Low-Pressure RF Discharge in the Free-Flight Regime

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Abstract—**The self-consistent equations system for low-pressure RF discharge in the free-flight regime is formulated. The expressions for the electron energy diffusion coefficient due to electron-neutral collisions and to the electron collisions with the plasma–space charge moving boundary (stochastic heating) are derived. If electron–neutral elastic collisions frequency exceeds the inelastic one, the generalization of conventional two-term approximation for the electron distribution function (EDF) is possible and the space-time-averaged electron kinetic equation can be reduced to the one-dimensional energy diffusion one. The fast electrons escaped to the electrodes can be also accounted for in this equation. It's shown, that in the cases of: (a) spatially uniform ion profile, (b) for frequencies that are small compared with the electron bounce frequency, and (c) for frequencies exceeding the electron plasma one in the sheath, the stochastic heating vanishes.**

I. Introduction

In this paper we present self-consistent equations system for description of the RF discharge in the free-flight regime, when the electron mean-free path λ exceeds the discharge gap $2L_0$. As it was pointed out in [1], in such discharges the stochastic electron heating [2] can be effective and unambiguously identified.

This problem was explored analytically in [3] and numerically in [4]–[7]. In an attempt of self-consistent calculation [3], the inelastic collisions and average plasma field $\phi(x)$ were neglected. So it was impossible to describe plasma inhomogeneity and slow electrons' behavior. In [4] also it was set, $\phi(x) = 0$. It implies the uniform ion-density profile $n_i(x)$. But in real inhomogeneous quasi-neutral plasma, the ambipolar potential $\phi(x)$ always exists. The $\phi(x)$ profile corresponds to the potential well for electrons. The low-energy electrons are trapped by this field and do not achieve the peripheral region, where the stochastic heating occurs. The heating of these electrons is produced by low bulk plasma fields and infrequent collisions. So the low-energy electrons that form the substantial part of the total electron density in the central region have essentially a smaller energy diffusion coefficient than high-energy ones. So there are two different energy scales of the electron distribution and EDF (electron distribution function) in this regime (and in the collisional nonlocal case [1]) is enriched by the slow and fast electrons [8]. The ion-density profile form is also very important for the stochastic heating's correct calculation. For example, if ion concentration $n_i(x)$ is uniform, the speed of the plasma–sheath boundary is $V_{\text{sh}}(t) = j(t)/en_i$, where j is the discharge current density. In the noninertial reference frame moving with the plasma–sheath boundary, the electric force is fully compensated by the electron inertia and stochastic heating in the free-flight limit disappears. For consistent description of this interesting phenomenon the form of $n_i(x)$ profile is important.

The field profile in the sheath in [3] was treated in rather crude approximation. The obtained expression for the energy diffusion coefficient is applicable only if the electron plasma frequency exceeds a discharge of one ω.

In numerical simulations [5], [6], the bounce frequency $\Omega = L_0/v$ was comparable with the collision one. So it was difficult to separate the collisional and stochastic mechanisms. In [7], the totally collisionless one-dimensional case was investigated. In atomic gases the electron–atomic collisions can be unambiguously subdivided into elastic with transport frequency ν, and inelastic ones that are switched on at energy threshold ϵ^* and are characterized by collision frequency ν^*. At not too high RF field intensities, the distribution tail at energies that are exceeding ϵ^* contains a small part of the total electron number. So in inelastic collision the fast electron loses almost all its energy. For not too high electron energy, the inelastic collision frequency ν^* is small compared with the elastic one ν. That's why during its energy relaxation time ν^{*-1} electron undergoes many elastic collisions, and the EDF tail is close to isotropic (contrary to the model adopted in [3], [7]). The EDF body (at energies less than ϵ^*) energy relaxation time far exceeds ν^{*-1}; so we can also consider EDF here as isotropic.

The electron movement in calculations [7] was not stochastic, because the chaotic motion criterion (see (1) and (29)) was not fulfilled. So the energy diffusion was absent and the electron energy could not exceed significantly $\sim mV_{\text{sh}}^2$. For the self-sustained discharge, the sheath velocity was to be

$$V_{\text{sh}} \sim \sqrt{\frac{2\epsilon_i}{m}}$$

where ϵ_i is the ionization potential. Close values were obtained in [7].

In this paper, the self-consistent equation system for the free-flight-regime RF discharge is formulated. The space-time-averaged electron kinetic equation as a rule can be reduced to a form of the one-dimensional energy diffusion equation. The fast electrons losses caused by their attachment to the electrode surface and by the inelastic collisions are also accounted for.

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The slow electrons are trapped by the stationary electric field and do not reach the moving of the space-charge boundary. So their energy diffusion coefficient D_ϵ is small—it is determined by the low oscillatory field in the discharge center and by rare collisions. The D_ϵ for the fast electrons is connected primarily with the stochastic collisionless heating [2], [3] if

$$\omega \geq \Omega(M/m)^{1/4} \tag{1}$$

where M, m are the ion and electron masses, respectively. The stochastic heating switching can be treated as instantaneous. The boundary thickness is of the order of local Debye radius. Accordingly, this heating mechanism switches off if the field frequency ω exceeds ω_{0e}, the local electron plasma one in the sheath. If the opposite to (1) inequality holds, the stochastic heating is absent and D_ϵ is determined by collisions.

In Section IV, ion motion at $\omega_{0i} \ll \omega$ is discussed (ω_{0i} is the ion-plasma frequency). The modified Bohm criterion for the plasma–sheath boundary position for this case is derived.

