Two-dimensional high-frequency discharge at intermediate pressures

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It was previously discovered that a high-frequency (hf) capacitive discharge may be characterized by two values of the normal current density. The causes of the inhomogeneity of an hf discharge across the current flow at intermediate pressures under the low-current regime have been investigated. It has been shown that the stability of a discharge is strongly dependent on the type of electrodes: in the case of metallic electrodes the stabilizing influence of the sheaths at the electrode boundaries is four times weaker than in the case of electrodes coated with a thin dielectric layer. Perturbations extending along the current, which do not affect the sheaths at the electrode boundaries, are most dangerous for instability associated with stepwise ionization. Both these findings indicate that an assessment of the stability on the basis of the slope of the current–voltage characteristic may be erroneous. It has been concluded on the basis of a comparison of calculations and experiments that the low-current regime in argon and nitrogen is associated with superheated instability and that the main type of instability in helium is not thermal. It has been shown on the basis of an analysis of the boundary between the plasma region and the periphery that in the case of superheated instability of a discharge with dielectric electrodes, the normal current density is 1.5 times greater than the value of the current density at the minimum of the current–voltage characteristic, while the corresponding voltages differ only slightly. The structure of the transition region is important when discharge regimes with two minima on the current–voltage characteristic are considered. In particular, it has been shown that under certain conditions the stable states may not adjoin one another, and the multiplicity of the forms of the normal current density may result in a dependence of the form of the discharge on its history.

1. INTRODUCTION

It is widely known that a high-frequency (hf) capacitive discharge at intermediate pressures can fill only part of the electrode gap in the transverse direction to the current and that it may be unstable toward transverse perturbations. The processes involved in the formation of two-dimensional structures in an hf discharge have been studied to a small extent. In some cases two minima corresponding to two values of the normal current density $j_{n1}<j_{n2}$ were observed experimentally on the current–voltage characteristic.[1] This raises the question of the nature of the instabilities which lead to specific spatial forms of a discharge. In a weakly ionized plasma of inert gases there are several instability mechanisms, which can, in principle, lead to confinement of a discharge. It is often difficult to single out one of them, since the regions for the existence of instabilities of different types may overlap.[2] At the same time, a quantitative description requires exact knowledge of the instability mechanism realized under given conditions. One of them, viz., the thermal mechanism of instability in an hf discharge was discussed in Ref. 3, but no comparison with experiment was made.

At very low concentrations of charged particles, at which a quasineutral plasma does not exist, an instability associated with a transition from a regime with free diffusion of the electrons to ambipolar diffusion can appear. This instability results, for example, in relaxational oscillations in gas-discharge tubes at low currents.[4] In an hf discharge this instability can results in an effect like a normal current density at ion concentrations $\sim T_e/4\pi eL_0^2$, where T_e is the electron temperature and $2L_0$ is the electrode spacing. However, in our experiments a normal current density with j_{n1} was observed at concentrations one to two orders of magnitude greater. Just as in dc discharges,[2] instability can be caused by instability of either the sheaths at the electrode boundaries or the plasma itself. The transition of a discharge to the regime with the high normal current density j_{n2} is accompanied by abrupt restructuring of the sheaths at the electrode boundaries. Therefore, it is naturally associated with the transition of the discharge from the α form to the γ form, under which the current–voltage characteristic of the sheaths becomes descending.[3]

The calculation method is described in the second section of this paper. In the third section it is concluded on the basis of a comparison of calculations with experiment that the low-current regime in Ar is associated with thermal instability of the bulk of the plasma. In He thermal instability gives a descending current-voltage characteristic at pressures several times greater than the experimental values. Therefore, the main mechanism of the instability observed in He is not thermal. In nitrogen the difference is less significant, but other instability mechanisms besides the thermal mechanism must also be taken into account to achieve agreement with experiment.

The constant displacement voltage V_{dc} between the plasma and an electrode in an hf discharge is known to be caused by the field in the space-charge sheaths at the electrode boundaries.[1] In an asymmetric discharge the characteristics of the sheaths are different. Consequently, a constant potential difference appears between electrodes not short-circuited at a constant current. If the electrodes are connected at a constant current, the sheath adjacent to the larger electrode undergoes significant restructuring. Here the electron concentration profile touches the smaller electrode once during each period. In the case of the larger electrode, the sharp (with a width of the order of the Debye radius) boundary of the electron profile oscillates far from the surface. Thus a region of ionic space charge is continually adjacent to the electrode. A similar situation arises, if the current density or the ion concentration profile is inhomogeneous along the surface of an electrode (in the Y direction). If a constant current cannot flow along the electrode (a split electrode or an electrode with a dielectric coating), a profile of constant potential $V_{dc}(y)$ relative to the plasma forms on its surface. This effect was observed experimentally in Ref. 5. In the case of a metallic electrode in a plasma that is inhomogeneous along y, the potential difference V_{dc} should not depend on y. Therefore, the electron profile touches the surface only at certain points. The parameters of the sheath at those points are the same as at the dielectric coating, and at other sites the sheath is thicker over the metallic surface than over the dielectric coating. Therefore, the stability of an hf discharge is greatly dependent on the type of electrode employed. It is shown below that in the case of an α discharge the contribution to the decrement caused by the stabilizing action of the sheaths is four times smaller in the case of a conducting surface than in the case of a dielectric surface. Therefore, the stability of a discharge cannot, in principle, be evaluated on the basis of its one-dimensional current–voltage characteristic (which does not depend on the properties of the boundary surface).

Since the sheaths have a stabilizing influence in the case under consideration, two-dimensional fluctuations in which $k_y \gg k_x$ and the perturbations of the sheaths are minimal may be most dangerous. They are also not subject to analysis on the basis of one-dimensional current–voltage characteristics.

As a result of the interaction between the plasma and the sheaths, complex two-dimensional structures,[6] which are not always confined to the phenomenon of a normal current density, can form in an hf discharge. For example, as the electrode spacing is smoothly diminished, the discharge column burning in an α form breaks up into several ($\sim 4-6$) columns of smaller size when the electrode spacing becomes comparable to the sheath thickness.

A discharge burning under the conditions of the low normal current density j_{n1}, which is governed by a thermal instability mechanism, is considered in the fourth section of this paper. The value of j_{n1} is found for the case of dielectric electrodes at small degrees of heating of the gas in the quasi-one-dimensional model (in which the characteristics of a discharge along the current are determined only by the local value of the current density and the voltage on the electrodes). It is shown that the value of the normal current density does not coincide with the minimum of the current–voltage characteristic. The difference between the current values reaches a factor of 1.5 [see (32)]. The values of the normal current density and the corresponding voltage should be determined from the condition for the existence of a smooth transition region between the center and periphery of the discharge, where the concentration of charged particles tends to zero. Since this transition is determined by diffusion and heat condition in the model, the parameters of the normal current density depend on the ratio between them. The transition itself may have a structure. For example, when there is little diffusion in comparison with recombination, the temperature profile of the gas "overshoots" the concentration profile, and its temperature remains significant in a region where the plasma concentration is nearly equal to zero (Fig. 4).

The analysis of the transition region is also important for analyzing the effects at high values of the current density $j=j_{n2}$. In particular, it is shown in the fifth section that under certain conditions transitions between stable states are forbidden. This places some restrictions on the form of a two-dimensional discharge and results in a dependence of the form of a discharge on its history.

2. MODELING OF A ONE-DIMENSIONAL hf DISCHARGE AT INTERMEDIATE PRESSURES

A. Calculation of discharge parameters

We performed quantitative calculations, which made it possible to evaluate the parameters of a discharge: the plasma concentration, the fields in the sheaths and in the plasma, the ratio between the displacement current and the conduction current, the sheath thickness, etc. The calculations were performed according to the method in Refs. 7 and 8 using the averaged motions of fast electrons. The averaged equation for the ion concentration has the form

$$\frac{d}{dx}\left(-D_{\text{eff}}\frac{dn}{dx}+Vn\right)=\langle I\rangle-\langle R\rangle, \tag{1}$$

where D_{eff} is the effective diffusion coefficient, which is equal to the sum of the ambipolar and high-frequency coefficients,[9] V is the velocity of the ions in the sheaths, which is governed by the mean field of the ionic space charge (3a), $\langle I\rangle=\langle n_e b_e \alpha E\rangle$ and $\langle R\rangle=\langle \beta n_e n_i\rangle$ are the time-averaged ionization and recombination rates, and α is the Townsend coefficient.

