Dielectric function and electron transport in collisional plasma

A. V. Brantov,[1,2] V. Yu. Bychenkov,[2,1] W. Rozmus,[1] and C. E. Capjack[3]

[1]*Theoretical Physics Institute, Department of Physics, University of Alberta, Edmonton T6G 2J1, Alberta, Canada*

[2]*P. N. Lebedev Physics Institute, Russian Academy of Science, Leninskii Prospect 53, Moscow 119991, Russia*

[3]*Department of Electrical Engineering and Computer Engineering, University of Alberta, Edmonton T6G 2V4, Alberta, Canada*

Abstract

A procedure is proposed for finding a solution to the linearized kinetic equation with the Landau collision integral included for charged particles in a plasma with a high degree of ionization. This procedure is used to obtain an expression for dielectric permittivity tensor for a collisional plasma over the entire range of frequencies and wave numbers as well as the collisionality parameter. This is transformed to the known expressions in the corresponding asymptotic strongly collisional and collisionless limits. Nonlocal linear transport equations for small perturbations are also formulated for arbitrary relations between the characteristic space, time and collision scales of the plasma.

PACS numbers:

I. INTRODUCTION

The dielectric response function of a plasma is the fundamental quantity considered in all textbooks on plasma physics. Still, an universal expression or an effective algorithm for its derivation in collisional plasmas has not been obtained over the entire range of wave numbers k and frequencies ω. This is due to the fact that even determining the linear plasma response involves the solution of an integro-differential kinetic equation for particles experiencing Coulomb collisions. The derivation of the plasma susceptibility function in a form that would permit its effective use in various applications has been formulated when the exact Landau collision integrals are replaced by model expressions. However, such simplifying assumptions may lead to a significant loss in the numerical accuracy.

One of the most widely used expression for the longitudinal permittivity of a collisional plasma has been derived by using the Bhatnagar-Gross-Krook (BGK) collision integral [1]. This simple approximation enables an effective description of dispersion properties of the plasma in the presence of collisions. However, the use of the BGK model can lead to a significant error in certain regions of $(\omega\,,k)$. Attempts to improve such a model description by introducing the velocity dependent effective collision frequency have not substantially improved the accuracy of the dielectric susceptibility [2, 3]. A noticeable improvement of the theory was obtained when the electrostatic plasma response was determined by using the Lorentz model with an exact Landau electron-ion collision integral [4–6]. However, it has been shown that neglecting the electron-electron collision integral still does not allow an accurate description to be obtained for the dielectric properties of plasmas over the entire range of frequencies and wavelengths [7, 8].

The same problem arises in the calculation of the transverse permittivity. Many applications simply rely on various approximations to the electron permittivity based on the Drude model or its modifications [2]. However, the actual limits of the validity of these approximations are often unknown. For example, one must account for significant variations of the electron-ion collision frequency ν_{ei}^T, which changes by factors of few as one moves from the ac regime, $\omega \gg \nu_{ei}^T$, to the dc regime, $\omega \ll \nu_{ei}^T$. Also, the nonlocality of the electron conductivity which depends on the collisionality parameter $k\lambda_{ei}$ should be taken into account (λ_{ei} is the electron-ion collision mean free path). A theory of the plasma response to electromagnetic perturbations has been developed in Ref. [9] based on the full solution to the Fokker-Planck equation in high-Z plasmas without electron-electron collisions. Those results have been compared to approximate expressions for the electron conductivity based on the Drude model and indicate a discrepancy by a factor of few in regions where the spatial dispersion is important.

In addition to theoretical models providing an accurate description of the dielectric susceptibility over the entire range of $(\omega\,,k)$, standard perturbation theory gives the correct asymptotic behavior of the dielectric function. Such approximations include the kinetic theory of weakly collisional plasmas [10] and the hydrodynamic-type theory for collisional plasmas [11]. These theories are appropriate within restricted regions of the parameters ω/ν_{ei}^T and $k\lambda_{ei}$. The most direct method for calculating the dielectric susceptibility for arbitrarily values of $(\omega\,,k)$ involves a numerical solution of the Fokker-Planck kinetic equation in Fourier space. However, a numerical solution to the kinetic equation is still a difficult task and the results are restricted to a particular set of parameters. Consequently, the construction of a theory which provides a universal method for obtaining the dielectric susceptibility tensor over the entire range of frequencies, wave numbers, and arbitrary plasma parameters $(\nu_{ei},\ \lambda_{ei})$ is important for many practical applications. Here we review our study which is devoted to the solution to this problem.

The problem of determining the dielectric susceptibility of a collisional plasma is closely related to the problem of nonlocal transport. Theoretical models of nonlocal transport in hot fully ionized plasmas have been developed for more than 20 years beginning with publications [12–15]. However, a further improvement of these models is required for the case where $\lambda_{ei}/L > 10^{-2}$. Classical strongly collisional transport theory does not apply [16, 17] in this limit. Such conditions are often encountered in inertial confinement fusion (ICF) experiments where the characteristic length L of plasma inhomogeneity in the region of laser energy absorption does not exceed one hundred electron mean free paths. Strong inverse dependence of a Coulomb collision frequency on the particle kinetic energy makes the nonlocality of the particle transport an essential feature of hydrodynamical models of fully ionized plasmas. For this reason, the interpretation of almost all laser produced plasma experiments requires the use of nonlocal transport theory.

Significant advances were made in the development of nonlocal transport theories by using the small perturbation model [12, 18–21]. Analytic solutions to linearized kinetic equations can be obtained relatively simply for high-Z plasmas and these are used to calculate electron fluxes. Most of these theories assume that the transport processes are sufficiently slow (quasi-stationary) so that the transport coefficients can be considered to be independent of time. In such a quasi-static approximation, the nonlocal hydrodynamic equations as derived in Refs. [19, 20] are completely equivalent to the linearized kinetic description of a plasma. However, the validity of the quasi-static nonlocal theory is restricted by the non-stationary nature of transport processes [7, 8, 22]. Even for small amplitude perturbations, effects of non-stationary transport are reflected in $\omega-$ terms appearing in the transport coefficients, which leads to a complicated frequency dependence of the dielectric function. In [4, 23], these effects were taken into account for weakly collisional and collisionless plasmas. The approach recently developed by the authors [7, 8] makes it possible to analyze transport properties of plasmas for arbitrary relations between the temporal, spatial, and collisional scale lengths.

Nonlocal hydrodynamics provides a reduced description of a plasma in terms of few hydrodynamical variables. Such equations are easier to solve than the equivalent kinetic model. Starting with the early 1990s, nonlocal models have been developed [24–27] with the objective of incorporating kinetic effects (such as Landau damping) into hydrodynamic equations. These theories dealt with collisionless and magnetized plasmas. A systematic procedure of deriving nonlocal closure relations for fluid equations is a necessary step in deriving a reduced plasma description.

In this paper, the derivation of transport equations for plasma perturbations and expressions for the dielectric susceptibility tensor are based on the solution to the initial value problem for the linearized kinetic equation for plasma particles [7, 8, 19, 20]. Independently the Ref. [28] has presented similar nonlocal closure for transport equation describing plasma evolution in response to initial perturbations. However, this work [28] has used simplified kinetic equation and applies only to limited range of plasma collisionality parameter. The method used in our paper for solving kinetic equation is valid for a plasma with a large ionic charge $Z \gg 1$. It is applicable for arbitrary relations between the perturbation inhomogeneity spatial scale length $L = k^{-1}$ and the electron mean free path, as well as the typical temporal perturbation time scale, $\tau = \omega^{-1}$, the electron collision time, and the free transit time (the time during which an electron with mean thermal velocity, v_{Te}, passes the distance equal to the characteristic scale length of plasma inhomogeneity, $1/kv_{Te}$). In this approach, a spherical harmonic expansion of the distribution function is used. All angular harmonics of the electron distribution function are summed, thus allowing a description of the continuous transition from the strongly collisional hydrodynamic limit to the collisionless case in the transport equations and in the expression for dielectric susceptibility. The solution to the initial value problem for perturbations of the distribution function [19, 20] is generalized to the non-stationary case [7, 8]. The transport equations are formulated in the form of relations between Fourier components of the electron fluxes and the generalized hydrodynamic forces (i.e., the density and temperature gradients, plasma velocity, and the electric field). Due to non-stationary response, all electron transport coefficients in the (ω ,k)-space contain imaginary components, which are missing in the quasi-stationary theory [12, 18–20]. The resulting complex longitudinal and transversal dielectric susceptibilities are analyzed over the entire (ω ,k) region as functions of the plasma collisionality parameters, $k\lambda_{ei}$, $\omega\lambda_{ei}/v_{Te}$. Relations between the dielectric susceptibility of a plasma and the non-stationary nonlocal transport coefficients are found.

II. KINETIC EQUATION

Consider a small perturbation of the homogeneous equilibrium plasma with electrons and ions described by the Maxwellian distribution functions f_M^a ($a = e, i$) which are characterized by particle densities n_a and temperatures T_a. The linearized equation for the spatial Fourier components $\delta f_a = f_a - f_M^a$ of the perturbation reads as follows:

$$(\frac{\partial}{\partial t} + i\,\mathbf{k}\cdot\mathbf{v})\delta f_a + \frac{e_a}{m_a}\mathbf{E}\frac{\partial f_M^a}{\partial \mathbf{v}} = \sum_b \left(C_{ab}[\delta f_a, f_b] + C_{ab}[f_a, \delta f_b]\right) \;, \tag{1}$$

where C_{aa} and C_{ab} are the Landau collision operators for particles of the same and different species, respectively and with a charge e_a and a mass m_a.

After taking the one-sided Fourier transformation in time we expand the function δf_a in spherical harmonics $Y_{lm}(\theta, \phi)$,

$$\delta f_a = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} f_{lm}^a(v) Y_{lm}(\theta,\phi)\;,\quad C_{ab}[\delta f_a, f_M^b] + C_{ab}[f_M^a, \delta f_b] = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} C_{ab}^{lm} Y_{lm}(\theta,\phi) \tag{2}$$

where θ and ϕ are the polar and azimuthal angles characterizing the direction of the particle velocity relative to the vector k. These operations reduce the kinetic equations (1) to an infinite system of equations for the angular harmonics of the distribution functions, f^a_{lm} :

$$-i\omega f^a_{lm} + ikv\sqrt{\frac{l^2-m^2}{4l^2-1}}f^a_{l-1,m} + ikv\sqrt{\frac{(l+1)^2-m^2}{4(l+1)^2-1}}f^a_{l+1,m} - C^{lm}_{aa} - C^{lm}_{ab} = S^a_{lm}\,, \tag{3}$$

Here the collision operators , C^{lm}_{ab} (both for $b=a$ and for $b\neq a$), have the form of Rosenbluth potentials:

$$\begin{aligned}\frac{C^{lm}_{ab}}{\nu_{ab}(v)} &= \frac{l(l+1)}{6}f^a_{lm}(I^0_2 - 3I^0_0 - 2J^0_{-1}) + \frac{v}{3}\frac{\partial}{\partial v}\left(v\frac{\partial f^a_{lm}}{\partial v}(I^0_2 + J^0_{-1})\right) + \\ &\frac{m_a}{m_b}v\frac{\partial}{\partial v}(f^a_{lm}I^0_0) + \frac{4\pi m_a}{n_e m_b}v^3 f^a_M f^b_{lm} + v\frac{\partial f^a_M}{\partial v}\frac{l\delta J^l_{-l-1} - (l+1)\delta I^l_l}{2l+1}\left(1-\frac{m_a}{m_b}\right) + \\ &\frac{v^2}{2(2l+1)}\frac{\partial^2 f^a_M}{\partial v^2}\left(\frac{l(1-l)}{2l-1}(\delta I^{lm}_l + \delta J^{lm}_{1-l}) + \frac{(1+l)(2+l)}{2l+3}(\delta I^{lm}_{l+2} + \delta J^{lm}_{-1-l})\right) + \\ &\frac{v}{2(2l+1)}\frac{\partial f^a_M}{\partial v}\left(\frac{(l^2+3l-2)\delta I^{lm}_l + l(l-1)\delta J^{lm}_{1-l}}{2l-1} - \frac{(l+1)(l+2)\delta I^{lm}_{l+2} + (l^2-l-4)\delta J^{lm}_{-1-l}}{2l+3}\right)\,,\end{aligned} \tag{4}$$

where $\nu_{ab}(v) = 4\pi n_b(e_a e_b)^2\Lambda_{ab}/m_a^2 v^3$ is the velocity dependent collision frequency between particles of the kind a with particles of the kind b, Λ_{ab} is the Coulomb logarithm, and

$$\left\{I^0_n;\delta I^{lm}_n\right\} = \frac{4\pi}{n_b v^n}\int_0^v \left\{f^b_M;f^b_{lm}\right\}v^{n+2}dv\,, \quad \left\{J^0_n;\delta J^{lm}_n\right\} = \frac{4\pi}{n_b v^n}\int_v^\infty \left\{f^b_M;f^b_{lm}\right\}v^{n+2}dv \tag{5}$$

are Rosenbluth potentials (J^0_n) and their perturbations (δJ^{lm}_n), defined in the standard way [11].

By assuming that initial perturbations of distribution functions $\delta f_a(t=0)$ have a Maxwellian form (i.e., they are characterized by initial perturbations of densities $\delta n_a(0)$ and temperatures $\delta T_a(0)$)

$$\delta f_a(v,t=0) = \left[\frac{\delta n(0)}{n_e} + \frac{\delta T(0)}{T_e}\left(\frac{v^2}{2v^2_{Te}} - \frac{3}{2}\right)\right]f^a_M(v)\,, \tag{6}$$

the source functions S^a_{lm} are specified by initial perturbations of distribution functions and by the Fourier components of the electric fields $\mathbf{E}$: $S^a_{00} = \sqrt{4\pi}\delta f_a(t=0)$, $S^a_{10} = \sqrt{4\pi/3}(e_a E_z/T)vf^a_M$ and $S^a_{1\pm1} = \sqrt{2\pi/3}(e_a(E_x \pm E_y)/T)vf^a_M$, where the vector $\mathbf{k}$ is assumed to be directed along the $z-$axis.

Since we are only interested in electron kinetic effects, we seek a solution to Eq. (3) for the electron distribution function (EDF). By assuming that ions have a large charge $Z \gg 1$, electron-electron (e-e) collisions can be neglected in the equations for higher harmonics of the electron distribution function, and are retained only in the equation for the symmetric part of the EDF. Further simplifications result from the expansion of the electron-ion collision integrals with terms involving the ratio of the characteristic ion velocity to the electron velocity being neglected. Contributions on the order of $\sim m_e/m_i$ in C_{ei}, which are responsible for the slow energy transfer from the electrons to the ions are also neglected. This is well justified, for example, in the case of a laser produced plasmas. Thus, for the electron-ion collision integral, we will use the expression:

$$C_{ei}^{lm} = -\frac{l(l+1)}{2}\nu_{ei} f_l^e + \delta_{l1}\frac{\nu_{ei} v}{v_{Te}^2}\left(\delta_{m0}\sqrt{\frac{4\pi}{3}}u_z^i + \delta_{m\pm1}\sqrt{\frac{2\pi}{3}}(u_x^i \mp u_y^i)\right) f_M^e\,, \tag{7}$$

where the terms proportional to the mean ion velocity, $\mathbf{u}^i$, (plasma velocity) give additional source terms.

