COLLISIONLESS ELECTRON HEATING IN RF GAS DISCHARGES: I. QUASILINEAR THEORY

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INTRODUCTION

Numerous applications of low-pressure gas discharges have recently prompted an interest in mechanisms of electron heating and power deposition in the plasma maintained by radio–frequency (rf) electric fields. A modern trend in plasma technology aims at decreasing the gas pressures down to the millitorr range. For these low pressures it is easier to maintain uniform plasmas with well controlled parameters. Due to the large value of the mean free path (MFP) the main mechanism of electron heating turns out to be a collisionless one rather than the conventional Joule heating dominant for higher pressures.

Being initially studied for a capacitively coupled plasma[1], this mechanism is now widely discussed in application to inductively coupled plasmas (ICP), ECR plasmas, etc. [2]. Initially collisionless heating was studied in a 'kick' model: electron obtains velocity kick in the strong electric field at the discharge periphery, then the phase of velocity kick is randomized either due to collisions in the bulk or due to non-linear mechanism of randomization (Fermi acceleration). As a result diffusion in velocity space arises, and this corresponds to collisionless heating.

In general case the electric field is presented in the whole discharge volume and the separation on periphery region where electron gains energy and a bulk without electric field is not applicapable. The other subject of investigation is the heating of trapped in the discharge center electrons in non-uniform plasma buy ambipolar electric field. These electrons also can be heated by collisionless mechanism by weak electric field in the plasma field. For this situation kick model is also not applicable. In this particular situation the use of rigorous quasi-linear theory is necessary. It is based on well known mechanism of collisionless power dissipation - Landau damping (see e.g.

[3]). The resonance particles moving with a velocity close to the wave phase velocity intensively interact with wave fields and receive or lose energy. The result of particle ⇔ wave interaction depends on the shape of the particle distribution function in velocity space. For Maxwellian distributions the wave amplitude decreases, while the particle energy increases. If many waves are excited the power dissipation should be summarized over the spectrum of waves. Such a theory was developed for a weak turbulence in order to explain the mechanisms of anomalous transport phenomena in hot plasma of thermonuclear fusion (see e.g.[4,5]).

But it can also be applied to stable plasmas, when small scale fields with a wide spectrum of wave numbers are excited by an external source. This situation is realized in the cases of Debye screening of longitudinal electric fields or cases of anomalous skin effects for transversal fields. A local spectrum of small scale Langmuir waves appears also in the region of plasma resonances for inhomogeneous plasmas interacting with electric fields capacitively coupled with plasmas (CCPs).

The only particles which are in resonance with a wave are heated by collisionless mechanism. It means that in the regime of collisionless dissipation the form of the electron energy distribution function (EEDF) is sensitive to wave spectrum. If the wave phase velocities are confined in some interval the plateau in the EEDF can be formed[5].

The problem of the plateau formation in the EEDF in the quasilinear theory was investigated first in[6,7]. In[8] it is shown that collisionless heating due to localized plasma oscillations results in plateau formation in the region of high electron energy. Collisions smooth out the plateau and lead to an increased number of high energy electrons[5]. The effect of collisions on electron heating by local plasma oscillations were investigated in [9]. The numerous investigations of EEDF by using a kinetic equation for electrons of a plasma slab with oscillating rigid walls have been performed in[10–12]. This treatment yields a power law energy dependence in the EEDFs. The experimental study of EDF for CCP have been carried out in[13,14].

A quasilinear approach to collisionless electron heating in the regions of plasma resonance in low pressure discharges, sustained by electromagnetic surface waves, is developed in[15,16]. In[17] it is demonstrated that — in view of experimental situations — self-consistent modeling has to account for nonlocal effects[18,19] as well as for collisionless heating. The classification of variuos regimes of collisionless heating were performed in [20].

In view of the history of successful use of the quasilinear approach as a well established method, particular in hot plasma physics, this paper aims at systematically applying this method also to study collisionless heating in various low temperature plasmas and to summarize existing results using a common basis as well as to gain extended new results. The paper intends to demonstrate that the quasilinear theory is a powerful and easily applicable method for investigations of these type of problems.

This paper is organized as follows: In section II, the derivation of the diffusion coefficient in energy space for uniform boundless plasmas, heated by localized high frequency field is presented. In section III the collisionless heating in inductively coupled plasmas (ICPs) is analyzed for semi–infinite and slab geometry. Both cases can be reduced to the boundless problem with specular reflecting walls. The profile of the electric field under the condition of the anomalous skin effect is found in analytical form and the electric field is shown to oscillate spatially with zero average value. As a result the diffusion coefficient of fast electrons is suppressed, but in the low energy region it is enhanced in comparison to the case of an exponential profile. For ICPs in slab geometry (of width L) it is demonstrated that the second boundary plays a similar role. In this case the corresponding Fourier spectrum of electric field is discret ($k_n = \pi n/L$) in contrast to a continuous one for semi–infinite geometry. In section IV the quasilinear theory is applied for capacitively coupled plasmas (CCPs). It is shown that the diffusion coefficient has a clear resonant structure with a main peak at $v = \omega L/\pi$. An analysis of the solutions of the kinetic equation for the EEDF in different regimes of plasma heating is presented. In section V the influence of the ambipolar electric field on the efficiency of collisionless heating is discussed. The general expression for the diffusion coefficient in energy space is obtained for cases both of ICPs and CCPs. The presence of ambipolar electric fields can strongly modify the condition for particle $\Leftrightarrow$ wave resonances. As a consequence the form of diffusion coefficient in energy space is changed, especially for low energy electrons. In a parabolic potential $\Phi = ax^2/2$ the bounce frequency of trapped electrons ($\sqrt{a/m}$) is the same for all energies. So the resonance condition ($\omega = \sqrt{a/m}$) can be fulfilled for one value of a only, and there is no collisionless heating for other values of a. In the general case of a non–parabolic potential the bounce frequency is energy dependent. Section VI contains the conclusion and outlook.

KINETIC THEORY OF COLLISIONLESS ELECTRON HEATING

Separation of the Space and Times scales in the Kinetic Description

First we discuss case when collisionless heating occurs in a small region at the discharge periphery of width $\delta \ll L$, L being the discharge gap. In the ICP, δ corresponds to the thickness of the skin layer. In a capacitively coupled plasma, this model corresponds to the sheath width smaller then an electron mean free path. For surface wave discharges it corresponds to colisionless dissipation in the narrow region of plasma resonance. In such low pressure discharges the space scale of the electric heating becomes small in comparison to the MFP λ. Under these conditions it is possible to simplify the kinetic description of the plasma by separation of the space scales.

The kinetic equation for the electron velocity distribution function $\mathcal{F}(\boldsymbol{r}, \boldsymbol{v}, t)$ is:

$$\frac{\partial \mathcal{F}}{\partial t} + \boldsymbol{v}\,\frac{\partial \mathcal{F}}{\partial \boldsymbol{r}} + \frac{e}{m}\left(\boldsymbol{\mathcal{E}} + \frac{1}{c}\boldsymbol{v} \times \boldsymbol{\mathcal{B}}\right)\frac{\partial \mathcal{F}}{\partial \boldsymbol{v}} = S(\mathcal{F}) \tag{1}$$

where e and m are the electron charge and mass, respectively, $S(F)$ is the collisional integral, $\boldsymbol{\mathcal{E}}(\boldsymbol{r}, t)$ is the electric and $\boldsymbol{\mathcal{B}}(\boldsymbol{r}, t)$ is the magnetic field.

