Formation of discontinuities in multistage evolution associated with diffusion of a multicomponent weakly ionized hot-electron plasma

I. D. Kaganovich and L. D. Tsendin

St. Petersburg State Technical University
(Submitted December 9, 1992; resubmitted March 4, 1993)
Fiz. Plazmy **19**, 1229–1240 (October 1993)

A qualitative difference in the way irregularities in two-component and multicomponent plasmas decay is demonstrated. In particular, it is shown that in a multicomponent plasma with hot electrons (electron temperature much greater than the ion temperature) ion density discontinuities can develop. The discontinuities can develop in a plasma with either positive or negative ions. Diffusive evolution in the nonlinear stage can occur in several stages; the final stage can be controlled by the low ion temperature.

1. INTRODUCTION

The evolution of ion and electron density irregularities in a two-component plasma (containing electrons and one positive ion species) is described by the ambipolar diffusion equation. This equation is linear in the plasma density and describes the decay of irregularities on the time scale L^2/D_a (where L is the characteristic length and D_a is the ambipolar diffusion coefficient). However, the linearity of the ambipolar diffusion equation results from complete cancellation of a number of nonlinear effects. When a current flows through the plasma and the mobility of the components depends on the field, the evolution of a nonuniform plasma is highly nonlinear and can be accompanied by the development of discontinuities.[1] A similar effect also occurs in a multicomponent current-carrying plasma. Extremely nonuniform time-independent density profiles in the current-free nonisothermal plasma of the positive column were obtained in electronegative gases.[2-6] Tsendin[4] interpreted this phenomenon as the formation of discontinuities, while Daniels et al.[5,6] used boundary-layer theory. It remains unclear how universal this phenomenon is, how such profiles develop during the process of evolution, and what the role is of the boundary conditions, plasma chemical reactions, and transport processes in their development. In the present work we show that the discontinuities can arise even in the absence of plasma chemical processes and far from the boundaries in a multicomponent current-free plasma when the electron temperature T_e is much greater than the ion temperature T_i. This situation is encountered, e.g., in gas discharges. The thickness of the discontinuities is determined by the ratio T_i/T_e. The initial ion and electron density profiles which give rise to a jump are determined by the condition that the time for formation of the jump be much less than the time required for an electron profile to smooth out. We show that discontinuities develop in three cases:

1) when the ion mobilities are very different (for a plasma with negative ions less mobile than the positive ions);

2) when the initial ion density gradients are larger than those of the electrons;

3) for a plasma containing negative ions the discontinuities develop near walls with zero boundary conditions.

The diffusive (current-free) evolution of a small perturbation in an m-component nonuniform plasma corresponds to $m-1$ values of the relaxation time. If these times are very different, then we can distinguish $m-1$ stages of the diffusive evolution.[1)] The nonlinearity can give rise to convective motion of the plasma irregularities, an increase in the nonuniformity of the ion component in the course of evolution, and also to new stages of the evolution.

2. FORMATION OF DISCONTINUITIES IN A MULTICOMPONENT PLASMA

Let us consider a quasineutral plasma containing two types of ions whose density will be denoted p and n. In this case the evolution is described by the system of equations

$$\frac{\partial n}{\partial t}=-\frac{\partial}{\partial x}\Gamma_n=\frac{\partial}{\partial x}\left(D_n\frac{\partial n}{\partial x}\pm b_n nE\right),$$

$$\frac{\partial p}{\partial t}=-\frac{\partial}{\partial x}\Gamma_p=\frac{\partial}{\partial x}\left(D_p\frac{\partial p}{\partial x}-b_p pE\right),$$

$$\frac{\partial n_e}{\partial t}=-\frac{\partial}{\partial x}\Gamma_e=\frac{\partial}{\partial x}\left(D_e\frac{\partial n_e}{dx}+b_e n_e E\right),$$

$$n_e=p\mp n; \tag{1}$$

everywhere in what follows the upper sign corresponds to the case in which the n ions are negative and the lower sign to that in which they are positive; n_e is the electron density.

From the condition for the absence of current in the system we have

$$E=\frac{D_p p'\mp D_n n'-D_e n_e'}{b_p p+b_n n+b_e n_e}. \tag{2}$$

From (2) it can be seen that if the electron density and its gradient are not too small, then the electrons are described by a Boltzmann distribution.