In contrast to Ref. 7, corrections of the order of $(\omega\tau)$, where ω is the frequency of the field and $\tau=\tau(x)=(4\pi e n_e(x) b_e)^{-1}$ is the electronic Maxwell time, were included in Eq. (1). These corrections were confined to the fact that the amplitude of the oscillator field $E_0(x)$ in the plasma region was calculated from the formula

$$E_0(x)=j_0 w/(b_e n_e); \quad w=((\omega\tau)^2+1)^{-1/2}. \tag{2}$$

Therefore, we used the following interpolation for $V(z)$, where $z=\omega t$, which is valid for $\omega\tau \ll 1$ and gives the correct expression for the sheath thickness when $\omega\tau \gg 1$,[7]

$$V(z)=4b_i j_0 w(\sin z - z\cos z)/\omega \tag{3a}$$

and the equation for the boundary of the spatial distribution of the electrons

TABLE I.

Gas	$b_i p$, 10^5 cgs units	$b_e p$, 10^8 cgs units	C, $cm^{-1}\cdot Torr^{-1}$	G, $(V/cm\cdot Torr)^{1/2}$	λ_0, $10^{-4} W/cm\cdot deg$
Ar	3	1.3	29.2	26.6	1.8
He	46	2.6	4.4	14	15
			A	B	
N_2	4.6	1.3	24.4	264	2.4

$$\frac{dz}{dx}\sin z=\frac{e\omega n(x)}{j_0 w}. \tag{3b}$$

The expression for ionization under the assumption of local field dependence (2) was written down in Ref. 7 [Eq. (42)]. Thus, the calculation gave self-consistent profiles of the ion and electron concentrations and the distributions of the fields in the plasma and in the sheaths. The errors in the model are due mainly to three factors: the assumption that the electronic distribution function has a local character, the choice of the approximate ionization model [see (5a) and (5b) below], and the assumption that the electron concentration profile has a sharp boundary. In other words, it was assumed that the potential drop in the sheath is much greater than the electron temperature, so that the Debye radius (over which n_e varies abruptly) is small compared with the sheath thickness.[6,7]

The calculations in Ref. 3 show that the nonuniform heating of the neutral gas must be taken into account already at comparatively small currents. Since the bulk of the energy imparted to the discharge is utilized to heat the neutral gas under the conditions considered, its temperature is described by the equation

$$\frac{d}{dx}\lambda(T)\frac{dT}{dx}+\langle Ej\rangle=0 \tag{4}$$

with the boundary conditions

$$T(x=\pm L_0)=T_0.$$

In (4) the evolution of Joule heat was calculated with consideration of the fact that the dependence of the plasma concentration profile on $T(x)$ is mainly due to the sharp dependence of $\alpha(E_0 T)$ [Eqs. (5a) and (5b)]. Therefore, in a zeroth approximation the product $E_0(x)T(x)$ was assumed to be constant, and the profile of $E(x)$ thus calculated was plugged into (4). The profile of $T(x)$ found from (4) was use to calculate $\langle I(x)\rangle$ in (1).

As we shall see below, to evaluate the instability increment it is very important to know the parameters of the sheaths with maximum accuracy. Therefore, the calculations based on Eqs. (1)–(3) were performed for the three simple gases He, Ar, and N_2.

The values of the parameters selected are listed in Table I. The approximations of the Townsend ionization coefficients for He and Ar were taken in the form

$$\alpha=\frac{CpT_0}{T}\exp\left(-\sqrt{\frac{G^2pT_0}{ET}}\right), \tag{5a}$$

and for nitrogen we used the usual approximation[3]

$$\alpha=\frac{ApT}{T}\exp\left(-\frac{BpT_0}{ET}\right), \tag{5b}$$

where T_0 corresponds to standard conditions.

The α regime was considered, and thus the ion–electron emission coefficient γ was set equal to zero. The thermal conductivity of the gas $\lambda=\lambda_0(T/T_0)^{1/2}$, where λ_0 is the thermal conductivity under standard conditions. Since the frequency of the field was much smaller than the relaxation frequency of the electron energy, an oscillator field of the form $E(x,t)=E_0(x)\sin\omega t$ was plugged into (5), and the result was averaged over the period of the high-frequency field after Ref. 7.

B. Calculation results

Calculations were performed for He and Ar at pressures from 50 to 600 Torr and $L_0=0.5$ and 1 cm and for N_2 at $p=4$ to 40 Torr and $L_0=0.5$ to 1 cm. These values corresponded to the experimental range of variation of the parameters. When these values of the parameters were taken, the exponential functions in Eqs. (5a) and (5b) were of the order of 10, and thus the ionization rate was a strong function of the field. It follows from (1) that the exponents are then weakly (logarithmically) dependent on the parameters of the discharge. The sheath thickness is determined by the sweep of the electrode-boundary trajectory of the electrons moving in the oscillating field $E_0\sin\omega t$.[8] Since the reduced field E_0T/p_0T_0 is weakly dependent on the conditions in the discharge, the amplitude of the electron oscillations, which equals

$$\frac{2v_{dr}}{\omega}=\frac{2b_e pE_0T/p_0T_0}{\omega},$$

also varies weakly with the current, the pressure, and the electrode spacing. It is proportional to ω^{-1} and depends on the kind of gas. The calculations gave the following values for the sheath thicknesses at a frequency of 13.6 MHz when $\omega\tau\ll 1$: $L\cong 1$ mm for He, $L\cong 4$ mm for N_2, and $L\cong 0.6$ mm for Ar.

These data are in satisfactory agreement with the experimental results in Ref. 7: $L\cong 1.7$ mm for He, $L\cong 3$ mm for air. When $\omega\tau\ll 1$ the current in the plasma is carried by electrons. There is a strong field in the sheaths, which draws ions to the electrodes and results in the formation of a profile of $n(x)$ decaying from the center toward the electrodes. However, in the plasma phase $nET=\text{const}(x)$. Therefore, according to (5), ionization increases strongly already when there is a small drop in the concentration.[8] As a result, the concentration at the plasma–sheath boundary generally drops to no less than half (see also Ref. 7) of the concentration in the center of the discharge. The concentration at an electrode is even smaller: for nitrogen [approximation (5)] the ratio of the concentration in the center of the discharge to the concentration at an electrode was equal to 2–2.5, while in the case of He and Ar [approximation (5a)] this ratio varied from 4 to 6. The oscillator fields in the plasma phase were stronger than the fields at the center of the plasma by roughly the same factor. This ratio is far smaller than the value at low pressures, at which the ionization process is concentrated at

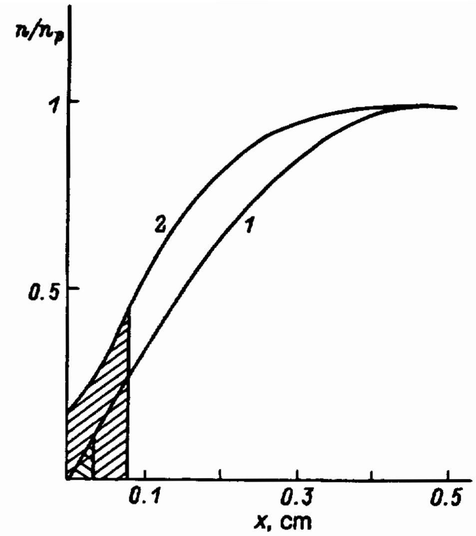

FIG. 1. Profile of the ion concentration normalized to the plasma concentration at the center of the discharge: n_p is the plasma concentration at the center of the discharge; the gas is helium; $p=100$ Torr; half of the electrode spacing $L_0=0.5$ cm; $j_0=1$ (*1*) and 3 mA/cm^2 (*2*); $n_p=1.5\times10^9$ (*1*) and $1.7\cdot10^{10}$ cm^{-3} (*2*); $\omega\tau=3.4$ (*1*) and 0.33 (*2*). The sheath regions are hatched.