The standard approach that is used to solve the infinite system of equations (3) is to assume that the higher angular harmonics are small and that reasonable accuracy can be obtained by retaining just two of them, f_{00}^e and f_{10}^e,. This procedure is fully justified in the strongly collisional limit. However, in order to describe transitions to the collisionless domain, a large number of angular harmonics of the distribution function δf_e must be taken into account. In fact the correct description of Landau damping requires the summation of the entire infinite series of angular harmonics. Such a summation procedure has been introduced before [14, 18, 20] in terms of the modified collision frequency ν_{lm}

$$f_{lm}^e = -i\sqrt{\frac{l^2-m^2}{4l^2-1}}\frac{kv}{\nu_{lm}} f_{l-1,m}^e \tag{8}$$

$$\nu_{lm} = -i\omega + \frac{1}{2}l(l+1)\nu_{ei} + ikv\sqrt{\frac{(l+1)^2-m^2}{4(l+1)^2-1}}\frac{f_{l+1,m}^e}{f_{lm}^e}$$

which satisfies the following recurrence relation

$$\nu_{lm} = -i\omega + \frac{1}{2}l(l+1)\nu_{ei} + \frac{(l+1)^2-m^2}{4(l+1)^2-1}\frac{k^2v^2}{\nu_{l+1,m}}\,. \tag{9}$$

Equation (9) can be also represented in terms of continuous fractions. Accurate calculations of the functions $\nu_{1,1}$ and $\nu_{1,0}$ for any practically interesting conditions usually involves 20 - 30 terms.

This summation procedure can be used to find an expression for the first angular harmonic of the EDF, $f_1^e = \sum_{-1}^{1} f_{1m}^e Y_{1m}$:

$$f_1^e = f_{10}^e\cos\theta\sqrt{\frac{3}{4\pi}} + \sqrt{\frac{3}{8\pi}}\sin\theta\sum_m f_{1m}^e e^{im\phi} = -\frac{i\mathbf{k}\cdot\mathbf{v}}{\nu_{10}} f_M^e + \tag{10}$$

$$\frac{1}{\nu_{10}}\frac{\partial f_M^e}{\partial v}\left(\frac{e\mathbf{E}_\parallel\cdot\mathbf{v}}{m_e v} - \nu_{ei}\frac{(\mathbf{u}_{i\parallel}\cdot\mathbf{v})}{v}\right) + \frac{1}{\nu_{11}}\frac{\partial f_M^e}{\partial v}\left(\frac{e\mathbf{E}_\perp\cdot\mathbf{v}}{m_e v} - \nu_{ei}\frac{(\mathbf{u}_{i\perp}\cdot\mathbf{v})}{v}\right)\,,$$

where the following notation has been introduced for the longitudinal and transverse components of a vector A:

$$\mathbf{A}_{\parallel} = \frac{\mathbf{k}(\mathbf{A}\cdot\mathbf{k})}{k^2}\,, \qquad \mathbf{A}_{\perp} = \frac{\mathbf{k}\times(\mathbf{A}\times\mathbf{k})}{k^2}\,. \tag{11}$$

The symmetric part of the perturbed EDF $f_0^e = f_{00}^e Y_{00} = f_{00}^e/\sqrt{4\pi}$ satisfies the kinetic equation

$$\left(-i\omega + \frac{k^2v^2}{3\nu_{10}}\right) f_0^e - C_{ee}[f_0^e] = \frac{iev^2}{3\nu_{10}}\frac{(\mathbf{E}\cdot\mathbf{k})}{T_e} f_M^e - i(\mathbf{k}\cdot\mathbf{u}_i)\frac{v^2\nu_{ei}}{3v_{Te}^2\nu_{10}} f_M^e + \delta f^e(v,t=0) \tag{12}$$

with the initial perturbation $\delta f_e(v,t=0)$ defined by the relation (6). Equation (12) is a linear inhomogeneous equation, whose general solution can be written as a linear combination of three basis functions [8, 20]:

$$f_0^e = i\frac{e(\mathbf{E}\cdot\mathbf{k})}{k^2T_e} f_M^e + \left(\frac{\delta n_e(0)}{n_e} - \omega\frac{e(\mathbf{E}\cdot\mathbf{k})}{k^2T_e}\right)\psi^N f_M^e + \frac{3}{2}\frac{\delta T_e(0)}{T_e}\psi^T f_M^e - i(\mathbf{k}\cdot\mathbf{u}_i)\psi^R f_M^e\,, \tag{13}$$

where the basis functions ψ_A satisfy three ($A = N, T, R$) kinetic equations with various source terms S_A

$$\left(-i\omega + \frac{k^2v^2}{3\nu_{10}}\right)\psi^A = (f_M^e)^{-1} C_{ee}[f_M^e\psi^A] + S_A\,. \tag{14}$$

The three velocity functions: $S_N = 1$, $S_T = v^2/3v_{Te}^2 - 1$ $\;S_R = v^2\nu_{ei}/3v_{Te}^2\nu_{10}$ are sources corresponding to the perturbations of the electron density (N), the electron temperature (T), and the ion velocity (R). Equation (14) has been analyzed in detail in Ref. [20]. It was solved numerically by expanding the solution in Sonine-Laguerre polynomials $L_n^{1/2}(v^2/2v_{Te}^2)$ and analytically in the strongly and weakly collisional limits.

The equation for the first azimuthal harmonic ($l = 1$, $m = \pm 1$) of the EDF (10) can be used directly to calculate transverse electron fluxes and transport coefficients. Calculation of the longitudinal electron fluxes requires elimination of the initial density and temperature perturbations from the expression for the symmetrical harmonics ($l = 0$) of the EDF (13). From the two first velocity moments of Eq. (13) for the perturbations of density $\delta n_e = 4\pi\int_0^\infty dv v^2 f_0^e$ and temperature $\delta T_e = 4\pi m_e/(3n_e)\int_0^\infty dv v^2(v^2 - 3v_{Te}^2) f_0^e$ at time t we obtain the following system of equations:

$$\begin{aligned}
\frac{\delta n_e}{n_e} &= i\frac{e(\mathbf{E}\cdot\mathbf{k})}{k^2T_e} + \left(\frac{\delta n_e(0)}{n_e} - \omega\frac{e(\mathbf{E}\cdot\mathbf{k})}{k^2T_e}\right) J_N^N + \frac{3}{2}\frac{\delta T_e(0)}{T_e} J_N^T - i(\mathbf{k}\cdot\mathbf{u}_i) J_N^R\,,\\
\frac{\delta T_e}{T_e} &= \left(\frac{\delta n_e(0)}{n_e} - \omega\frac{e(\mathbf{E}\cdot\mathbf{k})}{k^2T_e}\right) J_T^N + \frac{3}{2}\frac{\delta T_e(0)}{T_e} J_T^T - i(\mathbf{k}\cdot\mathbf{u}_i) J_T^R\,,
\end{aligned} \tag{15}$$

where we have introduced velocity moments of the basis functions

$$J_B^A = \frac{4\pi}{n_e} \int_0^\infty v^2 dv \psi^A f_M^e S_B \,, \tag{16}$$

where $A, B = N, T, R$. The matrix J_A^B is symmetric [20].

If the initial density and temperature perturbations are expressed in terms of their instantaneous values, (15) can be used to derive the following expression for the isotropic part of the EDF f_0^e:

$$\begin{aligned} f_0^e \;=\; & i\frac{e(\mathbf{E}\cdot\mathbf{k})}{k^2 T_e} f_M^e + \left(\frac{\delta n_e}{n_e} - i\frac{e(\mathbf{E}\cdot\mathbf{k})}{k^2 T_e}\right) \frac{J_T^T \psi^N - J_T^N \psi^T}{D_{NT}^{NT}} f_M^e + \\ & \frac{\delta T_e}{T_e} \frac{J_N^N \psi^T - J_N^T \psi^N}{D_{NT}^{NT}} f_M^e - i(\mathbf{k}\cdot\mathbf{u}_i)\left(\psi^R - \frac{D_{NT}^{RT}}{D_{NT}^{NT}}\psi^N - \frac{D_{NT}^{NR}}{D_{NT}^{NT}}\psi^T\right) f_M^e \,, \end{aligned} \tag{17}$$

where $D_{AB}^{CD} = J_A^C J_B^D - J_A^D J_B^C$. Equation (17) is written in terms of hydrodynamical moments and basis functions (14). The symmetric part of the EDF can be used to calculate the anisotropic perturbation to the distribution function f_1 and to derive closure relations for the system of hydrodynamic equations.

III. NONLOCAL HYDRODYNAMICS FOR ELECTRON PERTURBATIONS

We have proposed a new systematic closure procedure that expresses EDF in terms of its lower velocity moments (17). The well known closure strategy in the strongly collisional limit is the Chapman-Enskog procedure. However, this method only applies when the electron mean free path λ_{ei} and the characteristic perturbation scale length L satisfy the following inequality $\lambda_{ei}/L < 0.06/\sqrt{Z}$ [29]. Consequently, classical theory cannot be used for describing experiments such as those involving the interaction of laser radiation with matter in fusion studies, where small-scale perturbations are of particular interest. The range of applicability of the hydrodynamic equations in describing plasmas has been significantly expanded within the framework of nonlocal hydrodynamics [20]. At first, this theory was formulated for slow processes in the quasi-static approximation. In the previous sections we have summarized the generalization of the nonlocal hydrodynamics framework to the case of rapidly varying processes in a plasma for potential perturbations [8].

A. Nonlocal transport equations.

The first three moments of the kinetic equation (1) lead to equations of continuity, momentum and energy balance for electrons:

$$\begin{aligned}
&\frac{\partial \delta n_e}{\partial t} + n_e i\,\mathbf{k}\cdot\mathbf{u}_e = 0\,,\\
&\frac{\partial \mathbf{u}_e}{\partial t} = -\frac{e}{m_e}\mathbf{E}^* + \frac{1}{m_e n_e} i\,\mathbf{k}\cdot\hat{\mathbf{\Pi}}^e - \frac{1}{m_e n_e}\mathbf{R}_{ie}\,,\\
&\frac{\partial \delta T_e}{\partial t} + \frac{2}{3n_e} i\,\mathbf{k}\cdot\mathbf{q}_e + \frac{2}{3}T_e i\,\mathbf{k}\cdot\mathbf{u}_e = 0\,,
\end{aligned}\tag{18}$$

where $\mathbf{u}_e = \mathbf{u}_i - \mathbf{j}/en_e$ is the mean electron velocity and

$$\mathbf{j} = -e\int d^3v\mathbf{v}f_e\,, \qquad \mathbf{q} = m_e/2\int d^3v\mathbf{v}(v^2 - 5v_{Te}^2)f_e \tag{19}$$

are the electric current and the electron heat flux. We have also introduced an effective electric field $\mathbf{E}^*$ as follows

$$\mathbf{E}^* = \mathbf{E} + i\,\mathbf{k}\frac{T_e}{e}\left(\frac{\delta n_e}{n_e} + \frac{\delta T_e}{T_e}\right)\,. \tag{20}$$

As in the previous studies [8, 19, 20], we introduce an effective friction force $\mathbf{R}_{ie} = \mathcal{R}_{ie} - m_e n_e \nu_{ei}^T \mathbf{u}_i$ and the stress tensor $\hat{\mathbf{\Pi}}^e$

$$\mathcal{R}_{ie} = m_e\int d^3v\mathbf{v}\nu_{ei}f_e\,, \quad \hat{\mathbf{\Pi}}^e = m_e\int d^3v(\hat{\mathbf{I}}/3 - (\mathbf{v}-\mathbf{u}_e)(\mathbf{v}-\mathbf{u}_e))f_e\,, \tag{21}$$

where $\hat{\mathbf{I}}$ is the unit tensor and $\nu_{ei}^T = 3\sqrt{\pi/2}\,\nu_{ei}(v_{Te})$ is the averaged electron-ion collision frequency. Note that the electron momentum equation (the second in Eqs. (eq18)) can be used for defining the stress tensor $i\mathbf{k}\hat{\mathbf{\Pi}}^e = \mathbf{R}_{ei} + en_e\mathbf{E}^* - i\omega m_e n_e \mathbf{u}_e$. Two other equations from Eqs. (18), which involve only longitudinal electron fluxes are equivalent to the system (15).

Since the EDF contains terms that are proportional to the vectors $\mathbf{E}$ and $\mathbf{u}_i$, the electron flux will have components that are transverse and parallel to the vector $\mathbf{k}$. The longitudinal component, directed along $\mathbf{k}$ has the following form [8, 19, 20]:

$$\mathbf{j}_\parallel = \sigma\mathbf{E}_\parallel^* + \alpha i\mathbf{k}\delta T_e + \beta_j e n_e \mathbf{u}_{i\parallel}\,, \tag{22}$$

$$\begin{aligned}
\mathbf{q}_\parallel &= -\alpha T_e\mathbf{E}_\parallel^* - \chi i\mathbf{k}\delta T_e - \beta_q n_e T_e \mathbf{u}_{i\parallel}\,,\\
\mathbf{R}_{ie}^\parallel &= -(1-\beta_j)n_e e\mathbf{E}_\parallel^* + \beta_q n_e i\mathbf{k}\delta T_e - \nu_{ei}^T\beta_r m_e n_e \mathbf{u}_{i\parallel}\,,
\end{aligned}\tag{23}$$

where σ is the electrical conductivity, α is the thermoelectric coefficient, χ is the temperature conductivity and $\beta_{j,q,r}$ are the ion convection transport coefficients:

$$\sigma = \frac{e^2 n_e}{k^2 T_e}\left(\frac{J_T^T}{D_{NT}^{NT}} + i\omega\right), \quad \alpha = -\frac{en_e}{k^2 T_e}\left(\frac{J_T^N + J_T^T}{D_{NT}^{NT}} + i\omega\right), \quad \beta_j = 1 - \frac{D_{NT}^{RT}}{D_{NT}^{NT}},$$
$$\chi = \frac{n_e}{k^2}\left(\frac{2J_T^N + J_T^T + J_N^N}{D_{NT}^{NT}} + i\frac{5}{2}\omega\right), \qquad \beta_q = \frac{D_{NT}^{RT} + D_{NT}^{RN}}{D_{NT}^{NT}}, \tag{24}$$
$$\beta_r = 1 + k^2 v_{Te}\lambda_{ei}\left[J_R^R - (1-\beta_j)(J_R^N + J_R^T) + \beta_q J_R^T\right] - (2\pi)^{3/2}\frac{v_{Te}}{n_e}\int_0^\infty \frac{dvv\nu_{ei}}{\nu_{10}} f_M^e .$$

Transport coefficients (24) depend only on the moments J_A^B of the isotropic part of the basis functions ψ^A $(A = N, T)$. All additional moments of ψ^A, which are introduced as a result of the integration in (10) have been eliminated by using solutions of Eq. (14) and by taking into account the conservation of particle number and energy in e-e collisions:

$$\int_0^\infty d^3v C_{ee} = 0, \qquad \int_0^\infty d^3v v^2 C_{ee} = 0. \tag{25}$$

The transport relations (22) satisfy Onsager symmetries: the coefficient α is the same in the expressions for the electric current and the heat flux. Only one new coefficient, β_r, appears in the expression for the friction force. This is in agreement with the equalities $J_A^B = J_B^A$ $(A, B = N, T, R)$ and symmetry relations are satisfied for arbitrary $k\lambda_{ei}$, ω/ν_{ei}^T.

