Oleg V. Polomarov
Institute for Fusion Studies,
The University of Texas at Austin
opoloma@physics.utexas.edu

Fast Kinetic Modelling of Inductively Coupled Low-Pressure Discharges

Collaboration:
I. D. Kaganovich, PPPL,

C. E. Theodosiou, The University of Toledo, Ohio.

Motivation

- **To develop the fast self-consistent kinetic model of low-collisional ICP (the alternative to PIC-MCC),**

***I.D. Kaganovich and O.V. Polomarov*, Phys. Rev. E 68, 026411 (2003).**

- **Generalize the “non-local approach” to include kinetic and resonant effects.**

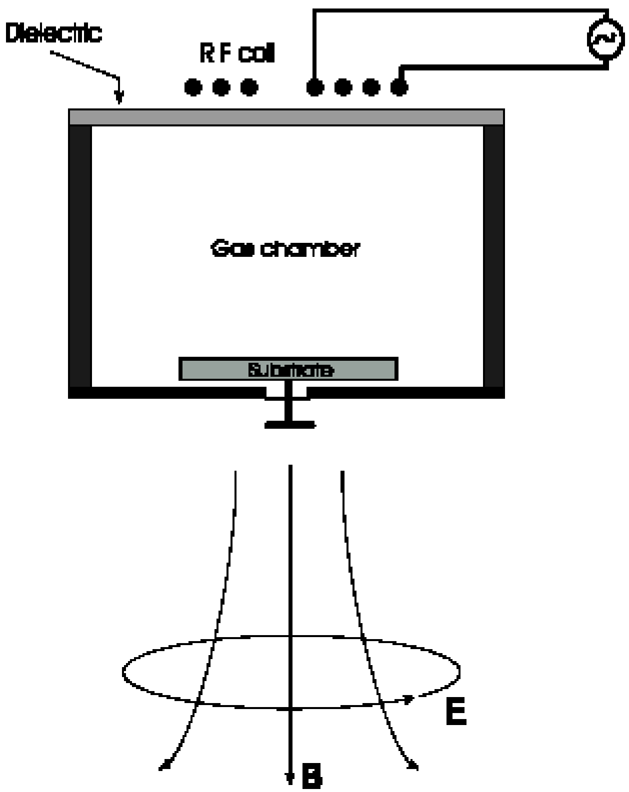
***I.D. Kaganovich, O.V. Polomarov and C.E. Theodosiou*, Phys. Plasmas 11, 2399 (2004).**

- **To study the effects of resonances, plasma non-uniformity and self-consistency in non-magnetized and magnetized ICP plasmas.**

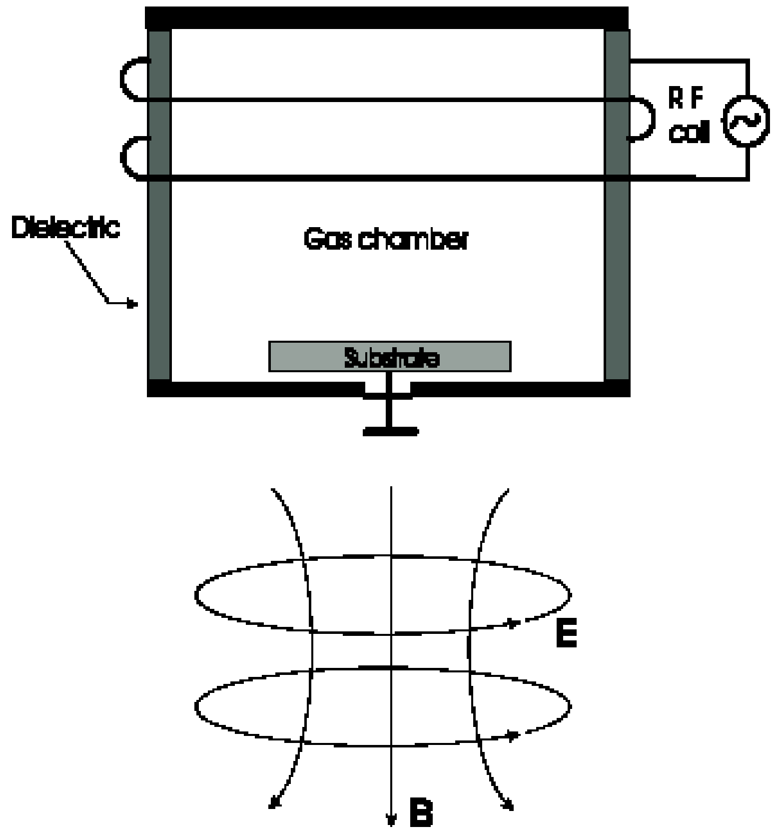
***O.V. Polomarov, C.E. Theodosiou, and I.D. Kaganovich*, accepted, PoP, August (2005).**
***O. V. Polomarov, C. E. Theodosiou, and I. D. Kaganovich*, accepted, PoP, September, (2005).**

ICP – the discharge supported by an electromagnetic wave

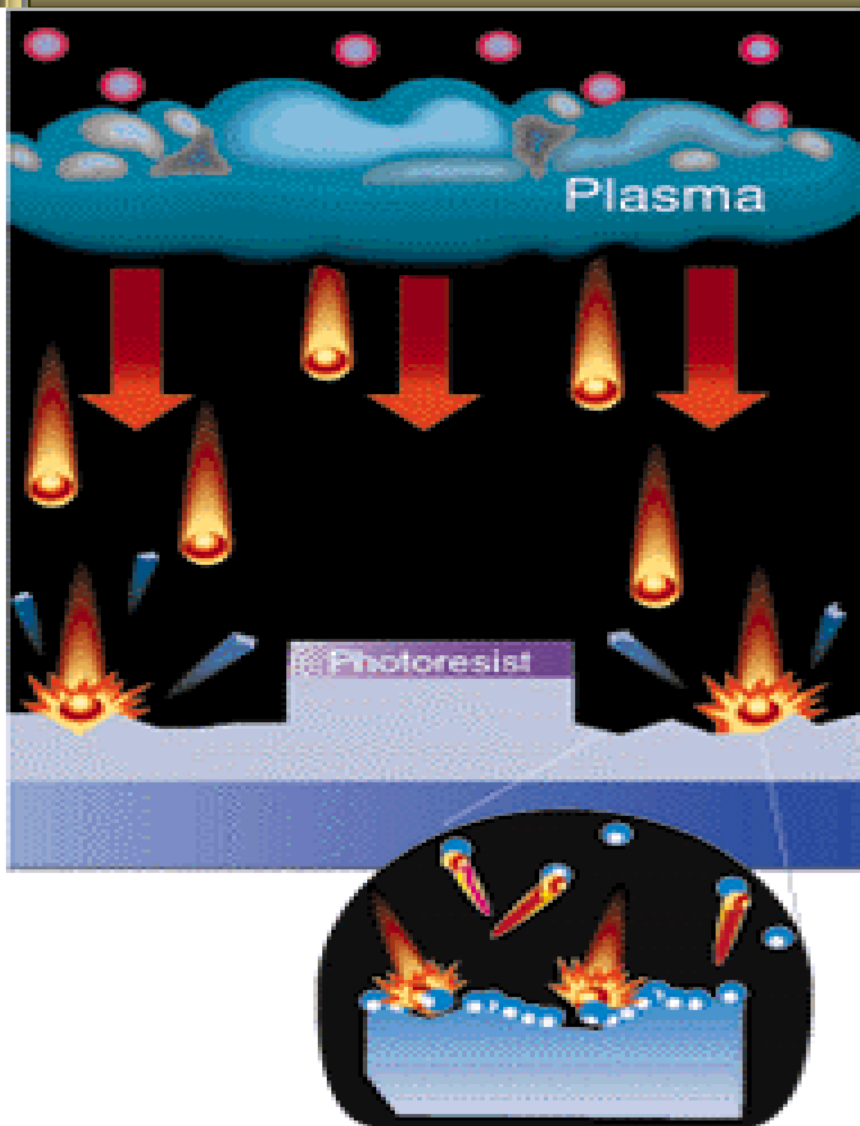
"Coil on top"