TABLE II.

Gas	$2L_0$, cm	j_0, mA/cm^2	p, Torr	V, (V)	V_R, (V)	V_c, (V)	$\omega\tau$	Q, W/cm^2	$\tan\phi$
Ar	1	13	200	320	305	70	0.5	1.5	0.7
				255*				1.3*	1.0*
He	1	8	100	165	85	142	0.1	–	1.6
				230*				–	0.9*
N_2	1	2.1	4	160	130	84	0.5	0.06	1.3
				200*					1.1*

Note. Here ϕ is the phase shift between the current and the voltage; Q is the power of the discharge per unit area; experimental data are marked by an asterisk.

the center due to its nonlocal nature and the values of n are tens of times smaller in the sheaths than at the center.[10] However, the simplifying model with a spatially homogeneous profile of $n(x)$ is too rough, and the decrease in concentration in the sheaths must be taken into account to calculate the sheath thicknesses. For example, for He the thickness of the layer of the sheaths with a constant ion concentration equal to the concentration at the center would be 3–4 times smaller than the value calculated with the real ion concentration profile. Typical ion concentration profiles are shown in Fig. 1.

We note that due to the exponential dependence of the ionization on the concentration, the profile of the latter in the bulk of the discharge is strongly flattened even without consideration of the ionization (curve *2* in Fig. 1 for He).

If $\omega\tau\gg1$, the discharge current in the plasma is mainly a displacement current. In addition, the field in the discharge is almost uniform, so that the ionization term in (1) is simply proportional to the concentration. Since the coefficient of hf diffusion is small for $\omega\tau\gg1$,[11] when L_0 is much greater than the sheath thickness L, Eq. (1) reduces to the usual equation of ambipolar diffusion with a nearly zero boundary condition at the electrodes. Therefore, the concentration in the sheaths drops practically to zero. The departure of ions in the sheaths is much slower than in the case of $\omega\tau\ll1$; therefore, the fields in the sheaths when $\omega\tau\gg1$ are weaker than the field in the plasma phase when $\omega\tau\ll1$. As a result, the sheath thickness is somewhat smaller in the case of $\omega\tau\gg1$ than in the opposite case. For example, in the case of helium at a pressure of 100 Torr, $L=0.3$ mm when $\omega\tau$ at the center equals 1.7, and $L=0.8$ mm when $\omega\tau=0.3$ (Fig. 1).

Table II presents a comparison of the experimental and calculated results. The discrepancies are apparently attributable to the inaccurate approximation of the Townsend coefficient in (5).

The discharge extinction current j_{ex} is an important characteristic. If $\omega\tau\gg1$, extinction occurs when the ionization frequency $\nu_i(E_0/p)$, which depends on the current density $[\nu_i(E_0/p)=\nu_i(4\pi j_{0ex}/p\omega)]$, becomes smaller than the frequency of the loss of charged particles $D_{\text{eff}}(2L_0/\pi)^2$. Since ν_i is strongly dependent on the reduced field (E_0/p), the quantity $j_{0ex}p/\omega=E_0/4\pi p$ varies weakly with the pressure and the frequency. Knowing j_{0ex}, we can find the relationship between $\omega\tau$ and the current density j_0. When $T\cong T_0$,

$$\omega\tau_{j_0}w\approx\text{const}(j_0)=j_{0\text{ex}} \tag{6}$$

Discharges with $\omega\tau\gg1$ burn in a narrow range of variation of the current density. When the latter decreases from $\sqrt{2}j_{0\text{ex}}$ to $j_{0\text{ex}}$, the value of $(\omega\tau)$ increases, according to (6), from unity to infinity. The calculation gives the amplitude values of $j_{0\text{ex}}$ at a frequency of 13.6 MHz and a pressure of 100 Torr, which are equal to 1 mA/cm^2 for He, 10 mA/cm^2 for N_2, and 2.3 mA/cm^2 for Ar. We note that this discharge is often unstable due to the lack of the stabilizing influence of the sheaths (see Sec. 3C).

3. DISCUSSION OF DISCHARGE INSTABILITY MECHANISMS ON THE BASIS OF A COMPARISON OF THE RESULTS OF CALCULATIONS AND EXPERIMENTS

A. Influence of the boundary conditions on the stability

In this section we shall also examine a discharge in the α form, in which the ionization by γ electrons in the sheaths may be neglected. Under these conditions the current–voltage characteristic of the sheaths is ascending,[1] and they are stable with respect to an increase in the transverse perturbations. Therefore, we attribute the instability of the discharge observed when $j\sim j_{n1}$ to the instability of the plasma itself. This is also confirmed experimentally, since the first normal current density corresponds to a discharge in the α form.[3]

The mechanisms of these instabilities have been thoroughly studied (see, for example, Ref. 2). They are associated with the dependence of the ionization frequency on the temperature of the gas or the concentration of metastables. The corresponding instability increments are presented in the Appendix. The instability decrement may be determined by various processes: heat conduction, recombination, diffusion, and the stabilizing influence of the sheaths.

Let us calculate the instability decrement associated with the last mechanism. Let the x axis be perpendicular to the electrodes, and let the y axis be parallel to them, the value $x=0$ corresponding to the center of the discharge gap. We assume for simplicity that the unperturbed profile $n(x)$ is homogeneous. Let us consider the evolution of a perturbation of the plasma concentration having the form

$$n' = \delta n(t)\cos(ky)\cos(\varkappa x), \tag{7}$$

under the assumption that the function $\delta n(t)$ is slow in comparison with the period of the hf field. Let the total current $I(t)$ (and the unperturbed field in the plasma) depend on the time as $\sin \omega t$. Then the current density in the sheaths $j=j_0 \sin(\omega t+\delta\psi)$ is also a harmonic function of the time, but it may be phase-shifted with respect to $I(t)$ by $\delta\psi(y)$. If $\omega\tau \ll 1$, the divergence of the electric current in the plasma equals zero:

$$-\operatorname{div}(n_0+n')(-\nabla \phi' + E_0 \sin \omega t)$$
$$= \Delta \phi' + \varkappa E_0 \frac{\delta n}{n_0} \sin \omega t \sin \varphi x \cos ky = 0, \tag{8}$$

where ϕ' is the perturbation of the hf potential.

The relationship between k and $\varkappa$ is determined by the conditions on the plasma-sheath boundary.