We assume $T_e \gg T_i$. Ion diffusion can be disregarded if the electron density gradient is not too small:

 1063-780X/93/10 0645-06 $10.00

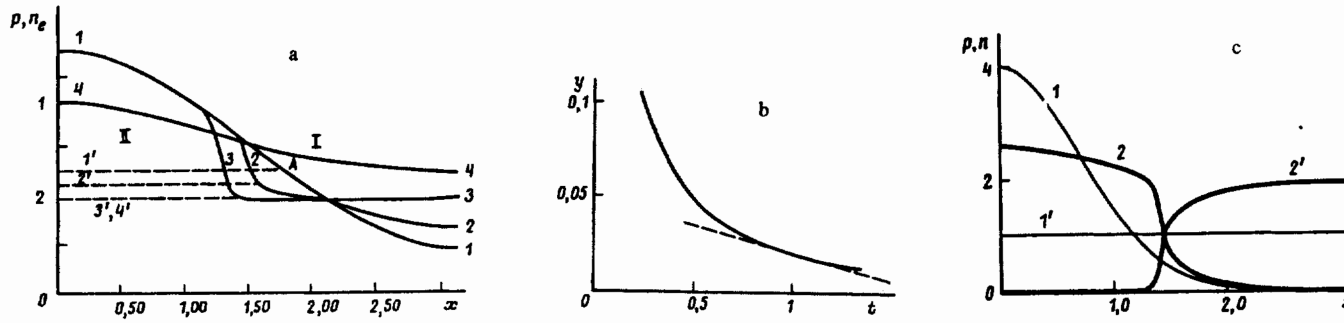

FIG. 1. a) Density profiles of the positive ions p (1, 2, 3, 4) and electrons n_e (1′, 2′, 3′, 4′) for $D = 0.1$, $T_i/T_e = 0.01$ at four times $t = 0$, 12.8, 20.5, 300. b) Plot of $y = [\max|d^2n_e/dx^2|]^{-1}$ as a function of time. The initial profiles were $p(x, 0) = 4(1 + \cos x)$, $n_e(x, 0) = n(x, 0) = 2(1 + \cos x)$. c) Evolution of a plasma with positive ions. The initial density profiles of the less mobile ions: 1) $p(x, 0) = 4\exp(-x^2)$; of the more mobile ions: 1′) $n(x, 0) = 1$; 2) $p(x, t)$ for $t = 5$; 2′) $n(x, t)$ for $t = 5$; the ratio of mobilities was $D = 0.1$.

$$b_n nE \sim \frac{T_e b_n n}{e n_e}\frac{dn_e}{dx} \gg D_n \frac{dn}{dx} = \frac{b_n T_i}{e}\frac{dn}{dx}.$$

Measuring the spatial coordinate in units of the characteristic length scale L of the electron density and the time in units of $T_i L^2/D_n T_e$, we find

$$\frac{\partial p}{\partial t} = \nabla D p \nabla \ln(n_e), \quad \frac{\partial n}{\partial t} = \nabla (Dp + n)\nabla \ln(n_e),$$
$$\frac{\partial n}{\partial t} = \mp \nabla n \nabla \ln(n_e), \quad n_e = p \mp n, \tag{3}$$

where $D = D_p/D_n$. We begin by considering the case $D \ll 1$.

2.1. Evolution in a Multicomponent Plasma with Very Different Ion Mobilities

a. Negative-ion plasma. From (3) the evolution of a small perturbation takes place in two stages.[8] During the first stage (for $t \ll 1$) the positive ions remain practically fixed, the negative ions are gathered together by the field near the maximum of n_e, and the electron profile becomes flattened out. The second stage is characterized by the slow time of the same order as the ion ambipolar time $(\sim DT_e/T_i)^{-1}$.

The case in which the average electron density $\langle n_e \rangle$ is greater than the initial positive ion density $p_0(x)$ at some point is of interest.[4] Then as the electron density evens out the negative ions leave this region and n_e approaches $p_0(x)$. Wherever $p_0(x) > \langle n_e \rangle$ holds the density n_e becomes essentially uniform (Fig. 1, traces 1 and 1′). Consequently, after a time $1/D \gg t \gg 1$ an n_e profile develops which consists of constant density regions (within which $n \neq 0$ holds) and regions where $n_e = p_0(x)$, $n = 0$. At the boundary between these there is a jump in the derivative $\partial n_e/\partial x$ (the point A in Fig. 1a). Let us estimate the time for such an n_e profile to form, assuming that the positive ions are stationary. In region II the density n_e equilibrates at a characteristic rate $\gamma_1 \sim (n/n_e)\partial^2 p_0/\partial x^2$ due to ambipolar diffusion of the electrons and negative ions. In region I, where $n_e \to p_0(x)$, the negative ion density approaches zero at a rate $\gamma_2 \sim \partial^2 p_0/\partial x^2$. At the boundary between these regions a kink in the electron density profile develops. Near this point (point A in Fig. 1a) the $p_0(x)$ profile can be replaced by the linear function $p_0(x) = 1 + b(x - x_A)$, and from Eq. (3) for the electrons we have

$$\frac{\partial n_e}{\partial t} = \frac{\partial}{\partial x}\left(1 - b(x - x_A) - \frac{n_e}{p_0(x_A)}\right)\frac{\partial n_e}{\partial x}.$$

The self-similar solution for $t \gg 1$ takes the form

$$1 - \frac{n_e}{p_0(x_0)} = b(x - x_A)\,\Phi\left(\frac{x - x_A}{t_0 - t}\right), \tag{4}$$

where $\Phi(-\infty) = 0$ and $\Phi(+\infty) = 1$. From (4) we see that the radius of curvature of the $n_e(x, t)$ profile must decrease linearly as a function of time. Figure 1b shows the time dependence of max $|\partial 2n_e/\partial x^2|$ obtained by solving the problem numerically. The linear portion is shown by the broken trace.