II. Space-Time-Averaged Kinetic Equation for the Electron-Energy Distribution Function in the Free-Flight Regime (HF Case ($\omega \gg \Omega \gg \nu$))

A. The Trapped Plasma Electrons' Collisional Heating

It is widely known [1] that the electric field in RF discharge is very inhomogeneous. It is small in the quasi-neutral plasma region where $n_e \approx n_i$ which occupies the central part of the discharge and steeply rises in the sheath region at the discharge periphery, where $n_e \approx 0$. This sharp transition region moves periodically between $x = L_p = (L_0 - L)$ and $x = L_0$, where L is the sheath thickness. We shall describe this boundary as a rigid moving wall (Fig. 1(a)). So in the arbitrary sheath point $L_p < x < L_0$, electrons are present only in the plasma phase (between the moments $t_1(x)$ and $t_2(x)$; Fig. 1(b)). In the plasma at $-L_p < x < L_p$, the time-averaged field is

$$\langle E(x)\rangle = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} E(x,t)\,dt \tag{2}$$

and the oscillatory part of the field is

$$\tilde{E}(x,t) = E(x,t) - \langle E(x)\rangle. \tag{3}$$

In the sheath, the averaged electric field that enters in the electron kinetic equation is

$$\langle E(x)\rangle = \int_{t_1(x)}^{t_2(x)} E(x,t)\,dt/(t_2 - t_1). \tag{2'}$$

Introducing potential energy $e\phi(x)$ that corresponds to the field ((2) and (2')), we can write the kinetic equation for the distribution function $f(v,x,t)$ in the form:

$$\frac{\partial f}{\partial t} + v_x\frac{\partial f}{\partial x} - \left(\frac{e\tilde{E}}{m} + \frac{e\,d\phi}{m\,dx}\right)\frac{\partial f}{\partial v_x} = S(f) + S^*(f) \tag{4}$$

where x is the coordinate in the current direction, v is the electron velocity, and $S(f)$, $S^*(f)$ are the elastic and inelastic

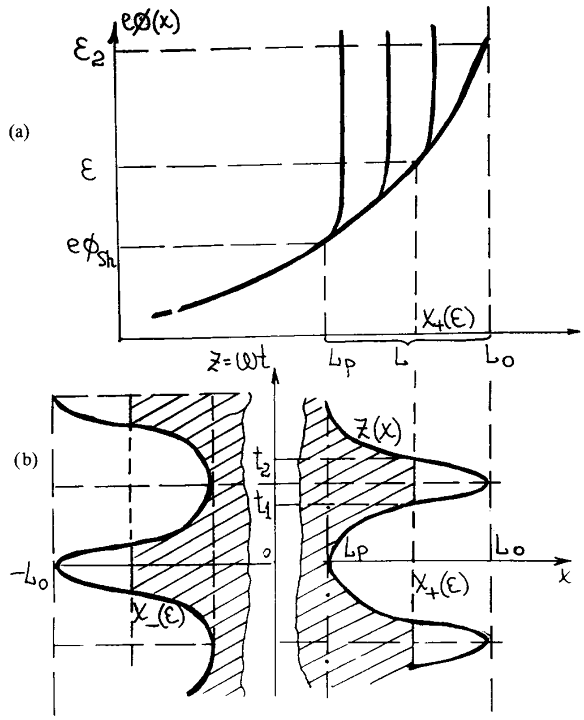

Fig. 1. (a) Schematic dependence of averaged electric plasma potential (curve) and position of the plasma space-charge boundary (vertical lines) in various moments. (b) The plasma space-charge boundary motion in the sheath.

collisional integrals, $\langle\tilde{E}\rangle = 0$. The averaging procedure described below represents a generalization of the method described in [9]. With the substitution

$$\epsilon_x = \frac{mv_x^2}{2} + e\phi, \qquad v_x = v_x(\epsilon_x, x) \tag{5}$$

(4) takes the form:

$$\frac{\partial f}{\partial t} + v_x\left(\frac{\partial f}{\partial t}\right)_{\epsilon_x} = e\tilde{E}v_x\frac{\partial f}{\partial \epsilon_x} + S(f) + S^*(f). \tag{6}$$

Separating the time-averaged and oscillating parts of (6), we have:

$$v_x\left(\frac{\partial\langle f\rangle}{\partial x}\right)_{\epsilon_x} = \left\langle e\tilde{E}v_x\frac{\partial\tilde{f}}{\partial\epsilon_x}\right\rangle + S(\langle f\rangle) + S^*(\langle f\rangle) \tag{7}$$

$$\frac{\partial\tilde{f}}{\partial t} + v_x\left(\frac{\partial\tilde{f}}{\partial x}\right)_{\epsilon_x} = e\tilde{E}v_x\frac{\partial\langle f\rangle}{\partial\epsilon_x} + S(\tilde{f}) + S^*(\tilde{f}) \tag{8}$$

where $\tilde{f} = f - \langle f\rangle, \langle f\rangle(x,\epsilon_x,v_y,v_z)$. In order to solve (8), we admit the widely used approximation for $S(\tilde{f})$ [10],

$$S(\tilde{f}) = -\nu(v)\tilde{f}$$

where ν is the transport collision frequency, $v = |\vec{v}|$. For the harmonical field $\tilde{E}(x,t) = E_0(x)e^{i\omega t}$, neglecting $S^*(\tilde{f})$ (as $\nu \gg \nu^*$), for large frequencies $\omega \gg \Omega$, (8) can be easily solved:

$$\tilde{f} = \frac{e\tilde{E}v_x}{(\nu + i\omega)}\frac{\partial\langle f\rangle}{\partial\epsilon_x}. \tag{9}$$

So the equation for $\langle f\rangle$ is

$$v_x\left(\frac{\partial\langle f\rangle}{\partial x}\right)_{\epsilon_x}=v_x\frac{\partial}{\partial\epsilon_x}\frac{e^2E_0^2(x)}{2}\frac{\nu v_x}{\nu^2+\omega^2}\frac{\partial\langle f\rangle}{\partial\epsilon_x}+S(\langle f\rangle)+S^*(\langle f\rangle). \tag{10}$$

In the free-flight regime $L\ll\lambda$, the left-hand side dominates. It vanishes if $\langle f\rangle$ is an arbitrary function from only three arguments: $\langle f\rangle=F(\epsilon_x,v_y,v_z)$.