We first consider the case in which an electrode is covered by a thin dielectric coating. Assuming that each sheath is one-dimensional ($kL \ll 1$) and that the plasma concentration in it is uniform, we find the potential difference between the plasma and the electrode

$$V_{\mathrm{sh}\pm} = \left(\frac{4\pi j_0}{\omega}\right)^2 \frac{(1 \pm \cos(\omega t + \delta\psi(y)))^2}{4\pi e n}$$
$$= \frac{2\pi e E_{x0}^2 b_2^2 n}{\omega^2}\left(\frac{3}{2} \pm 2\cos(\omega t + \delta\psi(y))\right). \tag{9}$$

If the higher harmonics are neglected here, E_{x0} is the amplitude of the x projection of the hf field in the x direction in the plasma at the electrode:

$$E_{x0} = E_0 - \frac{\partial \phi'}{\partial x},$$

The plus and minus signs correspond to the sheaths at positive and negative values of x, respectively. The total voltage on both sheaths at the fundamental frequency is

$$V_s = 8\pi e E_{x0}^2 b_e^2 n/\omega^2.$$

Perturbations of the concentration, the hf field, and the current density vary $V_{\mathrm{sh}\pm}$. Since the variable potential difference between the electrodes does not depend on y, an additional variable voltage appears on the plasma:

$$\phi'(x=L_0) - \phi'(x=-L_0)$$
$$= \frac{8\pi e b_e^2}{\omega^2}\left[(nE_{x0}^2)'|_{x=\pm L_0} \cos \omega t\right.$$
$$\left. + nE_0^2 \delta\psi \sin \omega t\right]. \tag{10}$$

The solution of (8) has the form[1)]

$$\phi' = \frac{\varkappa E_0}{\varkappa^2 + k^2}\frac{\delta n}{n_0} \sin \omega t \sin \varkappa x \cos ky + (A \operatorname{sh} kx$$
$$+ B \operatorname{ch} kx)\cos ky \cos \omega t. \tag{11}$$

Since each electrode is equipotential even in the presence of a perturbation, the boundary conditions which should be taken are

$$\phi'(x=-L_0) + V'_{\mathrm{sh}^-} = \phi'(x=L_0) - V'_{\mathrm{sh}^+} = 0, \tag{12}$$

where a prime indicates an addition due to the perturbation.

The values of the phase shift between j and I equal

$$\delta\psi(y) = -\operatorname{Im}\left(\frac{\partial \phi'}{\partial x} \Big/ E_0\right). \tag{13}$$

Substituting (11) and (13) into (12), we obtain the equations for A, B, and $\varkappa$. Owing to the symmetry of the problem, $A=0$, and the equation for k has the form

$$\frac{\varkappa \operatorname{tg} \varkappa L_0}{(k^2-\varkappa^2)kL_0^2 \operatorname{cth} kL_0} = \frac{V_s^2}{V_R^2}, \tag{14}$$

where U_s is the unperturbed value of (9) and $U_R = 2E_0L_0$ is the voltage drop on the plasma.

Linearizing Eq. (1) for the concentration, we easily obtain the expression for the decrement γ_d

$$\gamma_d = -\nu_i \hat{\nu}_i \varkappa^2/(k^2+\varkappa^2), \tag{15}$$

where ν_i is the ionization frequency and $\hat{\nu}_i = d \ln \nu_i / d \ln E_0$.

It follows from boundary condition (14) that when $kL_0 \ll 1$,

$$\varkappa L_0 = kL_0 V_s / \sqrt{V_s^2 + V_R^2} \tag{16}$$

and a one-dimensional analysis may be employed. When $kL_0 \gg 1$ and $U_s \sim U_R$, we have $\varkappa L_0 \cong \pi/2$. The decrement associated with the influence of the sheaths decreases proportionally to $(kL_0)^{-2}$. This is natural, since the concentration perturbation $\delta n \sim \cos(\varkappa x)$ tries not to "touch" the sheaths when $\varkappa L_0 \cong \pi/2$. If the increment would not depend on k (for example, in the case of an instability associated with stepwise ionization or with a mechanism of maxwellization of the electron distribution upon interelectronic collisions), the fluctuations with large values of k would be most dangerous. Consideration of essentially two-dimensional perturbations would then be necessary to analyze the stability. On the other hand, when $kL_0 \gg 1$, the increment of the thermal instability is proportional to k^{-2}, and in this case the perturbations with small values of k are most dangerous. If the dimension of the discharge across the current d is greater than the electrode spacing L_0, the minimal value $k_{\min} \sim 2\pi/d \ll L_0^{-1}$, and the one-dimensional approximation can be used to analyze the thermal instability.

Now let us consider the case of metallic electrodes. They are also equipotential at a constant potential. Therefore, due to the nonlinear nature of the sheaths, a concentration perturbation creates a perturbation of the quasistationary potential similar to (12):

$$\phi'_{dc} = \frac{6\pi}{\omega^2}(nE_{x0}^2)' \operatorname{ch} kx \cos ky / chkL_0. \tag{17}$$

The appearance of the field $-\nabla\phi'_{dc}$ induces a quasistationary electron current. The electrons drift toward the higher potential and tend to flow over into a region with a high plasma concentration. This current may reach an electrode only as a result of variation of the time-averaged electron flux incident to the electrode. The plasma–sheath boundary should then be additionally displaced over a distance $l(n;j_0)$ so as to satisfy the condition for the flow of current. When the displacement l is taken into account, Eq. (9) for $V_{\rm sh}$ is modified:

$$V_{\rm sh\pm}=\frac{2\pi}{en}(j_0/\omega)^2\left(1\pm\cos\,\omega t+\frac{2l+l_d}{L}\right)^2, \tag{18}$$

where L is the sheath thickness and L_d is the small (of the order of the Debye radius) thickness of the boundary between the plasma and the space charge, which ensures equality of the mean electron flux onto the electrode to the flux from the unperturbed plasma.[8] The mean change in the electron current during a period j' equals (Γ_i is the ion flux onto the electrode)

$$j'=-\Gamma_i\frac{V'_{\rm sh}(t=0)}{T_0}, \tag{19}$$

where $V'_{\rm sh}(t=0)$ is the change in the voltage due to the displacement l.

Using (17) and (19), we write the condition of equality between the electron currents in the plasma and at the electrode

$$b_ek\ \mathrm{th}\ kL_0\left(\phi'_{dc}+\frac{L}{l_d}V'_{\rm sh}(t=0)\right)=\Gamma_iV'_{\rm sh}(t=0)/T_e. \tag{20}$$

It follows from (20) that under the condition

$$\frac{l_d}{T_eb_ekL_0\ \mathrm{th}\ kL_0}\sim\frac{b_i(V_{\rm sh}/T_e)^{1/2}}{b_ekL_0\ \mathrm{th}\ kL_0}\gg 1 \tag{21}$$

the plasma is equipotential at a constant current. Since the ratio b_i/b_e is very small, inequality (21) is almost always satisfied.

We find the decrement of the instability in the quasi-one-dimensional case ($kL_{00}\ll 1$). From the condition that the electrode is equipotential at a constant current

$$\phi'_{dc}\frac{2\pi j_0^2}{en\omega^2}\left[\frac{1}{2}+\left(1+\frac{2(l+l_d)}{L}\right)^2\right]=\mathrm{const}(y) \tag{22}$$

it follows that the change in the amplitude of the hf potential at the fundamental frequency equals

$$\phi'=\frac{1}{4}\frac{4\pi eb_e^2}{\omega^2}(nE_{x0}^2)'. \tag{23}$$

Equation (23) gives a voltage change four times smaller than the change given by Eq. (9). This means that the instability decrement in the case of metallic electrodes is four times smaller than in the case of dielectric electrodes.

TABLE III.

	$2L_0$	p	j_0	V	V_R	V_s	$\omega\tau$	Q	$\tan\phi$
Calculation	1	150	12	280	260	74	0.34	1.2	1.4
Experiment	1	150	10.7	205				0.85	0.9
Calculation	2	105	11	290	270	94	0.26	1.1	1.2
Experiment	2	105	11.5	230					1.1
Calculation	1	250	18	340	323	104	0.37	2.0	1.3
Experiment	1	250	13.5	270				2.0	0.8

Note. Here ϕ is the phase shift between the current and the voltage; Q is the power of the discharge per units area.

B. Instability mechanism leading to the first normal current density

Owing to heat conduction, long-wavelength perturbations are most dangerous. The one-dimensional approximation can be used for perturbations with wave numbers across the current $k\ll L_0^{-1}$. When heat conduction and diffusion are disregarded, the instability corresponds to a descending current-voltage characteristic. The product (E_0T) is approximately constant in the plasma. As the current increases, the heating of the gas increases, its temperature increases, and the field accordingly weakens. This accounts for the descending current–voltage characteristic of the plasma. In the case of dielectric electrodes covered by a thin coating, the stability boundary is specified by the condition[3]

$$\frac{\partial}{\partial j_0}(V_R^2+V_s^2)=0. \tag{24}$$

The derivation of condition (24) and its generalization with the consideration of diffusion and heat conduction are presented in the appendix. Since the voltage in the sheaths increases with the current, it is seen from (24) that the sheaths have a stabilizing influence on a discharge.