The next intermediate stage of the evolution is characterized by the ambipolar time of the electrons and positive ions, $1/D$. In region II, where there are negative ions, $dn_e/dx \approx 0$ holds and the $p(x, t)$ profile remains essentially constant and equal to $p = p_0(x)$ as a function of time. In region I, where $n \approx 0$ holds (to the right of the point A in Fig. 1a), the positive ions diffuse according to the ambipolar diffusion equation. Let us consider what occurs at the boundary between regions I and II. The positive-ion flux to the left of the boundary (Fig. 1a), where $n \neq 0$ holds, is equal to zero, while on the right side we have $\Gamma_p = -D[(p/n_e)(\partial n_e/\partial x)] = -D\partial p/\partial x$. The discontinuity in the value of the flux is associated with the fact that at this point a jump develops in the densities of the positive and negative ions moving toward small values of Γ_p and large values of p (traces 2, 2′ and 3, 3′ in Fig. 1a). The velocity V with which the jump moves must be determined from the condition for flux conservation in the system moving with the jump. For $x = x_{\text{jump}}$ this flux is $-Vp - D\partial p/\partial x$ = const, from which we have

$$V = \frac{D\,\partial p/\partial x}{p_- - p_+}, \tag{5}$$

where $p_\pm = p(x_{\text{jump}} \pm 0)$, $p_N = p_0(x_{\text{jump}})$.

The electron density cannot be discontinuous, since its derivative determines the fluxes of positive and negative ions (a jump in n_e would yield infinite fluxes Γ_n and Γ_p). This means that the jumps in the positive and negative ion densities are equal: $p_+ - p_- = n_+ - n_-$. But at $x = x_{\text{jump}}$ the profile of n_e has a kink. The velocity (5) of the jump is proportional to the difference in the values of $\partial n_e/\partial x$ to the right and to the left of the kink.

As a result we find that the evolution of the positive-ion profile is described by the system of equations

$$\frac{\partial p}{\partial t} = D\frac{\partial^2 p}{\partial x^2} \quad \text{for } x > x_{\text{jump}}, \tag{6a}$$

$$p = p_0(x) \quad \text{for } x < x_{\text{jump}}.$$

An additional condition on $p_+(x_{\text{jump}})$ at the jump can be found from conservation of the total number of negative ions,

$$\frac{\partial}{\partial t}\int\limits_0^{x_{\text{jump}}} n\,dx = 0.$$

Noting that in region II we have $n = p_0(x) - n_e$ and in region I we have $n_e = p_+$, and taking into account expression (5) for $\partial x_{\text{jump}}/\partial t = V$, we find

$$D\frac{\partial p}{\partial x}\bigg|_{x=x_{\text{jump}}+0} + x_{\text{jump}}\frac{\partial}{\partial t}p_+ = 0. \tag{6b}$$

The numerical solution of the system (4), (5) is shown in Fig. 1a. We consider the initial stage of formation of a jump for the case in which the initial profile $n_e(x)$ already has a kink:

$$n_e(x, t=0) = \begin{cases} p_0(x_A) & \text{for} \quad x < x_A \\ p_0(x) & \text{for} \quad x > x_A, \end{cases}$$

and the $p_0(x)$ profile is smooth. At $t = 0$ we have $p_+ = p_-$, and the velocity (4) of the jump diverges. At early times $t \ll 1/D$ the location of the jump can easily be found analytically. Then the difference $p_+ - p_-$ grows essentially due to the change in p_-:

$$\frac{\partial}{\partial t}(p_- - p_+) = \frac{\partial p}{\partial x}(V + D/x_{\text{jump}}) \approx V\frac{\partial p}{\partial x} \text{ (since } V \gg D/x_{\text{jump}}).$$

The position of the jump is determined by the equations

$$V(p_- - p_+) = D\frac{\partial p}{\partial x},$$

$$\frac{\partial}{\partial t}(p_- - p_+) = V\frac{\partial p}{\partial x},$$

from which we have $x_{\text{jump}} = x_A - \sqrt{Dt}$.

