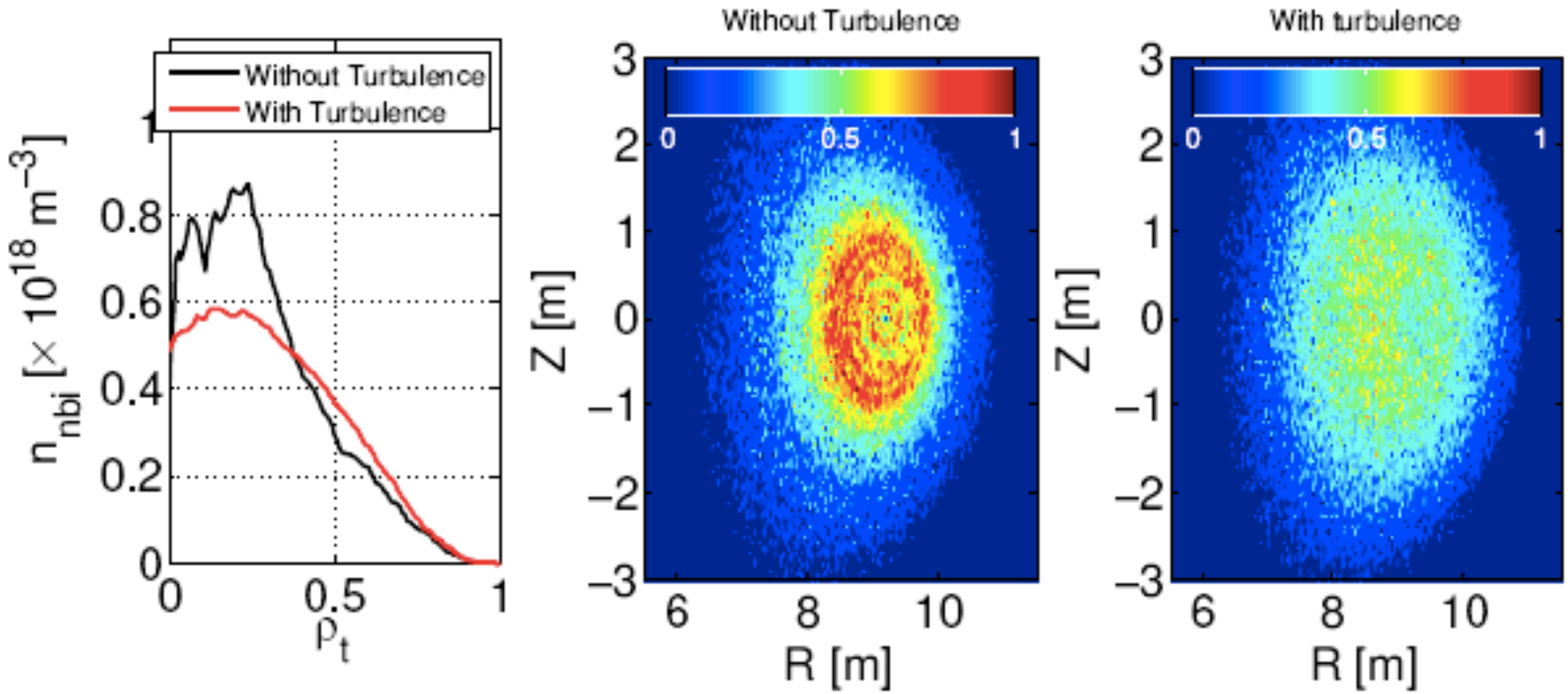


- Measurements during 5MW on-axis NBI found in good agreement with TRANSP simulations (assuming small anomalous transport)
- Clear disagreement between measurements and TRANSP simulations during 5MW off-axis NBI (analysis ongoing)

Expectations for ITER and DEMO

M. Albergante, 2011

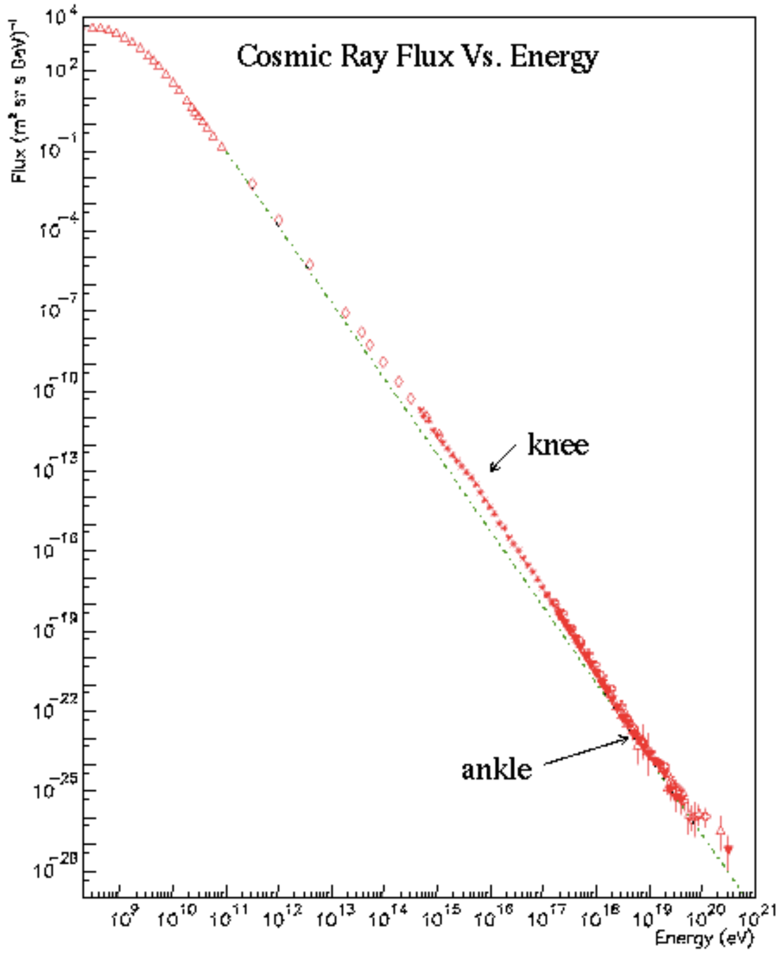
GENE / VENUS simulations indicate (again) that anomalous current redistribution is important up to $E_{NBI}/T_e \sim 20$ (see below)