The fields can be separated into two parts according to different space scales:

$$\boldsymbol{\mathcal{E}} = \boldsymbol{E} + \tilde{\boldsymbol{E}} \qquad \boldsymbol{\mathcal{B}} = \boldsymbol{B} + \tilde{\boldsymbol{B}} \tag{2}$$

where $\boldsymbol{E}$ and $\boldsymbol{B}$ have spatial scales large compared to λ, while $\tilde{\boldsymbol{E}}$ and $\tilde{\boldsymbol{B}}$ have spatial scales δ small compared to λ (i.e. $\delta \ll \lambda$). An analogous separation can be performed for the distribution function:

$$\mathcal{F}(\boldsymbol{r}, \boldsymbol{v}, t) = F(\boldsymbol{r}, \boldsymbol{v}, t) + \tilde{F}(\boldsymbol{r}, \boldsymbol{v}, t) \tag{3}$$

where F is averaged over a scale in the order of λ and $\tilde{F}$ describes the deviations of the distribution function on scales smaller than λ. Also the usual quasilinear approximation

$\tilde{F} \ll F$ is used[4,5]. As a result the kinetic equation (1) can be separated into two parts:

$$\frac{\partial \tilde{F}}{\partial t} + \boldsymbol{v}\frac{\partial \tilde{F}}{\partial \boldsymbol{r}} + \frac{e}{m}\left(\boldsymbol{E} + \frac{1}{c}\boldsymbol{v}\times\boldsymbol{B}\right)\frac{\partial \tilde{F}}{\partial \boldsymbol{v}} = -\frac{e}{m}\left(\tilde{\boldsymbol{E}} + \frac{1}{c}\boldsymbol{v}\times\tilde{\boldsymbol{B}}\right)\frac{\partial F}{\partial \boldsymbol{v}} \tag{4}$$

$$\frac{\partial F}{\partial t} + \boldsymbol{v}\frac{\partial F}{\partial \boldsymbol{r}} + \frac{e}{m}\left(\boldsymbol{E} + \frac{1}{c}\boldsymbol{v}\times\boldsymbol{B}\right)\frac{\partial F}{\partial \boldsymbol{v}} = -\frac{e}{m}\overline{\left(\tilde{\boldsymbol{E}} + \frac{1}{c}\boldsymbol{v}\times\tilde{\boldsymbol{B}}\right)\frac{\partial \tilde{F}}{\partial \boldsymbol{v}}} + S(F) \tag{5}$$

In (4) the collisionless term is omitted, since it is small compared to the second term on the left–hand side: $k\lambda \gg 1$. The bar in (5) denotes spatial averaging on scales smaller than λ.

The Form of Quasilinear Integral for Weakly Inhomogeneous Boundless Plasmas

First the case of weakly inhomogeneous boundless plasmas, when $\boldsymbol{E}$, $\boldsymbol{B} = 0$ and $\tilde{\boldsymbol{E}}$, $\tilde{\boldsymbol{B}}$ are only RF fields, is considered. It correspond to collisionless dissipation in narrow region with a gap width larger than a energy relaxation length, so influence of boundary can be neglected. The electric field is excited at fixed frequency:

$$\tilde{\boldsymbol{E}}(\boldsymbol{r},t) = \frac{1}{2}\left\{\boldsymbol{E}(\boldsymbol{r})\exp(-i\omega t) + \boldsymbol{E}^*(\boldsymbol{r})\exp(i\omega t)\right\}, \tag{6}$$

where * denotes the complex conjugation.

Under conditions, when the frequency of ineleastic collisions ν^* is small compared to the RF ω ($\nu^* \ll \omega$), the distribution function does not depend on time[19], i.e. $F = F(\boldsymbol{r},\boldsymbol{v})$, and (4), (5) can be simplified to

$$\frac{\partial \tilde{F}}{\partial t} + \boldsymbol{v}\frac{\partial \tilde{F}}{\partial \boldsymbol{r}} = -\frac{e}{m}\left(\tilde{\boldsymbol{E}} + \frac{1}{c}\boldsymbol{v}\times\tilde{\boldsymbol{B}}\right)\frac{\partial F}{\partial \boldsymbol{v}} \tag{7}$$

$$\boldsymbol{v}\frac{\partial F}{\partial \boldsymbol{r}} = S_{\mathrm{ql}}(F) + S(F) \tag{8}$$

The first term on the right hand side of (8) is well known in the theory of weak turbulent plasmas as the quasilinear collision integral, describing interaction of electrons with waves,

$$S_{\mathrm{ql}}(F) = -\frac{e}{m}\overline{\left\langle\left(\tilde{\boldsymbol{E}} + \frac{1}{c}\boldsymbol{v}\times\tilde{\boldsymbol{B}}\right)\frac{\partial \tilde{F}}{\partial \boldsymbol{v}}\right\rangle} \tag{9}$$

where the $\langle\ldots\rangle$ brackets indicate temporal averaging over the wave period $2\pi/\omega$.

Equation (7) can be solved by the Fourier method giving:

$$\tilde{F}(\boldsymbol{k}) = -\frac{ie}{m}\cdot\frac{\boldsymbol{E}(\boldsymbol{k}) + \frac{1}{c}\boldsymbol{v}\times\boldsymbol{B}(\boldsymbol{k})}{\omega - \boldsymbol{k}\boldsymbol{v}}\cdot\frac{\partial F}{\partial \boldsymbol{v}} \tag{10}$$

where $\tilde{F}(\boldsymbol{k})$, $\boldsymbol{E}(\boldsymbol{k})$ and $\boldsymbol{B}(\boldsymbol{k})$ are the Fourier transformations of the functions $\tilde{F}(\boldsymbol{r})$, $\boldsymbol{E}(\boldsymbol{r})$ and $\boldsymbol{B}(\boldsymbol{r})$, respectively.

By inserting (10) into (9) the quasilinear integral for the one–dimensional geometry can be written in the form:

$$S_{\mathrm{ql}} = \delta\left(x - x_0\right)\frac{\partial}{\partial v_i}D_{ij}(\boldsymbol{v})\frac{\partial F}{\partial v_j} \tag{11}$$

where the δ–function appears as a result of space averaging and reflects the fact of localization of the heating electric field in the region $x \approx x_0$ with a characteristic scale δ much smaller than λ and

$$D_{ij}(\boldsymbol{v}) = -\frac{e^2\pi}{m^2}\int dk \left[\boldsymbol{E}(k) + \frac{1}{c}\boldsymbol{v}\times\boldsymbol{B}(k)\right]_i \left[\boldsymbol{E}(k) + \frac{1}{c}\boldsymbol{v}\times\boldsymbol{B}(k)\right]_j \cdot \\ \mathrm{Im}\left(\frac{1}{\omega - kv_x}\right) \tag{12}$$

is the tensor of the diffusion coefficient in velocity space. By excluding the magnetic field from (12) with help of the Maxwell equation

$$\tilde{\boldsymbol{B}}(\boldsymbol{k}) = \frac{c}{\omega}\boldsymbol{k}\times\tilde{\boldsymbol{E}}(\boldsymbol{k}) \tag{13}$$

Thus in expression (12) the contribution of the Lorentz force is of the same order of magnitude as that of the electric field[21]. After substituting expression (13) for the magnetic field one gets:

$$D_{ij} = \frac{\pi(2\pi)^n e^2}{2m^2}\int d^n k \frac{k_i k_j}{\omega^2}|\boldsymbol{v}\tilde{\boldsymbol{E}}(k)|^2\delta(\omega - \boldsymbol{k}.\boldsymbol{v}) \tag{14}$$

From (12,14) it can be seen that the role of the Lorentz force is changing the direction of diffusion in velocity space. Without accounting for the Lorentz force the diffusion is in the direction of the electric field, but with accounting for the Lorentz force it is in the direction of wave propagation given by $\boldsymbol{k}$. The same result in an one–particle approach was discussed in[21]. The formula (14) has a general form: it is valid for longitudinal waves, too ($\boldsymbol{E}\times\boldsymbol{k} = \mathbf{o}$). This stems from the fact that the electrons receive impulse from the wave impulse directed along $\boldsymbol{k}$ independent of the type of heating field (transverse or longitudinal).

The relation

$$\mathrm{Im}\left(\frac{1}{\omega - kv_x + i\mathrm{o}}\right) = -\pi\delta(\omega - kv_x), \tag{15}$$

corresponding to the phase resonance between traveling wave and moving electron, was used to obtain expression (14).

It should be stressed that D_{ij} is a nonlocal function of the heating electric field due to the integral representation

$$\boldsymbol{E}(k) = \int \frac{dx}{2\pi}\exp(-ikx)\tilde{\boldsymbol{E}}(x)\,. \tag{16}$$

This type of nonlocality is also discussed in[22].