Let us consider the trapped electrons with $\epsilon_x<e\phi_{\rm sh}=e\phi(L_p)$ being the potential of the plasma–sheath transition (see below, Section IV). The condition $v_x(\epsilon_x,x)=0$ determines two turning points, $x_\pm(\epsilon_x)$ (Fig. 1). Integrating (9) from x_- to x_+ corresponds to the space-averaging over the part of the discharge cross section available for a given electron. After this integration of (9), the equation for function $F(\epsilon_x,v_y,v_z)$ results:

$$\hat{L}F=\int_{x_-(\epsilon_x)}^{x_+(\epsilon_x)}dx\frac{\partial}{\partial\epsilon_x}\frac{e^2E_0^2(x)}{2}\frac{\nu v_x}{\omega^2+\nu^2}\frac{\partial F}{\partial\epsilon_x}+\int_{x_-}^{x_+}S^*(F)\,dx/v_x \tag{11}$$

where

$$\hat{L}F=-\int_{x_-(\epsilon_x)}^{x_+(\epsilon_x)}S(F)\,dx/v_x.$$

If the elastic-collision frequency ν exceeds the inelastic one, $-S(F)\sim\nu F$ and $\hat{L}F$ dominates in (11). So the main part of F depends from the sole argument $\epsilon=\epsilon_x+m(v_y^2+v_z^2)/2 and F(\epsilon_x,v_y,v_z)=f_0(\epsilon)+\delta F(\epsilon_x,v_y,v_z)$, where $\delta F/F\ll 1;\hat{L}F=0,\hat{L}F\simeq 2m/Mf_0$. As a rule, at low pressure the energy losses in elastic collisions are negligible. Consequently, the equation for $f_0(\epsilon)$ is

$$\iint_{m/2(v_y^2+v_z^2)<\epsilon}dv_y\,dv_z\left[\int_{x_-(\epsilon_x)}^{x_+(\epsilon_x)}dx\frac{\partial}{\partial\epsilon_x}\left[\frac{e_x^2E_0^2}{2}\frac{(x)\nu v_x}{\nu^2+\omega^2}\frac{\partial}{\partial\epsilon}f_0(\epsilon)\right]+\int_{x_-}^{x_+}\frac{dx}{v_x}S^*(f_0)\right]=0.$$

Changing the integrating order, we have:

$$\frac{\partial}{\partial\epsilon}\int_{x_-(\epsilon)}^{x_+(\epsilon)}dx\int_0^{\sqrt{2(\epsilon-e\phi(x))/m}}dv_xD_c(v,x)v^2m\frac{\partial}{\partial\epsilon}f_0(\epsilon)+\int_{x_-(\epsilon)}^{x_+(\epsilon)}dx\int_0^{\sqrt{2(\epsilon-e\phi)/m}}dv_xS^*(f_0)=0 \tag{12}$$

where

$$D_c(v,x)=\frac{(eE_0(x))^2}{\nu^2+\omega^2}\frac{\nu}{2m^2} \tag{13}$$

is the collisional velocity diffusion coefficient.

If in the inelastic collision integral only one excitation level with threshold ϵ^* is important,

$$S^*(f_0)=-\nu(w)f_0(\epsilon)+\frac{\sqrt{w+\epsilon^*}}{\sqrt{w}}\nu^*(w+\epsilon^*)f_0(\epsilon+\epsilon^*)$$

where $w=w(\epsilon,x)=mv^2/2$. So the equation for f_0 coincides formally with the conventional local one [1]:

$$\frac{\partial}{\partial\epsilon}\overline{\sqrt{w}D_{\epsilon c}}\frac{\partial f_0}{\partial\epsilon}=\overline{\sqrt{w}\nu^*}f_0(\epsilon)-\overline{\sqrt{w+\epsilon^*}\nu^*(w+\epsilon^*)}f_0(\epsilon+\epsilon^*) \tag{14}$$

where the energy diffusion coefficient is

$$D_{\epsilon c}=\frac{\nu}{\nu^2+\omega^2}\frac{(eE_0)^2w}{3m}$$

and the upper dash denotes spatial averaging:

$$\overline{G}=\int_{x_-(\epsilon)}^{x_+(\epsilon)}G\,dx/(2L_0).$$

B. Fast Electrons' Escape to the Electrode Surface

For high-energy electrons $\epsilon>e\varphi_{\rm sh}$ that contribute to the electron density in the sheath during the plasma phase of it, the main part of the EDF is also a function of ϵ. But the space-time averaging of $\tilde{E}^2$ and of the inelastic collision frequency in (14) are to be fulfilled over the complicated area in the (x,t) plane (it is bounded by thick lines in Fig. 1(b), where ϵ_2 is the minimal electrode potential (Fig. 1(a))).