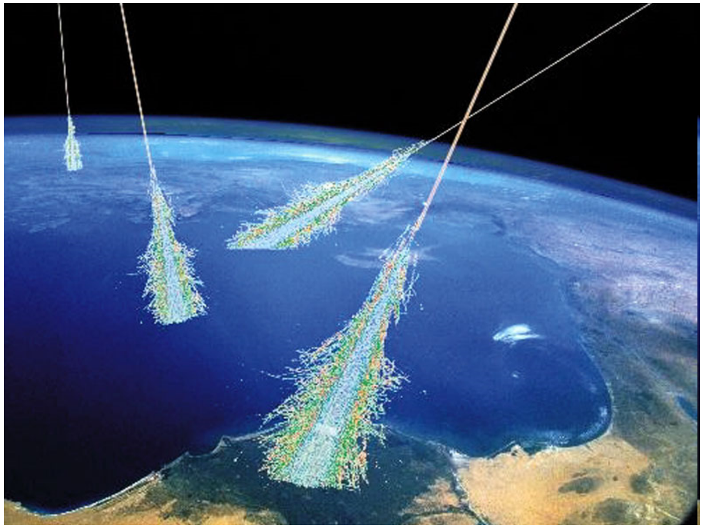


Both ITER and DEMO probably need to ensure $E_{NBI}/T_e \gg 20$ (and $\beta/\beta_{crit} \ll 1$)

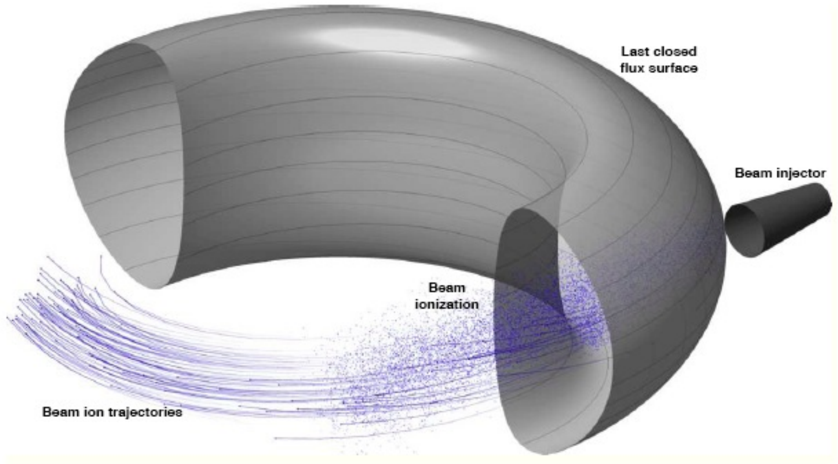
Application to CR transport theory

Acceleration and propagation of cosmic rays in astrophysical plasmas





Beam ions in tokamaks



Brief history of CR transport theory

Discovery of CRs by Victor Hess (Nobel Prize in 1936) 100 years ago

A few examples for applications of CR transport theory:

- Shock-wave acceleration in supernova remnants
- Propagation in the solar wind and in the interstellar medium
- Cross-field transport of energetic electrons in coronal loops

Theoretical description:

- Quasilinear theory (Jokipii 1966)

  evaluate Taylor relation for unperturbed orbits

  significant deviations from test particle simulations (!)

- Nonlinear theories (Owens 1974, Bieber 1997, Matthaeus 2003, etc.)

  develop adequate models for Lagrangian a.c. function

Some analytic expressions

Particle fieldline diffusion (with FLR effects):

$$D_\perp = \left(\frac{\tilde{B}_\perp}{B_0}\right)^\gamma \frac{\mu}{(1-\mu^2)^{\gamma/4}} c^{1-\gamma/2} \left(\frac{eB}{4\sqrt{\pi} m_0}\right)^{\gamma/2}$$
$$\times 1.73^{2-\gamma} \lambda_\parallel^{\gamma-1} \lambda_\perp^{2-\gamma/2} \frac{(\kappa^2-1)^{1/2-\gamma/4}}{\kappa}$$

Perpendicular decorrelation:

$$D_\perp = \left(\frac{\tilde{B}_\perp}{B_0}\right)^\gamma \mu^\gamma (1-\mu^2)^{1/2-\gamma/2} c \lambda_\perp \sqrt{1-1/\kappa^2}$$

...with… $\mu \equiv v_\parallel / v = \sqrt{E_\parallel / E}$ $\qquad \kappa \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{E}{m_0 c^2} + 1$

Astrophysical Journal **711**, 997 (2010)
Invited Talk, European Cosmic Ray Symposium 2010

Summary and outlook

Trying to tackle plasma turbulence

Ab initio simulations will remain very challenging (although invaluable), despite continuing growth in computer power

Quasilinear models can be extremely useful but fail to capture important nonlinear effects; thus, they must be complemented (or replaced) by nonlinear simulations

This motivates the search for reliable but minimal models; Large Eddy Simulations represent one such line of research

Turbulent transport of energetic particles offers an interesting opportunity for interdisciplinary cross-fertilization

In general, we are in need of a still better understanding of plasma turbulence in order to model it efficiently