The simple physical meaning of expression (14) shall be illustrated by the example of plasma heating due to the localized field of a longitudinal wave $\tilde{\boldsymbol{E}}$ $(E_x, 0, 0)$. In this case one obtains from (14):

$$D_{xx} = \frac{\pi^2 e^2}{m^2}\frac{1}{|v_x|}\left|E\left(k = \frac{\omega}{v_x}\right)\right|^2 \tag{17}$$

Expression (17) can be presented in the form

$$D_{xx} = \frac{|v_x|}{4}(\Delta v)^2 \tag{18}$$

where Δv is the amplitude of the velocity kick after interaction with the wave field:

$$\Delta v = \int_{-\infty}^{+\infty} e\tilde{E}_x(x = v_x t) \exp(-i\omega t) dt = \frac{2\pi e}{v_x} E\left(k = \frac{\omega}{v_x}\right) \tag{19}$$

Kinetic Equations for the EEDF

In gas discharge plasmas the anisotropy of the distribution function in velocity space is small due to the elastic collision frequency ν being large compared to the inelastic one (ν^*),[19] i.e. $\nu \gg \nu^*$ *. In this case the conventional two–term approach is applicable and for the spherically symmetric part of the EEDF F_0 one has the following equation for one–dimensional geometry:

$$\frac{v^2}{3\nu(v)} \frac{\partial^2 F_0}{\partial x^2} - S^0_{\mathrm{ql}}(F_0) - S^*(F_0) = 0 \tag{20}$$

where

$$S^0_{\mathrm{ql}}(F_0) = \frac{1}{v^2}\frac{\partial}{\partial v}\left(v^2 \mathcal{D}(v) \frac{\partial F_0}{\partial v}\right) \cdot \delta\left(x - x_0\right) \tag{21}$$

with

$$\mathcal{D}(v) = \frac{1}{v^2}\frac{\pi^2 e^2}{m^2} \int_{-\infty}^{+\infty} dk \int_0^{\pi} \sin\vartheta \, d\vartheta \int_0^{2\pi} \frac{d\tilde{\Phi}}{4\pi} |\boldsymbol{v}\boldsymbol{E}(k)|^2 \delta(\omega - k v_x) \tag{22}$$

is the averaged (over the velocity angles) quasilinear collision integral S_{ql} (11) and $S^*(F_0)$ is the inelastic collision integral.

Expression (22) like (14) has a general form for all types of fields. The Lorentz force in (22) for an isotropic EEDF has vanished and the resulting diffusion coefficient (14) is the same with or without accounting for the Lorentz force. Thus the wave instability discussed in[3,21], which is equivalent to electron cooling, is essential only for anisotropic or non monotonic EEDFs, where the role of the Lorentz force is essential.

Below we summarize the forms of the diffusion coefficients in velocity space for various types of discharges:

1. For the case of transverse em waves ($\boldsymbol{k} \perp \boldsymbol{E}(\boldsymbol{k})$, $\boldsymbol{k} = k\boldsymbol{e}_x$, $\boldsymbol{E}_y = \boldsymbol{e}_y E(\boldsymbol{k})$) — corresponding to an inductive discharge — (22) yields

$$\mathcal{D}(v) = \frac{1}{v^2}\frac{\pi^2 e^2}{m^2} \int_0^{\pi}\int_0^{2\pi} \frac{v_y^2 \left|E(k = \frac{\omega}{v_x})\right|^2}{|v_x|} \frac{\sin\theta \, d\theta \, d\tilde{\Phi}}{4\pi} . \tag{23}$$

After integration over $\tilde{\Phi}$ and introduction of $\mu = \cos\theta = v_x/v$, one gets

$$\mathcal{D}(v) = \frac{1}{v}\frac{\pi^2 e^2}{2m^2} \int_{-1}^{1} \frac{(1-\mu^2)\left|E(k = \frac{\omega}{v\mu})\right|^2}{2|\mu|} d\mu \, . \tag{24}$$

*It corresponds to the case, when a localized em field is excited with a broad spectrum of wavenumber Δk. So the corresponding time of diffusion through the region of wave–particle interaction ($\Delta v \approx {}^{\omega}/_{k^2}\, \Delta k) - t_i \approx (\Delta v)^2/\mathcal{D}$ is large compared with the collision time $\nu t_i \gg 1$. The steady state condition is established on a time $t \approx {\nu^*}^{-1}$. The opposite case corresponds to large anisotropy of the EEDF and can even lead to current drive[23].

2. For the case of a longitudinal electric field ($\boldsymbol{k} \parallel \boldsymbol{E}$, $\boldsymbol{k} = k\boldsymbol{e}_x$, $\boldsymbol{E} = E\boldsymbol{e}_x$) — corresponding to a capacitive discharge and plasma resonance heating in surface wave plasmas — (22) reduces to

$$\mathcal{D}(v) = \frac{1}{v}\frac{\pi^2 e^2}{m^2}\int_{-1}^{1}\frac{|\mu|}{2}\left|E\left((k = \frac{\omega}{v\mu}\right)\right|^2 \, d\mu\,. \tag{25}$$

3. For the case of a circularly polarized electric field ($\boldsymbol{k} = k_{y_0}\boldsymbol{e}_y + k_x\boldsymbol{e}_x$, $\boldsymbol{E} = E_x\boldsymbol{e_x} + E_y\boldsymbol{e}_y$, $E_y = iE_x = E_0$) — corresponding to SW sustained discharges — (22) gives

$$\mathcal{D}(v) = \frac{1}{v^2}\frac{\pi^2 e^2}{m^2}\int_0^{\pi}\int_0^{2\pi}\frac{(v_y^2 + v_x^2)\left|E_0\left(k = \frac{\omega - k_{y_0}v_y}{v_x}\right)\right|^2}{|v_x|}\frac{\sin\theta\, d\theta\, d\tilde{\Phi}}{4\pi}\,, \tag{26}$$

where k_{y_0} is the fixed wavenumber of the surface wave and $E(k)$ corresponds to the Fourier transformation of the electric field profile $E_0(x)$.

To obtain the EEDF, (20) should be integrated over a small vicinity of the point $x = x_0$. One gets as result:

$$-\frac{v^2}{3\nu(v)}\left.\frac{\partial F_0}{\partial x}\right|_{x=x_0+\mathrm{o}} = \frac{1}{2}\frac{1}{v^2}\frac{\partial}{\partial v}\left(v^2\mathcal{D}(v)\left.\frac{\partial F_0}{\partial v}\right|_{x=x_0+\mathrm{o}}\right) \tag{27}$$

In obtaining (27) symmetry of plasma heating and EEDF with respect to the space position $x = x_0$ is assumed. Relation (27) should be considered as boundary condition for the equation

$$\frac{v^2}{3\nu(v)}\frac{\partial^2 F_0}{\partial x^2} - S^*(F_0) = 0 \tag{28}$$

describing the space evaluation of the EEDF outside the region of RF power input.

By representing the collisional integral $S^*(F_0)$ in the form

$$S^*(F_0) = -\nu^*(v)F_0(v,x) \tag{29}$$

one obtains from (28):

$$F_0(v,x) = F_0(v)\exp\left(-\frac{x}{\lambda_\varepsilon}\right), \tag{30}$$

where $\lambda_\varepsilon = v/\sqrt{3\nu(v)\nu^*(v)}$ is the energy relaxation length. Substituting (30) into the boundary condition (27) now results in the equation for the EEDF:

$$v\sqrt{\frac{\nu^*(v)}{3\nu(v)}}\,F_0(v) = \frac{1}{2}\frac{1}{v^2}\frac{\partial}{\partial v}\left(v^2\mathcal{D}(v)\frac{\partial F_0}{\partial v}\right) \tag{31}$$

Equation (31) has previously been analyzed for the case of localized longitudinal RF fields in the ionosphere[9] and SW produced plasmas[15]. The character of the solutions of (31) essentially depends on the diffusion coefficient $\mathcal{D}(v)$. For its determination the space spectra of the heating EM fields have to be known.

In contrast to boundless plasmas considered above gas discharge plasmas are always bounded. Thus, in the next section a procedure of reducing the bounded problem to a boundless one will be developed.