"Cylindrical"



The Low Pressure Inductively Coupled Discharges Produce Quiet, Stable Plasmas



Discharge frequency f=0.45-29 MHz

Argon gas pressures of 0.3-1**0** mTorr

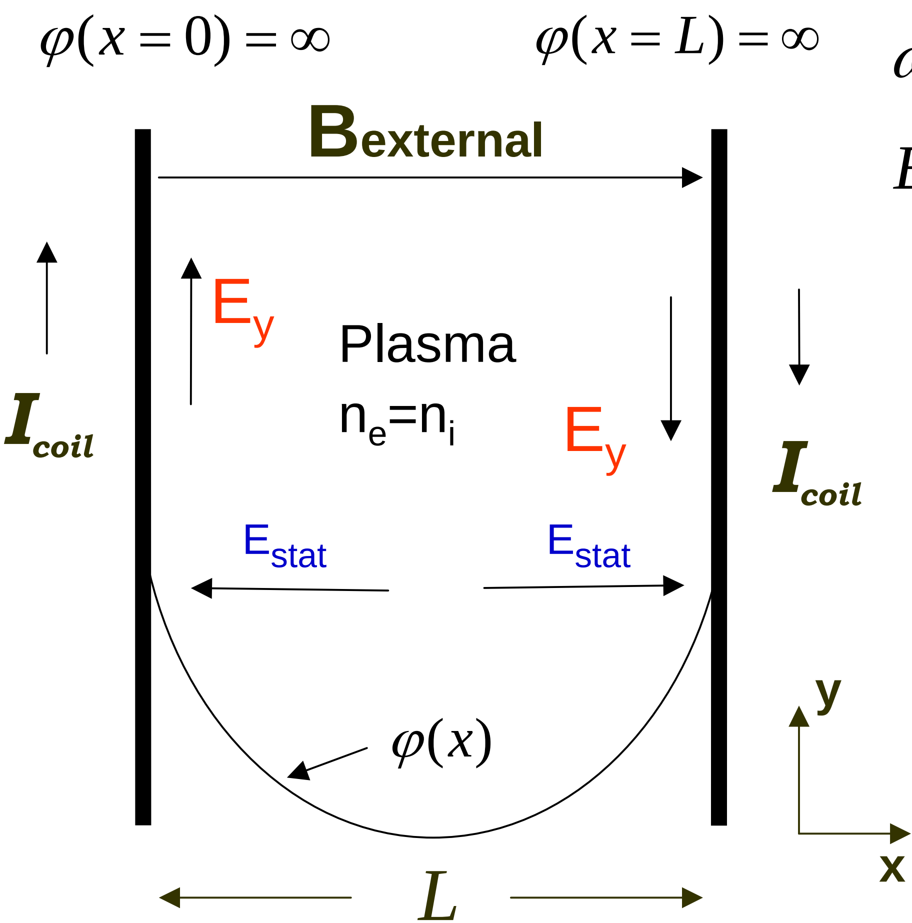
Plasma density n = 10^9 - 10^{13} cm^{-3}

Electron temperature T_e = few eV

RF power dissipation in the plasma 6-400 W**.**

Plasma size L~10 cm

1-D Plasma Slab Of Width L



Assumptions:

$$\omega \ll \omega_p\ ,\quad \lambda_D \ll L\ ,\quad n_e(x)=n_i(x)$$

$$E_y(x,t)\sim e^{-i\omega t},\ I_{coil}\sim e^{-i\omega t}$$

a) *The plasma is driven by coil current* I_{coil} *, that produces the transverse wave with* E_y *that excites plasma,*

b) E_{stat} *is produced by the ambipolar potential* $\varphi(x)$*,*

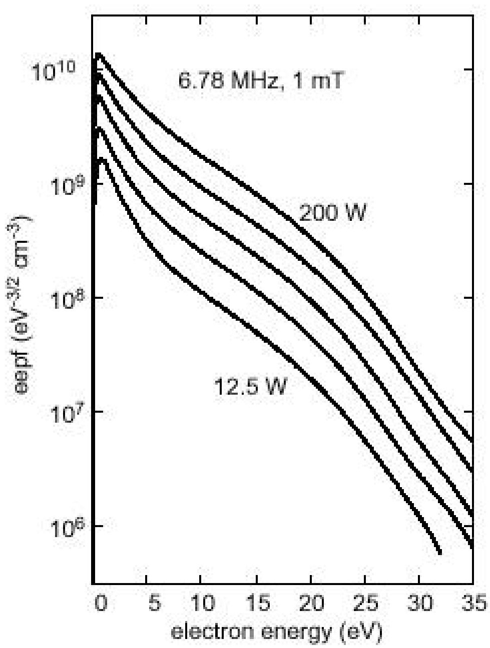
c) *Reflection from the walls is specular,*

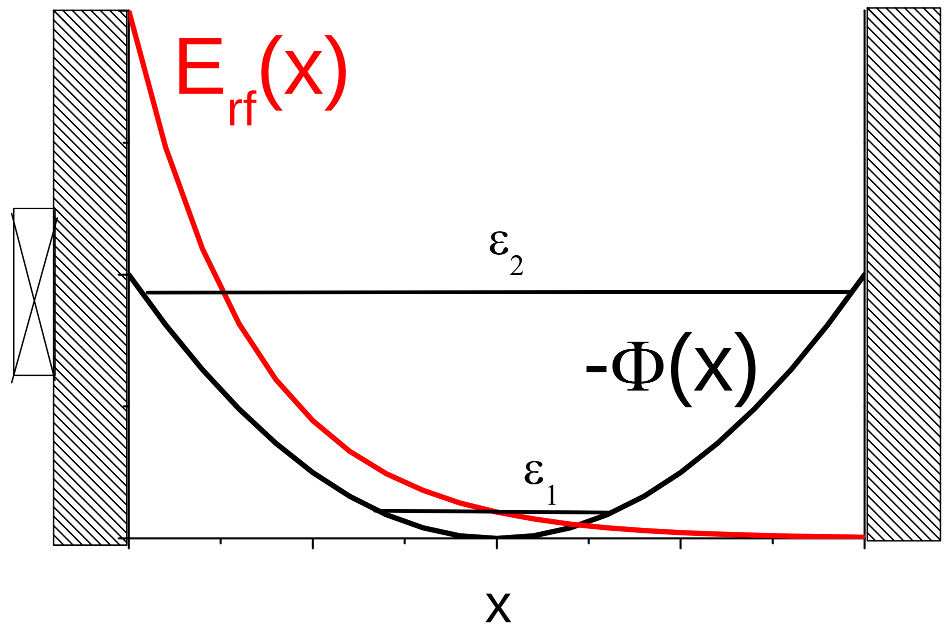
d) *One electrode can be grounded.*

Characteristic features of low-pressure, inductive plasma

- **Non-Maxwellian Electron Energy Distribution Function (EEDF)**
- **Collisionless heating**
- **Anomalous skin effect**