If we neglect the higher harmonics generation (according to [11] and [12] for harmonic dependence $j=j_0\sin\omega t$, the higher voltage harmonics are of an order of several percent), the voltage between the discharge center and electrode can be approximated as

$$U(t)=U_0(1-\cos\omega t)+\epsilon_2. \tag{15}$$

For the high-energy electrons it is necessary to introduce two additional terms in (14) that describe the fast electrons' escape to the electrons at $\epsilon>\epsilon_2$, and the stochastic heating at $\epsilon>e\phi_{\rm sh}$.

At energies $\epsilon>\epsilon_2$, electrons abandon plasma. To do so for an electron in a given place x in a moment t, it is necessary to get after scattering into the escape cone. Accordingly, the EDF at $\epsilon>\epsilon_2$ becomes anisotropic $f(\epsilon,\epsilon_x)$. The escape cone is determined by $\epsilon_x>\epsilon_2$. If

$$\frac{\epsilon-\epsilon_2}{\epsilon_2}\ll -1$$

this cone is small, and if the electron scattering is isotropic, the EDF outside the escape cone is also isotropic, $f(\epsilon,\epsilon_x)=f_0(\epsilon)$. Income to the escape cone term in the kinetic equation for $f(\epsilon,\epsilon_x)$ is $\overline{\nu(w)}f_0(\epsilon)$. Electrons with energy $\epsilon_x>\epsilon_2$ at $\omega t\cong 2\pi n$ ($n=$ integer) abandon plasma with frequency:

$$\nu_{\rm es}(\epsilon_x,x)=\left[\int_x^{L_0}\frac{dx}{\sqrt{2(\epsilon_x-e\phi(x))/m}}\right]^{1}$$

in the moments when $\epsilon_x > U(t)$. The space- and time-averaged escape frequency is $\overline{\langle\nu_{es}\rangle} = \overline{\nu_{es}(\epsilon_x, x)}\ 2 \arcsin$

$$\frac{(\epsilon_x - \epsilon_2)^{1/2}}{\sqrt{U_0}}.$$

So the EDF in the escape cone is

$$f(\epsilon, \epsilon_x) = f_0(\epsilon)\frac{\overline{\nu(w)}}{\overline{\langle\nu_{es}(\epsilon_x)\rangle} + \overline{\nu(w)}}. \tag{16}$$

The average escape rate is

$$\frac{1}{2L_0}\int_{-L_0}^{L_0} dx \int_{\epsilon_2}^{\epsilon} \frac{d\epsilon_x}{\sqrt{\epsilon - e\phi(x)}} f(\epsilon, \epsilon_x)\langle\nu_{es}(\epsilon_x, x)\rangle = f_0(\epsilon)\left[\frac{\sqrt{w}}{\tau_{es}}\right] \tag{17}$$

where

$$\left[\frac{\sqrt{w}}{\tau_{es}}\right] = \frac{\overline{\nu}\ \overline{\nu_{es}}}{\overline{\nu_{es}} + \overline{\nu}} = \frac{\overline{\nu}}{2L_0}\int_{-L_0}^{L_0} dx \int_{\epsilon_2}^{\epsilon} \frac{\overline{\langle\nu_{es}\rangle} d\epsilon_x}{(\langle\nu_{es}\rangle + \nu)\sqrt{\epsilon - e\phi(x)}}.$$

At $\epsilon > \epsilon_2$, this expression is to be added to the first term in the right-hand side of (14):

$$\overline{\sqrt{w}\ \nu^*} f_0(\epsilon) \to \left[\overline{\sqrt{w}\ \nu^*} + \overline{(\sqrt{w}/\tau_{es})}\right] f_0(\epsilon). \tag{18}$$

The value of ϵ_2 can be found from:

$$\frac{4\sqrt{2}\ \pi L_0}{m^{3/2}}\int_{\epsilon 2}^{\infty} f_0(\epsilon)\left[\frac{\sqrt{w}}{\tau_{es}}\right] d\epsilon = \Gamma_i \tag{19}$$

that expresses the equality of electron and ion fluxes from plasma; the ion flux Γ_i will be calculated in Section IV.

III. Stochastic Heating in the Nonuniform Free-Flight RF Discharge Plasma

A. Stochastic Energy Diffusion Coefficient

The fast electrons which collide with the moving plasma–sheath boundary can be heated by another heating mechanism—stochastic heating. In this section we shall discuss the main distinctions between stochastic heating in the simple Ulam model [13] and in real self-consistent fields.

As the ion concentration decreases steeply in the sheath, the majority of electrons are reflected by the strong field only during relatively short time intervals $[-t_1(\epsilon); t_1(\epsilon)]$ (Fig. 1(b)). So the value of the sheath velocity

$$V_{sh} = \frac{dx_{sh}}{dt}$$

in the moments of electron collision with the strong field boundary (and the intensity of stochastic heating) grows with ϵ. It is known [13] that the electronic motion becomes chaotic if the electron kinetic energy in the collision moment with a harmonically moving sharp boundary does not exceed $m\omega L_0 V_{sh}^{(0)}$. If the EDF anisotropy is small, the kinetic energy in the collision moment $(\epsilon - e\phi_{sh})$ is determined by the isotropic motion and exceeds $mV_{sh}^2/2$ that is determined by electron drift. It follows that considerable stochastic heating is possible only if

$$\omega \gg \Omega: \qquad L_0 \gg L. \tag{20}$$

In the absence of the plasma oscillating field $\tilde{E}(x, t)(3)$, the stochastic velocity diffusion coefficient in this case is [3]

$$D_{st}(\epsilon_x) = 2\omega\Omega(\epsilon_x)\int_0^{t_1(\epsilon_x)} V_{sh}^2(t')\, dt' \tag{21}$$

where the exact expression for bounce frequency is

$$\Omega(\epsilon_x) = \left[\oint \frac{dx}{\sqrt{2(\epsilon_x - e\phi(x))/m}}\right]^{-1} \cdot 2\pi.$$