The Semi–Infinite Plasma

In order to use the results of boundless plasmas, specular reflection of the electrons from the discharge wall ($x = 0$) is assumed:

$$\tilde{F}(v_x, v_y, v_z, x < 0) = \tilde{F}(-v_x, x_y, v_z, x > 0) \tag{32}$$

To reduce the semi-infinite problem to boundless one the continuation of Eq. (7) into the complete x–range ($-\infty < x < \infty$) can be performed. Now the reflected electron is represented by one passing to the region $x < 0$). The kinetic equation is symmetric with respect to electron reflection if the fields are continued by the following *ansatz* see Fig.1:

$$\begin{aligned} \tilde{B}_y(x < 0) &= -\tilde{B}_y(x > 0) \\ \tilde{E}_y(x < 0) &= \tilde{E}_y(x > 0) \\ \tilde{E}_x(x < 0) &= -\tilde{E}_x(x > 0) \end{aligned} \tag{33}$$

and the problem reduces to the case of boundless plasmas *.

Now the possible regimes of shielding the transverse EM field by the semi–space plasma are addressed. In the case of weak collisions ($\omega > \nu$) the field penetrates into the overdense plasma ($\omega \ll \omega_p = (4\pi e^2 n/m)^{1/2}$) over the depth $\delta_0 = c/\omega_p$. For such a regime the frequency of the electric field should be sufficiently high ($\omega > v_T/\delta_0$, with v_T being the thermal electron velocity). A regime of collisionless plasma heating is possible, if the skin length δ_0 does not exceed the MFP λ, i.e. if $v_T > \nu \cdot \delta_0$. In this case one should use the following form for the electric field, continued into the complete x–range to calculate the diffusion coefficient for semi–infinite plasmas:

$$E_y(x) = E_0 \exp\left(-\frac{|x|}{\delta_0}\right), \tag{34}$$

with the Fourier spectrum given by

$$E(k) = \frac{E_0 \delta_0}{\pi(1 + k^2\delta_0^2)}. \tag{35}$$

It should be noted that result (34) corresponds to the absence of power dissipation, accounting for low power dissipation either due to small collisionality or colisionless dissipation results in weak spatial field oscillations and a small EM power flux into the plasma. Substitution of (35) into (22) yields the expression for the diffusion coefficient $\mathcal{D}(v)$:

$$\mathcal{D}(v) = \omega\delta_0 v_E^2 g\left(\frac{v}{\omega\delta_0}\right) \tag{36}$$

*It should be mentioned that the approximation of specular boundary reflection ($v_x = -v_x$, $x = 0$) may not be valid in real plasmas, where electrons reflect from the sheath region and penetrate into the sheath by a length $\Delta(v_T) \sim r_D$ (r_D being the Debye radius). If this length is small (i.e. $\Delta(v_T) < v_T/\omega \ll \delta$), the sheath region is not important and formulae for specular reflection are valid. For the opposite case $\delta > \Delta(v_T) \gtrsim v_T/\omega$ the velocity kick (and thus the energy dissipation) is smaller[24].

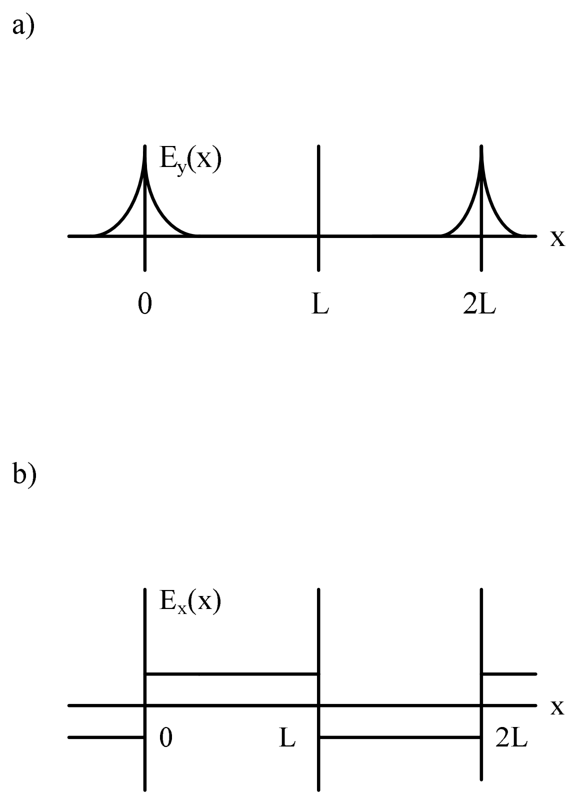


Figure 1. Scheme of electric field continuation: (a) transverse electric field; (b) longitudinal electric field.

where $v_E = eE_0/m\omega$ and

$$g(\tilde{v}) = \frac{1}{4\tilde{v}^3}[(2+\tilde{v}^2)\ln(1+\tilde{v}^2) - 2\tilde{v}^2] \tag{37}$$

which should be used in the kinetic equation (31) for obtaining the EEDF $F_0(v)$.

In the region of low electron energy ($v/\omega\delta_0 \ll 1 \to \tilde{v} \ll 1$) one has

$$g(\tilde{v}) \approx \frac{1}{24}\tilde{v}^3. \tag{38a}$$

In the limit of high electron energy ($v/\omega\delta \gg 1 \to \tilde{v} \gg 1$) one obtains:

$$g(\tilde{v}) \approx \frac{\ln(\tilde{v})}{2\tilde{v}} \tag{38b}$$

For lower frequencies $\omega^* < \omega < v_{T_e}/\delta_0$ the case of anomalous skin effect[3] is realized ($\omega^* = \nu^3 c^2/\omega_p^2 v_{\mathrm{Te}}$), independent of the relation between ω and ν. In this situation the depth of the electric field penetration becomes less than the MFP λ and electrons are heated in a collisionless manner.

If one uses a description of the electric field in the form

$$E_y(x) = E_0 \exp\left(-\frac{|x|}{\lambda_{\mathrm{sk}}}\right) \tag{39}$$

where $\lambda_{\mathrm{sk}} = (c\mathcal{Z})/(i\omega)$ is chosen from the condition of a correct representation for the power deposition into the plasma with the surface impedance $\mathcal{Z} = 2/3\delta(1 + {}^1\!/_3\sqrt{3}i)$, where $\delta = b^{-1/3}$ is effective skin depth see Eq.(44) the diffusion coefficient $\mathcal{D}(v)$ is:

$$\mathcal{D}(v) = \omega\delta v_E^2 g\left(\frac{v}{\omega\delta}\right) \tag{40a}$$

$$\begin{aligned} g(\tilde{v}) = \frac{4}{27}\frac{1}{\tilde{v}}\Bigg\langle \left(\frac{1}{2} + \frac{8}{27\tilde{v}^2}\right)\ln\left[\frac{3+\left(1+\frac{27}{8}\tilde{v}^2\right)^2}{4}\right] - \\ -\frac{1}{\sqrt{3}}\left(1-\frac{16}{27\tilde{v}^2}\right)\left\{\arctan\left[\frac{1+\frac{27}{8}\tilde{v}^2}{\sqrt{3}}\right] - \frac{\pi}{6}\right\} - 1\Bigg\rangle \end{aligned} \tag{40b}$$

The function $g(\tilde{v})$ has the following approximation in the region of high electron energy ($\tilde{v} = v/\omega\delta \gg 1$):

$$g(\tilde{v}) \approx \frac{4}{27}\frac{\ln(\tilde{v})}{\tilde{v}} \tag{41a}$$

and in that of low electron energy ($\tilde{v} \ll 1$):

$$g(\tilde{v}) = \frac{9}{128}\tilde{v}^3 \tag{41b}$$

The exact Fourier transformation of the electric field obtained from the kinetic approach[3] yields:

$$E(k) = \frac{E_0}{\pi L_{\mathrm{sk}}\left(k^2 - i\frac{b}{|k|}\right)} \tag{42}$$

where

$$b=\frac{4\pi\omega e^2}{c^2 m}\int v_y\frac{\partial F}{\partial v_y}\delta(v_x)d\boldsymbol{v} \tag{43}$$