The transverse electron fluxes are defined as a moment of the first harmonic $f_{1,\pm 1}$ and are expressed in terms of the transverse electric field and transverse mean ion velocity. The final expressions read [20]

$$\mathbf{j}_\perp = \sigma_\perp \mathbf{E}_\perp + en_e\beta_{\perp j}\mathbf{u}_{i\perp}, \quad \mathbf{q}_\perp = -\alpha_\perp T_e \mathbf{E}_\perp - n_e T_e \beta_{\perp q}\mathbf{u}_{i\perp}, \tag{26}$$
$$\mathbf{R}_{ie}^\perp = -(1-\beta_{\perp j})n_e e\mathbf{E}_\perp - \nu_{ei}^T m_e n_e \beta_{\perp r}\mathbf{u}_{i\perp} \tag{27}$$

where we have introduced the transversal transport coefficients as follows:

$$\sigma_\perp = \frac{4\pi e^2}{3T_e}\int_0^\infty dvv^4\frac{f_M^e}{\nu_{11}}, \quad \beta_{\perp j} = 1 - \frac{4\pi}{3n_e v_{Te}^2}\int_0^\infty dvv^4\frac{\nu_{ie}}{\nu_{11}}f_M^e, \tag{28}$$
$$\alpha_\perp = \frac{4\pi e}{3T_e}\int dv\left(\frac{v^2}{2v_{Te}^2} - \frac{5}{2}\right)v^4\frac{f_M^e}{\nu_{11}}, \quad \beta_{\perp q} = \frac{4\pi}{3n_e}\frac{1}{v_{Te}^2}\int dv\left(\frac{5}{2} - \frac{v^2}{2v_{Te}^2}\right)v^4\frac{\nu_{ei}}{\nu_{11}}f_M^e,$$
$$\beta_{\perp r} = 1 - \frac{4\pi}{n_e}v_{Te}\sqrt{\frac{\pi}{2}}\int dvv\frac{\nu_{ei}}{\nu_{11}}f_M^e.$$

These coefficients also satisfy relations similar to Onsager symmetries (coefficient $\beta_{\perp j}$ appears both in the electric current and in the friction force).

All electron transport coefficients have real and imaginary parts which when presented in dimensionless form, can be parametrized by $k\lambda_{ei}$, ω/ν_{ei}^T and Z similarly to classical expressions. The longitudinal transport coefficients (24) were analyzed in Ref. [8] and all transport coefficient in the static limit $\omega = 0$ were studied in Ref. [20].

B. Potential components of the electron fluxes

Consider first the limit of slow processes, such that $\omega \ll k^2 v_{Te}^2/\nu_{ei}, \nu_{ei}$, and the frequency, ω, can be neglected in the kinetic equations. In this approximation we substitute the solution of Eqs. (14) into Eqs. (24) and find the longitudinal transport coefficients to be purely real [19, 20] and with a dependence only on $k\lambda_{ei}$ and Z. Results of these calculations are shown in Fig.1. In the strongly collisional limit, $k\lambda_{ei} < 0.06/\sqrt{Z}$, the ion convective coefficients vanish as $k^2\lambda_{ei}^2$ while the remaining transport coefficients converge to their classical values:

$$\sigma_{SH} = \frac{32 n_e e^2}{3\pi m_e \nu_{ei}^T}\,, \quad \alpha_{SH} = \frac{16 n_e e}{\pi m_e \nu_{ei}^T}\,, \quad \chi_{SH} = \frac{200}{3\pi} n_e v_{Te} \lambda_{ei}\,. \tag{29}$$

All of these coefficients have similar long-wavelength asymptotic representations:

$$\begin{aligned} &\sigma = \sigma_0(1 - 19 Z k^2\lambda_{ei}^2), \quad \alpha = \alpha_0(1 - 107 Z k^2\lambda_{ei}^2), \quad \chi = \chi_0(1 - 239 Z k^2\lambda_{ei}^2), \\ &\beta_j = 22 k^2\lambda_{ei}^2, \qquad \beta_q = 88 k^2\lambda_{ei}^2, \qquad \beta_r = 2.4 k^2\lambda_{ei}^2\,. \end{aligned} \tag{30}$$

Note that in this limit, the ion convection coefficients do not explicitly depend on the ion charge.

In the short-wavelength limit $k\lambda_{ei} \gg 1$, the coefficients β_j and β_r approach unity and the coefficient β_q vanishes. All other transport coefficient are inversely proportional to the wave number and exhibit a fractional-power dependence on k in the weakly collisional regime $k\lambda_{ei} \gg 1/\sqrt{Z}$, similarly to results of Refs. [12, 30]:

$$\begin{aligned} &\sigma = \frac{5 e^2 n_e v_{Te}}{\sqrt{8\pi} k T_e} \frac{1 + 9/5\xi}{1 + 2\xi}\,, \quad \alpha = -\frac{e n_e v_{Te}}{\sqrt{2\pi} k T_e} \frac{1}{1 + 2\xi}\,, \quad \chi = \frac{4 n_e v_{Te}}{\sqrt{2\pi} k} \frac{1}{1 + 2\xi}\,, \\ &\beta_j = 1 - \frac{0.4 \ln(k\lambda_{ei}) - 0.1}{k\lambda_{ei}}\,, \quad \beta_q = \frac{1.4 \ln(k\lambda_{ei}) - 2.6}{k\lambda_{ei}}\,, \quad \beta_r = 1 - \frac{13 \ln(k\lambda_{ei}) - 41}{k\lambda_{ei}}. \end{aligned} \tag{31}$$

The function $\xi = 1.9 Z^{2/7} (k\lambda_{ei})^{-3/7}$ has been found in Ref. [30] from an asymptotic solution of the equation for the basis function (14) in the range $Z k^2\lambda_{ei}^2 \gg 1$.

We note that the electrical conductivity is almost independent of the ion charge and quickly converges to the asymptotic expression at large values of $k\lambda_{ei}$. The simple expression

$$\sigma = \sigma_{SH}\left(1 + \frac{128}{15\sqrt{2\pi}}k\lambda_{ei}\right)^{-1} \tag{32}$$

is a good approximation for the electric conductivity over the entire range of the $k\lambda_{ei}$.

The temperature conductivity is the most sensitive function of the ion charge Z and the dimensionless inhomogeneity scale length $k\lambda_{ei}$. Deviation from the classical limit occurs at $k\lambda_{ei} \sim 0.06/\sqrt{Z}$. We introduce the following approximation for the temperature conductivity coefficient:

$$\chi = \chi_{SH}\left[1 + k\lambda_{ei}\frac{100(1+\xi)Z^{0.25}(k\lambda_{ei})^{0.5}}{3\sqrt{2\pi}(40 + Zk^2\lambda_{ei}^2)^{0.25}}\right]^{-1}, \tag{33}$$

which works well over the entire range of the collisionality parameter.

The most unusual dependence on the wavelength is exhibited by the thermoelectric coefficient. It changes sign in the intermediate range, $k\lambda_{ei} \sim 1 - 10$. In the range $k\lambda_{ei} \lesssim 1$ the coefficient α is almost independent of the ionic charge and can be characterized by the simple expression;

$$\alpha = \frac{\alpha_{SH}}{1 + 35(k\lambda_{ei})^{1.2}}, \qquad k\lambda_{ei} \lesssim 1. \tag{34}$$

In the range $k\lambda_{ei} > 1$ the thermoelectric coefficient changes sign at a value of k which depends on Z. For example, α passes through zero at $k\lambda_{ei} = 2.6$ and 5 for $Z = 8$ and $Z = 64$, respectively.

The applicability of the static transport coefficients in the classical strongly collisional limit ($k\lambda_{ei} < 0.06/\sqrt{Z}$) requires small values of frequency ω as compared to the electron-ion collision frequency, $\omega \ll \nu_{ei}^T$ [11]. In this limit transport coefficients are determined primarily by the electron-ion collision frequency, ν_{ei}^T. Effects associated with electron-electron collisions represent small corrections of the order $\mathrm{O}(Z^{-1})$ [16, 17]. The limits of validity for the localized form of these classical coefficients are determined by the electron energy delocalization length $\lambda_\epsilon = \sqrt{Z}\lambda_{ei}$ [14, 15]. For $k\lambda_\epsilon \approx 1$, electron-electron collisions begin to affect transport coefficients and to modify the symmetric part of the distribution function, which in turn determines the anisotropic perturbations of the EDF (see Eq. (10)) as well as the electron fluxes. As $k\lambda_\epsilon$ increases, the role of low energy electrons (electrons which are characterized by the velocity $v^* \lesssim v_{Te}$) becomes dominant in the evolution of the symmetric part of the EDF. These slowly moving particles are strongly affected by electron-electron collisions [12]. For example, for $k\lambda_\epsilon \gg 1$, the characteristic electron velocity $v^* \sim v_{Te}/(Zk^2\lambda_{ei}^2)^{1/7}$ becomes noticeably lower than the thermal velocity [12, 20]. Thus, the region of validity of the static approximation for the transport coefficients for moderate gradients $0.06/\sqrt{Z} < k\lambda_{ei} < 6Z^{2/3}$ is determined by the conditions $\omega \ll \nu_{ee}^T\,,\ \nu_{ei}^T\,(k\lambda_{ei})^{4/7}/Z^{5/7}$ [12, 20], where $\nu_{ee}^T = 2\nu_{ee}(v_{Te})/(3\sqrt{2\pi})$. For higher gradients where $k\lambda_{ei} > 6Z^{2/3}$, all angular harmonics must be taken into account in order to obtain a correct description of the particle transport. In this collisionless regime the validity condition for the static approximation is defined in the usual way as $\omega \ll kv_{Te}$. In summary, the static approximation applies under the following conditions [8]

$$\omega \ll \begin{cases} \nu_{ei}^T\,, & k\lambda_{ei} < 0.06/\sqrt{Z} \\ \nu_{ee}^T\,, \nu_{ei}^T\,(k\lambda_{ei})^{4/7}/Z^{5/7}\,, & 0.06/\sqrt{Z} < k\lambda_{ei} < 6Z^{2/3} \\ kv_{Te}\,, & k\lambda_{ei} > 6Z^{2/3} \end{cases} \tag{35}$$

The jump in Eq. (35) at $k\lambda_{ei} \sim 6Z^{2/3}$ indicates the transition to the collisionless regime where all spherical harmonics should be taken into account.

The validity of the local transport theory can be extended into high frequency regime by using a simple exact solution to the kinetic equation [11] for $\mid \omega + i\nu_{ei}^T \mid \gg kv_{Te}$. Our nonlocal theory shows that nonlocal effects are insignificant for $k\lambda_{ei} < 0.06/\sqrt{Z},\ 0.1\omega/\nu_{ei}^T$. In this case, Eq. (10) for the two first harmonics of the EDF ($l = 0, 1$) can be solved by using the first two Laguerre polynomials $\psi^A = C_0^A + C_1^A(v^2/3v_{Te}^2 - 1)$ in the expansion of the basis functions. In this approximation the effective collision frequency satisfies the following expression: $\nu_{10} = \nu_{ei} - i\omega$. As a result, transport coefficients are given by [11]:

$$\begin{aligned} \frac{\sigma}{\sigma_{SH}} &= \frac{1}{48}\int_0^\infty dx x^6 Q(x)\,, & \frac{\alpha}{\alpha_{SH}} &= \frac{1}{144}\int_0^\infty dx x^6 (x^2-5) Q(x)\,, \\ \frac{\chi}{\chi_{SH}} &= \frac{1}{1200}\int_0^\infty dx x^6 (x^2-5)^2 Q(x)\,, & \beta_r &= 1 - \int_0^\infty dx Q(x)\,, \\ \beta_j &= 1 - \sqrt{\frac{2}{9\pi}}\int_0^\infty dx x^3 Q(x)\,, & \beta_q &= \sqrt{\frac{1}{18\pi}}\int_0^\infty dx x^3 (5-x^2) Q(x)\,, \end{aligned} \tag{36}$$

where we have introduced the notation $Q(x) = \nu_{ei}^T x \exp{(-x^2/2)}/(\nu_{ei}^T - i\sqrt{2/9\pi}\,\omega x^3)$, ($x = v/v_{Te}$). These expressions for transport coefficients are independent of the wave number and correspond to the local limit, including the hydrodynamic (static) limit. Figure 2 shows transport coefficients as functions of frequency. In the limit of strong collisions and low frequency $\omega \ll \nu_{ei}^T$, expressions (36) lead to classical transport coefficients [16] with small imaginary corrections

$$\frac{\sigma}{\sigma_{SH}} = 1 + i\frac{105}{16}\frac{\omega}{\nu_{ei}^T}\,, \quad \frac{\alpha}{\alpha_{SH}} = 1 + i\frac{105}{8}\frac{\omega}{\nu_{ei}^T}, \quad \frac{\chi}{\chi_{SH}} = 1 + i\frac{609}{40}\frac{\omega}{\nu_{ei}^T}\,,$$
$$\beta_j = -i\frac{32}{3\pi}\frac{\omega}{\nu_{ei}^T}\,, \qquad \beta_q = -i\frac{32}{2\pi}\frac{\omega}{\nu_{ei}^T}\,, \qquad \beta_r = -i\frac{\omega}{\nu_{ei}^T}\,. \tag{37}$$

In the high-frequency limit $\omega \gg \nu_{ei}^T$, coefficients β_j and β_r have small imaginary components and these coefficients tend to unity. The coefficient β_q has a small absolute value and has a real component that is smaller than the imaginary component. In the same limit, the transport coefficients σ, α and χ become purely imaginary and independent of Z and have small real corrections:

$$\sigma = \frac{i\,e^2 n_e}{m_e\omega}\left(1 - i\frac{\nu_{ei}^T}{\omega}\right)\,, \qquad \chi = \frac{i\,T_e n_e}{m_e\omega}\left(\frac{5}{2} - i\frac{13\nu_{ei}^T}{4\omega}\right)\,,$$
$$\alpha = \frac{i\,en_e}{m_e\omega}\left(\frac{5}{2}\left(\frac{\pi^8(\nu_{ei}^T)^5}{36\omega^5}\right)^{1/6} + i\frac{3\nu_{ei}^T}{2\omega}\right)\,, \qquad \beta_j = 1 - i\frac{\nu_{ei}^T}{\omega}\,, \tag{38}$$
$$\beta_q = -i\frac{3\nu_{ei}^T}{2\omega}\,, \qquad \beta_r = 1 - \left(\frac{9\pi(\nu_{ei}^T)^2}{2\omega^2}\right)^{1/3}\frac{\pi}{3\sqrt{3}}(1 + i\sqrt{3})\,.$$

As the the collision parameter $k\lambda_{ei}$ is increased, the nature of the frequency dependence of transport coefficients changes. For example, coefficients α and χ show a nonmonotonic frequency dependence, which can be clearly seen in Fig.3. First, the real part of the temperature conductivity increases with frequency ω as compared to the static case, and then decreases for $\omega/\nu_{ei}^T > 1$. At the same time, the imaginary part of the temperature conductivity is first negative and decreases to its minimal value; then begins to increase, changes sign, reaches its maximal value, and then decreases again. Both, the imaginary and real parts of the temperature conductivity have a maximum for $\omega/\nu_{ei}^T \sim 1$ for $k\lambda_{ei} = 1$ (see Fig.3). An even more complex frequency dependence appears for the thermocurrent coefficient α whose imaginary and real parts each have three local extreme points. The real part of α reverses its sign upon an increase in ω.

