Stochastic electron heating in bounded radio-frequency plasmas

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The mechanisms of electron heating in low-pressure bounded rf plasmas are analyzed. These processes are determined by the combined effect of electron interaction with the rf electric field, reflections from the walls and collisions. It is shown that when the discharge gap is small with respect to the electron mean-free path the finite size of the plasmas is crucial for the stochastic heating. A classification of heating regimes is performed and expressions for the power deposition are derived. In many cases, even though in a semi-infinite plasma heating exists, in a bounded plasma the electron motion is regular and there is no collisionless heating. [S0003-6951(96)04249-0]

Introduction of novel plasma sources have recently stimulated active investigation of collisionless (stochastic) electron heating in gas discharges. Being initially explored for a capacitively coupled plasma (CCP),[1] this mechanism is now widely discussed in application to inductively coupled plasmas (ICP),[2] ECR plasma;[3] (see Refs. 4–6). It is shown in this letter that the finite size of a plasma may be crucial for the electron heating and power deposition. This is due to the fact that the effects of the electron interactions with the rf field, reflections from the boundaries and collisions are combined in a rather complex way. Using model examples, we introduce a classification of different heating regimes (collisional, purely collisionless, and ''hybrid'') and examine peculiarities of the heating process in a bounded plasma.

The CCP is sustained by longitudinal, i.e., directed perpendicular along the boundary, electric field. The ICP is sustained by the electric field induced by a time-varying magnetic field. This electric field is solenoidal and usually is directed along the plasma boundary. It results in transversal velocity kick with respect to discharge gap. Thus, in general, a variety of heating regimes may arise.

Let us consider different mechanisms of electron heating using a simple model. Let L denote the gap length and $\delta \ll L$ is the layer thickness where electrons interact with the localized rf fields. For a CCP, such a model corresponds to a strongly asymmetric discharge with a large ratio of current densities on the powered electrode versus the grounded electrode. In an ICP, δ is the thickness of the skin layer. In addition to the rf fields, a static space-charge field is present that confines the majority of plasma electrons. We shall approximate its influence in model of rigid reflecting wall.

The electron motion is governed by three frequencies: the frequency of the rf field ω, the collision frequency ν, and the bounce frequency Ω. Depending on the ratio between these frequencies, different electron dynamics and a variety of heating regimes can be distinguished (Fig. 1). Region A corresponds to collision dominated electron motion (the mean-free-path λ is small as compared to δ) and to collisional (Joule) heating. In region B, $\delta < \lambda < L$, electrons return into the δ layer due to scattering with heavy particles, rather than due to reflection at the second plasma boundary at $x = L$. But contrary to the Joule heating, this hybrid heating is nonlocal: the place of electron interaction with the fields and the place where the phase randomization occurs are separated in space. Region C corresponds to rare collisions, $\lambda > L$, and multiple electron bounces from one plasma boundary and another between subsequent collisions, $\Omega > \nu$. This region is divided into subregions I, II, III and is treated below in more detail. Purely collisionless (stochastic) heating occurs only for longitudinal velocity kicks in region E, which in this case occupies a part of subregion I. In the region D, the energy change in a single passage through the δ layer is small. In metals, this case corresponds to a ''skin effect in the infrared region.''[7] For both longitudinal and transversal fields, the energy deposition in region D is small and will not be discussed here. It may be difficult to maintain a gas discharge in this regime.

The electron heating is adequately described in terms of the energy diffusion coefficient D_ϵ. It is an important quantity which determines microscopic characteristics of the electron ensemble such as the electron distribution function

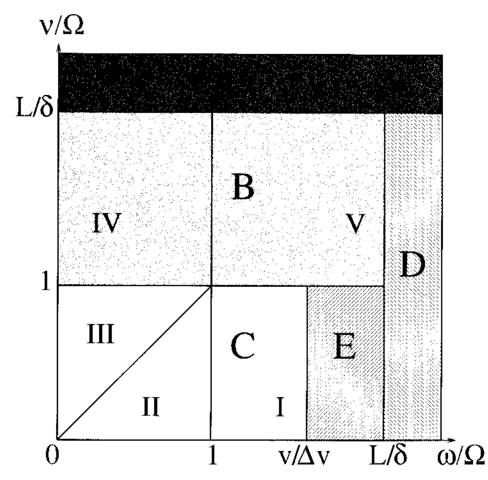

FIG. 1. A map of different heating regimes. A. Collisional (Joule) heating. B. Hybrid regime with frequent collisions covers subregimes IV and V. C. Hybrid regime with rare collisions covers subregimes I, II, and III. D. High-frequency limit-weak heating. E. Purely collisionless heating occurs in this part of subregime I for the longitudinal velocity kicks.