In the case of a Maxwellian EEDF one has:

$$b=\frac{2e^2 n\omega}{c^2}\sqrt{\frac{2\pi}{mT}} \tag{44}$$

where n is the electron density and

$$L_{\mathrm{sk}}=\frac{2(\sqrt{3}+i)}{3^{3/2}b^{1/3}}\,. \tag{45}$$

The space dependence of the electric field penetrating into the plasma over the region $0<x<v_T/\omega$ has the form:

$$\begin{aligned}E_y(x) = \frac{E_0}{\pi L_{\mathrm{sk}} b^{1/3}}\Bigg\{\frac{\pi}{3}\left[(\sqrt{3}\,i+1)\exp\left(i\frac{\sqrt{3}}{2}-\frac{1}{2}\right)xb^{1/3}+\exp\left(-xb^{1/3}\right)\right]\\ -2i\fint_0^\infty\frac{\xi\exp\left(-xb^{1/3}\xi\right)}{1-\xi^6}d\xi\Bigg\}\end{aligned} \tag{46}$$

The last term on the right–hand side of (46) represents the contribution of the branch point $k=0$ and the exponential terms are a result of the poles in the integration when the inverse Fourier transformation of expression (42) is performed. A plot of $E_y(x)$ is presented in Fig. 2. The electric field profile has a oscillatory structure which reflects the existence of power flux into the plasma due to collisionless dissipation. It should be noted that at large distances from the plasma surface ($x>v_{\mathrm{Te}}/\omega$) the influence of electron thermal motion is not important and the electric field penetration becomes purely exponential with a scale length $\delta_0=c/\omega_p$. But this effect of changing the regime of field penetration can be neglected because at this distance $E_y(x)$ has decreased to a small fraction of its value at the surface if the frequency ω is in the range $\omega<(v_T/c)\,\omega_p$ where the anomalous skin effect is applicable.

By substituting expression (42) into (22) one obtains:

$$\mathcal{D}(v)=\omega\delta v_E^2 g\left(\frac{v}{\omega\delta}\right) \tag{47}$$

where

$$g(\tilde{v})=\frac{9}{64\tilde{v}}\left\{\sqrt{3}\left[\arctan\left(\frac{2\tilde{v}^2-1}{\sqrt{3}}\right)+\frac{\pi}{6}\right]-\frac{1}{\tilde{v}^2}\ln(1+\tilde{v}^6)-\ln\left(\frac{1+\tilde{v}^2}{\sqrt{1-\tilde{v}^2+\tilde{v}^4}}\right)\right\}\,. \tag{48}$$

In the region of large values of the argument $\tilde{v}\gg 1$ the function $g(\tilde{v})$ decays and can be approximated by:

$$g(\tilde{v})\approx\frac{9\pi}{32\sqrt{3}}\frac{1}{\tilde{v}} \tag{49a}$$

For small values of $\tilde{v}\ll 1$ (44) goes to zero by the law:

$$g(\tilde{v})\approx\frac{9}{128}\tilde{v}^3 \tag{49b}$$

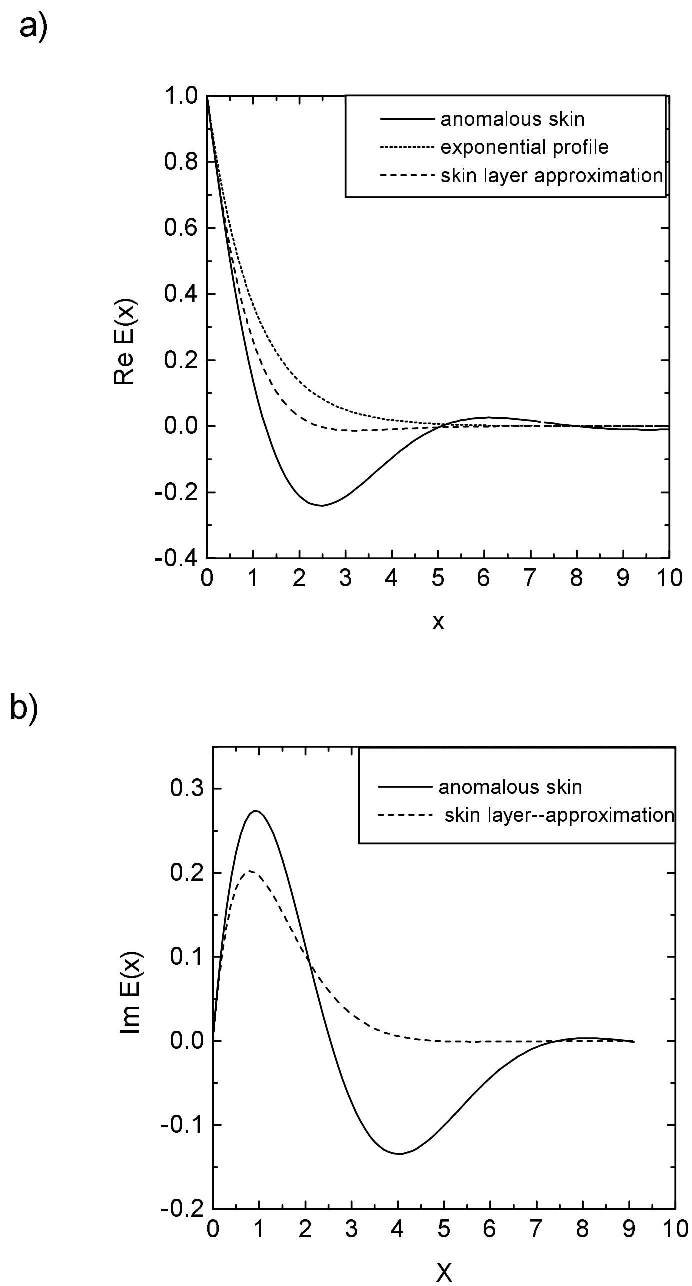


Figure 2. Plot of the electric field at anomalous skin effect as a function of normalized coordinates x/δ_0. The solid curve corresponds to the exact profile of the electric field (46), the dashed one to the exponential profile (34) and the small dashes represent the impedance approximation (39): (a) Real, (b) imaginary part of the electric field intensity.

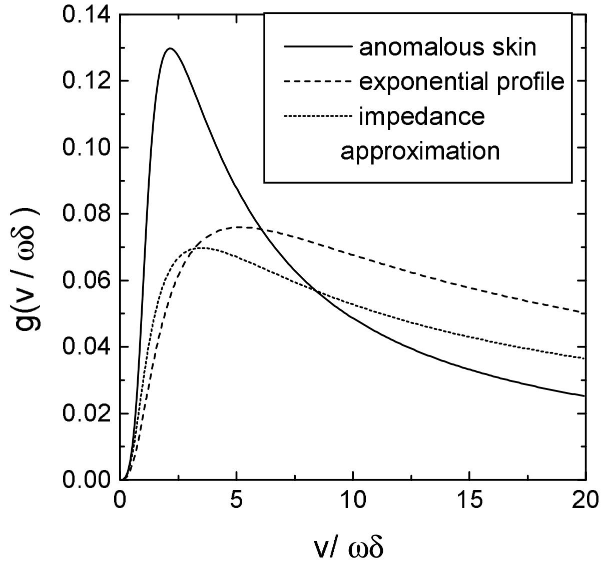


Figure 3. The normalized diffusion coefficient $g = \mathcal{D}/\frac{\delta_0}{2}\omega v_E^2$ as a function of the dimensionless velocity $\tilde{v} = v/\omega\delta_0$ for various models of electric field profile. The solid curve corresponds to the exact profile of the electric field (46), the dashed one to the exponential profile (34) and the small dashes represent the impedance approximation (39).

In Fig. 3 plot of $g(\tilde{v})$ for different cases of electric field representation is shown. The difference of the diffusion coefficients (49) to (38,41) (see Fig. 3) is essentially due to the oscillating structure of the em field penetrating into the plasma, which is very important for resonant interaction of particles with the field. The reason for this structure of the electric field is the nonlocality of the conductivity $\sigma(k)$. Due to the plasma conductivity an electron current arises and produces magnetic and electric fields in a direction opposite to that of the external em fields. In the local case (σ does not depend on k) this leads to a monotonic decrease of the sum of all em fields, while in the nonlocal case these em fields are shifted towards the plasma and as a result the oscillating structure of the em fields appears.

Another feature of exact description is that

$$\int_0^\infty E(x)\,dx = 0 \tag{50}$$

Note that it follows from (46) that the Fourier component $E(k = 0)$ should be equal to zero, which is in agreement with (42).