Solving system (1)–(3), we can construct the one-dimensional current–voltage characteristic of a discharge and find the condition under which instability condition (24) is satisfied. The characteristics of discharges were measured in the experiments at the normal current density (j_{n1}). In our calculations we identified j_{n1} with the current density corresponding to the minimum on the current–voltage characteristic. As we shall see below in Eq. (37), these values may differ by a factor of 1.5, which exceeds the errors of our calculations. A comparison of the discharge parameters for Ar is presented in Table III. It is seen that the agreement is fully satisfactory. In He and N_2 condition (24) was not satisfied in the experimental range of pressures. This raised the question of the exact determination of the cut-off pressure, below which condition (24) does not hold.

If the variations of the sheath thickness are neglected, the current-voltage characteristic is linear: $U_s\sim j_0L/\omega$. If the recombination rate in the plasma $\langle R\rangle$ in (1) is proportional to the square of the plasma concentration,

$$\frac{j_0}{V_R}\frac{\partial V_R}{\partial j_0}=-\frac{\Delta T}{T_0}+\frac{1}{\bar{\nu}_i}, \tag{25}$$

where $\bar{\nu}_i=d\ln\nu_i/d\ln E\gg 1$ and ΔT is the temperature drop between the center and the electrode.[2]

Condition (24) reduces to the quadratic equation

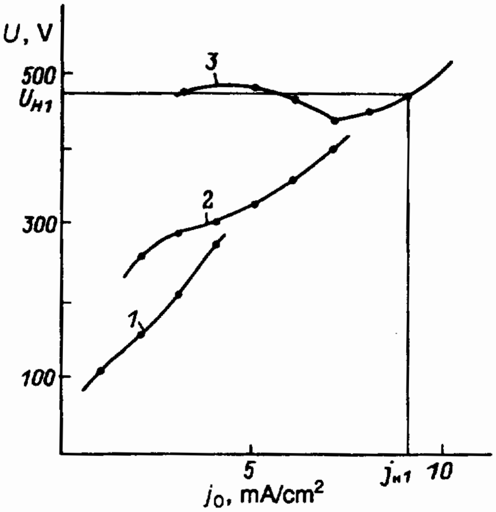

FIG. 2. Calculated current-voltage characteristics of an α discharge in nitrogen: half of the electrode spacing L_0=0.5 cm; p=4 (1), 14 (2), and 30 (3) Torr; U_{n1} and j_{n1} are the parameters of the normal current density.

$$\left(\frac{V_s}{V_R j_0}\right)^2 j_0^2 + \left(\frac{\Delta T}{T_0 j_0}\right) j_0 + \frac{1}{\hat{\nu}_i} = 0. \tag{26a}$$

A family of calculated current–voltage characteristics is presented in Fig. 2. At small values of P and L_0 the current–voltage characteristic has no descending branch. This is due to the stabilizing influence of the sheaths with an ascending current–voltage characteristic.

Instability should appear under the condition

$$\frac{\Delta T}{T_0} > \frac{2V_s}{V_R\sqrt{\hat{\nu}_i}}, \tag{27}$$

under which Eq. (26a) has two roots $j_{\min}$ and $j_{\max}$, which correspond to the minimum and the maximum of the current–voltage characteristic. In the case of dielectric electrodes, when $L_0 \gg L$, condition (27) reduces to

$$\hat{\nu}_i^{1/2} L_0^3 \omega (E/p)^2 / (8\pi^3 \lambda T_0 L) > 1. \tag{28}$$

In the case of metallic electrodes, the unity on the right-hand side should be replaced by 1/4. For example, under the conditions $2L_0$=1 cm and $2\pi\omega$=13.6 MHz the cut-off pressure in the case of a metallic electrode is p_b=200 Torr for He, p_b=50 Torr for Ar, and p_b=7.5 Torr for N_2. In the case of short discharges, in which $L_0 \sim L$ and $U_s/U_R \sim (\omega\tau)^{-1} L/L_0 \sim (\omega\tau)^{-1/2}/\sqrt{\hat{\nu}_i}$, the terms of order $(\omega\tau)^2$ must be taken into account to calculate the cut-off pressure. This was done in the appendix.

Confinement of the discharge was observed experimentally at pressures below the cut-off value given by (28): down to p_b=50 Torr for He, down to p_b=50 Torr for Ar, and down to 4 Torr for N_2. The difference is apparently attributable to the inadequate accuracy of the model of ionization and recombination. The discrepancy between the vibrational temperature and the gas temperature should apparently also be taken into account for nitrogen.[2] In He stepwise ionization and the influence of conversion in the ions on recombination may be significant. It has been reported that the recombination of He_2^+ with He occurs to a very small extent and involves a vibrational level with an energy of 0.6 eV.[12] Since the population of a vibrational level is strongly dependent on the gas temperature, this may also influence the confinement processes. In the case of He the descending current–voltage characteristic may be specified by the mechanism of maxwellization of the distribution function.[2] This requires[2] that $\nu_{een} \gtrsim \nu^* > \delta\nu$, where ν_{ee} is the frequency of interelectronic collisions, ν^* is the frequency of inelastic collisions, and $\delta\nu$ is the frequency of energy loss upon elastic collisions with the atoms. This criterion gives current densities orders of magnitude greater than the observed values. The purity of the gas in the experiments was $\sim 3\times10^{-4}$. If nitrogen or argon were present as an impurity, the ionization process might be determined by the impurity due to the large difference between the ionization potentials ($\sim$8 eV). Therefore, in contrast to the case of a pure gas, confinement can occur in such mixtures at electron concentrations approximately 3×10^3 smaller. However, even such a relaxed criterion gives excessively large values for the plasma concentration.

We note that at low pressures and plasma concentrations, at which recombination is insignificant in comparison with diffusion, the left-hand side of inequality (28) should contain the additional factor $(k_{\min}L/\pi)^{-2}$, where $k_{\min} \sim 1/d$ is determined by the transverse dimension of the discharge (for a cylindrical geometry $k_{\min}$=5.5/R, where R is the radius of the discharge). The stability of the discharge drops significantly at large values of $(L/\pi d)^2$.

C. Stability of a discharge when $\omega\tau \gg 1$

In the general case, regardless of the value of $\omega\tau$, equality (24) may be written in the form

$$\frac{\Delta T}{T_0} - \frac{L^2}{(\omega\tau)^2 L_0^2} - \frac{D_{\text{eff}} k_{\min}^2 + \beta n}{\nu_i \hat{\nu}_i} = 0. \tag{26b}$$

It is seen from Eq. (26b) that a small temperature increase ($\Delta T/T_0 \sim 0.1$) can destabilize a discharge. Inasmuch as the sheaths cease to stabilize the discharge when $\omega\tau > 1$, an increase in $(\omega\tau)$ is accompanied by a decrease in stability. Since $\hat{\nu}_i \sim 10$, when $\omega\tau \sim 1$, instability appears already with a small degree of warming $\Delta T/T \sim 10\%$. A regime with $\omega\tau > 1$ was not observed in our experiments, possibly due to the small value of (kL_0/π), which causes instability of the discharge upon even very small temperature increases.

Experimental verification of the different stabilities of discharges with metallic and dielectric electrodes requires the existence of a stable discharge with $(\omega\tau) < 1$. The stability limit of this discharge is determined by conditions (11) and (23). It follows from expression (26b) that a stable discharge with $\omega\tau < 1$ exists only under the condition $L/L_0 < ((D_{\text{eff}} k_{\min}^2 + \beta n)/\nu_i \hat{\nu}_i)^{1/2}$. Rewriting these conditions for the pressure, we find that

$$1 < p/p_b < \left(\frac{L_0 a}{L\sqrt{\hat{\nu}_i}}\right)^{1/2},$$

where $a=1$, if the loss of particles is determined by recombination (under the condition $(D_{rm\,\mathrm{eff}}\hat{\nu}_i b_e eE^2/\lambda T_0\beta)\ll 1$), and in the opposite case, in which the loss is determined by transverse diffusion, $a=k_{\min}L_0/\pi$.