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$f(\epsilon)$, which satisfies stationary kinetic equation:[8–10]

$$\frac{1}{\sqrt{\epsilon}}\frac{d}{d\epsilon}\frac{\sqrt{\epsilon}D_\epsilon}{2}\frac{df(\epsilon)}{d\epsilon}+St^*f(\epsilon)=0, \tag{1}$$

where $St^*f(\epsilon)$ is the inelastic collision integral. The macroscopic quantities such as the rate of power deposition into a unit volume of plasma P can be expressed in terms of D_ϵ and $f(\epsilon)$:[10]

$$P=\frac{2\sqrt{2}\pi}{m^{3/2}}\int_0^\infty f(\epsilon)\frac{d}{d\epsilon}(\sqrt{\epsilon}D_\epsilon(\epsilon))d\epsilon. \tag{2}$$

Thus, the energy diffusion coefficient contains all information about the electron heating. Since it is a stochastic process, the energy diffusion coefficient can be evaluated as a product of squared step of random walk in energy and a frequency of such steps. In what follows, we shall derive expressions for D_ϵ for different heating regimes shown in Fig. 1.

Region B corresponds to frequent collisions in the plasma bulk ($\delta\ll\lambda\ll L$). Since electrons return into the δ layer due to collisions, only electrons which are at the distance λ from the wall contribute to the heating. However, the spatially averaged energy diffusion coefficient depends on the finite size of the plasma. In subregion V the electrons return into the δ layer after collisions in random phases with respect to the rf field. It means that the spatially averaged energy diffusion coefficient is equal to

$$D_\epsilon=\frac{1}{2}(\Delta\epsilon)^2\nu_{\mathrm{eff}}, \tag{3}$$

where $\Delta\epsilon$, maximum kick in energy, ν_{eff} corresponds to the average frequency of electron-field interactions. This frequency can be calculated as the ratio of particle flux into the δ layer $\frac{1}{2}(v_x n)$ to the total number of particles nL. $\Omega(v_x)=v_x/(2L)$is the bounce frequency. The value of ν_{eff} does not depend on collision frequency. It follows from the fact that the maximum value of electron-field interaction frequency is of the order of ν and corresponds to the particles at a distance of the order of mean free path $\lambda=v/\nu$ from the wall. Averaging over all particles leads to $\nu_{\mathrm{eff}}=\Omega$. In spite of the fact, that the energy stochastization is due to collisions, the expression for energy diffusion coefficient does not depend explicitly on ν. The collisions lead also to isotropization; so the energy diffusion coefficient should be averaged over velocity directions.

Regions C and E. In the collisionless limit one can trace the electron velocity after each interaction with the rf field. Introducing the phase of the field $\phi_n=\omega t_n$ and the electron velocity $v_n=v(\phi_n)$ before the nth interaction, the mapping of electron motion can be obtained in the form:[11,12]

$$\phi_{n+1}=\phi_n+\omega/\Omega(v_{n+1}), \tag{4}$$

$$v_{n+1}=v_n+\Delta v\ \cos(\phi_n), \tag{5}$$

where v denotes v_y for the transversal kicks or v_x for the longitudinal kicks. For the transversal kicks, Ω does not dependent on v_n. For longitudinal kicks, $\Omega\sim v_{n+1}$ since the bounce time is inversely proportional to v_x.

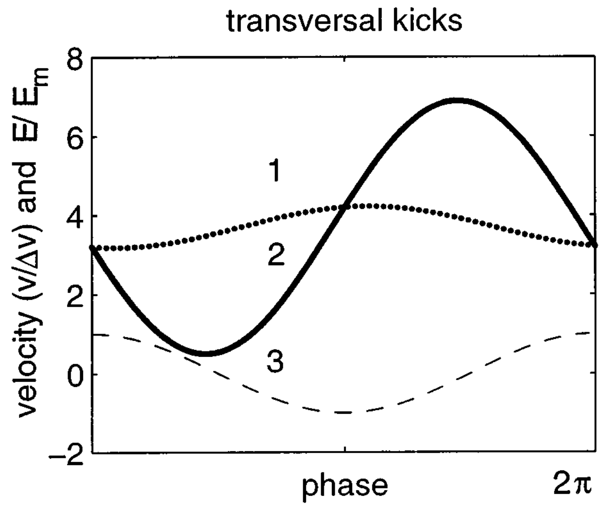

FIG. 2. Dimensionless electron velocity $v/\Delta v$ vs phase ϕ according to mapping (5,6) in region I of Fig. 2. Transversal velocity kicks: (curve 1) is for non-resonant electrons with $\omega/\Omega=10$, (curve 2) is for near-resonant electrons with $\omega/\Omega=0.95\times2\pi$, (curve 3) shows oscillations of dimensionless field, E/E_m, where E_m is the maximum value of the field.

For different ω, Δv, and v, various scenarios of the collisionless electron dynamics in phase space are possible. Figure 2 shows an example of phase portraits in subregion I for the transversal kicks. For the transversal kicks [Fig. 2], the motions along x and y are independent. The phases of subsequent interactions are strongly correlated, $d\phi_{n+1}/d\phi_n=1$. In this case there is no energy diffusion and no heating: the periods of electron acceleration and deceleration interchange. The velocity oscillates in a regular fashion. It means, in contrary to Refs. 2 and 4–6, that in generally accepted models of ICP discharge collisionless heating of nonresonant particles is absent. The amplitude of the velocity oscillations is rather large for resonant particles with $\omega\approx2\pi k\Omega$, where k is an integer [see curve 2 in Fig. 2].