Relatively simple equations for transport coefficients can be obtained in the (ω, k) region where e-e collisions are negligible [6], i.e., for $\omega \gg \nu_{ee}^T, \nu_{ei}^T(k\lambda_{ei})^{4/7}/Z^{5/7}$. In this case, we find solutions of the system (14) for the basis distribution functions in the form $\Psi^A = 3\nu_{10}S_0^A/(k^2v^2 - 3i\omega\nu_{10})$. We obtain the following expressions for velocity moments of basis functions:

$$J_N^N = \frac{3}{kv_{Te}} \int_0^\infty dx W(x)\,, \qquad J_N^T = \frac{1}{kv_{Te}} \int_0^\infty dx(x^2-3)W(x),$$
$$J_T^T = \frac{1}{3kv_{Te}} \int_0^\infty dx(x^2-3)^2 W(x)\,, \qquad J_R^N = \frac{3\nu_{ei}^T}{kv_{Te}}\sqrt{\frac{\pi}{2}} \int_0^\infty \frac{dxW(x)}{x\nu_{10}(x)}\,, \tag{39}$$
$$J_R^T = \frac{\nu_{ei}^T}{kv_{Te}}\sqrt{\frac{\pi}{2}} \int_0^\infty \frac{dxW(x)(x^2-3)}{x\nu_{10}(x)}\,, \quad J_R^R = \frac{3\pi(\nu_{ei}^T)^2}{2kv_{Te}} \int_0^\infty \frac{dxW(x)}{x^2\nu_{10}^2} - \int_0^\infty \frac{dxx\nu_{ei}e^{-x^2/2}}{k^2\lambda_{ei}v_{Te}\nu_{10}}\,,$$

where $W(x) = \sqrt{2/\pi}kv_{Te}\exp(-x^2/2)/(k^2v_{Te}^2/\nu_{10}(x) - 3i\omega/x^2)$. All transport coefficients (24) can easily be calculated in terms of these moments.

In the collisionless kinetic limit $k\lambda_{ei} \gg 1$, the transport coefficients are independent from the ion charge Z. In this limit, we have β_j, $\beta_r = 1$, $\beta_q = 0$, while the remaining coefficients are functions of parameter $p = \omega/kv_{Te}$ and can be obtained by using $\nu_{10} = kv_{Te}h_1$, as the effective frequency in Eqs. (39), where $h_{l-1} = -ip + x^2l^2/(4l^2-1)h_l$ (cf. Eq. (9)). In this case, we propose a simple approximate equation

$$h_1(x,p) = i(\pi(p - \sqrt{p^2 - x^2})/6 - p)\,, \tag{40}$$

for h_l. This has an accuracy which is within 1% of the exact value. By substituting expressions (39) calculated in this way into Eqs. (24), we obtain

$$\sigma = \frac{e^2n_ev_{Te}}{kT_e}\left(\frac{1}{\Delta}\int_0^\infty (\frac{x^4}{3} - 2x^2 + 3)W(x)dx + ip\right)\,, \tag{41}$$
$$\alpha = \frac{en_ev_{Te}}{kT_e}\left(\frac{1}{\Delta}\int_0^\infty (x^2 - \frac{x^4}{3})W(x)dx - ip\right)\,,\ \chi = \frac{n_ev_{Te}}{k}\left(\frac{1}{\Delta}\int_0^\infty \frac{x^4}{3}W(x)dx + i\frac{5}{2}p\right)\,,$$

where the following notation is used: $\Delta = \int x^4W(x)dx \int W(x)dx - (\int x^2W(x)dx)^2$. It should be noted that expressions for collisionless transport coefficients were also obtained in [23]. A different definition of the transport coefficients from those in (41)was used and included an explicit summation of infinite series. The collisionless transport coefficients can be calculated by solving the initial value problem for the Vlasov kinetic equation. This leads to the following expressions for moments, J_A^B:

$$J_N^N = \frac{i}{\omega}J_+(p)\,, \quad J_N^T = \frac{i}{3\omega}\left((p^2-1)J_+(p) - p^2\right)\,, \tag{42}$$
$$J_T^T = \frac{i}{9\omega}\left((p^4 - 2p^2 + 5)J_+(p) - p^4 + p^2\right)\,,$$

where $J_+(x) = x\exp(-x^2/2)\int_{i\infty}^x dt\exp(t^2/2)$ is the standard dispersion function used in the collisionless theory of plasmas [1]. The behavior of collisionless transport coefficients as functions of ω/kv_{Te} is illustrated in Fig.(4).

The expression for the heat flux is often written in terms of the temperature gradient and the electric current [17]. Accordingly, by eliminating the electric field from Eqs. (22), we obtain

$$\mathbf{q}_{\|} = -\frac{\alpha T_e}{\sigma}\mathbf{j}_{\|} - \kappa i\mathbf{k}\delta T_e - n_e T_e \beta \mathbf{u}_{i\|}, \quad \kappa = \chi - \frac{\alpha^2 T_e}{\sigma}, \quad \beta = \beta_q - \frac{e\alpha}{\sigma}\beta_j\,, \tag{43}$$

where the thermal conductivity κ and the ion convective transport coefficient β are introduced. Both coefficients have a sensitive dependence on the ion charge. In the strongly collisional limit $k\lambda_{ei} \ll 1$, κ transforms into the classical heat conductivity [16, 17] : $\kappa = 128 n_e v_{Te} \lambda_{ei}/3\pi$. Note that in the static limit (35) for $k\lambda_{ei} \lesssim 1$, the approximate formulas

$$\kappa = \frac{\kappa_0}{1 + (10\sqrt{Z}k\lambda_{ei})^{0.9}}\,, \quad \beta = \frac{55k^2\lambda_{ei}^2}{1 + 1.6(1 + 6k\lambda_{ei})(10\sqrt{Z}k\lambda_{ei})^{0.9}}\,, \tag{44}$$

provide a good description of nonlocal heat transport in a current free plasma. Figure (5) illustrates the dependence of these transport coefficients on $k\lambda_{ei}$ in the static limit (35) and on ω for $k\lambda_{ei} = 1$.

Formulas (43) for $j = 0$ (a no-current plasma) are directly related to the description of transport in an ICF plasma. It was shown in the hot spot relaxation problem [7, 22] that transient effects and nonlocal transport are important for $k\lambda_{ei} \gtrsim 0.1$. For such inhomogeneity scale lengths, the stationary approaches [12, 14, 15, 18–20] are not applicable. The equations of nonlocal hydrodynamics with nonstationary transport coefficients [8] enable description of a plasma for any spatial and temporal perturbation scales.

C. Nonpotential components of the electron fluxes

In discussing the nonpotential electron flux components, we recall that the transverse transport coefficients do not depend on e-e collisions or on the isotropic correction to the distribution function. Therefore, Eqs. (28) give explicit expressions for these coefficients, which are plotted in Fig.6 in the static limit. In this limit, all these transport coefficients are real. The applicability condition for the static approximation for the transverse transport coefficients reads $\omega \ll \nu_{ei}^T, kv_{Te}$. The static transverse transport coefficients have a long-wavelength asymptotic behavior which is similar to the behavior of the longitudinal transport coefficients. However, their deviation from classical values is determined by the small parameter $k^2\lambda_{ei}^2 \ll 1$, rather than $Zk^2\lambda_{ei}^2$, i.e.

$$\sigma_\perp = \sigma_{SH}(1-86k^2\lambda_{ei}^2)\,, \quad \alpha_\perp = \alpha_{SH}(1-314k^2\lambda_{ei}^2)\,, \quad k^2\lambda_{ei}^2 \ll 1 \tag{45}$$
$$\beta_{\perp j} = \frac{154}{3\pi}k^2\lambda_{ei}^2, \quad \beta_{\perp q} = \frac{616}{3\pi}k^2\lambda_{ei}^2, \quad \beta_{\perp r} = \frac{256}{45\pi}k^2\lambda_{ei}^2\,.$$

When the nonlocal behavior of transport is taken into account, the transversal transport coefficients differ from the longitudinal ones, i.e., the electron fluxes demonstrate an anisotropy. This vanishes in the local limit, $k\lambda_{ei} \ll 1, \omega/\nu_{ei}^T$, where the transverse transport coefficients have the same form as the longitudinal ones. By using $\nu_{11} = \nu_{ei} - i\omega$ in Eqs. (28), we obtain expressions (36) for all transverse transport coefficients.

In the weakly collisional limit $k\lambda_{ei} \gg 1$, the perpendicular transport coefficients are almost independent of the collision frequency ν_{ei}^T, which gives only small corrections. In this limit, the coefficients $\beta_{\perp j}$ and $\beta_{\perp r}$ tend to unity, while the coefficient $\beta_{\perp q}$ vanishes in accordance with

$$\beta_{\perp j} = 1 - \frac{2.95}{k\lambda_{ei}}, \quad \beta_{\perp q} = \frac{5.9}{k\lambda_{ei}}, \quad \beta_{\perp r} = 1 - \frac{2.76}{\sqrt{k\lambda_{ei}}}. \tag{46}$$

Note, that the transverse short-wavelength limit for the ion convective coefficients do not contain logarithmic terms as in Eqs. (31) because e-e collisions do not contribute to the transverse transport coefficient. In the same limit, the transverse coefficients $\alpha_\perp$ and $\sigma_\perp$ are functions of the parameter $p = \omega/kv_{Te}$. These can be represented in a similar way as the longitudinal ones, e.g. by using $\nu_{11} = kv_{Te}h_{11}$ similarly to the effective frequency in Eqs. (28) with $h_{l-1,1} = -ip + x^2(l^2-1)/(4l^2-1)h_{l,1}$. At the same time, the collisionless transverse transport coefficients can be calculated exactly by solving the initial value problem for the collisionless kinetic equation which gives

$$\sigma_\perp = i\frac{e^2n_e}{m_e\omega}J_+(p)\;, \quad \alpha_\perp = i\frac{en_e}{2m_e\omega}((p^2-1)J_+(p) - p)\,. \tag{47}$$

In the quasistatic collisionless limit, we recover the result

$$\sigma_\perp = \sqrt{\frac{\pi}{2}}\frac{e^2n_e}{m_e}\frac{1}{kv_{Te}}, \quad \alpha_\perp = -\sqrt{\frac{\pi}{8}}\frac{en_e}{m_e}\frac{1}{kv_{Te}}, \quad k^2\lambda_{ei}^2 \gg 1 \tag{48}$$

that corresponds to the free streaming transport limit. Figures 7 and 8 show frequency dependence of the above transport coefficients for several values of $k\lambda_{ei}$.

IV. DIELECTRIC TENSOR OF COLLISIONAL PLASMA

Since our transport equations contain both a potential part and a transverse one, the total dielectric permittivity of a plasma

$$\epsilon_{ij} = \frac{k_i k_j}{k^2}\epsilon^l + \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\epsilon^t \tag{49}$$

is determined by the longitudinal (ϵ^l) and transversal (ϵ^t) components. The hydrodynamic equations (18) are equivalent to a kinetic description and completely determine the linear response of a plasma to small perturbations over the entire range of parameters (ω, k). These equations can be used for deriving the permittivity $\epsilon(\omega, k)$ of a plasma. In order to calculate the longitudinal permittivity ,

$$\epsilon^l = 1 + 4\pi i \frac{j_\parallel}{\omega E_\parallel} \tag{50}$$

we eliminate the density and electron temperature perturbations from the expression for electric current by solving the system (18):

$$\begin{aligned}
\mathbf{j}_\parallel \; = & \left[1 - i\omega\left(\frac{e^2 n_e}{k^2 T_e \sigma} + \frac{2n_e(\sigma + e\alpha)^2}{\sigma^2(2k^2\kappa - 3i\omega n_e)}\right)\right]^{-1}\left\{-\frac{ie^2 n_e}{k^2 T_e}\omega \mathbf{E}_\parallel + \right. \\
& \left. e n_e \mathbf{u}_{i\parallel}\left[1 - i\omega\left(\frac{e^2 n_e \beta_j}{k^2 T_e \sigma} + \frac{2n_e(\sigma + e\alpha)(1-\beta)}{\sigma(2k^2\kappa - 3i\omega n)}\right)\right]\right\} \equiv \\
& -\frac{ie^2 n_e}{k^2 T_e}\omega(1 + i\omega J_N^N)\mathbf{E}_\parallel + e n_e \mathbf{u}_{i\parallel}\left(1 + i\omega J_N^R\right) .
\end{aligned} \tag{51}$$

For the transversal permittivity, $\epsilon^t = 1 + 4\pi i j_\perp/\omega E_\perp$, we can use Eq. (26). We first analyze a pure electron plasma in the limit of stationary (infinitely heavy) ions by assuming $u_i = 0$.

A. Longitudinal electron susceptibility

We will characterize the electron contribution $\delta\epsilon_e$ to the longitudinal permittivity ($\epsilon^l = 1 + \delta\epsilon_e$) by the function $\delta\epsilon \equiv k^2\lambda_{De}^2\delta\epsilon_e$, where λ_{De} is the electron Debye radius. By using relation (51), we obtain the following expression

$$\delta\epsilon = \left[1 - i\omega\left(\frac{e^2 n}{k^2 T\sigma} + \frac{2n(\sigma + e\alpha)^2}{\sigma^2(2k^2\kappa - 3i\omega n)}\right)\right]^{-1} \equiv 1 + i\omega J_N^N , \tag{52}$$

which enables contributions to ϵ^l from all transport coefficients to be found. However, the function ϵ^l is determined by only one specific moment J_N^N of the basis function.

In the local limit $k\lambda_{ei} < 0.06/\sqrt{Z}$, $0.1\omega/\nu_{ei}^T$, an analytic expression for $\delta\epsilon$ is obtained by substituting Eqs. (36) into formula (52). In the hydrodynamical limit of low frequencies ($\omega \ll \nu_{ei}^T$), this leads to the expression

$$\delta\epsilon = \frac{2x(8x - 3i\omega)}{16x^2 - 6\omega^2 - 47i\omega x}\,, \quad x = \frac{32k^2 v_{Te}^2}{3\pi\nu_{ei}^T}\,. \tag{53}$$

In the limit $\omega \gg x$, the electronic susceptibility is determined by the classical electrical conductivity $\delta\epsilon_e = 4\pi i\sigma_{SH}/\omega$. In the opposite case ($\omega \ll x$) the static permittivity describes Debye screening effect, Re $\epsilon^l = 1 + 1/(k^2\lambda_{De}^2)$; transport coefficients only determine the small imaginary correction Im $\epsilon^l = 41\omega/16x$, which includes comparable contributions from coefficients σ, α, and χ. The dispersion relation $\epsilon^l = 0$ in the static limit for $k\lambda_{De} \ll 1$ gives the entropy mode $\omega = 2ik^2\kappa_{SH}/3n_e$ with a classical thermal conductivity [16, 17]. For fast processes ($\omega \gg \nu_{ei}^T$), the permittivity is determined by the high-frequency electrical conductivity and is described by the well known expression $\epsilon = 1 - (\omega_{pe}^2/\omega^2)\,(1 - i\,\nu_{ei}^T/\omega)$ [10].

