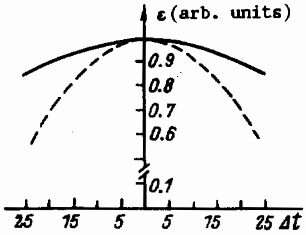

FIG. 2. Output radiation energy ε as a function of the firing nonsynchronism Δt of the thyratrons for a laser with the excitation scheme of Fig. 1a (solid line) and of Fig. 1b (dashed line).

pressure of $2.3\cdot 10^5$ Pa is 350 J, and the average power at a frequency of 100 Hz is 30 W.

When the laser was operated according to the scheme of Fig. 1a the radiated energy was 400 mJ and the average power at 100 Hz was 35 W. It should be noted that when the pulse repetition rate is changed the excitation scheme becomes desynchronized because of the difference in the starting losses in the thyratrons. In operation at a fixed frequency, however, the processes in the circuits remain steady for a long time.

In clusion we note that the two-circuit method of excitation may be used not only for increasing the pump energy, but also for optimizing the mass and size parameters of a laser or for attaining higher repetition rates. The results of this investigation of a two-circuit method of excitation gives a justification for expecting a very efficient and practical realization of a system of excitation with a large number of circuits.

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Collisionless electrode sheath in an rf discharge

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Interest in low-pressure rf discharges has increased considerably on account of their use in preparing epitaxial films. For these purposes it is important to know the dc voltage drop between the plasma and the electrode, the sheath thickness, and other of its characteristics. Estimates of these quantities have been published in Ref. 1. Numerical calculations for the sheath, by a solution of the complete system of equations[2] is quite difficult. An attempt has been made at an analytical calculation[3] based on an incorrect expression for the electron density. In the present investigation we obtain simple analytic expressions for the parameters of the sheath using the method of averaging over the fast electron motion.[4]

Let us assume that the sheath thickness L is small compared to the mean free path of the particles and the interelectrode gap and large compared to the Debye length r_D, and that the frequency of the field satisfies $\omega_i \ll \omega \ll \min(\omega_e, \nu_{Maxw})$, where ν_{Maxw} is the Maxwell time and $\omega_{i,e}$ are the ion and electron plasma frequency.

In this case the field in the plasma is much less than that in the sheath and $L \sim j/en\omega$. The displacement of the ions during a period of the field is small compared to L, since their motion is determined by the average field. The boundary of the electron profile (thickness on the order of r_D) can thus be considered sharp, and the potential drop in the sheath large compared to T. Ionization in the sehath and the initial velocities of the ions can be neglected. Then the ion flux in the sheath is conserved and $\Gamma_i = \gamma n_0 (T/M)^{\frac{1}{2}}$, where n_0 is the ion density ahead of the sheath, T is the electron temperature, and γ is a number of the order of unity and depends on the nature of the distribution function. The density of ions in the sheath is

$$n(x) = A/\sqrt{\varphi(x)}, \qquad (1)$$

where $A = \gamma n_0 \sqrt{T/2e}$ and $\phi(x)$ is the dc potential in the sheath.

Writing the current density as $j = -j_0 \sin \omega t$ and introducing the new variable $z(x) = \omega t(x)$, the phase at which the boundary of the plasma sheath reaches the point x, we find a closed system of equations[4] for $\phi(x)$:

$$\begin{aligned} d\varphi/dx &= 4j_0/\omega\,(\sin z - z\cos z), \\ \sin z\, dz/dx &= e\omega A/(j_0 \varphi^{1/2}), \end{aligned} \qquad (2)$$

where $z = 0$ corresponds to the plasma and $z = \pi$ corresponds to the electrode.

The solution is found in parametric form

$$\varphi(x)^{1/2} = 2j_0/(e\omega A)\left(z/2\,(1 + 1/2\cos 2z)^{-3/8} \sin 2z\right),$$

$$x = j_0^3/(6e^2\omega^3 A^2)\left[-z\cos z(3+2\cos^2 z)+5\sin z-{}^{11}\!/_3\sin^3 z\right]. \quad (3)$$

It is convenient to write the following relation between the sheath thickness and the dc potential drop ϕ_0 in the sheath.

$$L = {}^5\!/_9\left({}^2\!/_{3\pi}\right)^{1/2}\left(\varphi_0^3/e^2A^2\right)^{1/4}. \quad (4)$$

We note that as the discharge current increases the sheath thickness increases in proportion to j_0, and if the rf voltage is dropped mainly in the sheaths, then $L \sim U^{1/2} \sim j_0$.