For longitudinal velocity kicks, the electron motion is irregular at

$$\left|\frac{d\phi_{n+1}}{d\phi_n}-1\right|=\frac{\omega}{\Omega}\frac{\Delta v}{v}\geqslant1, \tag{6}$$

and regular in the opposite case.[11,12] From Eq. (6) it follows that slow electron dynamics is chaotic while fast electron dynamics is regular. Equation (6) is not held for typical conditions of rf discharges $[(\Delta v/v\sim\frac{1}{3},\ \omega\sim\Omega]$ purely collisionless (stochastic) heating (region E) disappears.

With the exception of longitudinal kicks in region E, the electron collisions with heavy particles are always important for the heating process. The corresponding heating regimes are referred below as ''hybrid'' regimes, to distinguish them from the local Joule heating (region A) and purely collisionless (stochastic) heating. In hybrid regimes an electron forgets the field phase due to collisions with heavy particles. The hybrid regime can be subdivided into the regimes with rare and frequent collisions.

In region C collisions are rare. In subregions I, II, III, when Eq. (7) is not fulfilled, there is no collisionless stochastization, and phase randomization and heating can occur only due to collisions. In subregion II, the electron passes the

δ layer many times during the field period. To find the amplitude of the velocity oscillations, one can use the smallness of the ratio ω/Ω to rewrite Eqs. (4) and (5) in the differential form:

$$\frac{dv}{d\phi}=\frac{\Omega\Delta v}{\omega}\cos\phi. \tag{7}$$

By virtue of Eq. (7), the amplitude of the velocity oscillations is $\Omega\Delta v/\omega$, which is much larger than a single velocity kick Δv. The phase shift between velocity oscillations and field oscillations is close to $\pi/2$ [see Fig. 2]. Thus plasma in the absence of collisions is purely inductive and has no active resistance. Even rare collisions can result in a substantial heating in this regime. Accounting for collisions results in the energy diffusion coefficient $D_\epsilon=(\nu/2)(\Delta\epsilon\Omega/\omega)^2$.

In subregion III an electron experiences Ω/ν kicks in the δ layer between subsequent collisions with heavy particles. Thus, the amplitude of the velocity oscillations is $(\Omega/\nu)\Delta v$, and the energy diffusion coefficient is $D_\epsilon=\frac{1}{2}(\Delta\epsilon\Omega/\nu)^2\nu$. The diffusion coefficient in subregions II and III can then be written by interpolation as

$$D_\epsilon\approx\frac{1}{2}\frac{\nu\Omega^2(\Delta\epsilon)^2}{\omega^2+\nu^2}. \tag{8}$$

At $\omega\gg\Omega$, (subregion I with the exception of E for longitudinal kicks) the amplitude of velocity oscillations coincides with the velocity kicks Δv. Thus, for nonresonant particles, the energy diffusion coefficient is [compare to Eq. (4)]:

$$D_\epsilon=\frac{1}{2}(\Delta\epsilon)^2\nu. \tag{9}$$

For the resonant particles, at $\omega\approx 2\pi k\Omega$, the diffusion coefficient is anomalously large. Since in the mapping (4) the phase shift $2\pi k$ can be omitted, the effective frequency $\omega-2\pi k\Omega$ is small and we can apply the expression (8) with the substitution $\omega\rightarrow 2\pi k\Omega$. It results in

$$D_\epsilon=\frac{1}{2}\frac{(\Delta\epsilon)^2\nu\Omega^2}{(\omega-2\pi k\Omega)^2+\nu^2}. \tag{10}$$

At $\nu\rightarrow 0$, the diffusion coefficient Eq. (10) transforms into the δ function: $D_\epsilon=\pi/2(\Delta\epsilon)^2\Omega^2\delta(\omega-2\pi k\Omega)$. Even a small number of resonant particles may contribute considerably to the energy deposition. As a result, for small values of ν/Ω the heating is produced mainly by resonant particles. If the fraction of these particles is small, the contribution of nonresonant particles is to be accounted for. For very small values of ν/Ω, the nonlinear effects should be incorporated and nonlinear resonance width added in denominator of Eq. (10). In this case D_ϵ is proportional to ν and tends to zero with $\nu\rightarrow 0$. For transversal kicks there is no nonlinear effects, and D_ϵ remains a constant at $\nu\rightarrow 0$. The heating obtained in PIC simulations[4] may be attributed to such resonant particles. The model of ICP plasma (Ref. 4) corresponds to transversal kicks (not to longitudinal, as proposed in Ref. 13) with electron velocities close to the first resonance $k=1$. In agreement with Eq. (10) the energy dissipation in Ref. 4 was found to be constant at $\nu\rightarrow 0$.

Thus, the electron heating always results from some process of the stochastization. In low-pressure bounded plasma, when the mean-free path exceeds the discharge gap various scenaria are possible. The situation is different for the cases of velocity kicks along and normal to the plasma boundary. For the case of kicks along the plasma boundary, the nonlinear stochastization, equivalent to the well-known Fermi mechanism, is absent, and only the collisional stochastization of resonant particles remains. If number of these particles is small, the stochastization and heating are strongly suppressed.

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