To describe the formation of the jump we rewrite Eq. (3) in terms of quantities which are conserved as we pass through the jump: n_e, $\partial n_e/\partial t$, and the electron flux Γ_e. Expressing $\partial n_e/\partial x$ in terms of Γ_e and substituting into the equations for the ions we find

$$\frac{\partial n_e}{\partial t} = \frac{\partial}{\partial x}\frac{Dp + n}{n}\frac{\partial n_e}{\partial x},$$

$$\frac{\partial n}{\partial t} = \frac{1}{Dp + n}\left[n\frac{\partial n_e}{\partial t} - \Gamma_e\frac{Dn_e\dfrac{\partial n}{\partial x} - Dn\dfrac{\partial n_e}{\partial x}}{Dp + n}\right]. \tag{7}$$

Near the point A the first and third terms on the right-hand side of the equation for n are small, since $n(x_A) \simeq 0$ holds. Near $x = x_A$ it can therefore be written in the form

$$\frac{\partial n}{\partial t} + U(n)\frac{\partial n}{\partial x} = 0, \tag{8}$$

where the signal propagation speed is

$$U(n) = -\frac{D\Gamma_e n_e}{[Dn_e + n(1 + D)]^2} \tag{9a}$$

This equation describes the steepening and breaking of the $n(x)$ profile.[9] The velocity $U(n)$ increases as n decreases, with the velocity U in the region of small $n \lesssim D$ being larger than the velocity in the region $n \sim 1$ by a factor $1/D^2 \gg 1$. Negative ions will therefore be gathered toward the center with a velocity which increases strongly near the periphery of the density profile $n(x)$, which is the reason for the formation of a jump. The time for the jump to form is

$$t_{\text{jump}} = \min\left[\frac{\partial U}{\partial n}\frac{\partial n_0}{\partial x}\right]^{-1},$$

where $n_0(x)$ is the negative-ion density profile after the region $n \simeq 0$ has formed. Using the expression (9) for the velocity U, we find for the time of formation of the jump

$$t_{\text{jump}} = \left[\frac{(1 + D)\,\Gamma_e\, dp_0/dx}{(Dp_0(x_A))^2}\right]^{-1}. \tag{10}$$

In the limit $D \ll 1$ the time for the kink in $n_e(x, t)$ to form is $t_{\text{jump}} \sim D \ll 1$, i.e., when a real profile evolves the jump in n and p develops simultaneously with the kink in the n_e profile.

The jump moves toward large values of $p_0(x)$. As time increases the kink in $n_e(x, t)$ [which according to (5) is responsible to its motion) decreases. At the end of the intermediate stage the n_e profile flattens out and the profile stops (traces 3 and 3′ in Fig. 1a).

The third slow stage corresponds to smearing out of the remaining irregularity of the ion density profile over a time on the order of the ion ambipolar time with an effective diffusion coefficient $D_{\text{eff}} = T_i b_p b_n(p + n)/(b_p p + b_n n)$.

b. Positive-ion plasma. In this case the situation is substantially similar to that described above. The main difference is that the highly mobile positive ions do not collect under the influence of the field (2) at the point where n_e is large, but rapidly leave these regions. After a time of order unity (the ambipolar time of the electrons and the mobile ions) an n_e profile can develop with a kink and a jump can arise in the ion densities. The intermediate stage of evolution is described by (5) and (9), with (9a) replaced by

$$U(n) = \frac{D\Gamma_e n_e}{[Dn_e + n(1 - D)]^2}. \tag{9b}$$

Figure 1c shows profiles with a jump that has developed at the end of the intermediate stage as a result of the evolution in a plasma with two types of positive ions with very different mobilities. These profiles are the initial conditions for ion–ion ambipolar diffusion.

2.2. Formation of Jumps in a Multicomponent Plasma with Ions of Comparable Mobility ($D \sim 1$)

From the system of equations (7) we see that jumps can also develop in the case $D \sim 1$. In this case, however, the time for the electron profile to flatten out is comparable with the time scale on which $p(x, t)$ evolves, so that the kink in n_e and the jump may be able to form only for a particular choice of the initial conditions. Over the characteristic time $t_{\text{ev}} \sim [Dd^2n_e/n_e dx^2]^{-1}$ the electron profile flattens out and the subsequent evolution of the ion profiles occurs only on account of ion ambipolar diffusion, which is not included in (7). Consequently, the condition $t_{\text{jump}} < t_{\text{ev}}$ for the formation of a jump after the substitution (9a) reduces to