Figure 9 shows the parametric (k, ω), plane divided into regions corresponding to different approximations for describing the permittivity beginning with the classical hydrodynamic limit (dashed region) to the collisionless kinetic limit (dotted region). The grey region between the fine solid curves in Fig.9 corresponds to strongly decaying perturbations, for which Im $\epsilon^l >$ Re ϵ^l. Under the unmarked bold solid curve, the real part of the permittivity corresponds to Debye screening, Re $\epsilon^l = 1 + 1/k^2\lambda_{De}^2$. The $\omega(k)$ boundary curve denoted by $e - e$ separates the quasistationary regime (35), for which electron-electron collisions are important, from the nonstationary regime. It should be noted that for $k\lambda_{ei} \ll 6Z^{2/3}$ in the quasi-static approximation, two angular harmonics (diffusion approximation) are sufficient for calculating the electron distribution function and, accordingly, all transport coefficients as well as the permittivity [12]. In this limit and for $k\lambda_{ei} \gg 1/\sqrt{Z}$, the approximate expression for the permittivity has the form [12]

$$\delta\epsilon^l = 1 + i\frac{\omega}{kv_{Te}}\left\{\sqrt{\frac{\pi}{2}} + 2.17\frac{Z^{2/7}}{(k\lambda_{ei})^{3/7}}\right\}\,, \tag{54}$$

which is close to the exact solution. Our analysis shows that the range of applicability of relation (54) is in fact $k\lambda_{ei} > 1$.

In the frequency range in which the e-e collisions can be neglected, we can reconstruct from relation (39) the permittivity obtained in [4–6], which leads to the well-known expression

$$\epsilon = 1 + \frac{1}{k^2\lambda_{De}^2}\left[1 - J_+\left(\frac{\omega}{kv_{Te}}\right)\right]. \tag{55}$$

in the collisionless limit. The general expression derived for permittivity is applicable for describing the plasma over the entire range of k and ω for any number of collisions in the plasma. The contribution of collisions to the permittivity of the plasma is often described by using a simplified collision integral in the Bhatnagar-Gross- Krook (BGK) form. The theory presented here makes it possible to determine the accuracy of this approximation. The best agreement is obtained by using the expression proposed in [3]:

$$\delta\epsilon^C = \frac{1 - J_+(y)}{1 - iJ_+(y)/yk\lambda_{ei}}, \quad y = \frac{\omega + i\nu_{ei}^T}{kv_{Te}}. \tag{56}$$

However, in spite of the fact that the behavior described by this expression is correct in general, it differs noticeably from the exact result for $k\lambda_{ei} < 1$ in a wide frequency range (see Fig. (10a). With increasing $k\lambda_{ei}$, the agreement is improved; however, it follows from Fig. (10b), that the formula (56) still differs from the exact solution by a factor of 2 to 3 in the range of frequencies $\omega \sim \nu_{ei}$.

B. Transverse electron susceptibility

The electron transverse permittivity is completely defined by the transverse electrical conductivity (28) [9]:

$$\epsilon^t = 1 + \frac{4\pi i\sigma_\perp}{\omega} = 1 - i\omega_{pe}^2\sqrt{\frac{2}{\pi}}\int_0^\infty dx\frac{x^4\exp(-x^2/2)}{3\nu_{11}\omega} \tag{57}$$

In the local regime, $k\lambda_{ei} \ll 1$, using $\nu_{11} = \nu_{ei} + i\omega$ as an effective frequency in Eq. (57) one obtains an expression for the transverse electron susceptibility known as the Drude model:

$$\epsilon^t = 1 - i\omega_{pe}^2\sqrt{\frac{2}{\pi}}\int_0^\infty dx\frac{x^4\exp(-x^2/2)}{3(\nu_{ei} - i\omega)\omega} \tag{58}$$

In the collisionless limit $k\lambda_{ei} \gg 1$, the dielectric permittivity agrees with the Vlasov theory result [1]

$$\epsilon^t = 1 - \frac{\omega_{pe}^2}{\omega^2}J_+\left(\frac{\omega}{kv_{Te}}\right) \tag{59}$$

There are also several limits where expressions for transverse susceptibility are well known. They follow from asymptotic representations for the electron conductivity (29,37,38,48). The collisional, static limit, $k\lambda_{ei} \ll 1$, $\omega \ll \nu_{ei}^T$, corresponds to the electron permittivity in the hydrodynamical approximation:

$$\epsilon^t = 1 - \frac{70}{\pi}\frac{\omega_{pe}^2}{(\nu_{ei}^T)^2} + i\frac{32}{3\pi}\frac{\omega_{pe}^2}{\omega\nu_{ei}^T} \tag{60}$$

. This is useful, for example, in the description of the normal skin effect. The collisionless static limit, $k\lambda_{ei} \gg 1, \omega/\nu_{ei}^T$, corresponds to resonant wave interaction with slow particles,

$$\epsilon^t = 1 - \frac{\omega_{pe}^2}{k^2 v_{Te}^2} + i\sqrt{\frac{\pi}{2}}\frac{\omega_{pe}^2}{\omega k v_{Te}} \tag{61}$$

and describes an anomalous skin effect. The high frequency limit, $\omega \gg \nu_{ei}^T, kv_{Te}$, is also well known,

$$\epsilon^t = 1 - \frac{\omega_{pe}^2}{\omega^2} + i\frac{\omega_{pe}^2 \nu_{ei}^T}{\omega^3}\,. \tag{62}$$

Figure 11 shows transverse permittivity in comparison with different models. It is clear that for $k\lambda_{ei} \geq 1$, the Drude model (58) gives a poor approximation in a wide frequency range.

The transverse susceptibility plotted in the parametric $(k,\, \omega)$ plane in the Fig.12 is simpler as compared to Fig. 9 for the longitudinal susceptibility. The region in which Im $\epsilon_\perp >$ Re $\epsilon_\perp$ (grey region) is roughly defined by the inequality $\omega < \nu_{ei}^T, kv_{Te}$. The Drude model (58), which corresponds to classical local description, is applicable for $k < 0.1/\lambda_{ei}, 0.1\omega/v_{Te}$ (dashed region in Fig.12). The dotted region corresponds to the collisionless kinetic model (59), which is valid for $k > 10/\lambda_{ei}$.

C. Ion contribution to permittivity. Damping of ion-acoustic wave.

In accordance with definition (50), elimination of the ion velocity from expression (51) allows ion contributions to plasma permittivity to be calculated. This will be accomplished in this Section by solving hydrodynamical equations and finding relation between $\mathbf{u}_i$ and the electric field. Strictly speaking, a complete kinetic description of ions is required in this case. However, according to the results obtained in [30], for fast perturbations ($\omega \gg kv_{Ti},$), where v_{Ti} is the ion thermal velocity, we can use hydrodynamic equations for ions, in which the ion viscosity and heat conductivity are taken into account by using the 21-moment approximation of the Grad method. This leads to the hydrodynamical equations for ions in the following form

$$-i\omega\mathbf{u}_i = \frac{eZ}{m_i}\mathbf{E} - i\mathbf{k}v_{Ti}^2\left(\frac{\delta n_i}{n_i}+\frac{\delta T_i}{T_i}\right) - \frac{4}{3}\frac{(\mathbf{k}\cdot\mathbf{u}_i)v_{Ti}^2}{\nu_i}\hat{\eta}_i\mathbf{k} + \frac{1}{n_i m_i}\mathcal{R}_{ie} - \nu_{ei}^T\mathbf{u}_i$$

$$-i\omega\delta T_i = -\frac{2}{3}i(\mathbf{k}\cdot\mathbf{u}_i)T_i - \frac{2}{3n_i}k^2\kappa_i\delta T_i, \qquad -i\omega\delta n_i + i(\mathbf{k}\cdot\mathbf{u}_i)n_i = 0\,. \tag{63}$$

The longitudinal component of the stress tensor is represented in terms of the ion viscosity $\mathbf{k\Pi}^i = i\mathbf{k}4n_iT_i\hat{\eta}_i(\mathbf{k}\cdot\mathbf{u}_i)/3\nu_i$, [30]

$$\hat{\eta}_i = \frac{i\nu_i(\omega+1.46i\nu_i)}{(\omega+1.20i\nu_i)(\omega+1.46i\nu_i)+0.23\nu_i^2}\,, \tag{64}$$

and the energy exchange during e-i collisions in the energy conservation equation is neglected. An expression for the ionic thermal flux $\mathbf{q}_i = -i\mathbf{k}\kappa_i$ is determined by the thermal conductivity [30]:

$$\kappa_i = \frac{5}{2}\frac{n_i v_{Ti}^2}{\nu_i}\frac{i(\omega+1.29i\nu_i)}{(\omega+0.8i\nu_i)(\omega+1.29i\nu_i)+0.21\nu_i^2}\,. \tag{65}$$

Here, the ion-ion collision frequency is introduced in the standard form, $\nu_i = 4\sqrt{\pi}e_i^4 n_i\Lambda/3T_i^{3/2}\sqrt{m_i}$.

By using relations (63-65), we can exclude the ion velocity from expressions for the electric current (26, 51) to describe the total permittivity of the plasma in the following form

$$\epsilon^l = 1 + \frac{1+i\omega J_N^N}{k^2\lambda_{De}^2} - \frac{c_s^2}{\lambda_{De}^2}\frac{(1+i\omega J_N^R)^2}{\Delta}\,,$$
$$\epsilon^t = 1 + \frac{4\pi i\sigma_\perp}{\omega} - \frac{\omega_{pi^2}}{\Delta_\perp}\beta_{j\perp}^2 \tag{66}$$

where

$$\Delta = \omega^2 + i\omega\nu_{ei}^T + i\omega k^2c_s^2\left(J_R^R + \frac{(2\pi)^{3/2}}{n_ek^2\lambda_{ei}}\int_0^\infty dvv\frac{\nu_{ei}}{\nu_{10}}f_M^e\right) - k^2v_{Ti}^2\left(\frac{4i\omega\hat{\eta}_i}{3\nu_i} - \frac{5n_i\omega+2ik^2\kappa_i}{3n_i\omega+2ik^2\kappa_i}\right)\,,$$
$$\Delta_\perp = \omega^2 + i\omega\nu_{ei}^T + i\omega\nu_{ei}^T\frac{Zm_e}{m_i}(\beta_{r\perp}-1)\,. \tag{67}$$

Thus, expression (66) defines the total permittivity of a plasma with negligibly small ion Landau damping, $\omega \gg kv_{Ti}$.

From the dispersion relation $\epsilon^l = 0$ in the quasineutral limit $k\lambda_D \ll 1$ we obtain a weakly damped solution $\omega = kc_s - i\Gamma_s$, which describes an ion-acoustic wave with a damping rate specified by the formula [20]

$$\frac{\Gamma_s}{kc_s} = \frac{n_ec_s}{2k}\left[\frac{(1-\beta)^2}{\kappa} + \frac{e^2\beta_j^2}{T_e\sigma} + \frac{\beta_r}{n_ev_{Te}\lambda_{ei}}\right] \tag{68}$$

where c_s is the ion-acoustic velocity. We note that all nonlocal transport coefficients contribute to the damping factor Γ_s. This result corresponds exactly to the numerical solution of the Fokker-Plank equation [18]. The dependence of Γ_s on k is shown in Fig. (13). The decay rate agrees with the hydrodynamic expression $\Gamma_s/kc_s = 3\pi c_s/256\gamma_k(Z)v_{Te}k\lambda_{ei}$ in the long wavelength limit $c_s/v_{Te} \ll k\lambda_{ei} \ll 1$ and with collisionless Landau damping rate of $\Gamma_s/kc_s = \sqrt{\pi/8}c_s/v_{Te}$ in the short-wavelength range $k\lambda_{ei} \gg 1$.

V. CONCLUSION

We have derived equations of nonlocal transport for small perturbations in the general case of arbitrary relations between the characteristic space, time, and collision time scales. Our hydrodynamic equations are equivalent to a kinetic description of a plasma in terms of the linearized Fokker-Planck equation. The nonstationary and nonlocal transport coefficients in a Fourier representation are calculated in the entire (ω, k) region. The theory that is developed generalizes earlier transport models to the case of arbitrary (ω, k) and describes all limiting transitions to known results.

We propose a practical algorithm for calculating the dielectric tensor of a Maxwellian plasma for arbitrary values of frequency and wave number. The expression for permittivity derived here describes a smooth transition from the hydrodynamic region of strong collisions to the collisionless kinetic region and from the static to the high-frequency limit. On the basis of our theory, it becomes possible to analyze the linear plasma response and dispersion relations for unmagnetized plasma modes over the entire region of wave numbers and frequencies.

The development of nonlocal hydrodynamics is especially important for describing heat transport which is fundamental process for the laser plasma interactions in ICF experiments. It is well known that traditional hydrodynamic codes with a thermal flux that is described by using classical theory or its simple heuristic modifications, fail to explain experimental data correctly. A model with nonlocal transport give a much better agreement with experimental results [32].

The direct application of nonlocal nonstationary linear transport coefficients may involve the theory of laser plasma instabilities. The importance of nonlocal effects on transport in the quasi-static limit of filamentation instability and stimulated Brillouin scattering was demonstrated in [33, 34]. Our approach can be used to investigate instability in a strongly nonstationary laser plasma. Another important application of the permittivity is associated with the calculation of the Thomson scattering cross section, which is widely used for diagnostics of plasmas [31, 32].

ACKNOWLEDGMENTS

This study was financed in part by the Russian Foundation for Basic Research and the Natural Science and Engineering Research Council of Canada. Two of us, A.V.B. and W.R., would like to acknowledges support from the Alberta Ingenuity Fund.

---

[1] A. F. Aleksandrov, L. S. Bogdankevich, A. A. Rukhadze, *Principles of Plasma Electrodinamics*, Springer, Berlin, 1984.

[2] E. M. Lifshitz and L. P. Pitaevskii, *Physical Kinetics*, Pergamon, Oxford, 1981.

[3] M. Opher, G. J. Morales, and J. N. Leboeuf, "Krook collisional models of the kinetic susceptibility of plasmas", *Phys. Rev. E*, vol. 66, pp. 016407-1 –016407-10, July 2002.

[4] V .Yu. Bychenkov, "Longitudinal permittivity of a collisional plasma", *Plasma Phys. Rep.*, vol. 24, pp. 801-803, Sept. 1998.

[5] R. A. Koch and W. Horton Jr.,"Effects of electron angle scattering in plasma waves", *Phys. Fluids*, vol. 18, pp.861-865, July 1975.

[6] J. R. Penano, G. J. Morales, and J. E. Maggs, "Properties of drift waves in a filamentary density depletion", *Phys. Plasmas*, vol. 4, 555-565, March 1997.

[7] A. V. Brantov, V. Yu. Bychenkov, W. Rozmus, C. E. Capjack, "Kinetic Susceptibility and Transport Theory of Collisional Plasmas", *Phys. Rev. Lett.* vol. 93, 125002, Dec. 2004;

[8] A. V. Brantov, V. Yu. Bychenkov, W. Rozmus, C. E. Capjack, "Permittivity of Plasma and Nonstationary Theory of Nonlocal Transport", *JETP*, vol. 100, pp. 1159-1174, June 2005;

[9] V. Yu. Bychenkov, V. T. Tikhonchuk, W. Rozmus, "Transverse electron susceptibility and the electromagnetic wave absorption in weakly collisional plasmas",*Phys. Plasmas*, vol. 4, pp. 4205-4209, Dec. 1997.

[10] V. P. Silin, A. A. Rukhadze, *Electromagnetic Properties of Plasma and Plasma-like Media*, Atomizdat, Moscow [in Russian], 1961, p. 110.