Electron Distribution Function is not Maxwellian





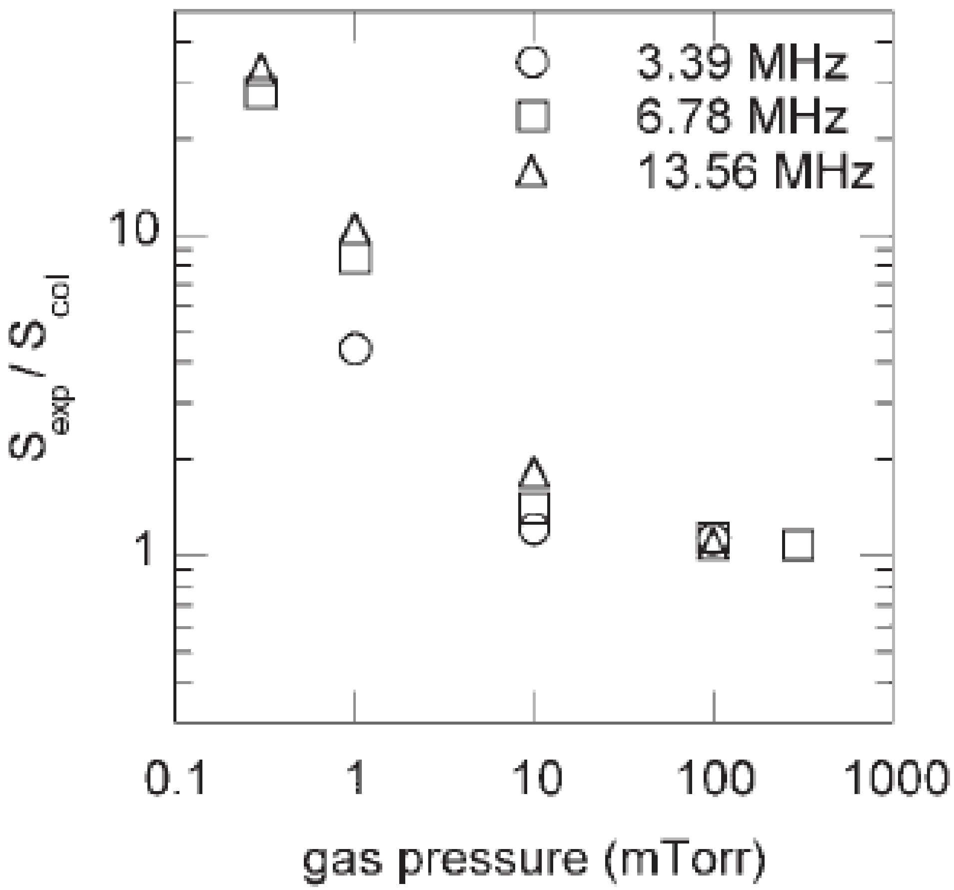
Measured EDF shows departure from a Maxwellian:

a) Depopulated by high-energy electrons - inelastic collisions.

b) Enriched by low-energy electrons - cold electrons are trapped in a small rf electric field.

Electron Heating Is Collisionless

The ratio of the total measured power S_{exp} to collisional S_{col} power dissipation at 100W.



$$S_{col} = < j_{col} E >$$

$$j_{col} = \frac{e^2 n E}{m(-i\omega + \nu)}$$

ν is the collision frequency

ω is the rf field frequency

The enhanced power dissipation is due to resonant wave-particle interaction

Collisionless Heating is Analogous to Landau Damping

Collisionless (Landau) plasma wave damping is larger than collisional damping for $\nu \ll \omega$

Analogy: a particle sits on the crest of the wave

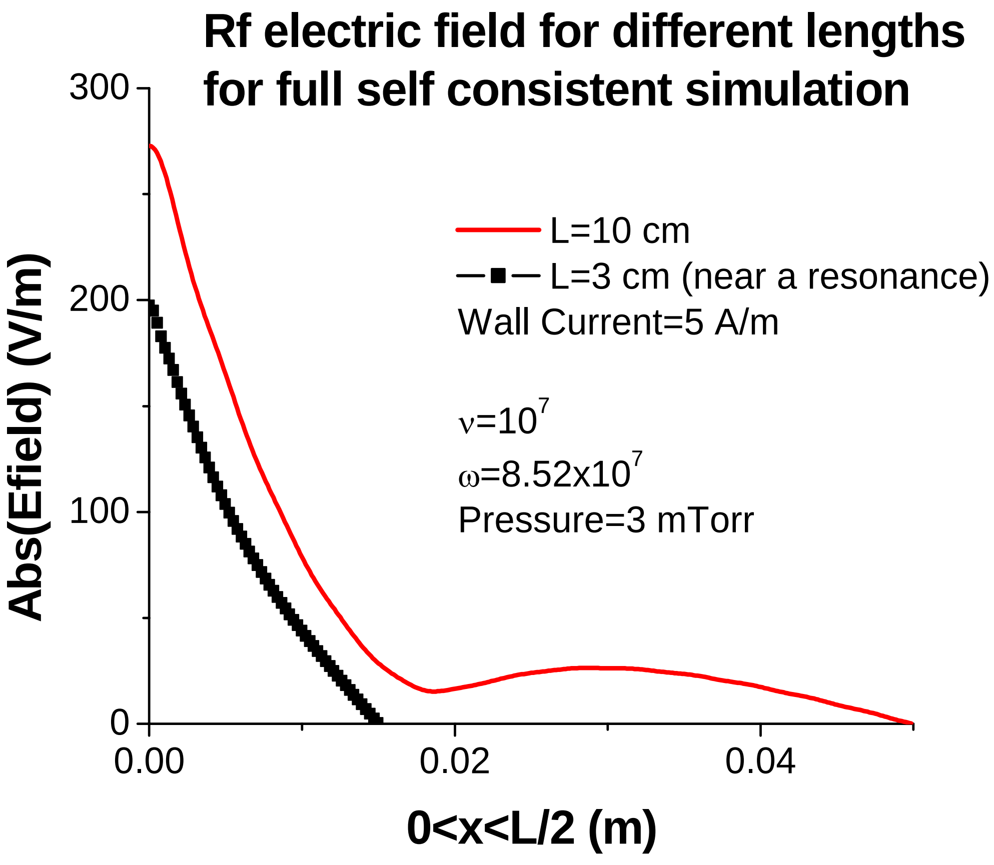
Inhomogeneous rf field => E(k) = > ω=> ω-vk

$$j_{col} = \frac{e^2 nE}{m(-i\omega + \nu)} \;\; => \;\; j(k) = \frac{e^2 nE(k)}{m} \int d\mathbf{v} \frac{f(\mathbf{v})}{-i(\omega - \mathbf{vk}) + \nu}$$

Due to resonance ω=vk

$$\mathbf{j(k)} > \mathbf{j_{col}}$$

Typical profile of rf electric field shows anomalous skin effect



$$\frac{d^2E_y}{dx^2} = -\frac{4\pi i\omega}{c^2}\, j(x)$$

$$Normal\ \ skin\ \ effect:$$

$$j(x) = \sigma E\ \ for$$

$$l_{mfp} < \delta$$

$$Anomalos\ \ skin\ \ effect:$$

$$j(x) = \int \sigma(x,y)E(y)dy$$

$$for\ \ l_{mfp} > \delta$$

Overview of Kinetic Model for Discharge Simulations

- Calculate nonlocal conductivity in non-uniform plasma.

- Find a non-Maxwellian electron energy distribution function driven by collisionless heating of resonant electrons.

- What to expect: self-consistent system for kinetic treatment of collisionless and nonlocal phenomena in inductive discharge.

The Model Consists Of 3 Blocks:

- **The *rf electric field* using Maxwell's equations and the non-local conductivity operator,**
- **The *electron distribution function (EEDF)* using the averaged over fast electron motions kinetic equation,**
- **The *electrostatic potential* using *quasi-neutrality condition* and the fluid equations for *ion density* and *ion momentum*.**

Sketch of the non-local approach

Electron Velocity Distribution Function for $\frac{\nu^*}{\omega} << 1$, $\frac{\nu^*}{\Omega_b} << 1$ and $\frac{\nu^*}{\nu} << 1$

is represented as $f(\vec{x},\vec{v},t) \sim f_0(\varepsilon) + f_1(\vec{x},\vec{v},t)$,

where $\varepsilon = \frac{mv^2}{2} - e\phi(x)$, $\nu - elastic\ collision\ frequency$, $\nu^* - inelastic\ collision\ frequency$,

$\Omega_b = \frac{\pi}{L}\sqrt{\frac{2T_e}{m_e}} - electron\ bounce\ frequency.$

The Boltzmann equations splits in 2 equations:

1) for the main isotropic part, or $EEDF$ $f_0(\varepsilon)$, which gives density: $\int f_0 d\vec{v} = n_e(\vec{x})$

2) for the small anisotropic part $f_1(\vec{x},\vec{v},t)$, which gives the non-local conductivity operator: $e\int \vec{v} f_1 d\vec{v} = \vec{j}(\vec{x})$.

Averaging used: over the rf period, over fast bouncing, over all velocities with given total energy ε.