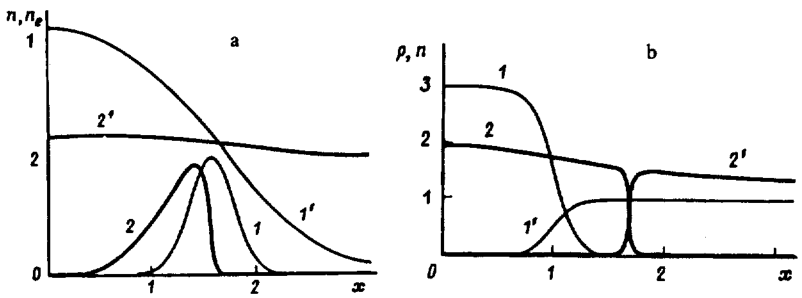

FIG. 2. a) Density profiles of the negative ions in an electron n_e. The initial density profiles: 1) negative ions $n(x, 0) = 2\exp(-11(x - \pi/2)^2)$; 1') electrons $n_e(x, 0) = 2(1.1 + \cos x)$; 2) profile of n at $t = 5$; 2') profile of n_e at $t = 5$. b) Density profiles for two species of positive ions (p are the less mobile, and n are more mobile, $D = 0.5$). The initial density profiles were: 1) $p = 1.5(1 - \tanh(5(x - 1)))$; 1') $n = 0.5(1 + \tanh(5(x - 1)))$; 2) p for $t = 2.1$; 2') n for $t = 2.1$.

$$\frac{\partial n_0}{\partial x} > \frac{n_e^2}{N_e} \frac{D^2}{(1 \pm D)}, \tag{11}$$

where n_e is the electron density at the point where a jump forms and N_e is the total number of electrons $\int n_e dx$. From condition (11) it follows that a jump can develop in three cases: 1) in the case $D \ll 1$ already treated; 2) for $D \sim 1$, provided that $dn_0/dx \gtrsim \partial n_e/\partial x$ holds; and 3) for $D \sim 1$, when n_e is small at the point where the jump forms.

From (9b) we see that jumps do not form for $D = 1$ in a plasma with two types of positive ions. This case corresponds to ordinary ambipolar diffusion. Figure 2a shows the evolution of the initial negative-ion and electron profiles $n_0(x) = 2\exp((-x - \pi/2)^2/0.1)$ and $n_e = 2(1 + \cos x)$ for $D = 1$. The condition for the absence of transmission (vanishing of the derivative) is imposed at the bounds. The negative ions are displaced toward the maximum in the electron density. Since the velocity $U(n)$ given by (9a) and (9b) decreases as a function of density, a jump develops at the trailing edge. Steepening of the trailing edge of the negative-ion profile and flattening out of the leading edge are clearly exhibited in Fig. 2a. As already noted, the system of equations (7) is unchanged to within signs under interchange of negative and positive ions. Jumps should therefore also form in a plasma containing two types of positive ions. Figure 2b shows the evolution of the initial positive-ion profile $p_1 = 1.5(1 - \tanh 5(x - 1))$, $p_2 = 1.5(1 + \tanh 5(x - 1))$, $b_{p1}/b_{p2} = 0.5$. Just as in the case when the ratio of the mobilities was small, the more mobile ions attempt to leave the region where the less mobile ions are located. In consequence, a plasma with two types of ions tends to separate into regions with only one type of ion.[3]

Thus far we have restricted ourselves for simplicity to the case of solid-wall boundary conditions $\partial n_e/\partial x = 0$. Let us consider the more realistic zero boundary conditions, which describe rapid plasma recombinations at the walls. According to (10), a jump should develop in a plasma with negative ions near the wall. Figure 3 shows the formation of such a jump. The initial ion density profiles have the form $p_0(x) = 2(1 + \cos x)$ and $n_0(x) = 1 + \cos x$ for $D = 0.5$.

Just as in the case of the problem of transport with a nonlinear velocity,[9,10] the thickness of the jump is determined by the small terms with the highest derivatives discarded in Eq. (3). They correspond to violation of quasineutrality and a small ion diffusion (determined by ion temperature $T_i \ll T_e$). Let us consider the case in which the Debye radius is small compared with the thickness of the jump, so that quasineutrality holds everywhere. Then in the coordinate frame moving with the jump velocity V, the ion fluxes are conserved at the jump, and taking into account ion diffusion we have

$$\Gamma_n = \frac{n}{n_e}\frac{\partial n_e}{\partial x} - \frac{T_i}{T_e}\frac{\partial n}{\partial x} + Vn, \quad n_e = p - n,$$

$$\Gamma_p = -\frac{Dp}{n_e}\frac{\partial n_e}{\partial x} - \frac{DT_i}{T_e}\frac{\partial p}{\partial x} + Vp. \tag{12}$$

The boundary conditions can be written in the form

$$n, p \to n_\pm;\ p_\pm \quad \text{for} \quad x \to \pm\infty,$$

$$\frac{\partial n_e}{\partial x} \to \left.\frac{\partial n_e}{\partial x}\right|_\pm \quad \text{for} \quad x \to \pm\infty.$$

From the boundary conditions it follows that the jump velocity should be determined by the values of the boundary densities and the derivatives $\partial n_e/\partial x$. To lowest order in T_i/T_e we find in analogy with (5)

$$V = -\frac{D}{Dp_- + n_-}\left.\frac{\partial n_e}{\partial x}\right|_+ = -\frac{D}{Dp_+ + n_+}\left.\frac{\partial n_e}{\partial x}\right|_-. \tag{13}$$