4. PARAMETERS OF A DISCHARGE BURNING UNDER THE LOW-CURRENT REGIME (DIELECTRIC ELECTRODES)

Let us consider the case in which a regime with a normal current density is realized with little heating of the gas at small values of $\omega\tau$. This situation is possible, if the thickness of the sheaths and the voltage drop in them are small and the gas pressure does not greatly exceed the threshold value given by (28). We assume that the characteristic scale of the variations across the current is smaller than the characteristic scale along the current. We write model equations for the concentration and temperature in the center of the discharge:

$$D_{\mathrm{eff}}\frac{d^2n}{dy^2}=n(\beta n-\nu_i(ET));\quad \nu_i(ET)$$

$$=\nu_i(\varepsilon T)\exp\left(\hat{\nu}_i\left(\frac{E}{\varepsilon}-1+\frac{T}{T_0}-1\right)\right);$$

$$\lambda\frac{d^2T}{dy^2}=\lambda\left(\frac{\pi}{2L_0}\right)^2(T-T_0)-\frac{j_0E_0}{2}. \tag{29}$$

The electric field ε corresponds to the extinction current

$$\nu_i(\varepsilon T)=\pi^2 D_{\mathrm{eff}}/(4L_0^2),$$

and the concentration

$$n_0=\lambda_0 T_0\pi^2/(2eb_e\varepsilon^2L_0^2)$$

corresponds to a temperature difference $\Delta T\sim T_0$. Since $\omega\tau\ll 1$, the voltage drop in the sheaths $V_s=2\sqrt{2}\varepsilon L_0 n/n_1$, where the characteristic value of the concentration

$$n_1=\frac{\omega}{\sqrt{2}\pi e^2 b_e}$$

corresponds to $\omega\tau=L/(L_0 2\sqrt{2})$.

Taking into account the condition $U_s\ll U$, we rewrite the expression for E_0 in the form

$$E_0=\frac{U}{2L_0}-(n/n_1)^2;$$

when diffusion is neglected, Eqs. (29) take on the forms

$$\exp[\hat{\nu}_i(A-(n/n_1)^2+T/T_0-1)]-n\beta/\nu_i(\varepsilon T)=0,$$

$$-(2L_0/\pi)^2\frac{d^2T}{dy^2}=nT_0/n_0-(T-T_0), \tag{30}$$

where A is determined by the voltage on the discharge:

$$A=\left(\frac{U}{L_0}-\varepsilon\right)/\varepsilon.$$

The current–voltage characteristic of the one-dimensional discharge analyzed in the preceding section corresponds to the homogeneous solution of (30)

$$A=-n/n_0+(n/n_1)^2+\frac{1}{\hat{\nu}_i}\ln(n\beta/\nu_i(\varepsilon T)). \tag{31}$$

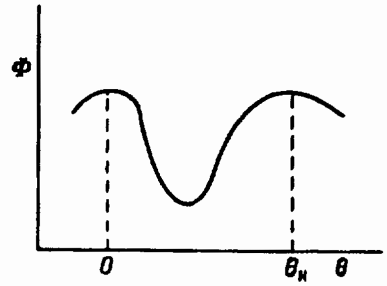

FIG. 3. Dependence of the "potential" Φ on θ for discharge parameters corresponding to the normal current density.

Under the condition

$$\left(\frac{n}{n_0}\right)^2\hat{\nu}_i\gg 1 \tag{32}$$

the last term in (31) may be neglected, and the minimum of the one-dimensional current–voltage characteristic corresponds to

$$n_{\min}=n_1^2/2n_0;\quad A_{\min}=-(n_1/2n_0)^2. \tag{33}$$

Thus, if $n_1\ll n_0$, in the vicinity of $n=n_{\min}$ the increase in the gas temperature in the discharge and the voltage drop in the sheaths are small. The normal current density j_{n1} and the corresponding values of U_{n1} and n_{n1} must be determined on the basis of the condition that a stationary solution of (30) corresponding to a smooth transition from n_{n1} to zero exists.

Expressing n in terms of T from the first of Eqs. (30) and introducing the dimensionless variable

$$\Theta=\frac{T-T_0}{T_0},$$

we obtain the equation for Θ

$$\left(\frac{2L_0}{\pi}\right)^2\frac{d^2\Theta}{dy^2}=Q(\Theta), \tag{34}$$

where

$$Q(\Theta)=\frac{d\Phi}{d\Theta}=\Theta-n(\Theta)/n_0.$$

Under condition (32)

$$n(\Theta)=\begin{cases} n_1\sqrt{A+\Theta} & \text{when } \Theta>-A,\\ 0 & \text{when } \Theta<-A.\end{cases} \tag{35}$$

Equation (34) is completely analogous to the equation of motion of a particle, the role of the time being played by the coordinate, the role of the force by $+Q$, and the role of the potential by Φ. The choice of A is determined by the condition

$$\left.\frac{d\Theta}{dy}\right|_{y=\pm\infty}=0$$

along with the remaining derivatives, i.e., the "potential" Φ in Eq. (34) should have the form schematically shown in Fig. 3, and

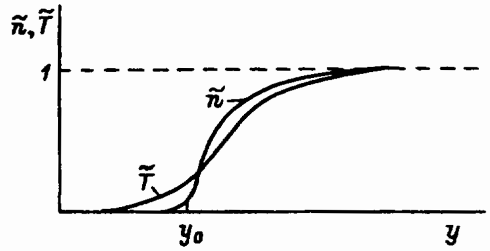

FIG. 4. Schematic representation of the profile of the reduced concentration $\tilde{n}=n/n_H$ and temperature $\tilde{T}=\theta/\theta_H$. The profile of $\tilde{T}$ "overshoots" the profile of $\tilde{n}$.

$$\Phi(0)-\phi(\Theta_{n1})=\int_0^{\Theta_{n1}}(\Theta_n(\Theta)/n_0)d\Theta=0, \tag{36}$$

where Θ_{n1} is the root of the integrand.

In other words, the maxima of the "potential" $\Phi(\Theta)$ should have identical heights. From (36) we find that under the low-current regime

$$A_{n1}=-\frac{3}{16}\left(\frac{n}{n_0}\right)^2; \quad n_{n1}=\frac{3}{4}\frac{n_1^2}{n_0}=\frac{3}{2}n_{\min}. \tag{37}$$

It is seen from a comparison of (33) and (37) that the voltage at the normal current density differs only slightly (by ~10%) from the minimal value, while the plasma concentration n_{n1} and thus the current density are 1.5 times greater than the values for the corresponding minimum on the current–voltage characteristic. The transition region from $n=n_{n1}$ to $n=0$ corresponding to the solution of (34) and (35) is schematically shown in Fig. 4, which presents concentration and temperature profiles. A similar situation is observed in a dc discharge.[12]

We note one interesting feature of the solution of (34) and (35). The plasma concentration on the periphery of the conducting region is strictly equal to zero at a finite gas temperature. This situation can easily be understood on the basis of the following reasoning: in the region where $n=0$ the field is insufficient for self-sustaining of the discharge. An increase in T increases ionization, while an increase in n diminishes it according to (30). Therefore, up to $\Theta=-A$ the concentration is practically equal to zero (point y_0 in Fig. 4).

The discharge burns in a certain region so as to pass the current assigned by the generator, and in the remaining region $n=0$. The states with the plasma concentrations $n=n_{n1}$ and $n=0$ and the voltage $U=U_{n1}$ are stable. The region in which an abrupt transition from n_{n1} to zero occurs should also be stationary. The values of n_{n1} and U_{n1} are also determined from this condition.