1) The transverse RF Electric Field

$$\frac{d^2E_y}{dx^2}+\frac{\omega^2}{c^2}E_y=-\frac{4\pi i\omega}{c^2}\left[j(x)+I\delta(x)-\delta_{anti}I\delta(x-L)\right]$$

Using spectral method

$$E_y=\sum_{s=0}^{\infty}\Xi_s cos(k_s x)$$, where: $k_s=(2s+1)\pi/L$,

it gives:

$$\left(-k_s{}^2+\frac{\omega^2}{c^2}\right)\Xi_s=-\frac{4\pi i\omega}{c^2}\left[j_s+\frac{I(1+\delta_{anti})}{L}\right]$$

1a) Current Density

The non-local conductivity operator:

$$j_s = \frac{e^2}{m}\frac{1}{(2s+1)\Omega_{bT}}\sum_{l=0}^{\infty}\Xi_l Z_{s,l}^{gen}\left(\frac{\omega+i\nu}{(2s+1)\Omega_{bT}}\right)$$

The generalized dispersion function:

$$Z_{s,l}^{gen}(\zeta) \equiv \sqrt{\frac{2}{m}}\frac{(2s+1)\pi\Omega_{bT}}{L}\sum_{n=-\infty}^{\infty}\int_0^{\infty}\frac{\Gamma(\varepsilon)}{n\Omega_b(\varepsilon)-(2s+1)\Omega_{bT}\zeta}\frac{G_{s,n}(\varepsilon)G_{l,n}(\varepsilon)}{\Omega_b(\varepsilon)}d\varepsilon\ ,$$

where $G_{l,n}(\varepsilon) = \frac{1}{T}\left[\int_0^T \cos[k_l x(\tau)]\cos\left(\frac{\pi n\tau}{T}\right)d\tau\right]$

can be effectively computed using Fast Fourier Transform (FFT).

2) Kinetic Equation Is Averaged over Fast Electron Bouncing in Potential Well

$$-\frac{d}{d\varepsilon}(D_\varepsilon + \overline{D_{ee}})\frac{df_0}{d\varepsilon} - \frac{d}{d\varepsilon}\overline{V_{ee}}f_0 = \sum_k \left[\overline{\nu_k^*(u+\varepsilon_k^*)\frac{\sqrt{(u+\varepsilon_k^*)}}{\sqrt{u}} f_0(\varepsilon+\varepsilon_k^*)} - \overline{\nu_k^*} f_0 \right],$$

D_{ee} V_{ee} are from the electron-electron collision integral,

ν^* is inelastic collision frequency and upper bar denotes space averaging with constant total energy.

$$D_\varepsilon = \frac{\pi e^2}{4m^2}\sum_{n=-\infty}^{\infty}\int_0^\varepsilon d\varepsilon_x \left|E_{yn}(\varepsilon_x)\right|^2 \frac{\varepsilon-\varepsilon_x}{\Omega_b(\varepsilon_x)} \frac{\nu}{[\Omega_b(\varepsilon_x)n-\omega]^2+\nu^2}$$

Energy diffusion D_e coefficient is a function of the rf electric field E_y and the plasma potential φ*(x)*.

3) The ambipolar potential

The **quasineutrality condition**

$$n_{ion}(x) = n_{electron}(x) = \int_{\varphi(x)}^{\infty} f_0(\varepsilon)\sqrt{\varepsilon - \varphi(x)}\, d\varepsilon,$$

gives the **electrostatic potential:**

$$\frac{d\varphi(\varepsilon)}{dx} = -T_e^{scr}(x)\frac{d\ln[n_{ion}(x)]}{dx},$$

where T_e^{scr} is the electron "screening temperature".

3a) The ion density profile

The **ion density profile** $n_{ion}(x)$ is given by:

$$\frac{d(n_{ion}u)}{dx} = R_{ion}$$

$$u_+\frac{du_+}{dx} = -\frac{eT_e^{scr}(x)}{M}\frac{d\ln n_{ion}}{dx} - \nu_{ion}(u_+)u_+ - \frac{R_{ion}}{n_{ion}}u_+$$

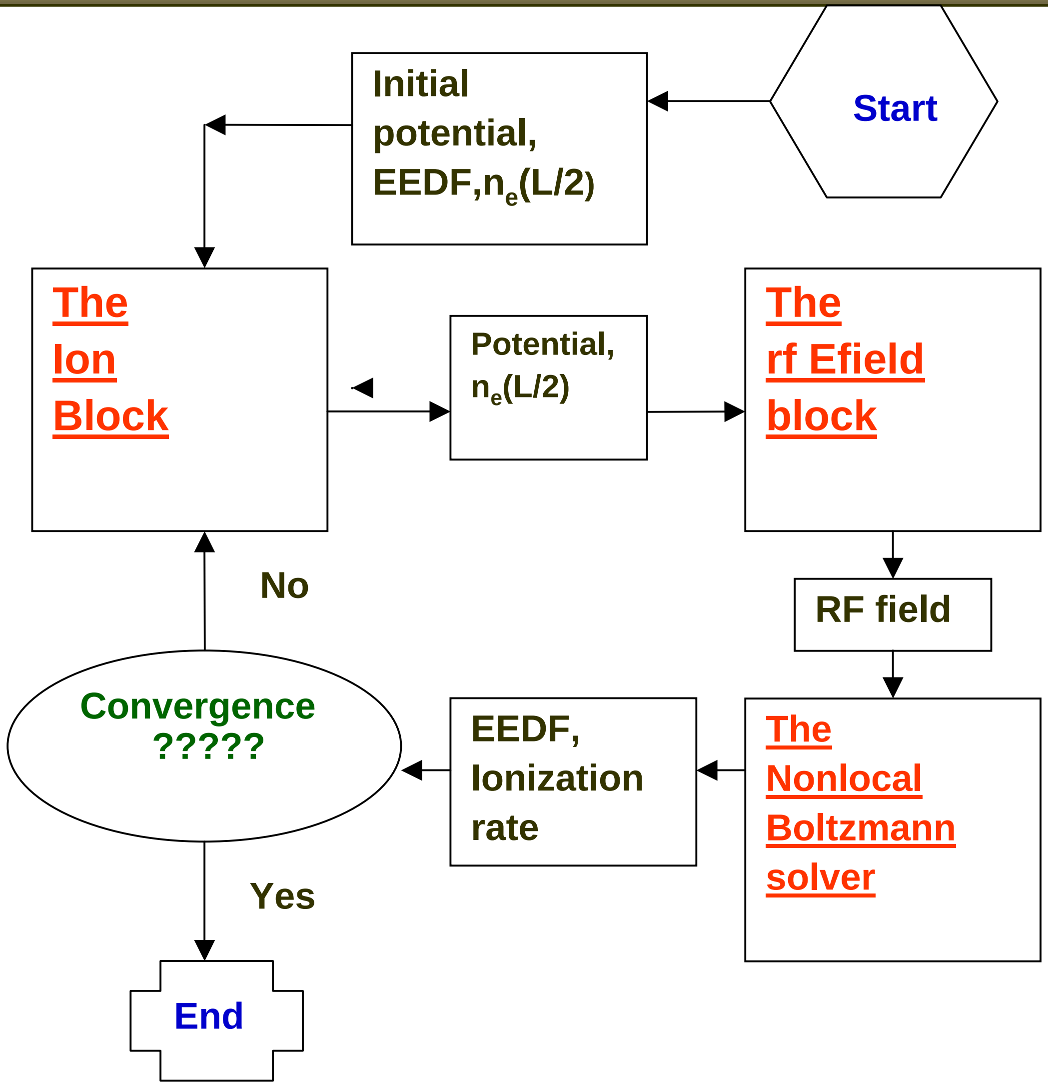
where $$R_{ion}(x) = N_{gas}\sqrt{\frac{2e}{m}}\int_{\varphi(x)}^{\infty}\sigma_{ioniz}(u)(\varepsilon-\varphi(x))\,f_0(\varepsilon)\,d\varepsilon$$

is the ionization rate, and $\nu_{ion}(u)$

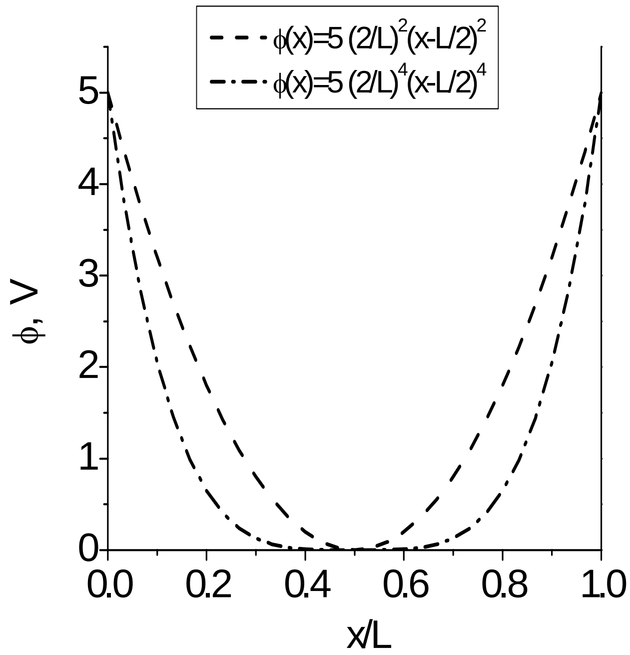
is the ion-neutral and ion charge transfer frequency.