[11] I. P. Shkarofsky, T. W. Johnston, and M. P. Bachynski, *The Particle Kinetics of Plasmas*,

Addison-Wesley, Reading, 1966.

[12] V. P. Silin ,"Kinetics of weakly collisional plasma ", *Phys. Usp.*, vol. 45, pp. 955-976, Aug 2002.

[13] A. R. Bell, "Electron energy transport in ion waves and its relevance to laser-produced plasmas", *Phys. Fluids*, vol. 26, pp. 279-284, Jan. 1983.

[14] J. F. Luciani, P. Mora, and J. Virmont, "Nonlocal Heat Transport Due to Steep Temperature Gradients", *Phys. Rev. Lett.*, vol. 51, pp.1664-1667, Oct. 1983.

[15] J. R. Albritton, E. A. Williams, I. B. Bernstein, and K. P. Swartz, "Nonlocal Electron Heat Transport by Not Quite Maxwell-Boltzmann Distributions", *Phys. Rev. Lett.*, vol. 57, pp. 1887-1890, Oct. 1986.

[16] L. Spitzer and R. Härm, "Transport Phenomena in a Completely Ionized Gas", *Phys. Rev.*, vol. 89, pp. 977-981, March 1953.

[17] S. I. Braginskii, Reviews of Plasma Physics, **1**, 205 (1965).

[18] E. M. Epperlein, "Effect of electron collisions on ion-acoustic waves and heat flow", *Phys. Plasmas*, vol. 1, pp. 109-115, Jan. 1994.

[19] V. Yu. Bychenkov, W. Rozmus, V. T. Tikhonchuk and A. V. Brantov,"Nonlocal Electron Transport in a Plasma", *Phys. Rev. Lett.*, vol. 75, pp. 4405-4408, Dec. 1995.

[20] A. V. Brantov, V. Yu. Bychenkov, W. Rozmus, V. T. Tikhonchuk, "Nonlocal electron hydrodynamics of semicollisional plasma", *JETP*, vol. 83, pp. 716-730, Apr. 1996.

[21] V. Yu. Bychenkov, V. N. Novikov, V. T. Tikhonchuk, "Theory of nonlocal transport for small perturbations in a plasma", *JETP*, vol. 87, pp. 916-925, Nov. 1998.

[22] S. Brunner, E. Valeo, and J. A. Krommes, "Linear delta-f simulations of nonlocal electron heat transport", *Phys. Plasmas*, vol. 7, pp. 2810-2823, July 2000.

[23] A. Bendib, G. Matthieussent and F. Bouzid, "Nonstationary closure relations of the collisionless fluid equations", *Phys. Plasmas*, vol. 9, pp. 35-46, Jan. 2002.

[24] G. W. Hammett, F. W. Perkins, "Fluid moment models for Landau damping with application to the ion-temperature-gradient instability", *Phys. Rev. Lett.*, vol. 64, pp.3019-3022, June 1990;

[25] G. W. Hammett, W. Dorland, F. W. Perkins, "Fluid models of phase mixing, Landau damping, and nonlinear gyrokinetic dynamics", *Phys. Fluids B*, vol. 4, pp. 2052-2061, July 1992.

[26] Z. Chang, J. D. Callen, "Unified fluid/kinetic description of plasma microinstabilities. Part I:

Basic equations in a sheared slab geometry", *Phys. Fluids B*, vol. 4, pp. 1167-1181, May 1992.

[27] N. Mattor, S. E. Parker,"Nonlinear Kinetic Fluid Equations", *Phys. Rev. Lett.*, vol. 79, pp. 3419-3422, Nov. 1997.

[28] R. D. Hazeltine, "Transport theory in the collisionless limit", *Phys. Plasmas*, vol. 5, pp. 3282-3286, Sept. 1998.

[29] J. F. Myatt, W. Rozmus, V. Yu. Bychenkov, and V. T. Tikhonchuk, "Kinetic theory of ion acoustic waves in a plasma with collisional electrons", *Phys. Rev. E*, vol. 52, pp. 6759-6776, Dec. 1995.

[30] V. Yu. Bychenkov, J. Myatt, W. Rozmus and V. T. Tikhonchuk, "Quasihydrodynamic description of ion acoustic waves in a collisional plasma ", *Phys. Plasmas*, vol. 1, pp. 2419-2429, Aug. 1994.

[31] J. F. Myatt, W. Rozmus, V. Yu. Bychenkov, and V. T. Tikhonchuk, "Thomson scattering from ion acoustic waves in laser plasmas", *Phys. Rev. E*, vol. 57, pp. 3383-3391, March 1998.

[32] G. Gregory, S. H. Glenser, J. Knight et. al., "Effect of Nonlocal Transport on Heat-Wave Propagation", *Phys. Rev. Lett.*, vol. 92, pp. 205006-1–205006-4, May 2004.

[33] E. M. Epperlein, "Kinetic theory of laser filamentation in plasmas", *Phys. Rev. Lett.*, vol. 65, pp. 2145-2148, Oct. 1990.

[34] V. Yu. Bychenkov, W. Rozmus, A. V. Brantov, and V. T. Tikhonchuk, "Theory of filamentation instability and stimulated Brillouin scattering with nonlocal hydrodynamics", *Phys. Plasmas*, vol. 7, pp. 1511-1519, May 2000.

FIG. 1: Dependence of the longitudinal transport coefficients σ, α , χ and $\beta_{j,q,r}$ on $k\lambda_{ei}$ in the static limit (35) for a plasma with Z=8 (small dots) and Z=64 (large dots). The solid curves correspond to the proposed approximation. The dots lines correspond to the classical strongly collision asymptotic behavior and the dashed lines correspond to the collisionless limit.

FIG. 2: Dependence of the real and imaginary parts of transport coefficients σ, α , χ and $\beta_{j,q,r}$ on ω/ν_{ei}^T in the long-wave limit $k\lambda_{ei} < 0.06/\sqrt{Z}$, $0.1\omega/\nu_{ei}^T$. Dashed curves correspond to the static limit (37).

FIG. 3: Dependence of the real and imaginary parts of longitudinal transport coefficients σ, α , χ and $\beta_{j,q,r}$ on ω/ν_{ei}^T for $k\lambda_{ei} = 1$ and for a plasma with $Z = 8$ (solid lines) and $Z = 64$ (dots).

FIG. 4: Dependence of the real and imaginary parts of longitudinal transport coefficients σ, α , χ on ω/kv_{Te} , calculated using formula (41) (dots), in comparison with the exact collisionless theory (42) (solid curve).

FIG. 5: Dependence of the real and imaginary parts of longitudinal transport coefficients κ, β on $k\lambda_{ei}$ in the static limit (35) (imaginary part =0) for a plasma with Z=8 (small dots) and Z=64 (large dots) and on ω/ν_{ei}^T for $k\lambda_{ei} = 1$ and for a plasma with $Z = 8$ (solid lines on second panel) and $Z = 64$ (dots). The solid curves on first panel correspond to the proposed approximation. The dots lines correspond to the classical strongly collision asymptotic behavior and the dashed lines correspond to the collisionless limit.

FIG. 6: Dependence of the transverse transport coefficients $\sigma_\perp$, $\alpha_\perp$, and $\beta_{j\perp,q\perp,r\perp}$ on $k\lambda_{ei}$ in the static limit (35) (dots). The dots lines correspond to the classical strongly collision asymptotic behavior and the dashed lines correspond to the collisionless limit.

FIG. 7: Dependence of the real and imaginary parts of transverse transport coefficients $\sigma_\perp$, $\alpha_\perp$, and $\beta_{j\perp,q\perp,r\perp}$ on ω/ν_{ei}^T for $k\lambda_{ei} = 1$.

FIG. 8: Dependence of the real and imaginary parts of transverse transport coefficients $\sigma_\perp$, $\alpha_\perp$ on ω/kv_{Te} (dots) for $k\lambda_{ei} = 10$ in comparison with the exact collisionless theory (47) (solid curve).

FIG. 9: Parametric (k, ω) plane for the longitudinal permittivity of plasmas. Dotted curves describe the spectra corresponding to the Langmuir (*epw*) and ionic-acoustic waves (*iaw*). References are given in the brackets.

FIG. 10: Dependence of the real and imaginary parts of $\delta\epsilon(\omega, k)$ (52) (dots) on ω/ν_{ei}^T for $k\lambda_{ei} = 0.25$ (a) and 2.25 (b) in comparison with the theory disregarding the electron-electron collisions (solid curves) [9] and the BGK model (56) (dashed curves) [3].

FIG. 11: Dependence of the real and imaginary parts of the transversal permittivity $(\epsilon^t - 1)\omega^2/\omega_{pe}^2$ (57) (dots) on ω/ν_{ei}^T for $k\lambda_{ei} = 1$ in comparison with Drude model (58). Dashed lines correspond to collisionless theory (59).

FIG. 12: Parametric (k, ω) plane for the transversal permittivity of plasmas. References are given in the brackets.

FIG. 13: Dependence of the ion-acoustic damping factor Γ_s on $k\lambda_{ei}$ for a plasma with $Z = 8$ and $Z = 64$ in comparison with a numerical solution (dots) of the Fokker-Plank equation [18]

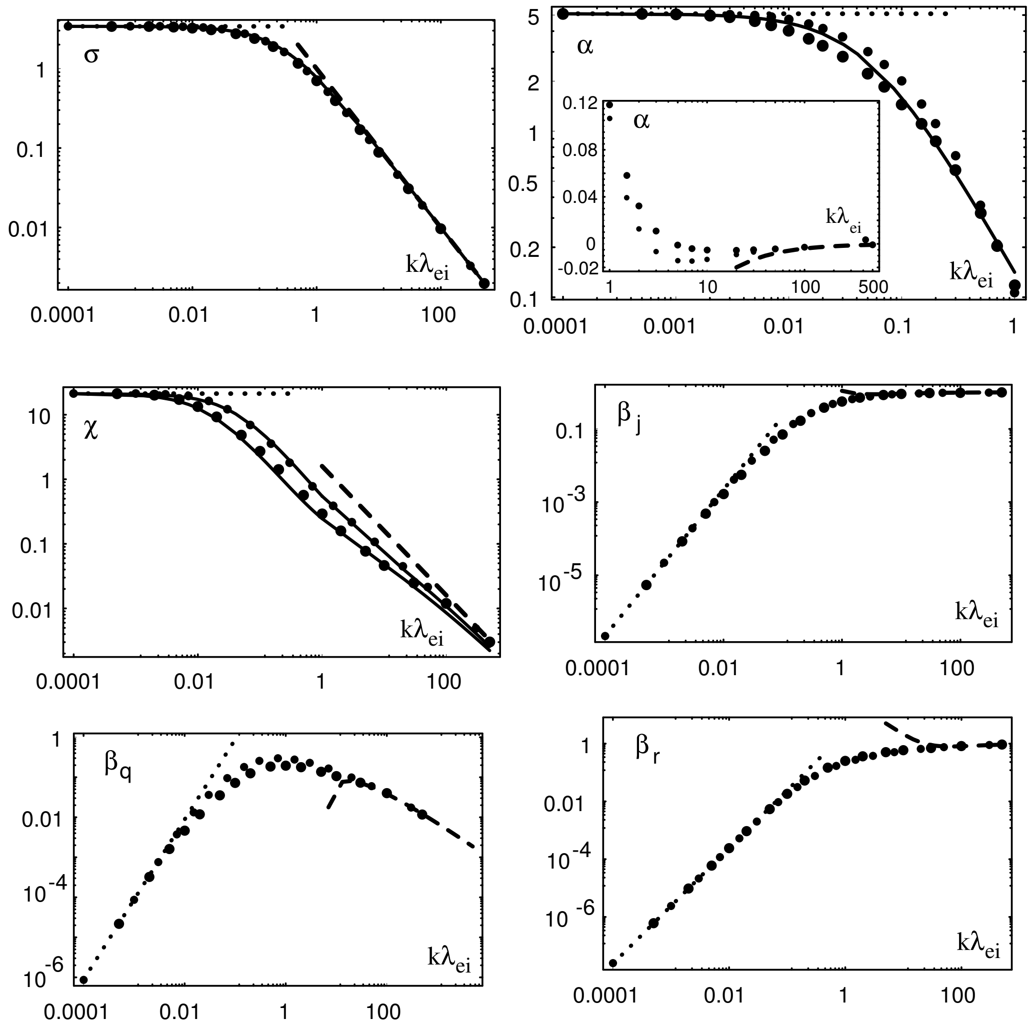

σ
α
χ
β j
β q
β r
kλei
0.0001
0.01
1
100
0.001
0.1
5
2
0.5
0.2
10
0.12
0.08
0.04
0
-0.02
500
10-3
10-5
10-4
10-6