The expression for D_{st} (21) is to be added to $D_c(v)$ in the integrand of (12). As the energy losses are connected with inelastic collisions and fast electrons escape, the electron energy variation is slow compared with the elastic collision frequency (1), and the EDF is close to isotropic in this case also. So the argument of the EDF is ϵ and (12) can be averaged over v_x, v_y, x to give the total averaged energy diffusion coefficient:

$$\begin{aligned}\overline{\sqrt{w}D_{\epsilon\Sigma}} &= \overline{\sqrt{w}(D_{\epsilon c} + D_{\epsilon st})} \\ &= \frac{1}{L_0}\int_0^L dx \int_0^{\epsilon} d\epsilon_x \left(\sqrt{2m(\epsilon_x - e\phi)}\right)^{-1} \\ &\quad \cdot \left[\frac{\nu}{\nu^2 + \omega^2}\frac{(eE_0(x))^2 w}{m^2} + D_{st}(\epsilon_x)\right]. \end{aligned}\tag{22}$$

As in the free-flight regime, the electron collisions with sheath are more frequent than with molecules, $D_{st} \gg D_c$.

For uniform ion density in the sheath $(\phi(|x| > L_p) = \text{const})$, integrations in (21) are trivial and we have:

$$D_{\epsilon st}(\epsilon) = \frac{m^2}{8L_0}\left[\frac{\omega L}{2}\right]^2\left[\frac{2\epsilon}{m}\right]^{3/2} \tag{23}$$

the result obtained earlier in [3]. But due to the ion-density profile inhomogeneity, the real boundary motion is very anharmonic [11], [12]. For the collisionless sheath,

$$x_{sh}(z = \omega t) - L_p = \frac{L}{5\pi}\left[-z(3 + 2\cos^2 z)\cos z + 5\sin z - \frac{11}{3}\sin^3 z\right]. \tag{24}$$

The boundary displacement during $-\pi/2 < z < \pi/2$ is small:

$$(x_{sh}(\pm\pi/2) - L_p) = L/9.$$

On the contrary, the sheath motion (24) in the second half-period is fast and the maximal sheath acceleration value 3.6 times exceeds $(\omega^2 L/2)$— its value for harmonic law. In the inhomogeneous plasma the electron motion at a given total energy ϵ is restricted by the rather complex boundary that consists of stationary and moving parts (Fig. 1(b)). Only the electron reflections from its moving parts may result in the energy diffusion. The averaging (21) and (22) can be easily fulfilled only for high-energy electrons $\epsilon \gg e\varphi_{\rm sh}$. For these electrons the whole boundary is oscillating (24), and averaging for the harmonic current dependence gives an additional factor of 1.58 in (23). The third significant distinction between the real situation and the conventional rigid wall model consists in the finite boundary thickness. If the field profile in this region is exponential, with thickness equal to the local Debye radius [14], the energy gain in single reflection is $\Delta\epsilon = 2mV_{\rm sh}(z)v/ch(\omega/\omega_{0e}(x_{\rm sh}))$ decreases with the frequency growth ($\omega_{0e}(x_{\rm sh})$, the local electron plasma frequency in the reflection point). So the problem of the stochasticity criterion is rather complicated and will be discussed in detail elsewhere. In (21), the oscillatory plasma field $\tilde{E}$ was neglected. In numerical calculations [6] it was demonstrated that the influence of $\tilde{E}$ leads to the reduction of $D_{\epsilon{\rm st}}$ up to 20%. In order to reveal the origin of this reduction, let's consider the case of constant ion density in the sheaths and the inhomogeneous profile in the plasma $n_i(x)$. The oscillatory field in the laboratory frame is

$$\tilde{E}(x,t) = -\frac{m}{en_i(x)}\frac{dj(t)}{dt}.$$

As the sheath velocity $V_{\rm sh}(t) = j(t)/(en_i(x_{\rm sh}(t)))$, in the frame moving with the boundary the effective field is

$$\tilde{E}'(x,t) = \frac{m}{e}\frac{dV_{\rm sh}}{dt} + \tilde{E} = \frac{m}{e}\left[\frac{1}{n_i(x_{\rm sh})} - \frac{1}{n_i(x)}\right]\frac{dj}{dt}. \quad (25)$$

The mapping of electron velocities at successive reflections is

$$\Delta v = v_{n+1} - v_n = \frac{e}{m}\int_{t_n}^{t_{n+1}} \tilde{E}'(x(t),t)\, dt$$

where $x(t)$ is the electron motion in the average field $\phi(x)$. So field $\tilde{E}$ slightly lowers the velocity variation Δv, and consequently the stochastic velocity diffusion coefficient.

B. Absence of Stochastic Heating for Trapped Electrons and High-Frequency $\omega > \omega_{pe}$ Fields

For trapped electrons with $\epsilon < e\phi_{\rm sh}$, the problem of stochastic heating is more complicated. Interaction between the RF field $\tilde{E}$ and the stationary one $d\phi/dx$ can in principle lead to stochastic heating. It is evident that the movement in the "rigid wall" potential,

$$\phi(x) = \begin{cases} 0, & x \in [0,L] \\ \infty, & x = [0,L] \end{cases} \quad (26)$$

and RF field $\tilde{E} = E_0 \sin\omega t$ is equivalent to the classical Ulam model, where the free particle is colliding with the walls which are oscillating with velocity,

$$V = \frac{e}{m}\int \tilde{E}\, dt.$$

It is well-known that the stochastic heating arises in this system.