The integral of the electric field $\int_0^\infty E\,dx \to 0$, since the electron current has to vanish at large x, where the average electron velocity is determined by this integral.

Diminishing of the diffusion coefficient (47) in regions of a high electron velocity $v > \omega\delta$ has a transparent physical meaning. To the electron with such a high velocity the electric field appears as a stationary one. The increase of the electron energy by interacting with the localized electric field is equal to zero, since the spatial integral over electric field is zero.

In the region of high electron energy the diffusion coefficients $\mathcal{D}(v)$ (36), (40) decays much less than in the case, when the electric field is described exactly (48). This is a consequence of the fact that in the former two cases the long wave part of the Fourier spectrum of the electric field does not vanish and $E(k = 0) \neq 0$.

The Slab Geometry

Now the case of a uniform plasma slab (of width L) bounded by two plane walls with specular reflection of electrons is considered. For the case $L \gg \lambda$ collisionless heating of overdense plasmas occurs only in the nearest vicinity of the walls, while between then the usual Joule heating takes place. By using the method of continuation considered in the previous section the following solution for the EEDF is obtained

$$F_0(v,x) = F_0(v) \cdot \frac{\cosh\left(\frac{x-\frac{L}{2}}{\lambda_\varepsilon}\right)}{\cosh\left(\frac{L}{2\lambda_\varepsilon}\right)} \tag{51}$$

generalizing the result (30) for the case of plasma slabs with symmetrical heating on both walls. The kinetic equation for the EEDF $F_0(v)$ instead of (31) now has the form:

$$v\sqrt{\frac{\nu^*}{3\nu}}\, F_0(v) = \frac{\coth\left(\frac{L}{2\lambda_\varepsilon}\right)}{2v^2} \frac{\partial}{\partial v}\left(v^2 \mathcal{D}(v)\, \frac{\partial F_0(v)}{\partial v}\right) \tag{52}$$

The diffusion coefficient $\mathcal{D}(v)$ in (52) is defined by (22). The limit $L \gg \lambda_\varepsilon$ corresponds to the case of semi–infinite plasmas and (52) changes to (27). In the opposite limit of thin slabs ($L \ll 2\lambda_\varepsilon$), on the right-hand side of (52) a large parameter $2\lambda_\varepsilon/L \gg 1$ arises corresponding to more effective heating. In this case (52) takes the form:

$$\nu^* F_0(v) = \frac{1}{v^2}\frac{\partial}{\partial v}\left(\frac{v^2\mathcal{D}(v)}{L}\, \frac{\partial F_0(v)}{\partial v}\right) \tag{53}$$

Equation (52) expresses the fact that losses of electron energy by inelastic collisions in the plasma volume balances the collisionless heating. It should be noted that in this limit the EEDF is spatially uniform and is equal to $F_0(\varepsilon)$ in spite of the localization of energy input regions near the walls. This limit corresponds to the so called "nonlocal" heating regime of plasmas with space dimensions less than the length of of electron energy relaxation λ_ε[18,19].

If the slab width does not exceed the MFP ($L < \lambda < \lambda_\varepsilon$), the kinetic equation for the EEDF has a form identical to (53) but with the diffusion coefficient

$$\mathcal{D}(v) = \frac{\pi e^2 L}{2v^2 m^2} \sum_{n=-\infty}^{\infty} \int_0^\pi \sin\vartheta\, d\vartheta \cdot \int_0^{2\pi} \frac{d\tilde{\Phi}}{4\pi} \cdot (v_y)^2 |E_n|^2 \cdot \delta\left(\omega - \frac{\pi n}{L} v_x\right) \tag{54}$$

where

$$E_n = \frac{1}{L}\int_0^L E_y(x) \cos\left(\frac{\pi}{L} n x\right)\, dx \tag{55}$$

is the Fourier transformation of the periodically continued—in accordance with (33)—electric field $E_y(x)$ (with period L).

Expression (54) shows that only particles being in resonance ($v_x = \omega L/\pi n$) contribute to the collisionless heating in contrast to semi–infinite geometry, where *all* particles contribute to the heating, as can be seen from (14). The reason for such a discrimination lies in the way of stochastization of the em field phase[20]. In the semi–infinite case all electrons undergo collisions in the periphery of the heating region, so subsequent interactions with the em field region happen after several collisions and randomly.

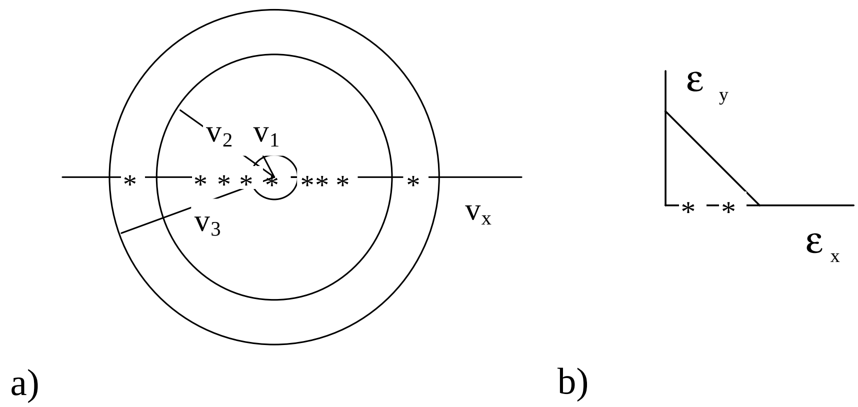


Figure 4. (a) Scheme of velocity space, area of averaging and resonance points. Small velocity v_1 corresponds to the region of many resonances, medium velocity v_2 lies in the region of the second resonance, and large velocity v_3 lies in the region of the first resonance. (b) Scheme of energy space, area of averaging and resonance points with accounting for the ambipolar electric field; no resonance points occur at small energies.

Thus all electrons participate in the heating. In the slab geometry ($L < \lambda$) electrons return to the same place in a time much smaller than the time between collisions. So these interactions are correlated and no diffusion in velocity space occurs. The collision frequency does not enter in the expressions for the diffusion coefficient (22) and (54), but the presence of collisions (or other stochastization mechanisms) is necessary for collisionless heating. The situation is similar to the problem of Landau–damping. The different ways of stochastization in semi–infinite and slab geometry result in different diffusion coefficients (22) and (54). For a given velocity v the sum in (54) is over the resonance velocities $v_x = \omega L/\pi n < v$ (see Fig. 4). If $v \ll \omega L/\pi$, only higher n contribute to the integral. For small v the approximation of hard wall and specular reflection can be not valid and the real profile of the potential sheath should be considered (see comment after Eq.(33). It can be shown that the spectrum E_n is always proportional to $1/n^2$, independent of the form of dependence of $E_y(x)$. This behavior is connected to the symmetric continuation of $E_y(x)$ and originates from the jump of the magnetic field component $B_y = -\frac{ic}{\omega}\frac{\partial E_y}{\partial x}$ and is easy to check by twice partially integrating (55). For such a spectrum the sum in (54) can be changed to an integral for low energy electrons ($v < \omega L/\pi$) and the diffusion coefficient takes the form of (22) with the function $E(k)$ corresponding to the Fourier spectrum of the electric field

$$E_y(x) = \begin{cases} E_y(|x|) & -L < x < L \\ 0 & |x| > L \end{cases} \tag{56}$$

In the region of high electron energy $v \geq \omega L/\pi$) the sum in (54) cannot be transformed into an integral. To demonstrate this the simple case of an exponential profile of the penetrating electric field with skin depth $\delta_0 \leq L$ is considered. The spectrum E_n for this case is

$$E_n = \frac{E_0 \frac{\delta_0}{L}}{1 + \left(\frac{n}{n^*}\right)^2} \tag{57}$$

(with $n^* = L/\pi\delta_0$) and the diffusion coefficient (54)

$$\mathcal{D}(v) = \omega \frac{\delta_0}{2} v_E^2 \, g\left(\frac{v}{\omega\delta_0}\right) \tag{58}$$

is obtained, where

$$g(\tilde{v}) = \frac{1}{2\tilde{v}} \sum_{n > \frac{n^*}{\tilde{v}}}^{+\infty} \frac{1 - \left(\frac{n^*}{n\tilde{v}}\right)^2}{\left[1 + \left(\frac{n}{n^*}\right)^2\right]^2 \cdot n} . \tag{59}$$

For small $\tilde{v}$ ($\tilde{v} < n^*$) the main contribution to the sum in (59) stems from large n so that the sum can be represented by an integral and the result coincides with the result (36) for the case of semi–infinite plasmas.