Simulation flow chart.

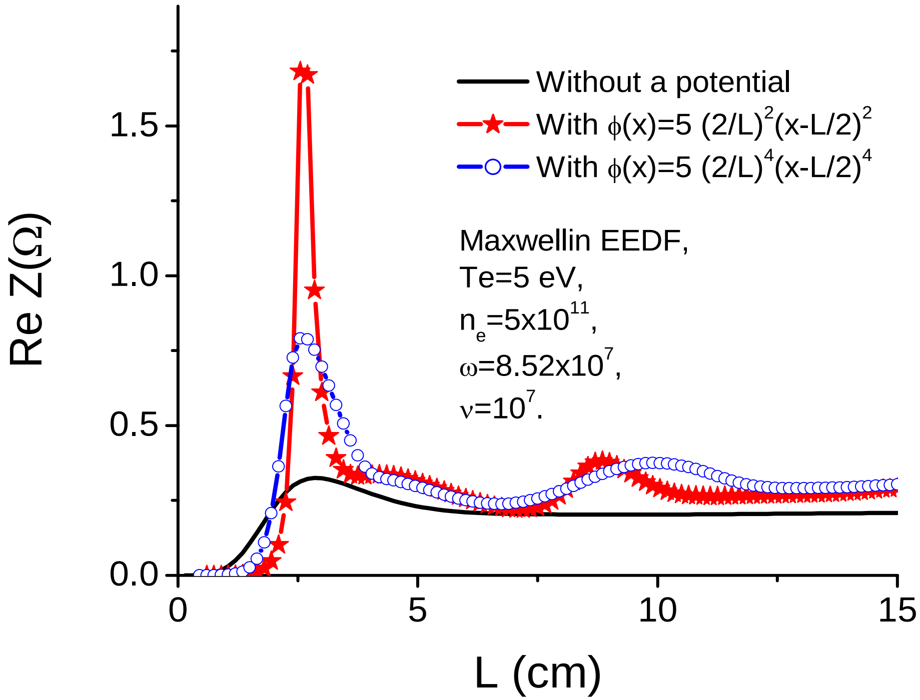
Each block depends on the results of other blocks – iterations.



The role of plasma non-uniformity at the bounce resonance : $\omega = \Omega_b(\varepsilon_x)$

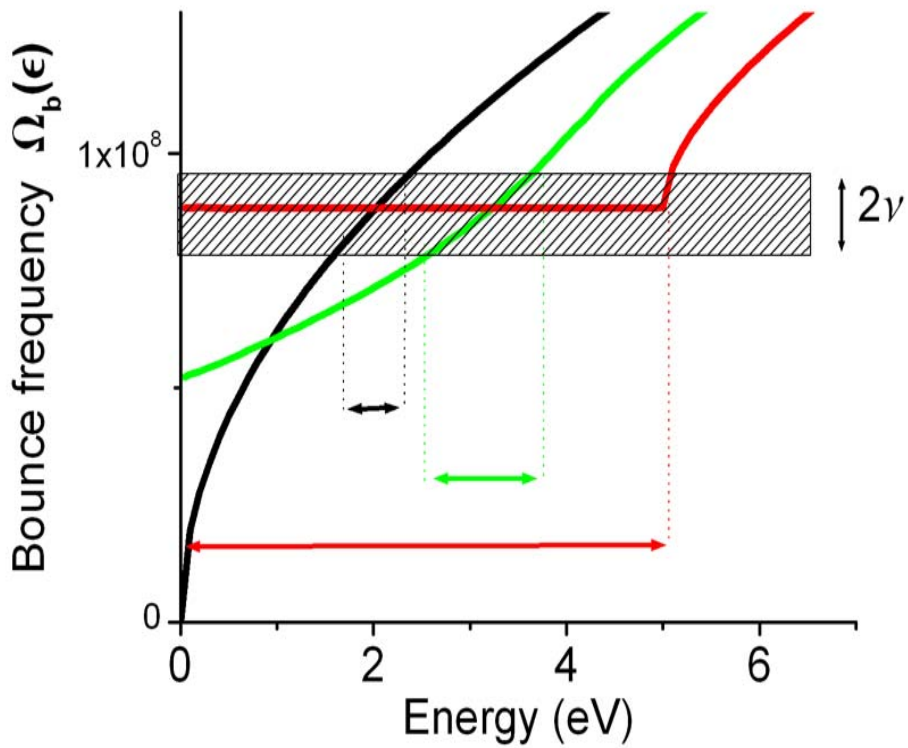


Re z (Ω) for uniform and non-uniform plasma



***O. V. Polomarov, C. E. Theodosiou, and I. D. Kaganovich*, "Enhanced collisionless heating in non-uniform plasma at the bounce resonance condition"**. Accepted **PoP August (2005).**

Influence of ambipolar potential on the bounce frequency and the corresponding enlargement of the number of resonant electrons



Solid lines -

***The bounce frequency vs energy*:**

Black - no ambipolar potential,

Green - realistic potential,

Red - quadratic potential.

Dashed box -

the resonant region

Arrows - ***the width of corresponding energy ranges***

The resonant electrons are the electrons with bounce frequency:

$$\Omega_b(\varepsilon_x)n \in [\omega-\nu, \omega+\nu]$$

As it follows from

$$D_e(\varepsilon) \sim \sum_{n=-\infty}^{\infty} \int_0^{\varepsilon} d\varepsilon_x [\ldots] \frac{\nu}{(\Omega_b(\varepsilon_x)n-\omega)^2+\nu^2}$$

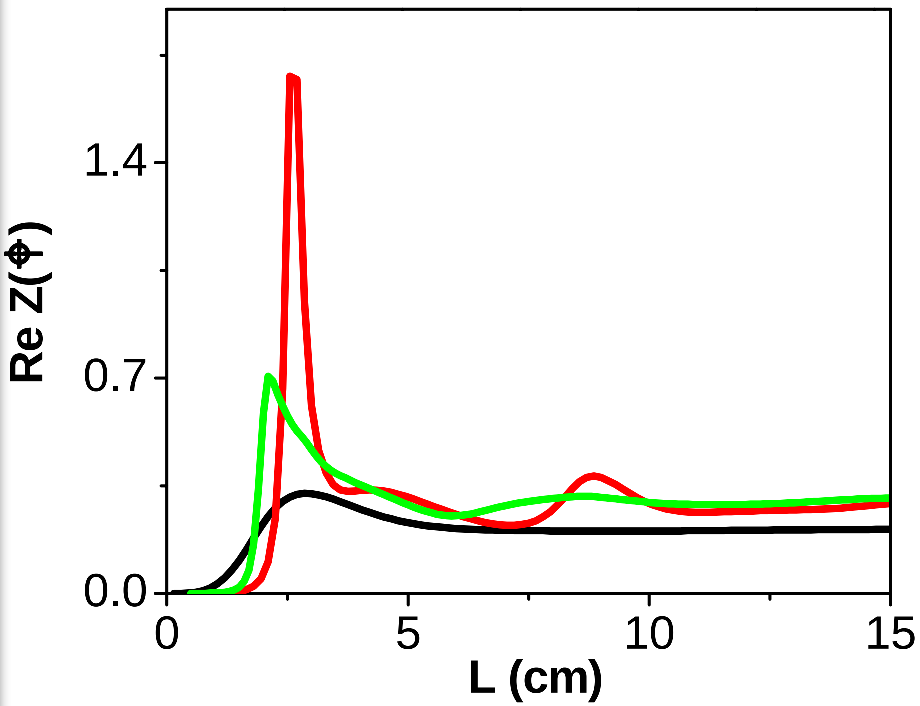
The number of resonant electrons increases if the ambipolar potential is taken into account.