Figure 1

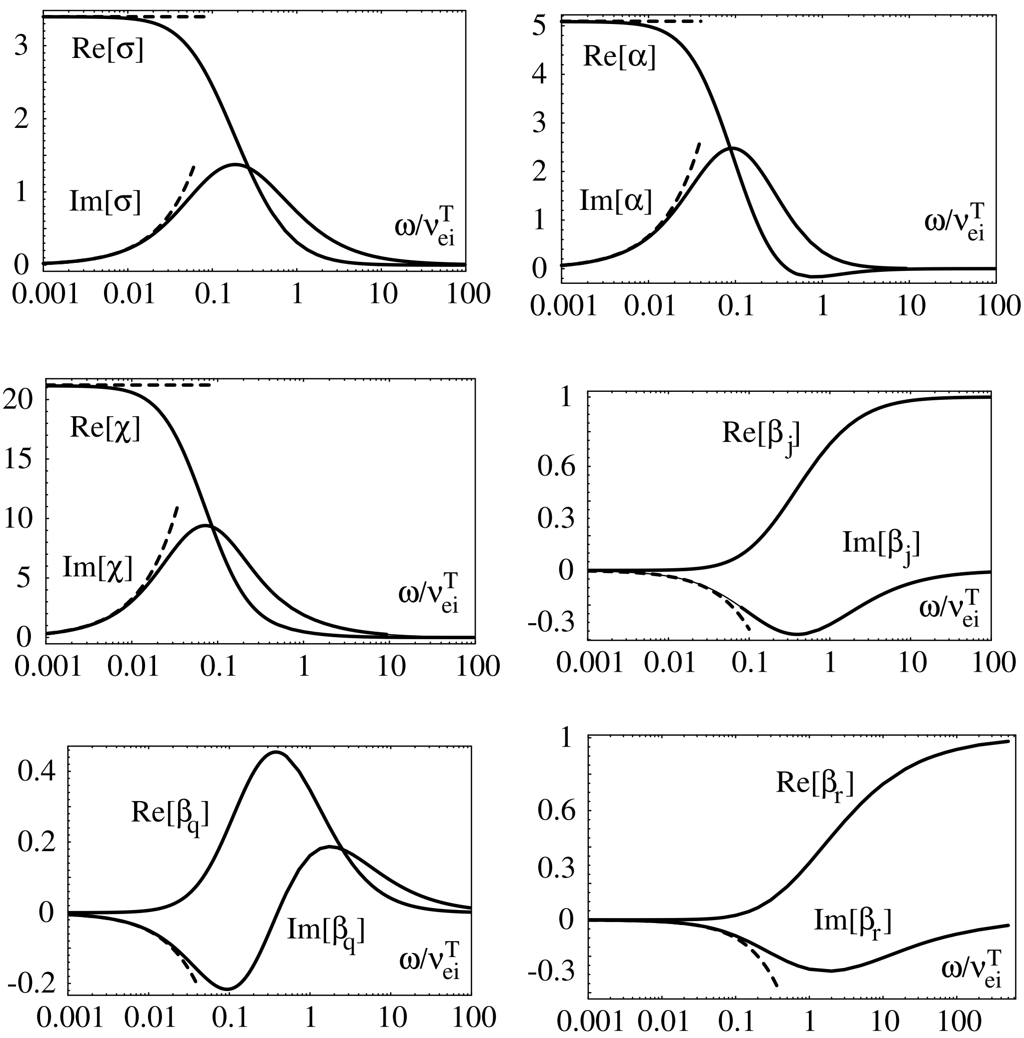


Figure 2

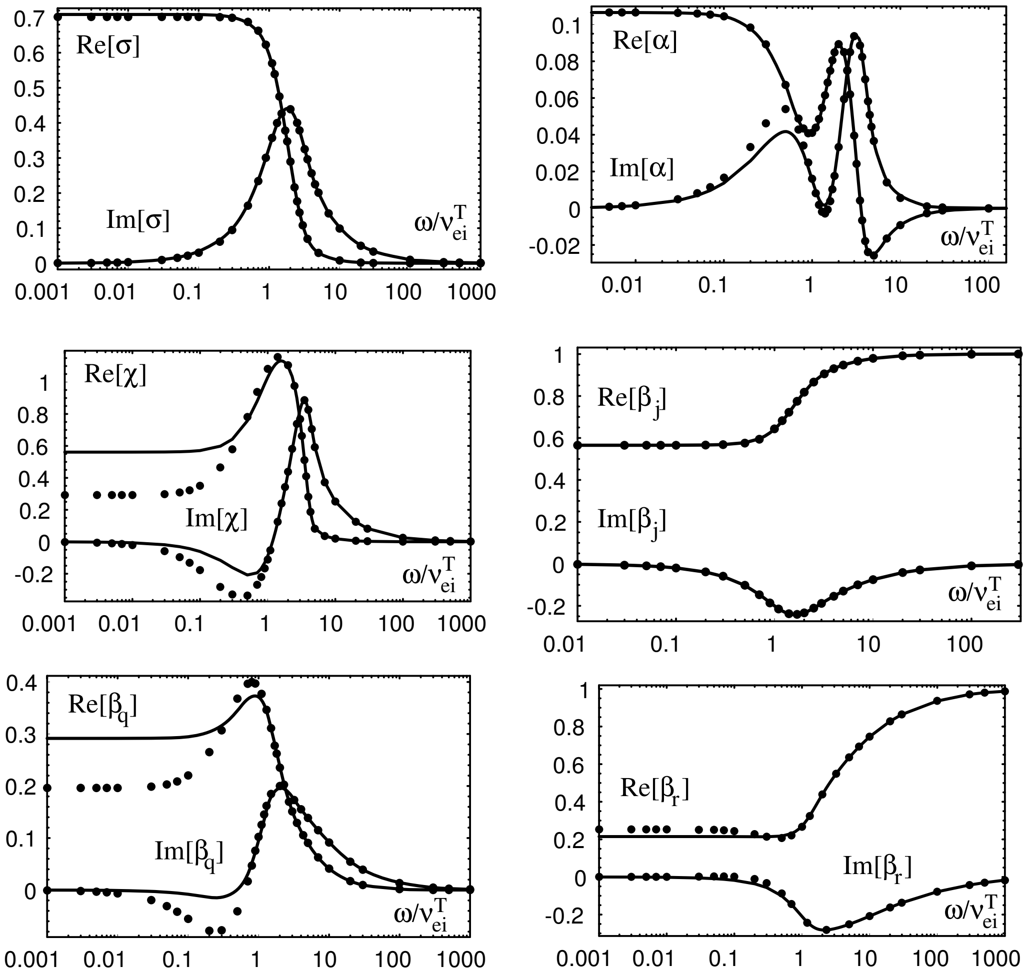


Figure 3

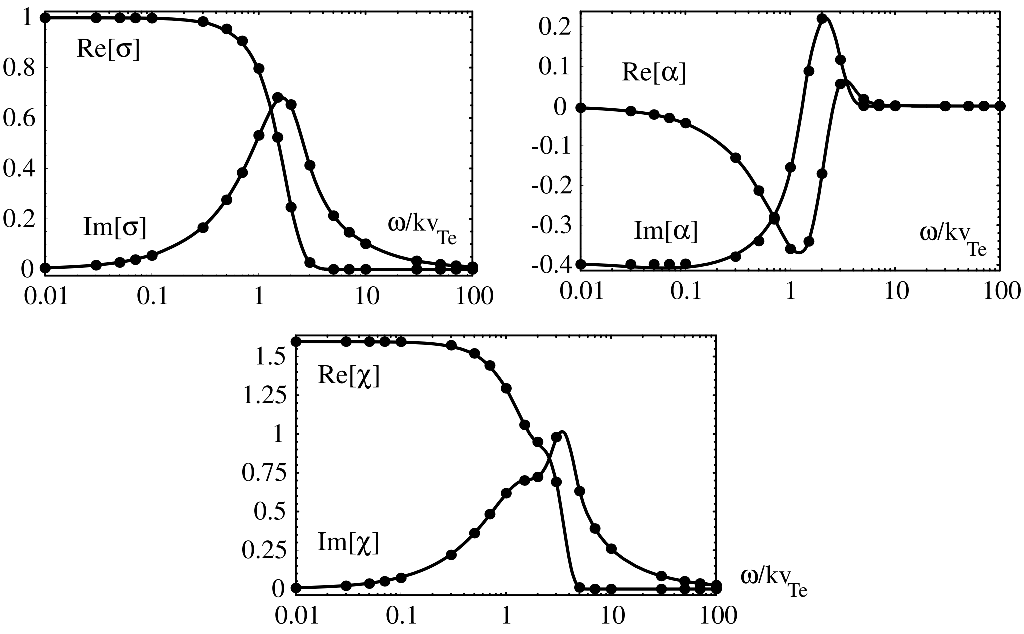
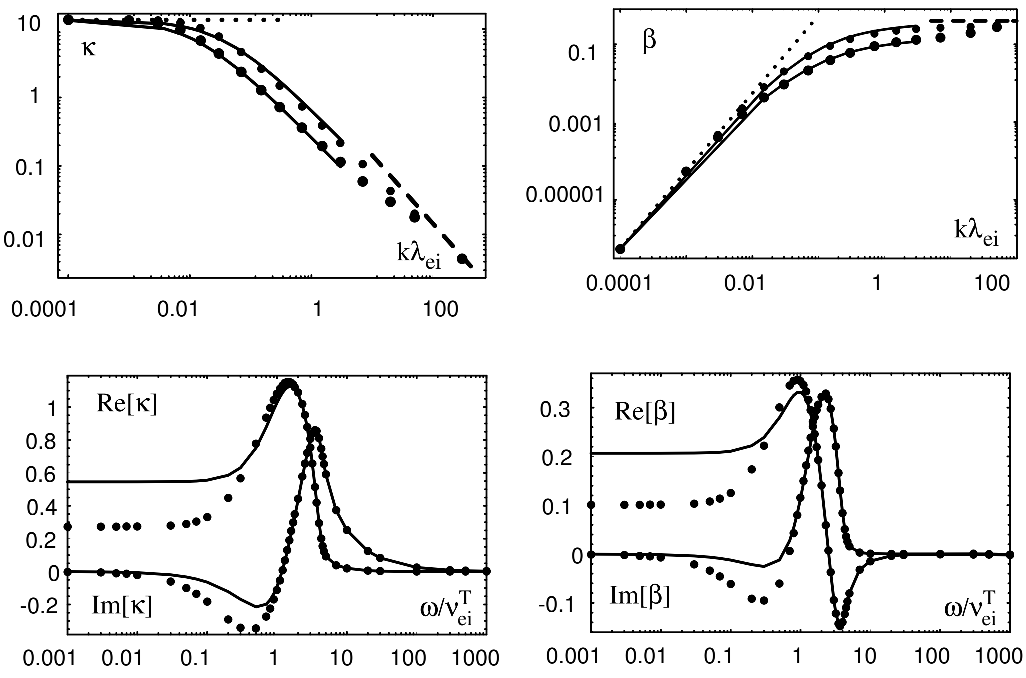

Re[σ]
Im[σ]
ω/kv_Te
1
0.8
0.6
0.4
0.2
0
0.01
0.1
1
10
100
Re[α]
Im[α]
ω/kv_Te
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0.01
0.1
1
10
100
Re[χ]
Im[χ]
ω/kv_Te
1.5
1.25
1
0.75
0.5
0.25
0
0.01
0.1
1
10
100

Figure 4



κ
10
1
0.1
0.01
kλei
0.0001
0.01
1
100
β
0.1
0.001
0.00001
kλei
0.0001
0.01
1
100
Re[κ]
Im[κ]
1
0.8
0.6
0.4
0.2
0
-0.2
ω/νeiT
0.001
0.01
0.1
1
10
100
1000
Re[β]
Im[β]
0.3
0.2
0.1
0
-0.1
ω/νeiT
0.001
0.01
0.1
1
10
100
1000

Figure 5

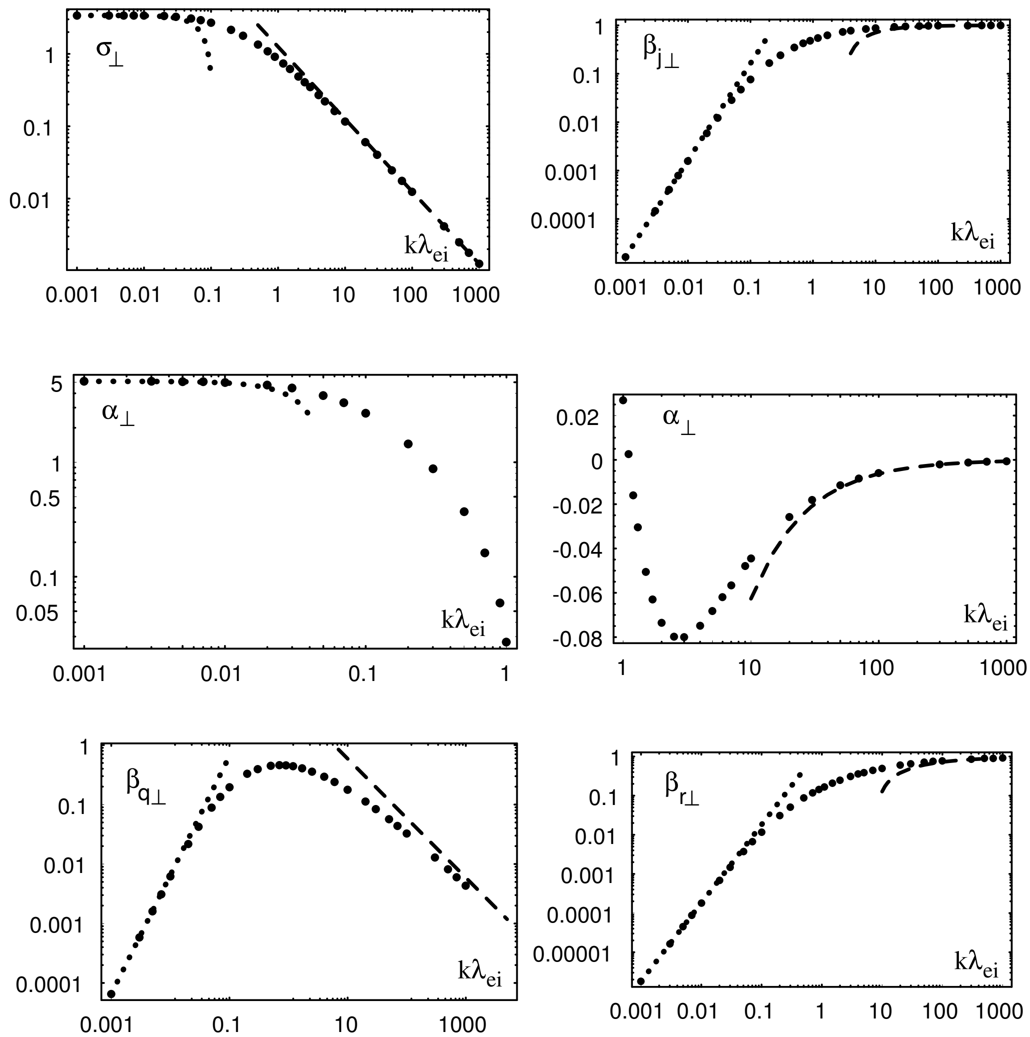

σ⊥
kλei
0.001 0.01 0.1 1 10 100 1000
1 0.1 0.01
βj⊥
1 0.1 0.01 0.001 0.0001
α⊥
5 1 0.5 0.1 0.05
α⊥
0.02 0 -0.02 -0.04 -0.06 -0.08
1 10 100 1000
βq⊥
1 0.1 0.01 0.001 0.0001
0.001 0.1 10 1000
βr⊥
1 0.1 0.01 0.001 0.0001 0.00001

Figure 6

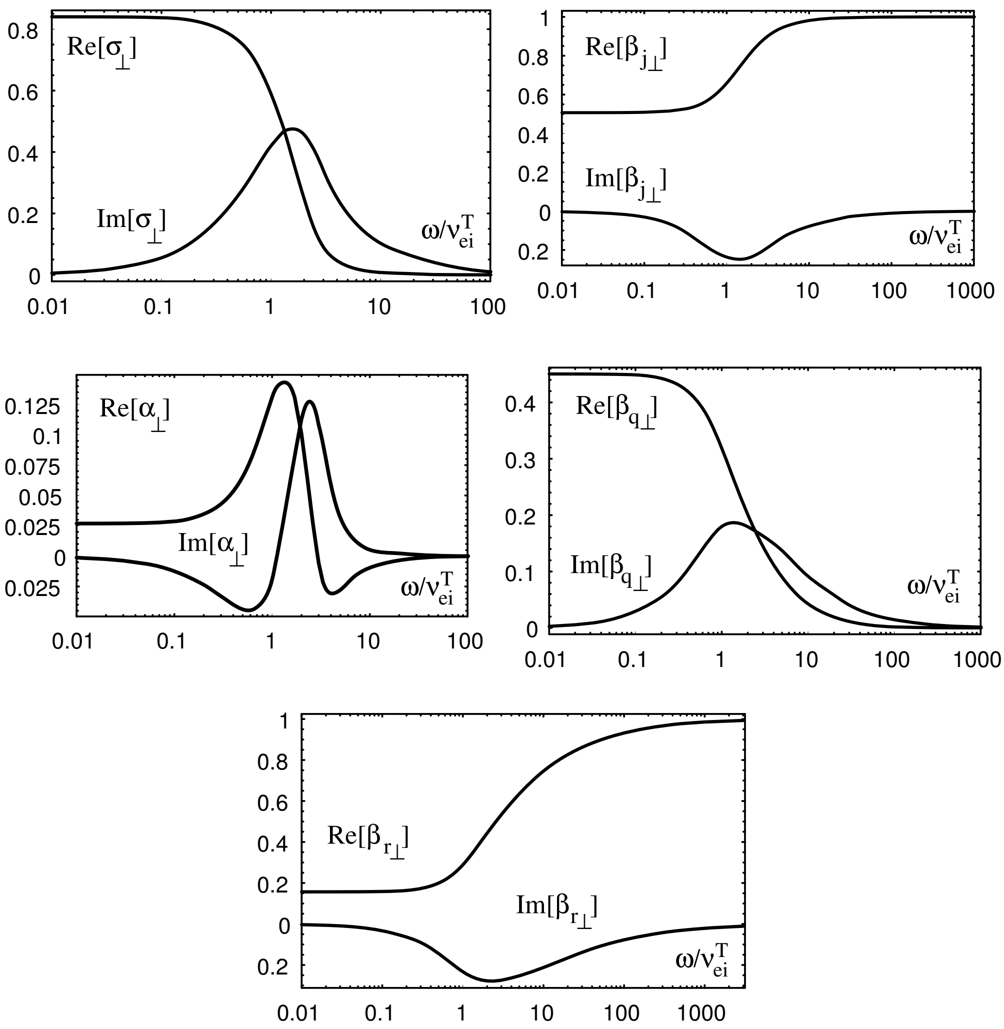
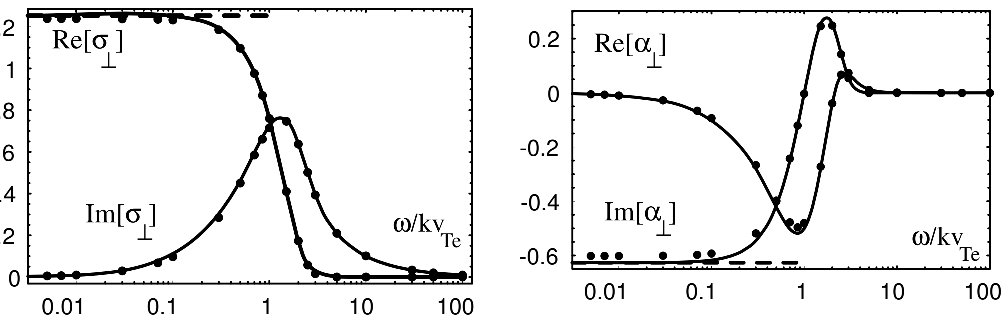