For large $\tilde{v}$ ($\tilde{v} > n^*$) the sum in (54) yields a smaller diffusion coefficient compared with that in the case of semi–infinite plasmas. This is connected to the effective cut–off at small $k \leq \pi/L$ for the slab geometry.

From the point of view of the one particle approach formula (54) can be interpreted as follows. Passing forward and backward through the slab, an electron obtains a velocity kick. Subsequent kicks are correlated, and the sequence of this kicks leads to no diffusion. The diffusion in the energy space is only due to resonance electrons which randomize the phase of velocity kick due to rare collisions. And the diffusion coefficient reads:

$$\mathcal{D}(v) = L \sum_{n=-\infty}^{\infty} \int_0^{\pi} \int_0^{2\pi} \frac{1}{2} \frac{|v_x|}{2L} \frac{(\Delta \boldsymbol{v_n} \cdot \boldsymbol{v})^2}{2v^2} \delta\left(n - \frac{\omega L}{\pi v_x}\right) \frac{\sin\vartheta}{4\pi} d\vartheta d\tilde{\Phi}, \tag{60}$$

$$\Delta v_n = \int_{-\frac{L}{v_x}}^{\frac{L}{v_x}} \frac{e}{m} \tilde{E}(x = v_x t) \exp(-i\omega t)\, dt = \frac{2eL}{m|v_x|} E_n \tag{61}$$

It should be noted that (60) is only valid with accounting for collisions. Without collisions the stochastization for $\omega \gtrsim {}^{\pi v}/_{L}$ usually does not exist, even due to nonlinear effects[20]. The collisions play two important roles:

1. They stochastizate the phase of electron motion with respect to the phase of the electric field and

2. they transfer electrons from non–resonance velocity to resonance and vice versa.

This complex process of electron motion can be considered as diffusion with diffusion coefficient (60). The identity of this diffusion coefficient with the one from the one particle approach is fortuitous, but accounting for nonlinear effects, the results are different[20]. The plot of the normalized diffusion coefficient function $g(v)$ is shown in Fig. 5. The reduction of the diffusion coefficient due to the second boundary for ${}^{\pi v}/_{\omega L} \gtrsim 1$, where correlation effects are important, are clearly seen.

The vanishing of correlation effects for $v > \omega L$ was proposed first in[26], but the conclusion that it can occur without collisions is not correct[20,25]. The decrease of the real part of the surface impedance in the slab geometry compared to the semi–infinite

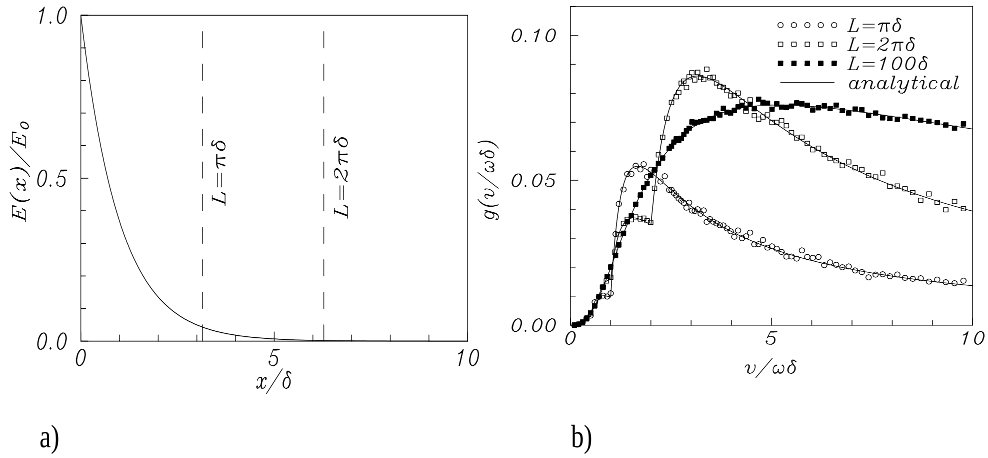


Figure 5. Influence of the second boundary on collisionless heating. (a) profile of the inductive electric field $E = E_0 \exp(-x/\delta)$ with second boundary at $x \equiv L = (\pi;\, 2\pi)\delta$; (b) normalized diffusion coefficient for an inductive exponential electric field $g = \mathcal{D}/\frac{\delta}{2}\omega v_E^2$, as a function of the normalised velocity $\tilde{v} = v/\omega\delta$ for different slab widths L. Solid curves correspond to the the analytical formulae (54), symbols to Monte Carlo simulations[25]: black squares: $L \to \infty$ (100δ); circles: $L = \pi\delta$; squares: $L = 2\pi\delta$.

one was found in[27] for a case, when the electron density drops exponentially with the width a and a skin depth much smaller than a. The reduction of collisionless heating due to the influence of the second boundary was observed in numerical simulations[28].

For a homogeneous electric field ($E_{n\geq 1} = 0$) the collisionless heating vani-shes[29]. In the sum in (54) there is no wave–particle resonance at all for $n = 0$. This can also be understood from the one–particle approach. Since the electric field is continued symmetrically, the motion of the electron in the case of slab geometry can be substituted by the motion in infinite space with a homogeneous electric field, where no condition for collisionless heating occurs. This situation is different for a longitudinal electric field.

CAPACITIVELY COUPLED PLASMAS

CCPs have a more complicated structure than ICPs due to the presence of large oscillating sheaths. For such a situation the approximation of the sheath potential as a moving rigid wall has repeatedly been used (see e.g.[22]). In this model the sheath velocity is $v_{\rm sh} = v_E \cos(\omega t + \tilde{\Phi})$. The electric field in the plasma bulk is neglected.

When MFP $\lambda < L$ we can apply Eq. (18) with velocity kick in the sheath $\Delta v = 2v_E$. After averaging over velocity angle the diffusion coefficient reads

$$\mathcal{D}(v) = \frac{v}{4}(v_E)^2 \tag{62}$$

If the energy relaxation length is larger than discharge gap $\lambda^* \gg L$ the kinetic equation has a form (53). The physical meaning of expression (62) can be explained as follows. After reflection from the wall an electron obtains a velocity kick $2v_{\rm sh}$. If the next kick can be considered as randomly with respect to the previous one, diffusion in velocity space and electron heating appear. The randomization can originate from two mechanisms: collisions, when the mean free path of the electrons λ is smaller than the slab width ($\lambda \leq L$), and the nonlinear mechanisms of collisionless stochastization[30]. In this case the averaged diffusion coefficient from two sheaths is[12]:

$$\frac{\mathcal{D}}{L} = \frac{1}{2}\frac{1}{4}\frac{v}{L}\left\langle (2v_{\mathrm{sh}}(t))^2 \right\rangle \tag{63}$$

Therefore $\frac{\mathcal{D}}{L}$ is the product of a factor $^1/_2$, the bounce frequency, the squared velocity kick averaged in time and a coefficient $^1/_4$ resulting from averaging over the velocity angle.

For $\lambda > L$ the correlation between two following kicks are important. In this case Eq. (60) can be applied. After integration the diffusion coefficient reads:

$$\mathcal{D}(v) = \frac{\omega L}{2\pi} v_E^2\, g\left(\frac{\pi v}{\omega L}\right) \tag{64}$$

with

$$g(\tilde{v}) = \frac{4}{\tilde{v}^3} \sum_{\substack{n=1,3,5,\ldots \\ n > 1/\tilde{v}}} n^{-5} \tag{65}$$

For small $\tilde{v}$ ($v < \omega L/\pi$) the sum in (64) can be replaced by an integral and $g(\tilde{v})$ is approximated by

$$g(\tilde{v}) = \frac{\tilde{v}}{2} \tag{66}$$

so that

$$\mathcal{D}(v) = \frac{v v_E^2}{4}. \tag{67}$$

Thus in the low energy region the expression for the diffusion coefficient (62) is identical to the result obtained in[12] with an one–particle approach.