More resonant electrons-larger dissipated power.

The corresponding surface resistance for Maxwellian EEDF.

$$\omega = \pi V_T / L$$

ω **=13.56 MHz,**



Solid lines -

***The bounce frequency vs energy*:**

Black - no ambipolar potential,

Green - realistic potential,

Red - quadratic potential.

Dashed box -

the resonant region

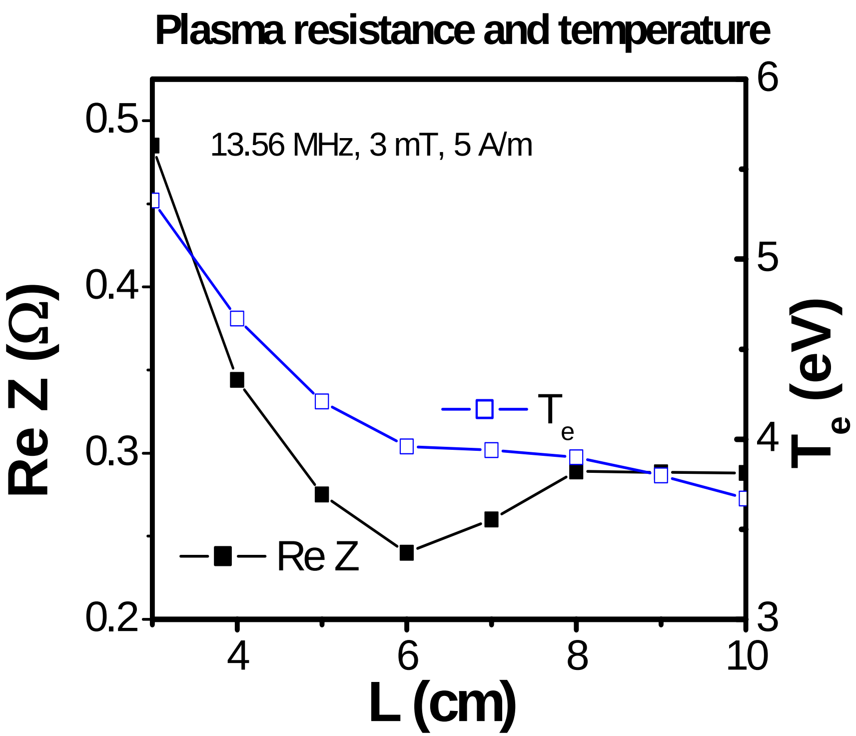
Arrows - ***the width of corresponding energy ranges***

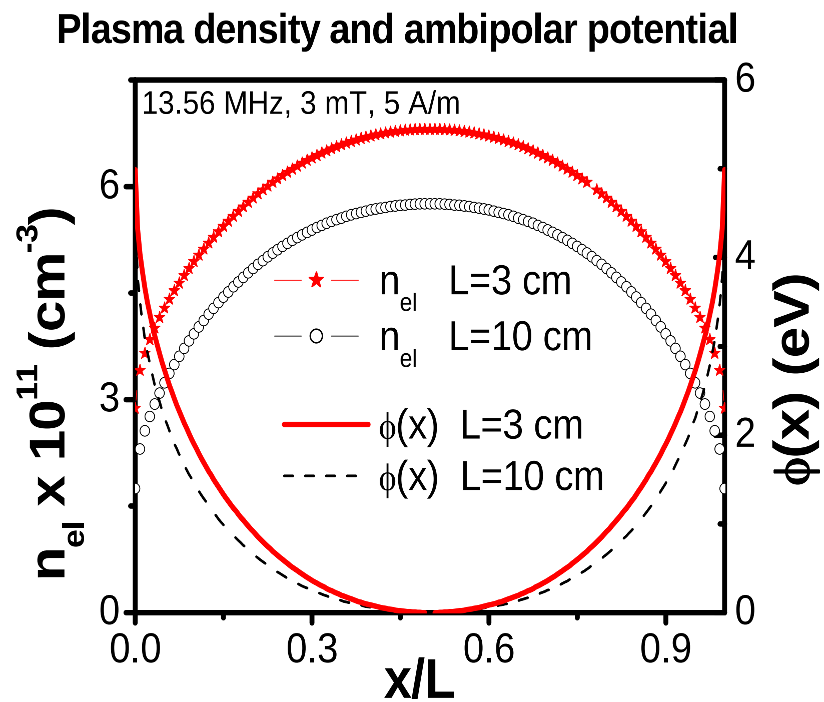
The resonant length ~ 3 cm

Plasma resistance, Te, density and ambipolar potential profiles

Full self-consistent simulation

Resonant length ~ 3 cm





1-D simulation of magnetized ICP discharges

1) The dc magnetic field is applied perpendicularly to boundaries.

where $E^{\pm}$ is left and right polarized electric fields, respectively.

2) $$\frac{d^2E^{\pm}}{dx^2}+\frac{\omega^2}{c^2}E^{\pm}=-\frac{4\pi i\omega}{c^2}(j^{\pm}(x)+I\delta(x)-\delta_{anti}I\delta(x-L))$$

3) $$D_{\varepsilon}=\frac{\pi e^2}{8m^2}\sum_{n=-\infty}^{\infty}\int_0^{\varepsilon}d\varepsilon_x\left|E^{+}{}_{n}(\varepsilon_x)\right|^2\frac{\varepsilon-\varepsilon_x}{\Omega_b(\varepsilon_x)}\frac{\nu}{[\Omega_b(\varepsilon_x)n-(\omega+\Omega_c)]^2+\nu^2}+$$

$$+\frac{\pi e^2}{8m^2}\sum_{n=-\infty}^{\infty}\int_0^{\varepsilon}d\varepsilon_x\left|E^{-}{}_{n}(\varepsilon_x)\right|^2\frac{\varepsilon-\varepsilon_x}{\Omega_b(\varepsilon_x)}\frac{\nu}{[\Omega_b(\varepsilon_x)n-(\omega-\Omega_c)]^2+\nu^2}$$

where Ω_c is the electron-cyclotron frequency.

O. V. Polomarov, C. E. Theodosiou, and I. D. Kaganovich at all. "Self-consistent kinetic modeling of low-pressure inductively coupled plasmas." submitted to IEEE(2005)

Resonances in magnetized plasmas

O. V. Polomarov, C. E. Theodosiou, I. D. Kaganovich **at all, "Effectiveness of electron-cyclotron and transmission resonance heating in ICP plasmas", accepted, PoP, September, (2005).**