But the real potential $\phi(x)$ does not have such a strong peculiarity as (26), and as a result there is no stochastic heating for trapped electrons. In order to prove it we use the resonances overlap criterion [13].

The trapped electrons move in the potential $e\phi(x)+U(t,x)$, where $U(t,x) = e\int \tilde{E}(x,t)\, dx$. It is convenient to introduce the action-phase variables for Hamiltonian,

$$H_0(I) = \frac{mv_x^2}{2} + e\phi(x)$$

which are

$$H_0(I) = \int \Omega(I)dI; \qquad \dot{\theta} = \Omega(I).$$

In these variables,

$$U(t,x) = \sum_{k=1}^{\infty} \frac{A_k}{k}\sin(k\theta \pm \omega t).$$

The movement equation for Hamiltonian $H = H_0(I)+U(t,x)$ becomes:

$$\begin{cases} \dot{I} = \dfrac{\partial U}{\partial e} = \sum\limits_k A_k \cos(k\theta \pm \omega t) \\ \dot{\theta} = \Omega(I) + \dfrac{\partial U}{\partial I} \end{cases} \quad (27)$$

For phase randomization, the fulfillment of $\omega \gtrsim \Omega$ is necessary. So the resonances occur at $k_{\rm res} = \omega/\Omega(I) \gtrsim 1$. The resonance width is

$$\Delta I = \left[A_{k_{\rm res}} \bigg/ \left(k_{\rm res}\frac{d\Omega}{dI}\right)\right]^{1/2}.$$

From $k\Omega(I_{\rm res}) = \omega$, the distance between neighboring resonances is

$$\delta I = \Omega \bigg/ \left(k_{\rm res}\frac{d\Omega}{dI}\right). \quad (28)$$

The motion is chaotic when $\Delta I \gtrsim \delta I$ [13]:

$$A_{k_{\rm res}} \gtrsim \frac{\Omega^2}{k_{\rm res}(d\Omega/dI)}.$$

For example, for the potential (26)

$$A_k = \frac{2eE_0L_0}{k}, \qquad \Omega(I) = \frac{2\pi^2 I}{2mL_0^2}, \qquad I = \frac{2mvL_0}{2\pi},$$
$$L = V/\omega, \qquad V = eE_0/m\omega$$

and (28) in variables $V_{\rm sh}, L, L_0, v_x$ can be rewritten in well-known form [13]:

$$v_x \lesssim V_{\rm sh}(L_0/L)^{1/2}. \quad (29)$$

For the smooth potential $\phi(x)$, coefficients A_k are proportional to e^{-k^*}/k, where $k^* \sim 1$ [13]. It is evident that (28) can be fulfilled only for very low values of Ω that correspond to very slow electrons. So the chaotic motion of trapped electrons in the potential $\phi(x)$ is practically absent and the stochastic velocity diffusion coefficient (21) $D_{\rm st}(\epsilon_x) = 0$ for $\epsilon_x < e\phi_{\rm sh}$. The transition scale between the plasma and space-charge regions is of the order of the Debye radius. So at frequencies $\omega \geq \omega_{0e}$, the corresponding potential becomes soft and, as was mentioned above, the stochastic heating also "switches off."

C. Combination of Electron–Atom and Electron–Sheath Collisions

If the stochasticity condition (29) is not fulfilled but $\omega \gg \Omega, L_0 \ll \lambda$, the combined mechanism [4], [15] of the fast electron heating exists. The electron with $\epsilon_x > e\phi_{\rm sh}$ acquires directed velocity $\Delta v_x = \pm 2V_{\rm sh}$ in the moments of collisions with the space-charge boundary. The randomization occurs in the electron–neutral collisions. It leads to the velocity diffusion with $D_1 \sim V_{\rm sh}^2\nu$. As the ion density in the sheathes is small compared with the central one, the values of $V_{\rm sh}$ exceed the oscillatory velocity in the bulk plasma, and D_1 can exceed the conventional collisional coefficient.

The stochastic heating in the collisional regime $L_0 \gg \lambda$ manifests itself as the so-called "wave-riding" phenomenon that was found in Monte Carlo simulations [4], [15]. If $\Delta(\epsilon) = (x_+(\epsilon) - L_p)$—the penetration depth of the electron with energy ϵ into the sheath—is great compared to λ, the collisional heating dominates. As the value of $V_{\rm sh}(\Delta(\epsilon))$ coincides with the oscillatory electron velocity in the plasma phase, and interactions with the boundary are seldom compared to the electron–neutral ones, the total energy diffusion coefficient is determined by collisions [1]. So the "wave-riding" is significant only if $\Delta(\epsilon) \ll \lambda$—the collisions in the sheathes where $\tilde{E}$ is great are almost absent. In this case, the great directed velocity acquired in sheathes is dissipated in the plasma volume where the value of $\tilde{E}$ (and conventional collisional heating) is small. If gap L_0 is small compared to the fast electrons' energy relaxation length

$$\lambda^* = \lambda\sqrt{\nu/\nu^{*-}}$$

the corresponding term is to be added to the averaged energy diffusion coefficient in (22) [3], and EDF depends only on ϵ. In the opposite case, the EDF is enriched by fast electrons at distances $\leq \lambda^*$ from electrodes [4], [15].