Figure 1 (curve 1) shows the profile of the dc potential in the sheath (3) as compared with the resutls of numerical calculation [5] (curve 2).

Although the calculation in Ref. 5 was carried out for the case of a given sinusoidal voltage on a single sheath, the calculated $\phi(x)$ and the thickness of the sheath for various value sof A and ϕ_0 differ little from (3) and (4). Formula (3) is accurate up to terms of order $(T/e\phi)^{1/2}$. It can be seen from Fig. 1 that the function $\phi(x)/\phi_0 = (x/L)^{10/7}$ (curve 3), which is an accurate solution for small x, gives a good approximation to the profile over the whole sheath.

With the potential profile $\phi(x)$ that has been found it is easy to find the ac field in the sheath

$$E(x,t) = \left(4\pi j_0/\omega\right)\cdot(1+\cos\omega t) - 4\pi e\int_x^L n\,dx \quad (5)$$

or with the use of (2)

$$E(x,t) = \left(4\pi j_0/\omega\right)(\cos\omega t - \cos z(x)), \quad (6)$$

whre z varies with ωt to π.

Integrating (6) we find that in a discharge between identical plane-parallel electrodes (symmetric case) the total rf voltage drop in the sheaths is

$$U(\omega t) = {}^{4\pi j_0}\!/_{\omega}\left[\left(\int_{\omega t}^{\pi}(\cos\omega t-\cos z)+\int_{(\pi-\omega t)}^{\pi}(\cos\omega t+\cos z)\right){dx(z)}/{dz}\,dz\right]. \quad (7)$$

A curve of $U(\omega t)$ is plotted in Fig. 2. It can be seen that $U(\omega t)$ is an anharmonic function. Since $U(\omega t)$ is an even function and $U(\pi/2 - \omega t) = -U(\pi/2+\omega t)$, we have expanded $U(\omega t)$ in the functions $\cos[(2n + 1)\omega t]$. For example, the amplitude of the third harmonic (n = 1) is 2.3% of that of the first harmonic. The voltage in the discharge is 3.3 times higher than the dc drop in the sheath. This is very close to the value π predicted and verified experimentally in Ref. 1.

Result (7) is not difficult to generalize to the case of an arbitrary nonsinusoidal periodic variation of the current density. If we assume that $j = j_0 f(z)$, where $f(z) = -f(-z)$, we obtain instead of (7) the result

$$U(\omega t) = 2j_0^3/e^2\omega^3\left[\int_{\omega t}^{\pi}\{F(\omega t)-F(z)\}A(z)\,dz - \int_{(\pi-\omega t)}^{\pi}\{F(\pi-\omega t)-F(z)\}A(z)\,dz\right],\quad F(z) = -\int_z^{\pi} f(z)\,dz,$$

$$A(z) = -f(z)\int_0^z f(z')\,dz\int_0^{z'} z'' f(z'')\,dz'', \quad (8)$$

It can be seen that it is a much more complicated matter to detrmine f(z) for a sinusoidal U(z) then to find U(z) for a sinusoidal f(z) [expression (7)].

The case of a highly asymmetric discharge has been studied in detail in Refs. 2 and 5. In these cases practically all the voltage U is dropped in the sheath adjacent to the smaller electrode. In the case of a sinusoidal variation

$$U(\omega t) = U_{DC} + U_0\cos\omega t,$$

$$\text{где}\quad U_{DC} = -U_0 + (T/2e)\ln(2\pi m\gamma^2/M) + (T/2e)\ln(U_0 e 2\pi/T), \quad (9)$$

the current density will be highly nonsinusoidal. Let us examine qualitatively the form of this dependence.