Using the value (13) for the velocity, we can easily obtain an equation from (12) for the change in n at the jump:

$$\frac{T_i}{T_e}\frac{\partial n}{\partial x} = \frac{(1 + D)\,\partial n_e/\partial x|_+}{Dp_- + n_-}\frac{(n - n_+)(n_- - n)}{p + n}. \tag{14}$$

For the cases considered above we have $n_+ = 0$, and the length scale at the jump is equal to

$$l(x) = \frac{n}{\partial n/\partial x} = \left.\frac{T_i n_e(n + n_e)}{T_e n_- \partial n_e/\partial x}\right|_+.$$

In our calculations (see Figs. 1-3) the thickness of the jump is generally determined by numerical diffusion. The steep portions of the profiles correspond to from one to three grid points. Since at the jump n varies from n_- to 0, for $n_- \gg n_e$ the length scale $l(x)$ decreases sharply as n approaches zero. Hence the function $n(x)$ becomes markedly steeper. It is possible in principle for jumps to exist in which $n_+ \neq 0$ holds. However, such jumps result only for special initial conditions, when the initial density profiles are very nonuniform.

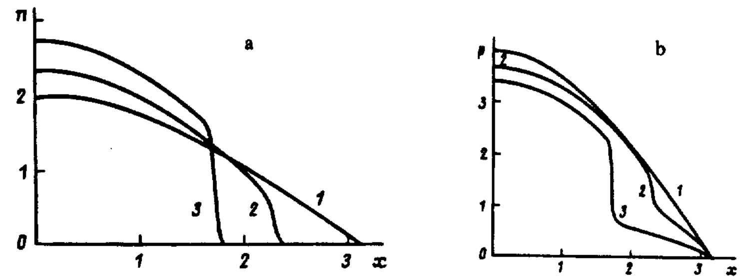

FIG. 3. a) Density profiles of the negative ions. The initial density profiles were: negative ions $n = (1 + \cos x)$; positive ions $p = 2(1 + \cos x)$; ratio of mobilities $D = 0.5$; zero boundary conditions were used. The numbers label the density profiles of the negative ions at times (t): 1) 0; 2) 1; 3) 5. b) Density profiles of the positive ions; conditions as in Fig. 3a.

3. EFFECT OF PLASMA CHEMICAL PROCESSES ON THE POSSIBILITY OF JUMP FORMATION

In many practical applications it is important to take into account processes by which electrons and ions are converted into one another. These processes tend to establish a plasma with equilibrium composition. At the jumps the electron density remains discontinuous, while the ion densities break up. The processes by which ions and electrons are converted into one another, so whether or not a jump can form depends on which takes place more rapidly, profile steepening due to transport on the time scale (10) or establishment of a plasma chemical equilibrium.

Let us consider the effect of the processes encountered most frequently in electronegative gases: attachment and detachment of electrons. Equations (7) take the form

$$\frac{\partial p}{\partial t} = D\frac{\partial}{\partial x}\frac{p}{n_e}\frac{\partial n_e}{\partial x},$$
$$\frac{\partial n}{\partial t} = -\frac{\partial}{\partial x}\frac{n}{n_e}\frac{\partial n_e}{\partial x} + \alpha n_e - \beta n, \qquad (15)$$

where α and β are the electron attachment and detachment probabilities. Expressing $\partial n_e/\partial x$ from the first equation, substituting into the second, and discarding terms not responsible for the formation of a jump, we find [similar to (8) and (9a)]

$$\frac{\partial n}{\partial t} = -U(n)\frac{\partial n}{\partial x} + \alpha p - (\alpha + \beta)\, n. \qquad (16)$$

The first term describes the formation of the jump, while the second describes the establishment of equilibrium between electrons and negative ions over a time of order $(\beta + \alpha)^{-1}$ with constant negative-ion density. Let us consider the formation of moderate-sized jumps, from which n differs little from the equilibrium value $n_p = (\beta + \alpha)p/\alpha$. Then for a small deviation $\tilde{n} = n - n_p$ Eq. (16) can be rewritten in the form

$$\frac{\partial \tilde{n}}{\partial t} = -\left(U(n_p) + \frac{\partial U}{\partial n}\Big|_{n=n_p}\tilde{n}\right)\frac{\partial \tilde{n}}{\partial x} - (\alpha + \beta)\,\tilde{n}. \qquad (17)$$

Using the substitution $x = \tilde{x} + U(n_p)t$, $\tilde{n} = ce^{-(\alpha+\beta)t}$ and $\tau = (1 - e^{-(\alpha+\beta)t})/(\alpha + \beta)$ we find from (17) and equation for $c(x, \tau)$:

$$\frac{\partial c}{\partial \tau} = -\frac{1}{2}\frac{\partial U}{\partial n}\Big|_{n=n_p}\frac{\partial c^2}{\partial \tilde{x}}. \qquad (18)$$