- **Electron-cyclotron resonance:**

$$\Omega_c - \omega \leq k_s V_T$$

- **Transmission resonances for B>B$_c$:**

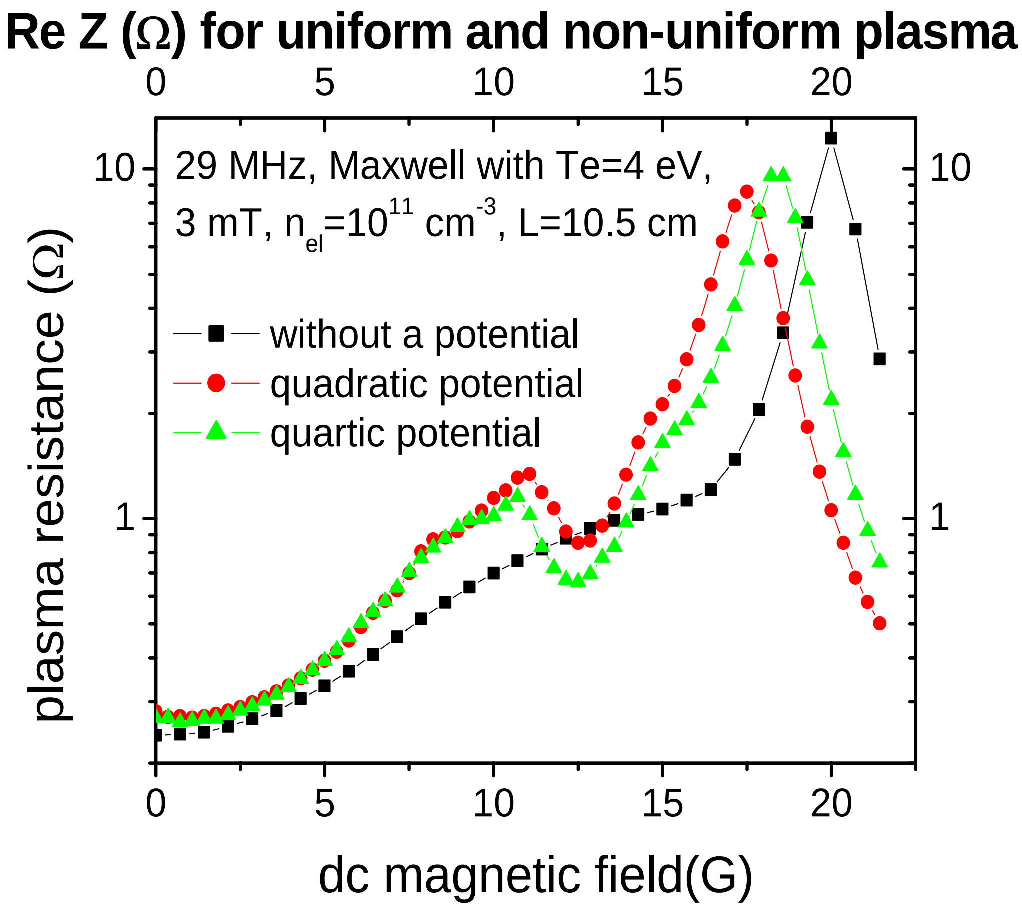
$$I)\quad k_s^2 - \frac{\omega_{pe}^2}{c^2}\frac{\omega}{k_s V_T}\operatorname{Re} Z_M\left(\frac{\omega - \Omega_c + i\nu}{k_s V_T}\right) = 0,$$

$$II)\quad \Omega_c - \omega >> k_s V_T$$

Where $\Omega_c = \frac{eB}{mc}$, $k_s = (2s+1)\frac{\pi}{L}$, $V_T = \sqrt{\frac{2T_e}{m}}$, $\omega_{pe} = \sqrt{\frac{4\pi e^2 n_e}{m}}$

and $Z_M(\xi)$ is the "standart" plasma dispersion function.

Resistance of uniform and non-uniform plasmas for *Maxwellian EEDF*



$ECR\ resonance:$

$B \sim 10\,G\ Transmission$

$resonances:$

$B \sim 20, 30\,G$

$$Power = \mathrm{Re}\,Z\,I^2$$

Self-consistent simulation of 1-D ICP for fixed coil current

Input parameters of the discharge:

- Coil current, or power
- Pressure
- Frequency and Length

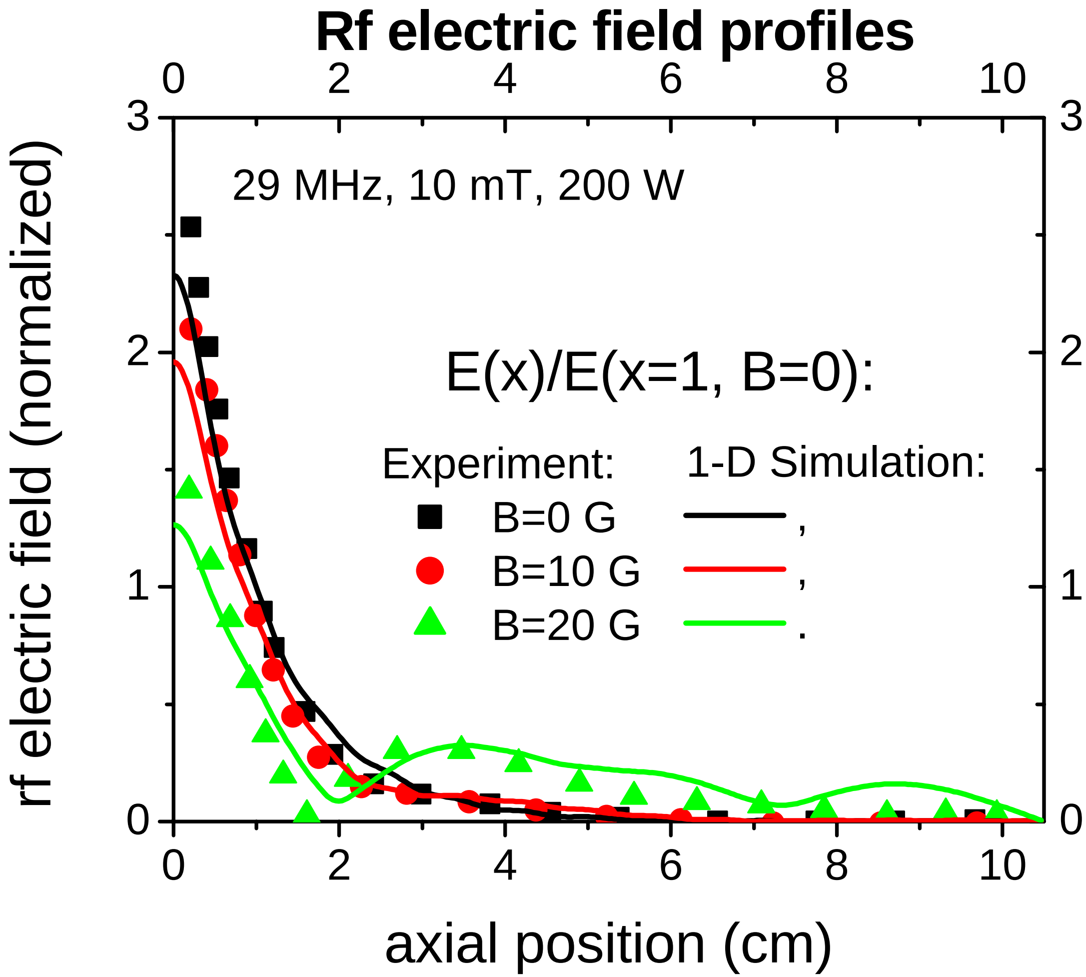
Self-consistent simulation of the magnetized ICP and comparison with the experiment.

$$\omega = 29\ MHz, \quad L = 10,5\ cm$$

Experimental data are taken from:

V. A. Godyak and B. M. Alexandrovich, Phys. Plasmas 11, 3553 (2004).

Rf electric field



$ECR\ resonance:$

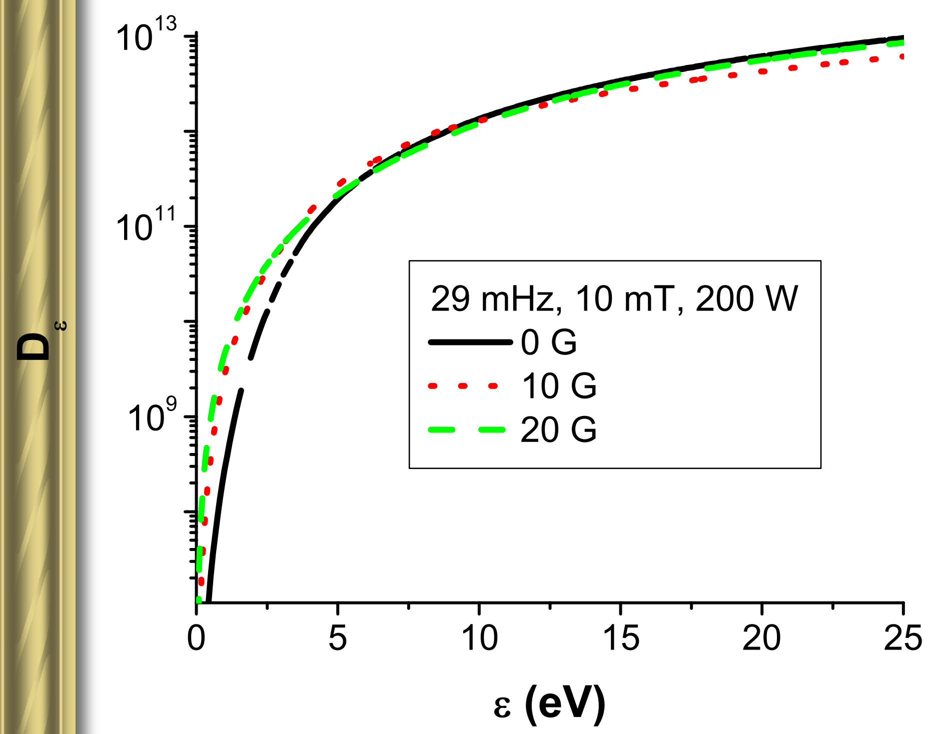
$B \sim 10G$

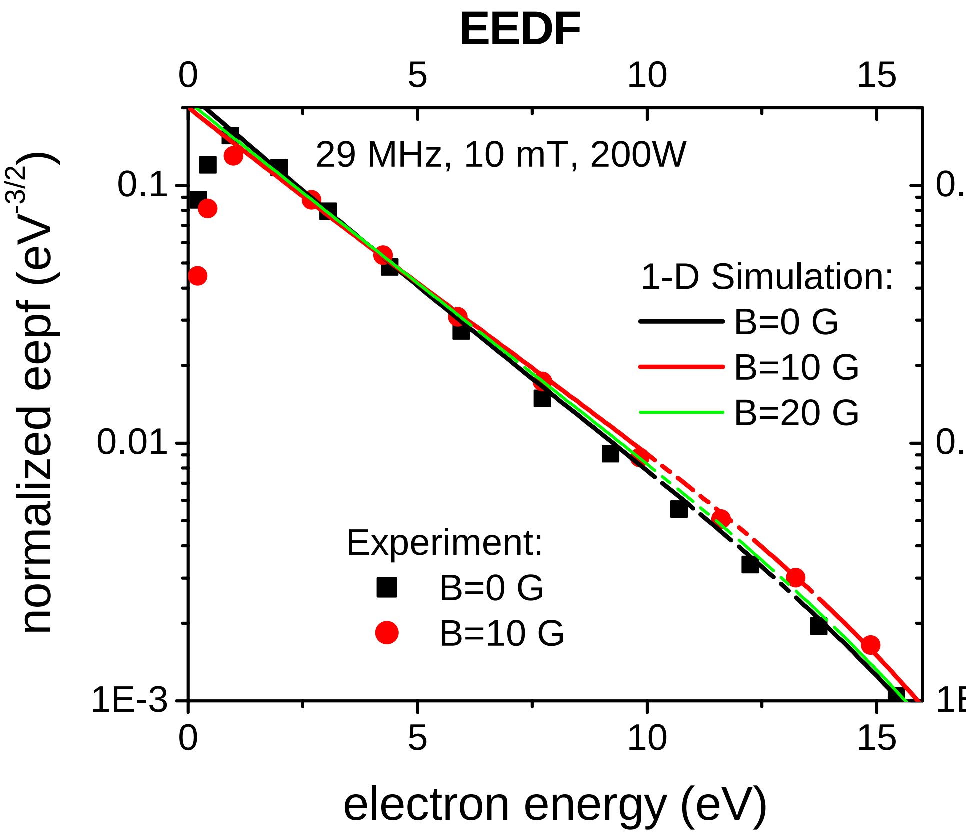
$Transmission$

$resonances:$

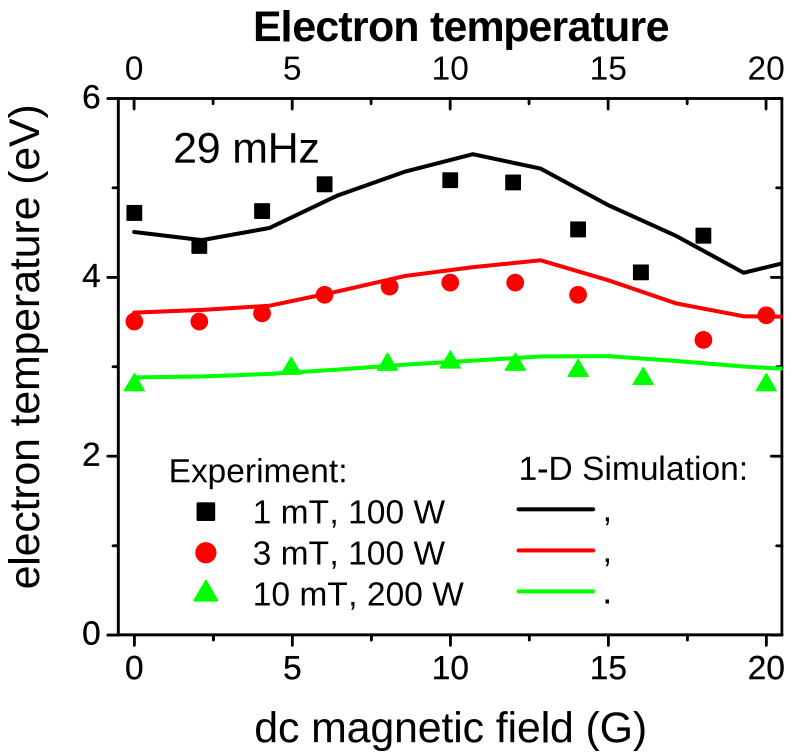
$B \sim 20G$

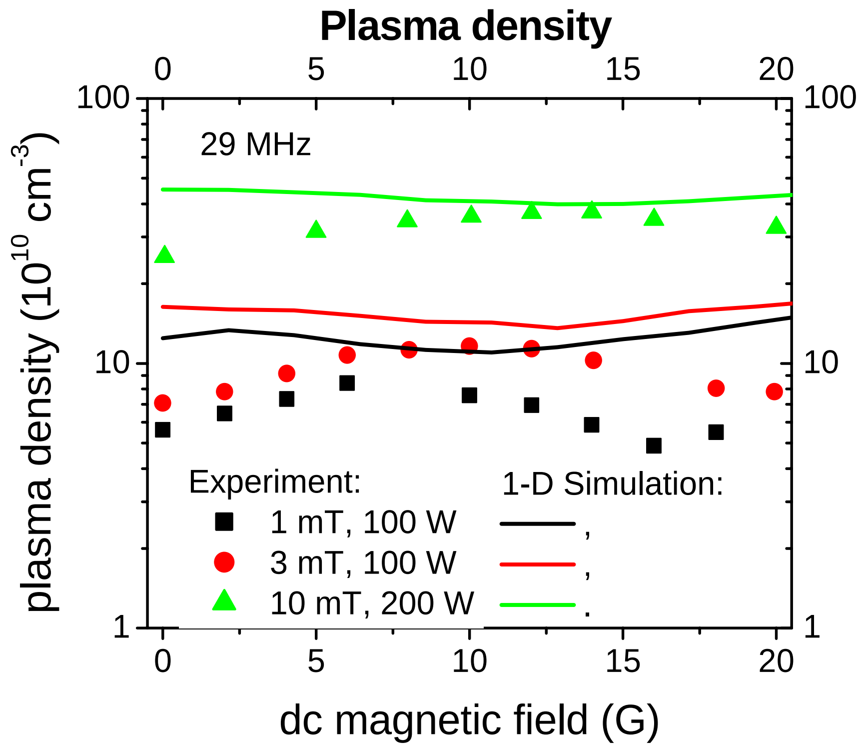
The energy diffusion coefficient and EEDF





Electron temperature and density





Conclusion

- **The self-consistent system of equations is derived for description of plasma heating and anomalous skin effect in non-uniform non-magnetized and magnetized plasmas.**

- **The robust kinetic code was developed for fast modeling of ICP discharges.**