For $\pi/2 < \omega t < 3\pi/2$ the potential drop in the sheath is large and the boundary of the electron profile is located where n(x) falls off sharply (1). The distance from the smaller electrode to this boundary differs little from the total thickness L of the sheath. Therefore the displacement current density to the electrode is $\sim(I/L)\partial U/\partial t$.

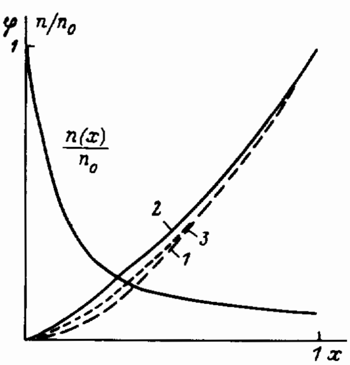

FIG. 1. dc potential and density in the sheath.

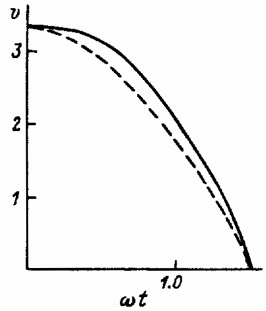

FIG. 2. rf potential drop in the sheath of a symmetric discharge. The dashed line corresponds to a sinusoidal dependence.

It can be seen from the numerical calculations of Ref. 2 that near the phase $\omega t = 0$, where $eU \sim T$ and the electrons approach a distance $\sim r_D$ from the electrode, features appear in the displacement current behavior. On this scale, the ion density can be considered constant $n_i = n_i(x = L)$. By integrating Poisson's equation we find the electric field at the electrode

$$E(t) = \pm\left(4\pi e n_i \left(T/e\left(e^{\frac{e\varphi(t)}{T}} - 1\right) - \varphi(t)\right)\right)^{1/2}, \tag{10}$$

where $\varphi(t) = U(t) + \left(T/2e\right)\ln\left(2eU_0/r^2T\right)$ is the potential difference between the electrode and the plasma, and $U(t)$ is given by formula (10).

The displacement current at the electrode is

$$j(t) = \left(\frac{e n_i U_0}{4\pi}\right)^{1/2} \omega G(t),$$

$$G(t) = \pm\frac{\left(\frac{eU_0}{T}\right)^{1/2}\left(1 - \exp\left(e\varphi(t)/T\right)\right)}{\left(\exp\left(e\varphi(t)/T\right) - 1 - e\varphi(t)/T\right)^{1/2}}. \tag{11}$$

The function $G(t)$ is plotted in Fig. 3. Far from $\omega t = 0$ the function $j(t)$ depends only weakly on t, but near the time that $\omega t = 0$, for $eU_0/T < (M/2m)^{1/2}/2\pi$ the displacement current increases sharply in a time of the order of

$$\omega t_0 = \left(\frac{2\varphi_3}{U_0\omega^2}\right)^{1/2}, \quad \text{where } \varphi_3 = \begin{cases} \Phi(t=0) & \text{for } |e\varphi(t=0)| > T \\ T/e & \text{for } |e\varphi(t=0)| \leqslant T \end{cases} \tag{12}$$

in agreement with the resutls of Ref. 2 (see Fig. 3). In the opposite case expression (11) also gives a sharp increase in the displacement current for $\omega t \approx 0$. However, at this intant the electrons are conducting, so that the displacement current at the electrode is much less than the electron current.

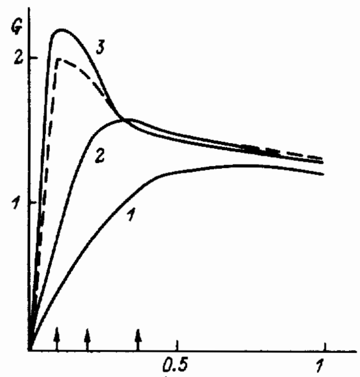

FIG. 3. Dimensionless displacement current (6) as a function of ωt in xenon for the following values of eU_0/T: 1) 17; 2) 48; 3) 199. The solid lines show the calculation from formula (11) and the dashed line is the calculation from the formula of Ref. 2. In cases 1 and 2 the resuts of (11) and of Ref. 2 coincide. The arrows show the times defined by (12).

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