Since $\tau < 1/(\alpha + \beta)$ holds, a jump can form only if we have

$$-\frac{\partial U}{\partial n}\Big|_{n=n_p}\frac{\partial n_0}{\partial x} > \alpha + \beta.$$

Thus, in a three-component plasma plasma chemical processes compete with breaking of the profile and cause the jumps to decrease or disappear. When more than two types of ions are present in the plasma and plasma chemical processes can be separated into fast and slow processes, more complicated jumps can develop, similar to shock waves in a gas with chemical reactions.[8,10] The structure of a jump is then determined by ion diffusion and the fast plasma chemical processes. Situations are possible in which the density profiles in the jump consist of steep and gentle sections (similar to the viscous jump and relaxation zone of a shock wave in a reactive gas). This situation was considered by Tsendin et al.[1,8] for the case of a semiconductor plasma with a current consisting of four types of charged particles (electrons, holes, charge centers, and traps).

4. STEADY PROFILES IN A BOUNDED MULTI-COMPONENT PLASMA WITH PLASMA CHEMICAL REACTIONS

It is well known that smooth quasisteady solutions containing regions of sharp variation can develop in two cases: because of nonlinearity in the dependence of the flux on density and because of the effect of boundary conditions. In the former case these regions were called jumps, and in the latter case boundary layers. However, intermediate cases are also possible, in which a region develops where the density changes abruptly due to both factors. Such solutions were found by Tsendin[4] and Daniels et al.[5,6] in analyzing the positive column of a negative-ion discharge. Just as in the case of a jump, the scale length on which the functions change within this region is shorter than in the surrounding plasma. On the other hand, like a boundary layer, these regions result from the presence of boundaries.

Consider the structure of such regions for the case of the steady solutions (15) including the ionization Zn_e in the right-hand side of the first equation (15). These equations can be conveniently rewritten if we distinguish derivatives only of n_e and n_e/p. Multiplying Eq. (15) by the weight coefficients $1/D$ and 1, expressing $\partial n_e/\partial x$ in terms of the positive ion flux $\Gamma_p = \int Zn_e dx$, and substituting these into the equation for the negative ions, we find

$$\frac{d^2n_e}{dx^2} = (\alpha + Z/D)\, n_e - \beta n, \tag{19}$$

$$\frac{\Gamma_p n_e^2}{Dp^2}\frac{d}{dx}\left(\frac{p}{n_e}\right) = \left(\alpha + \frac{nZ}{pD}\right) n_e - \beta n. \tag{20}$$

Let us begin by considering the case $\beta >> \alpha >> 1$. Then we have $Z \approx 1$ (Ref. 4) and $n << n_e$ holds everywhere, so that (20) can be rewritten in the form[6]

$$\frac{n_e^2}{p^2}\frac{d}{dx}\left(\frac{n}{n_e}\right) = \frac{\beta n n_e}{p\Gamma_n}\left[\frac{\alpha}{\beta} - \frac{n}{n_e}\right]. \tag{21}$$

This equation describes a boundary layer consisting of an electron–ion plasma with a scale length

$$l \sim \frac{\Gamma_p}{\alpha p} \sim (\alpha l)^{-1} \sim \alpha^{-1/2}, \tag{22}$$

corresponding to the transition from $n/n_e = 0$ at the boundary to $n/n_e = \alpha/\beta$ in the central region of the column. As α increases the thickness decreases, and for $\alpha \sim \beta$ the nonlinearity of the drift velocity [the coefficient on the left-hand side of (21)] becomes important. It leads to steepening of the n_e/n profile as we move toward the central part of the plasma. In the limit $\alpha >> \beta >> 1$, Z the thickness of the boundary layer is of order $\alpha^{-1/2}$, as before.

But the scale of the transition decreases sharply in accordance with (22) when the ratio n_e/p approaches $\beta/\alpha << 1$ as we move away from the wall. This part of the boundary layer was therefore treated in Ref. 4 as a jump. The right-hand side of (19) decreases sharply, which corresponds to the occurrence of a kink in dn_e/dx, similar to that treated above in Secs. 2 and 3. Calculations[6] also show that a kink in dn_e/dx occurs in this case for $x \approx \alpha^{-1/2}$.