Re[σ⊥]
Im[σ⊥]
ω/ν_ei^T
0.01 0.1 1 10 100
0 0.2 0.4 0.6 0.8
Re[β_j⊥]
Im[β_j⊥]
ω/ν_ei^T
0.01 0.1 1 10 100 1000
0.2 0 0.2 0.4 0.6 0.8 1
Re[α⊥]
Im[α⊥]
ω/ν_ei^T
0.01 0.1 1 10 100
0.025 0 0.025 0.05 0.075 0.1 0.125
Re[β_q⊥]
Im[β_q⊥]
ω/ν_ei^T
0.01 0.1 1 10 100 1000
0 0.1 0.2 0.3 0.4
Re[β_r⊥]
Im[β_r⊥]
ω/ν_ei^T
0.01 0.1 1 10 100 1000
0.2 0 0.2 0.4 0.6 0.8 1

Figure 7



Re[σ⊥]
Im[σ⊥]
ω/kvTe
1.2
1
0.8
0.6
0.4
0.2
0
0.01
0.1
1
10
100
Re[α⊥]
Im[α⊥]
ω/kvTe
0.2
0
-0.2
-0.4
-0.6
0.01
0.1
1
10
100

Figure 8

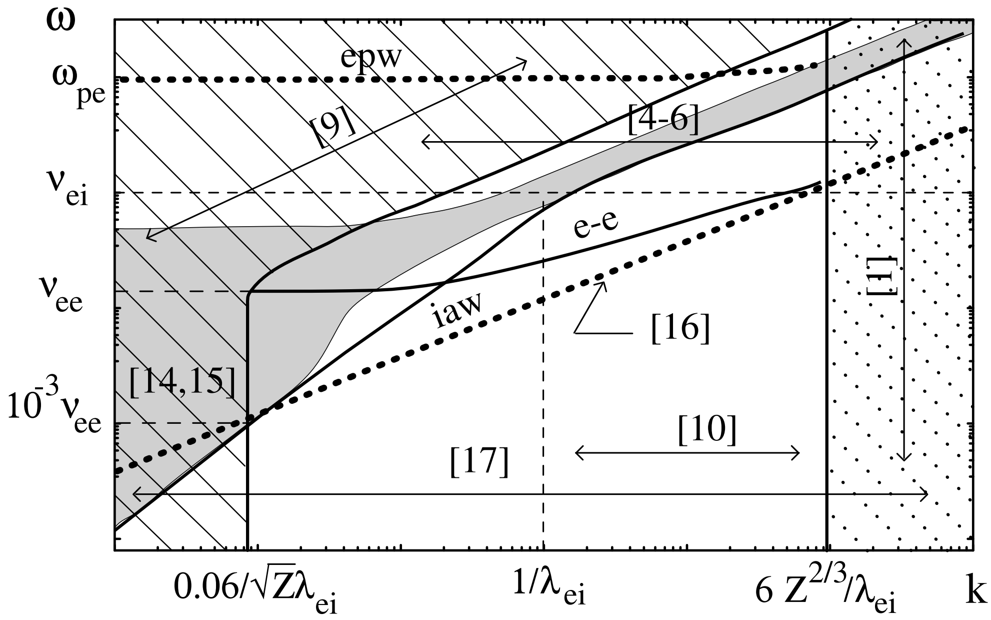
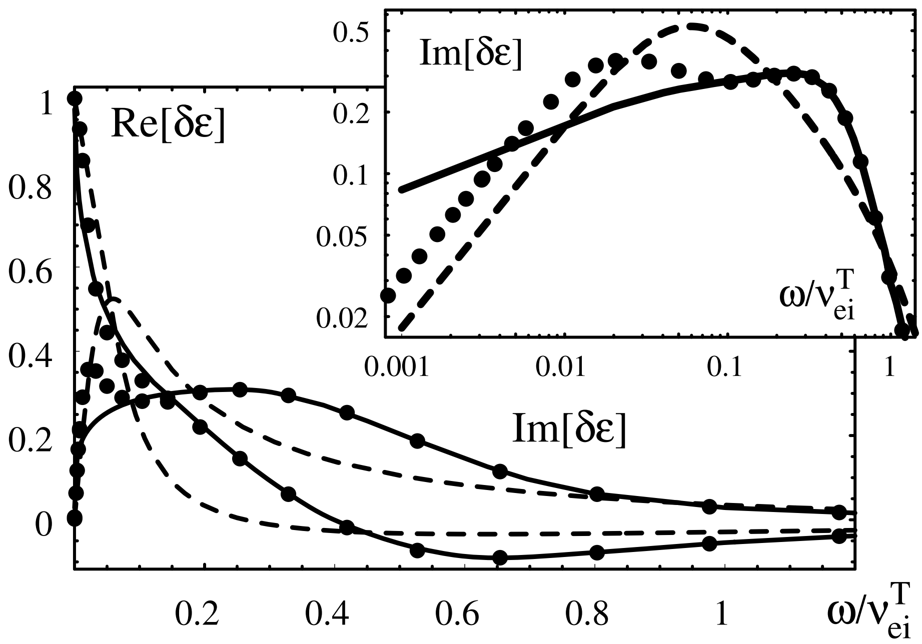
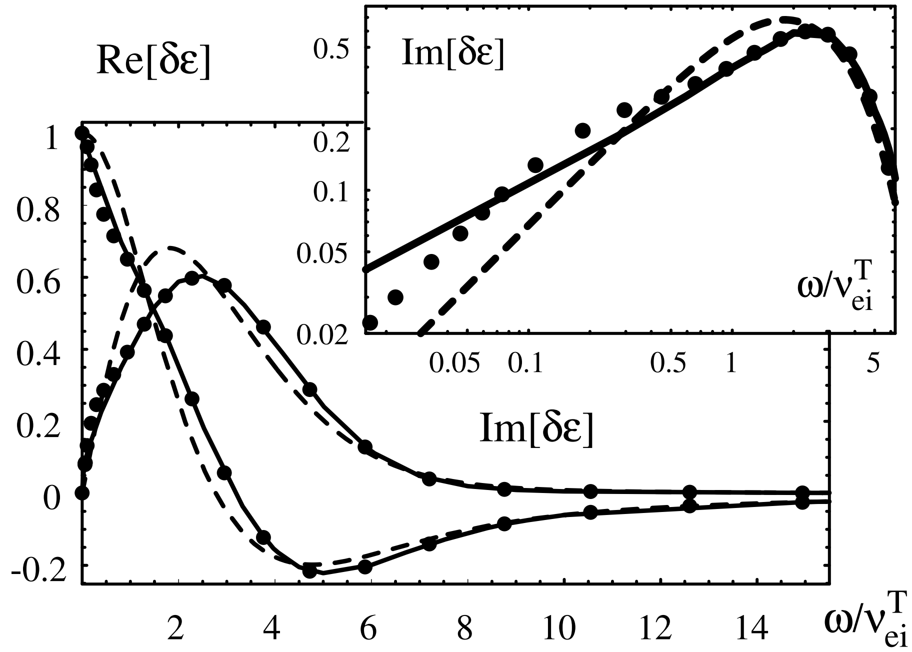

ω
epw
ω_{pe}
[9]
[4-6]
ν_{ei}
e-e
ν_{ee}
iaw
[16]
[1]
[14,15]
$10^{-3}\nu_{ee}$
[10]
[17]
$0.06/\sqrt{Z}\lambda_{ei}$
$1/\lambda_{ei}$
$6\,Z^{2/3}/\lambda_{ei}$
k

Figure 9


Re[δε]
Im[δε]
Im[δε]
ω/ν_ei^T
0.5
0.2
0.1
0.05
0.02
0.001
0.01
0.1
1
1
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1
ω/ν_ei^T

(a)


Re[δε]
Im[δε]
Im[δε]
ω/ν_ei^T
0.5
0.2
0.1
0.05
0.02
0.05
0.1
0.5
1
5
1
0.8
0.6
0.4
0.2
0
-0.2
2
4
6
8
10
12
14
ω/ν_ei^T

(b)

Figure 10

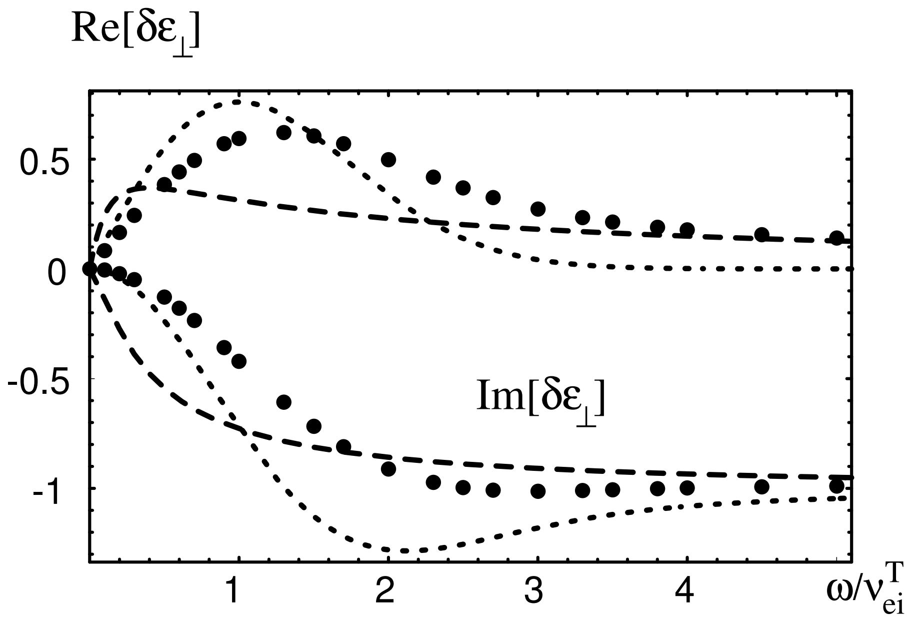


Figure 11

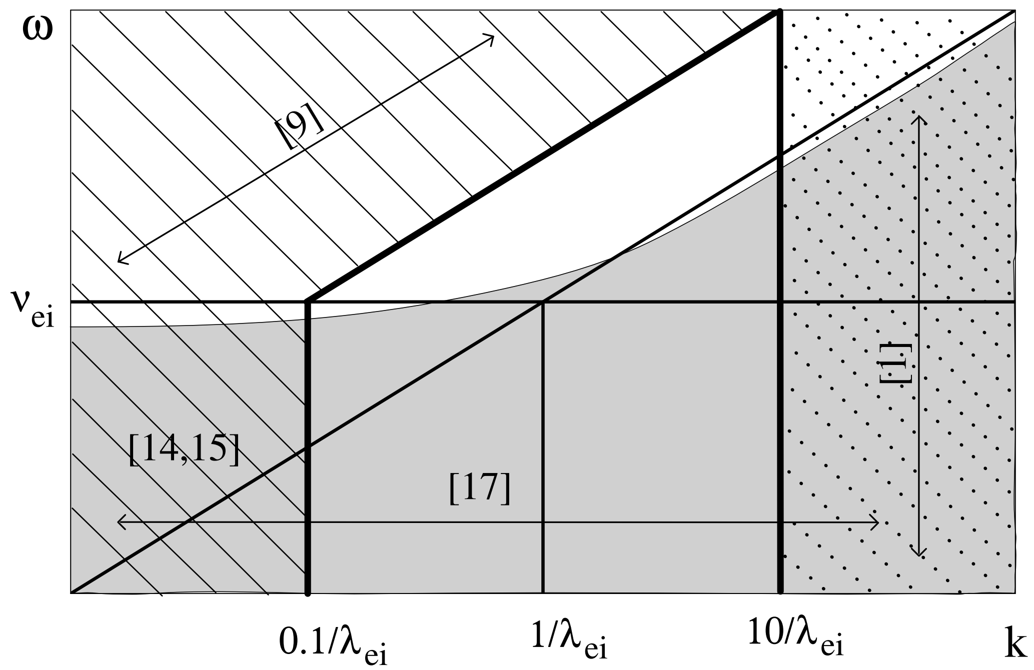


Figure 12

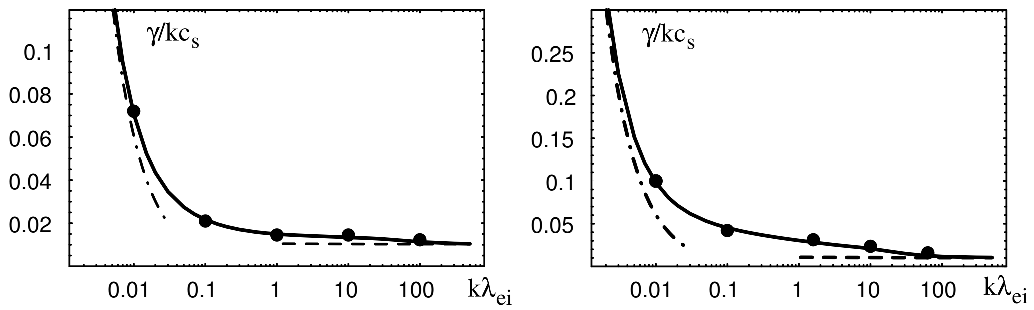

γ/kc_s
0.1
0.08
0.06
0.04
0.02
0.01
0.1
1
10
100
kλ_ei
γ/kc_s
0.25
0.2
0.15
0.1
0.05
0.01
0.1
1
10
100
kλ_ei

Figure 13