At low frequencies $\omega < \Omega$, the subsequent electron reflections from the space-charge boundary are correlated and the stochastic heating vanishes. The usual collisional mechanism is also strongly suppressed in this case.

IV. Ion Density Profile and Bohm Criterion

The EDF can be calculated if the values of ϵ_2 (19), (34), $\phi_{\rm sh}$ (33), and profiles $\phi(x), \tilde{E}(x,t)$ are given. But these values are to be found self-consistently via EDF. The $\phi(x)$ profile at $\omega \ll \omega_{0e}$ can be found from the quasi-neutrality condition:

$$n_e(x) = \frac{4\pi(2)^{1/2}}{m^{3/2}}\int_{e\phi(x)}^{\infty} f_0(\epsilon)(\epsilon - e\phi(x))^{1/2}d\epsilon = n_i(x). \quad (30)$$

The spatial dependence of $\tilde{E}(x,t)$ is determined by (9) and RF current conservation:

$$\begin{aligned} \tilde{j}(t) &= e\int v_x dv_x dv_y dv_z \tilde{f} \\ &= \frac{4\pi e^2\tilde{E}(x,t)}{3m}\int_{e\phi(x)}^{\infty}\frac{\partial f_0}{\partial \epsilon}\frac{v^3}{\nu + i\omega}d\epsilon. \end{aligned} \quad (31)$$

The expression for ion density $n_i(x)$ is to be substituted into (30):

$$n_i(x) = \int_0^x \sqrt{\frac{M}{2e}}\frac{I(x')dx'}{(\phi(x) - \phi(x'))^{1/2}} \quad (32)$$

where $I(x)$ is the ionization rate in point x.

The potential $\phi_{\rm sh}$ of the plasma—sheath transition can be determined approximately by generalization of the well-known Bohm criterion. It states that $\phi_{\rm sh}$ corresponds to the point where for the quasi-neutral $\phi(x)$ profile (30), (32) holds

$$\frac{dx}{d\phi} = 0.$$

Performing the Abel transformation and differentiating (30) and (32), we have:

$$\int_0^{e\phi_{\rm sh}} \frac{dN(\phi')}{d\phi'}\frac{d\phi'}{(1 - \phi'/\phi_{\rm sh})^{1/2}} = -1 \quad (33)$$

where $N(\phi) = n_e(\phi)/n_e(0)$ is given by (30). At $\phi > \phi_{\rm sh}$ the sheath begins, where (32) is not fulfilled and the ion motion is determined by the averaged field in the space-charge phase that far exceeds the plasma field $\phi(x)$. In the nonlocal case, the excitations and direct ionization are concentrated in the central region [1]. So the significant simplification in the sheath analysis is possible, if ionization here is negligible. This being the case, results of [11] and [12] are applicable with the accounting of the non-Maxwellian EDF.

As the radical in (33) is of the order of unity, it follows that $N(\phi_{\rm sh}) \sim 1$; i.e., the contribution of fast electrons with $\epsilon > e\phi_{\rm sh}$ in the central electron density is comparable with the density of slow ones. The ion flux density at the plasma–sheath boundary is given by

$$\Gamma_i = \frac{2(2e)^{1/2}}{\pi(M)^{1/2}}n(0)\int_0^{\phi_{\rm sh}}\frac{dN(\phi')}{d\phi'}\frac{\phi'\, d\phi'}{(\phi_{\rm sh} - \phi')^{1/2}}. \quad (34)$$

The central plasma density $n(0)$ and scaling factor for $\tilde{E}(x,t)$ can be found from:

$$\Gamma_i = \int_{-L_0}^{L_0} dx \int_{e\phi(x)}^{\infty} d\epsilon \sqrt{\frac{2(\epsilon - e\phi)}{m}}\nu_i(\epsilon - e\phi)f_0(\epsilon)/m \quad (35)$$

that expresses the charged particles conservation ($\nu_i(w)$ is the ionization frequency).

V. A Simple Model and Analytic Estimates

As the values of $D_{\epsilon\Sigma}$(22) rise steeply with energy, let us consider a simple model with zero condition for EDF at $\epsilon = \epsilon^*$ [3], and

$$\sqrt{w}D_{\epsilon\Sigma}(\epsilon) = \begin{cases} D_1(\epsilon), & \text{at } \epsilon < e\phi_{\rm sh} \\ D_2(\epsilon), & \text{at } \epsilon > e\phi_{\rm sh}. \end{cases} \quad (36)$$

In this case the simple estimate of $e\phi_{\rm sh}/\epsilon^*$ can be found. From (14), it follows that

$$f(\epsilon) = A \int_{\epsilon}^{\epsilon^*} \frac{d\epsilon'}{\sqrt{w} D_{\epsilon\Sigma}(\epsilon')}. \qquad (37)$$

If $\phi_{\rm sh} \ll \epsilon^*$, it follows from (33)–(37) that in the lowest order in $\phi_{\rm sh}/\epsilon^*$,

$$\phi_{\rm sh}^{5/2} \int_0^1 \frac{dt}{D_1(\phi_{\rm sh} t)} \left[t^{1/2} + (1-t) \ln \frac{(1-t)^{1/2}}{(t^{1/2}+1)} - \frac{2}{3} t^{3/2} \right] = \frac{2}{3} (\epsilon^*)^{5/2} \int_0^1 \frac{t^{3/2} dt}{D_2(\epsilon^* t)}. \qquad (38)$$