The structure of this jump which separates the boundary layer from the attachment–detachment equilibrium region can easily be found as follows. Taking n_e to be constant and neglecting terms proportional to Z, we find from Eqs. (19) and (20) the equation

$$\frac{n_e}{p} + \frac{\beta}{\alpha}\ln\left(\frac{\alpha n_e}{p} + \beta\right) = \frac{\alpha n_e D}{\Gamma_p} x. \tag{23}$$

Its solution describes an exponential approach to the equilibrium $n_e/p = \beta/\alpha$ over a distance of order $\beta l/\alpha$ given by Eq. (22) and a power-law approach [when it is possible to disregard the variation in n_e/p in the argument of the logarithm of Eq. (23)] to an ordinary boundary layer for $p/n_e \sim 1$, where now it is necessary to take into account the variation of n_e.

More complicated structure develops in the case $\beta << 1$, $\alpha << 1$ (Refs. 4 and 5). A linear estimate shows that the boundary layers of an electron–ion plasma in this case should merge and occupy the whole discharge cross section, similar to the behavior observed when Poisseulle flow develops in a tube. However, the vanishing of Γ_p at the center of the discharge implies that the drift velocity [the coefficient on the left-hand side of (20)] becomes very small, a region occurs in which $n(x)$ increases abruptly, and an ion–ion plasma forms. The physical reason for this is that the densities averaged over the cross section must satisfy $\langle n\rangle/\langle n_e\rangle = \alpha/\beta$, while nearly the entire volume of the column is occupied by an electron–ion plasma. Daniels et al.[5] termed this phenomenon column contraction. It should be noted, however, that the current density, electron density, and rates of ionization and attachment are distributed uniformly over the cross section – only the n and p profiles are highly peaked. The boundary between the electron–ion and ion–ion plasmas in this case can also be treated as a jump.[4] Its structure is quite complicated.[4,5] The negative-ion density in the center is $n_0 \sim \alpha n_e/(\beta r_0)$, where r_0 is the radius of the ion–ion plasma. From (20) in the center of the column we have $\beta n_0 = rn_e/D$, whence $r_0 \sim \alpha << 1$. Over distances of order r_0 we can treat n_e as constant. Then Eq. (20) has the solution[5]

$$x^{1-\bar\beta+\bar\alpha} = \frac{\bar\alpha\left(\frac{n_e}{p} - \beta\right)^{\bar\beta}}{\left(1 + \bar\alpha - \frac{n_e}{p}\right)^{1+\bar\alpha}}, \tag{24}$$

where $\bar\beta = \beta D/Z$ and $\bar\alpha = \alpha D/Z$.

The solution of (24) describes a plateau for small values of x:

$$n = n_e/(\beta + (x/\bar\alpha)^{1/\bar\beta}) \tag{25}$$

and an abrupt transition at $x \sim \bar\alpha$ to the solution which describes the region $x > r_0$:

$$n = \frac{\bar\alpha n_e(x)}{1-\bar\beta}\left(\frac{1}{(\sin x)^{1-\bar\beta}} - 1\right).$$

The typical width of the transition region is $n/dn/dx \sim \bar\beta\bar\alpha << r_0$.

We are indebted to E. L. Levin and V. A. Shveigert for assistance in performing the numerical calculations, Yu. E. Gorbachev and V. A. Rozhanskii for fruitful discussions, and to the Russian Fund for Fundamental Research, Project No. 93-02-16873, for financial support.

[1)] We will assume that quasineutrality develops over a time $\sim(4\pi\sigma)^{-1}$, where σ is the plasma conductivity (cf. Ref. 7), which is small in comparison with the diffusion time; this stage will not be treated here.

[1] A. P. Dmitriev, V. A. Rozhanskii, and L. D. Tsendin, *Usp. Fiz. Nauk* **146**, 237 (1985) [*Sov. Phys. Usp.* **28**, 467 (1985)].

[2] H. Sabadil, *Beitr. Plasmaphys.* **13**, 235 (1973).

[3] C. M. Ferreira, B. Gousset, and M. Touzeau, *J. Phys. D: Appl. Phys.* **21**, 1403 (1988).

[4] L. D. Tsendin, *Zh. Tekh. Fiz.* **59**, 21 (1989) [*Sov. Phys. Tech. Phys.* **34**, 11 (1989)].

[5] P. G. Daniels, R. N. Franklin, and J. Snell, *J. Phys. D: Appl. Phys.* **23**, 823 (1990).

[6] P. G. Daniels and R. N. Franklin, *J. Phys. D: Appl. Phys.* **22**, 780 (1989).

[7] E. M. Lifshitz and L. P. Pitaevskii, *Physical Kinetics*, Pergamon Press, Oxford (1981).

[8] V. A. Rozhanskii and L. D. Tsendin, *Collisional Transport in a Partially Ionized Plasma* [in Russian], Énergoatomizdat, Moscow (1988).

[9] G. B. Whitham, *Linear and Nonlinear Waves*, Wiley, New York (1974).

[10] Ya. B. Zel'dovich and Yu. P. Raizer, *The Physics of Shock Waves in High-Temperature Hydrodynamic Phenomena*, Academic, New York (1966).

Translated by David L. Book