Since Eq. (34) corresponds to variation of the temperature with a transverse scale of the order of the electrode spacing, the quasi-one-dimensional model describes the structure of the transition region only with accuracy to the order of magnitude. However, the qualitative results should be correct. The parameters of a discharge with the normal current density (U_{n1},n_{n1}) should be somewhat greater than the parameters corresponding to the minimum on the current-voltage characteristic. They are determined on the basis of the condition that the transition region from $n=n_{n1}$ to $n=0$ is stationary, i.e., that the integral sources of particles and energy for this region equal zero. If the temperature increase in a discharge with a normal current density is large ($\Delta T \gtrsim T_0$), the problem takes on an essentially two-dimensional character, since the bending of the force lines must be taken into account (in the case of little heating, the transverse fields contribute second-order terms with respect to $\Delta T/T_0$ to the absolute value of the total field). This problem can be solved only numerically.

The difference between the minimal and "normal" voltages and current densities can be determined experimentally in the following manner. The current under the normal-current regime is increased until a current at which the discharge fills the entire electrode gap is attained. Then, decreasing the current by moving along the stability branch of the ascending current-voltage characteristic (Fig. 2), we can reach a point on the current–voltage characteristic with $U<U_{n1}$ and $j<j_{n1}$. The possibility of such a transition is closely related to the influence of the boundary conditions with respect to y. This regime should have been observed in a coaxial geometry, where there are no edge effects. In a planar geometry this effect may be strongly suppressed due to the influence of the electrode edges.

5. CONFINEMENT OF A DISCHARGE UPON TRANSITION FROM THE α FORM TO THE γ FORM

Contraction of a discharge upon the transition from the α form to the γ form was observed in the experiments in Refs. 5 and 6. When the ionization in the sheaths begins to be sustained by γ electrons, the sheath thickness may vary greatly. If the value of L under the low-current regime is greater than the energy relaxation length of the fast γ electrons, the ionization rate may be assumed, as before, to be a local function of the field. In this case ionization by γ electrons increases the ion concentration in the sheaths.[2,7] The values of the field in the plasma phase of the sheaths decrease accordingly. This results in a decrease in the derivative $\partial V_s^2/\partial$ in comparison with the discharge in the α form, in which $L\approx$const. Under certain conditions the derivative can even change sign, and the regions next to the electrodes may become unstable. As a result, an additional minimum associated with the transition from the α form to the γ form may appear on the one-dimensional current–voltage characteristic, and the discharge may undergo a transition to a new two-dimensional form. In the experiments in Refs. 5 and 6, different types of burning of a two-dimensional discharge were observed, depending on its characteristics. In a narrow range of parameters ($L\sim L_0$) the α discharge divided up into five or six "spots," which "wandered" over the electrode surface. In some cases, in which the width of the discharge in the transverse direction to the current was comparable to L_0, the discharge burned in the form of a truncated cone in the regions next to the electrodes. However, in most cases, the discharge burned under a normal-current regime. Upon the transition from the α regime to the γ regime, a bright spot with a current density corresponding to the γ regime of the sheaths at the electrode boundaries ($n=n_{n2}$) appeared, and in

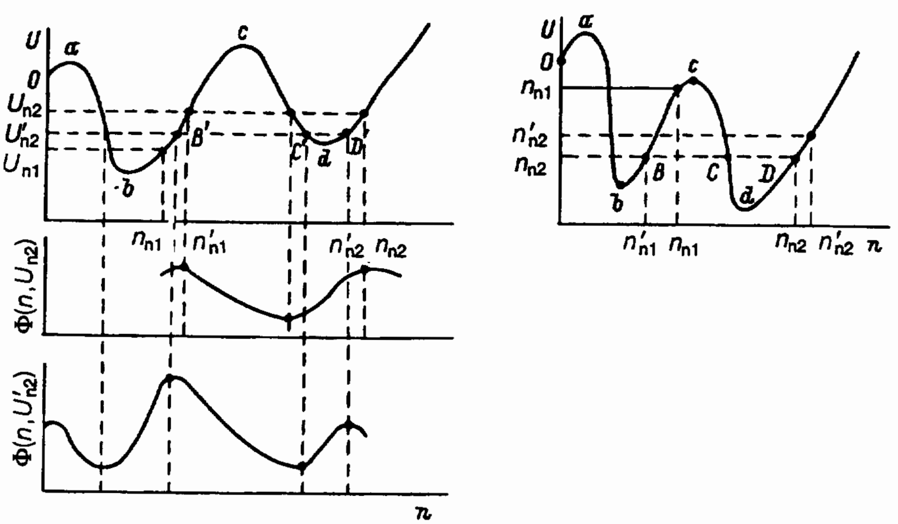

FIG. 5. Current-voltage characteristics of a discharge when $U(n)$ has two minima and corresponding plots of the "potential" $\Phi(n)$. The possible values under a normal-current regime are indicated along the axes.

some cases the remainder of the discharge gap was filled by an α discharge, while in other cases there was no burning of the plasma ($n=0$) on the periphery.

The discharge consisted of two (of the three possible) "phases" with the concentrations n_{n2}, n_{n1}, or 0 and a narrow transition region between them. Let us consider the necessary conditions for the coexistence of these phases on the basis of a model equation like (34). In a homogeneous discharge $Q(n,U)=0$:

$$\frac{d^2n}{dy^2}=-\frac{d\Phi}{dn}=Q(n,V). \tag{38}$$

Figure 5a contains a schematic representation of the dependence of the discharge voltage on the plasma concentration at the center of the discharge n (n is proportional to the electron current, which equals the total current under the conditions of an $\alpha\rightarrow\gamma$ transition, under which $\omega\tau\ll 1$. Segments ab and cd are unstable. Therefore, when the current increases at points a and c or when it decreases at points b and d, the discharge should undergo a transition to a new inhomogeneous state. When the current at point a increases, the discharge undergoes a transition to the regime with the first normal current density, the voltage U_{n1}, and the concentration at the center of the discharge n_{n1}. This transition was considered above in Sec. 3B.

When the current at point c increases, the discharge may undergo a transition to different inhomogeneous states in the transverse direction to the current. The voltage U_{n2} or U'_{n2} may be established on the discharge, and the discharge itself may consist of regions with the concentrations n_{n2} and n'_1 or n'_{n2} and 0. The number of different forms of the discharge increased, since the number of stable branches of the current–voltage characteristic increased. The state with the lowest concentration may now have a zero concentration, or it may be one of the states on branch bc. The discharge can, in principle, exist in any of these forms; the specific form realized depends on the properties of the "potential" $\Phi(n,U)$.

The state with the normal current density corresponds to motion of the "particles" between "potential" humps of equal height (Fig. 5b). The equality of their heights ensures a smooth transition between the equilibrium states, where $dn/dy\rightarrow 0$, i.e., under the normal-current regime the voltage is determined by the condition of the equality $\Phi(n'_1,U_{n2}) = \Phi(n_{n2},U_{n2})$, i.e., the maximum values of the "potential," which is equivalent to the condition $\int_{n'_1}^{n_{n2}}Q\,dn = 0$. This condition is an exact analog to the area rule used to construct van der Waals isotherms or the jump in a shock wave. In cases in which the current–voltage characteristic $U(n)$ has two minima, some transitions may be forbidden. For example, when $U_{n1}<U_{n2}$, the $n'_2\rightarrow 0$ transition is not allowed. In fact, let us examine the potential Φ for the $n_{n2}\rightarrow n'_1$ and $n'_{n2}\rightarrow 0$ transitions (Fig. 5b). Since the condition $\partial Q/\partial U>0$ nearly always holds, the points on the current-voltage characteristic $U=U(n)$ (Fig. 5) where $U>U(n)$ correspond to $Q>0$, and vice versa. It is not difficult to see that under these conditions the voltage U'_{n2} for the $n'_2\rightarrow 0$ transition must be between U_{n1} and U_{n2}. Then the integral $\int_0^{n_{eq}}Q(V',n)dn$, where $n_{eq}(U')$ is the point of the equality $U(n_{eq})=U'$ on branch bc, is greater than zero when $U'_{n2} > U_{n2}$. On the other hand, if $U'_{n2} < U_{n2}$ when $U_{n2}<U_{n1}$, an insurmountable barrier appears. Physically, this means that the creation of particles on segment $D'C'$ cannot exceed their destruction on segment $B'C'$. Thus, when $U_{n2}>U_{n1}$, only the $n_{n2}\rightarrow n'_{1,2}$ transitions are possible, and the $n'_{n2}\rightarrow 0$ transition corresponding to the "usual" normal current density is forbidden. If $U_{n2}>U_{n1}$, the $n_{n2}\rightarrow n'_1$ transitions may be forbidden when the minimum on the current-voltage characteristic on segment CD is sufficiently deep and the creation of particles on this segment is greater than on segment BC (even when point B coincides with point b).