So we have for the power dependencies of D_1, D_2 on ϵ an estimate:

$$\phi_{\rm sh}/\epsilon^* \sim \sqrt{D_1/D_2} \ll 1. \qquad (39)$$

For the numerical estimate, let us consider an example with constant ion densities in the sheath ($n_{\rm sh}$) and plasma (n_0) : $n_{\rm sh} < n_0 . \omega \gg \nu$. It corresponds to

$$\begin{aligned} D_1 &= \frac{\nu(\epsilon)}{3} \left(\frac{eE_0}{m\omega} \right)^2 \epsilon^{3/2} / \sqrt{2m} \\ D_2 &= \frac{v_{\rm sh}^2 \epsilon^2}{4\sqrt{2} m^{3/2} L_0}, \qquad \text{at } \epsilon \gg e\phi_{\rm sh}. \end{aligned} \qquad (40)$$

Such a model for $D_1 = 0$ was considered in [3]. Substituting (40) into (38) and introducing approximation

$$\nu(\epsilon) = \begin{cases} \nu_0 (\epsilon/\epsilon_0)^{1/2}, & \text{at } \epsilon < \epsilon_0 \\ \nu_0 (\epsilon/\epsilon_0)^{3/2}, & \text{at } \epsilon > \epsilon_0 \end{cases}$$
$$\nu_0 = 0.42 * 10^9 p : \qquad \epsilon_0 = 1 \text{eV}$$

that roughly describes scattering in Ar, we obtain for $e\phi_{\rm sh} \leq \epsilon_0$:

$$e\phi_{\rm sh} = 3.9 \epsilon^* \sqrt{D_1/D_2}.$$

It is to be noted that in real low-pressure discharges, the energy scale of the EDF tail can be considerable ($\gtrsim \epsilon_0 : e\phi_{\rm sh}$). In this case the return term in the kinetic equation (last term in (14)) is to be accounted for.

As a rule, several inelastic collisions correspond to one ionizing one. So the electron-energy relaxation time that is of an order of

$$\frac{\epsilon^{*2}}{D_{\epsilon\Sigma}(\epsilon)}$$

is comparable with the ionization time and ion lifetime:

$$\frac{\epsilon^{*2}}{D_{\epsilon\Sigma}(\epsilon^*)} \lesssim \sqrt{\epsilon^*/M} / L_p. \qquad (41)$$

From (41) we can estimate $V_{\rm sh}$. Substituting (41) in (29), we obtain the stochastic motion criterion in the form of (1).

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REFERENCES

[1] I. D. Kaganovich and L. D. Tsendin, "The space-time-averaging procedure and modeling of the RF discharge, part II: model of collisional low-pressure RF discharge," *IEEE Trans. Plasma Sci.*, this issue, pp. 66–75.
[2] V. A. Godyak, *Soviet Radio Frequency Discharge Research*. Falls Church, VA: Delphic, 1986.
[3] C. G. Goedde, A. J. Lichtenberg, and M. A. Lieberman, "Self-consistent stochastic electron heating in RF discharges," *J. Appl. Phys.*, vol. 64, no. 9, pp. 4375–4382, 1989.
[4] M. Kushner, "Mechanisms for power deposition in A/SiH$_4$ capacitive coupled RF discharges," *IEEE Trans. Plasma Sci.*, vol. PS-14, pp. 188–196, 1986.
[5] D. B. Graves and M. Surendra, "Self-consistent kinetic level simulations of radio-frequency glow discharges," in *Proc. SPIB* (Dubrovnik, Yugoslavia), 1990.
[6] T. J. Sommerer, W. N. G. Hitchon, and J. E. Lawler, "Electron heating mechanisms in helium RF glow discharge: a self-consistent kinetic calculation," *Phys. Rev. Lett.*, vol. 63, pp. 2361–2364, 1989.
[7] D. Vender and R. W. Boswell, "Numerical modeling of low-pressure RF plasmas," *IEEE Trans. Plasma Sci.*, vol. 18, pp. 725–730, Aug. 1990.
[8] V. A. Godyak and R. B. Piejak, "Abnormally low electron energy and heating mode transition in a low-pressure argon RF discharge at 13.56 MHz," *Phys. Rev. Lett.*, vol. 65, pp. 996–999, 1990.
[9] L. D. Tsendin and Yu. B. Golubovsky, "Theory of low-pressure discharge positive column at low electron densities," *Sov. Phys.-Tech. Phys.*, vol. 22, no. 9, pp. 1066–1073, 1977.
[10] J. H. Ferziger and H. G. Kaper, *Mathematical Theory of Transport Processes in Gases*. Amsterdam & London: North-Holland, 1972.
[11] M. A. Lieberman, "Dynamics of a collisional, capacitive RF sheath," *IEEE Trans. Plasma Sci.*, vol. 17, pp. 338–341, 1989.
[12] I. D. Kaganovich and L. D. Tsendin, "Collisionless sheath of RF discharge," *Sov. Tech. Phys. Lett.*, no. 1, pp. 43–45, 1990.
[13] A. J. Lichtenberg and M. A. Lieberman, *Regular and Stochastic Motion*. New York: Springer-Verlag, 1983.
[14] L. D. Tsendin, "Electron motion and average field in electrode sheathes of RF discharge," *Sov. Phys.-Tech. Phys.*, vol. 35, no. 8, pp. 904–907, 1990.
[15] J. P. Boeuf and P. H. Benlenguer, "Nonequilibrium processes in partially ionized gases" (NATO Advanced Study Instit. series B, vol. 20), M. Capitelly and G. N. Bardsley, Eds. New York: Plenum, 1990, p. 155.

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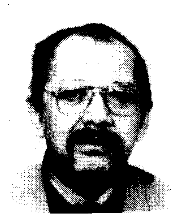

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