Two-dimensional discharge forms corresponding to the $n_{n2}\rightarrow n'_1$ transition when $U_{n2}>U_{n1}$ and to the $n'_{n2}\rightarrow 0$ transition when $U'_{n2} < U_{n1}$ were observed in the experiments in Ref. 5 and 6, in agreement with the theoretical conclusions. Under the condition $U'_{n2} < U_{n1}$ with a not excessively large difference between them (in the sense indicated above) two discharge states with $U = U'_{n2}$ and U_{n2} are possible. One has always been observed in experiments, i.e., nature selects the state with the greater voltage, violating the usual minimum principle, which prefers the state with the smallest voltage.

6. CONCLUSIONS

To analyze the stability of a discharge, the interaction of the stable and unstable regions of the discharge must be investigated, and the use of von Engle and Steenbeck's phenomenological principal of minimal power is inadequate, since in some cases it gives incorrect quantitative results, and sometimes incorrect qualitative results.

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APPENDIX

Let us examine the most dangerous long-wave perturbations, under which the ion concentration varies adiabatically with the current density in the longitudinal direction, the wavelength in the transverse direction $2\pi/k$ is maximal, $k \sim 2\pi/d$, and $kL_0 \ll 1$. Then integral characteristics, viz., the resistance of the plasma, the capacitance of the sheaths, etc., can be used to describe the electrical properties of the discharge.

We consider a discharge when $\omega\tau \ll 1$. An electron current proportional to the plasma concentration then flows in the volume. Let us consider the fluctuation of the concentration $n' e^{iky}$. This fluctuation causes variation of the electron current $j'_\parallel e^{iky}$ ($\parallel$ indicates that this fluctuation of the current is in phase with the original current, and $\perp$ indicates that it is ahead by $\pi/2$). We shall omit the exponential factor below. Since the voltage on the electrodes is constant, the variation of the potential in the sheaths $\Phi'_s(j)$ under the action of $j'_\parallel$, $\Phi'_\perp$, should induce a current $j'_\perp$ in the plasma with a phase shift of $\pi/2$. Due to $j'_\perp$, the phase of the total current $j_0 + j'_\perp$ varies, and a varying potential $\Phi'_{s\parallel}$ appears in the sheaths.

Due to the perturbation of the concentration n' in the plasma, the voltage drop varies. Let us find it. We introduce the quantities $\Phi'_{R\perp}$ and $\Phi'_{R\parallel}$, which denote the variations of the potential in the plasma, as well as $\Phi'_{s\perp}$ and $\Phi'_{s\parallel}$, which denote the variations of the potential in the sheaths. The quantities satisfy the following relations

$$j'_\parallel/j_0 = n'/n_0 + \Phi'_{R\parallel}/V_R, \quad \Phi''_{R\parallel} = -\Phi'_{s\parallel} = j'_\perp V_s/j_0;$$

$$j'_\perp/j_0 = \Phi'_{R\perp}/V_R, \quad \Phi'_{R\perp} = -\Phi'_{s\perp} = -j'_\parallel V_s/j_0, \qquad \text{(A1)}$$

where U_s is the voltage on the sheaths, and U_R is the voltage on the plasma.

The first relations follow from the condition that the electron current $j \sim nU_R/L_0$ flows in the plasma, the second follow from the constancy of the voltage on the electrodes, and the third follow from relations in the sheaths. Solving system (A1), we find that

$$\Phi'_\parallel = n'/n_0 - \alpha V_R/(1+\alpha), \quad \alpha = \frac{V_s j_0}{V_R^2}\frac{\partial V_s}{\partial j_0}. \qquad \text{(A2)}$$

Linearizing Eq. (29) for the plasma temperature and concentration, we find the relationship between the quantities when the instability increment equals zero:

$$\Phi'_\parallel/V_R + T'/T_0 - n'/(\hat{\nu}_i n_0) = 0,$$

$$-((2kL_0/\pi)^2+1)T'/\Delta T + n'/n_0 + \Phi'_{R\parallel}/V_R = 0, \qquad \text{(A3)}$$

where ΔT is the increase in the temperature of the gas in the unperturbed state relative to the walls.

From (A2) and (A3) we find that instability develops under the condition

$$-((2kL_0/\pi)^2+1)(1+\alpha(1+\hat{\nu}_i)) + \hat{\nu}_i\Delta T/T_0 > 0. \quad \text{(A4)}$$

If $k=0$, (A4) becomes the usual condition

$$\frac{\partial}{\partial j_0}(V_s^2 + V_R^2) < 0, \qquad \text{(A5)}$$

where

$$\frac{\partial V_R}{\partial j_0} = \frac{V_R}{j_0}\frac{1-\hat{\nu}_i\Delta T/T_0}{1+\hat{\nu}_i}. \qquad \text{(A6)}$$

It is seen from (A4) that if kL_0 becomes comparable to unity, heat condition can greatly stabilize the discharge. This can probably account for the absence of the normal current density effect in the experiments in Ref. 13, where $kL_0 \sim 1$, and explains the difference between the critical currents at which contraction occurs in tubes ($kL_0 \approx 2.4$) and in an hf discharge between planar electrodes ($kL_0 \to 0$). In the experiments in Ref. 12 and 14 contraction occurred in He when $p = 100$ Torr and $j = 32$ mA/cm and in an hf discharge when $j_0 = 8.2$ mA/cm^2.

If expression (A5) is rewritten with consideration of the dependences of ΔT and $\partial U_s/\partial j$ on the current, we obtain

$$\frac{\partial}{\partial j}(V_R^2 + V_s^2) = \frac{V_R^2 L^2}{4 j_0 L_0^2}\left\{-\frac{2q}{\omega\tau} + (\omega\tau)^{-2} \times [1 - (1+(\omega\tau)^{-2})^{-1/2}] + \hat{\nu}_i^{-1}\right\}, \quad \text{(A7)}$$

where

$$q = V_R^2 L_0^2 \omega/(8\pi^3 \lambda T_0 L^2). \qquad \text{(A8)}$$

If $L_0 \geq L\sqrt{\hat{\nu}_i/4}$, expression (A7) has a maximum at $\omega\tau = 1/q \leq 1$, and the instability threshold is determined from the condition

$$q > \frac{2L_0}{L\sqrt{\hat{\nu}_i}}. \qquad \text{(A9)}$$

If $L_0 \sim L$, the maximum of (A7) appears when $(\omega\tau) = q^{-1/3} \sim 1$ and the following condition on q is satisfied:

$$q > \left(\frac{8L_0^2}{3\hat{\nu}_i L^2}\right)^{3/4}. \qquad \text{(A10)}$$

[1)]The substitution of the solution of homogeneous equation (9) that is proportional to $\sin \omega t$ into Eq. (1) for balancing the concentrations of the charged particles would result in the appearance of density perturbations proportional to $\sinh kx$ and $\cosh kx$, which do not have form (8). In addition, the perturbation n' (8) would not be an eigenfunction of the problem.

Therefore, only the solution of the homogeneous problem which is proportional to cos ωt should be left.

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