In the high velocity region ($v > \omega L/\pi$) the main contribution in the sum of (65) is due to the first resonance and one obtains

$$g(\tilde{v}) = \frac{4}{\tilde{v}^3} \tag{68}$$

and correspondingly the diffusion coefficient decays $\sim v^{-3}$:

$$\mathcal{D}(v) = \frac{2\omega^4 L^2 v_E^2}{\pi^4}\frac{1}{v^3} \tag{69}$$

From Fig. 6 the large deviation of the quasilinear diffusion coefficient (solid line) from the one obtained without account correlation effect (dashed line) in the region of high electron velocity is evident. It is also possible to transfer the problem into a noninertial system moving with the sheath velocity v_{sh}. In this system there is a stationary specularly reflecting boundary and an alternating electric field connected to the inertial force:

$$E_x(t) = -\frac{m}{e}\frac{dv_{\mathrm{sh}}}{dt} = E_0 \sin(\omega t + \tilde{\Phi})\,, \tag{70}$$

Now the quasilinear approach is easily applied and with the help of the continuation method described above (32), (33) it yields the same expression for the diffusion coefficient (54) in ICP.

$$\mathcal{D}(v) = \frac{1}{v^2}\frac{\pi e^2}{2m^2}\sum_{n=-\infty}^{\infty}\int_0^{\pi} \sin\vartheta\, d\vartheta \int_0^{2\pi}\frac{d\tilde{\Phi}}{4\pi} v_x^2 |E_n|^2 \cdot \delta\left(\omega - \frac{\pi n}{L} v_x\right) \tag{71}$$

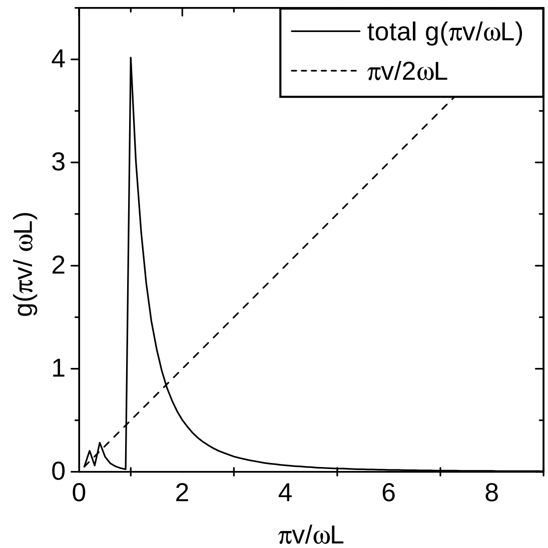


Figure 6. The normalized diffusion coefficient $g = \mathcal{D}/\frac{L}{2\pi}\omega v_E^2$ as a function of the dimensionless velocity $\tilde{v} = \pi v/\omega L$ for CCPs. The dashed curve represents $g = \tilde{v}/2$.

where

$$E_n = \frac{1}{L}\int_0^L E_x(x) \sin\left(\frac{\pi n x}{L}\right)\, dx. \tag{72}$$

In the case considered here $E_x(x) = E_0 =$ constant and Eq.(71) coincide with Eq.(64).

The interesting difference between capacitively and inductively coupled discharges is the different influence of the boundary on collisionless heating. For inductively coupled discharges the reflection from the boundary does not change the correlation between particle motion and electric field, since the influence of the electric field is the same before and after reflection and vice versa. For capacitively coupled discharges when the electric field diminishes the absolute value of velocity before reflection, but increases it after reflection. Consequently there is no collisionless heating with uniform electric fields for slab geometry in the case of inductively coupled discharges, but it does occur in the case of capacitively coupled discharges.

The difference in the nature of the heating field for CCPs and ICPs rests also in the structure of the diffusion coefficient near resonances. In the case of CCPs every resonance gives a well defined peak near $v_n = \pi\omega/Ln$ (see Fig. 6). With increasing v an additional resonance v_n yields at first a considerable contribution, followed, however, by a rapid decrease $\sim v^{-3}$. As a consequence at $v > \pi\omega/3L$ (i.e., $n \leq 3$) the diffusion coefficient is practically determined by the lowest resonance ($n = 1$) and its curve resembles a sequence of peaks (see Fig. 6). For ICPs the peak structure is smoothed out, since every new resonance results only in a small contribution $\sim (v - v_n)^2$ and the decrease of the resonance contributions is also slower than for CCPs ($\sim v^{-1}$). Therefore only the peak of the first resonance is pronounced in the net diffusion coefficient compare Fig.5 and Fig 6.

THE DIFFUSION COEFFICIENTS FOR INHOMOGENEOUS PLASMAS ACCOUNTING FOR STATIONARY AMBIPOLAR ELECTRIC FIELDS

In inhomogeneous plasmas the stationary ambipolar electric field appears which tends to trap electrons in the plasma volume. As a consequence low energy electrons cannot reach the periphery of the discharge, where the RF field is large. Therefore taking into account the ambipolar potential leads to a decrease in the efficiency of heating of low energy electrons. The diffusion coefficient for these electrons is small and the effective temperature of the low energy part of the EEDF can be considerably smaller than that of the higher energy part. For instance, in CCPs EEDFs resembling bi–Maxwellian ones have been observed, with a temperature of low energy electrons ($\varepsilon < 2$eV) ten times smaller than that of high energy electrons ($\varepsilon > 2$eV)[13]. Trapped electrons are heated by the bulk rf electric field. If the mean free path is larger than the plasma slab width ($\lambda > L$), the heating of trapped electrons tends to be collisionless and thus an appropriate quasilinear theory has to be developed.

The presence of the ambipolar electric field results not only in quantitative effects, but in qualitative ones, as we will see it changes the distribution of resonance particles over energy (see Fig. 4b).

First the case of a collisionless slab for CCPs ($\lambda \gg L$) will be considered. The kinetic equation (4) for the fast varying part of the distribution function $\tilde{F}(x, \boldsymbol{v})$ with the ambipolar potential $\Phi(x)$ takes the form:

$$-i\omega\tilde{F} + v_x\frac{\partial\tilde{F}}{\partial x} - \frac{e}{m}\frac{\partial\Phi}{\partial x}\frac{\partial\tilde{F}}{\partial v_x} = -\frac{e}{m}\tilde{E}_x\frac{\partial F}{\partial v_x} - \nu\tilde{F} \tag{73}$$

Equation (73) can be simplified by introducing new variables: instead of v_x and x now ε_x and x, where $\varepsilon_x = mv_x^2/2 + e\Phi(x)$[18,19].

The corresponding boundary condition for the distribution function is

$$\tilde{F}(v_x) = \tilde{F}(-v_x), \quad x = x_\pm(\varepsilon_x) \tag{74}$$

The solution of (73) is given by (see e.g.[31])

$$\begin{aligned}\tilde{F}(x,\varepsilon_x) = e\frac{\partial F(\varepsilon)}{\partial\varepsilon}\exp(i\Omega)\Bigg[&-\frac{\int_{x_-}^{x_+}\sin(\Omega^* - \Omega(x'))E_0(x')\,dx'}{\sin\Omega^*}\\ &+\int_{x_-}^{x}\exp(-\Omega(x'))E_0(x')\,dx'\Bigg]\end{aligned} \tag{75}$$

where

$$\Omega(x,\varepsilon_x) = \text{sign } v_x\int_{x_-(\varepsilon_x)}^{x}\frac{\omega + i\nu\,dx'}{\sqrt{\frac{2}{m}(\varepsilon_x - e\Phi(x'))}} \tag{76}$$

$\Omega^* = \Omega(x_+, \varepsilon_x)$ and the turning points $x_\pm(\varepsilon_x)$ are defined by the relation (see Fig.7)

$$e\Phi(x_\pm) = \varepsilon_x \tag{77}$$

Due to isotropization in collisions the main part of the distribution function $F = F(\varepsilon)$ is the function of the total energy $\varepsilon = e\Phi + {}^1\!/\!_2 mv^2$. After averaging over both x and velocity angle the kinetic equation reads:

$$\frac{d}{d\varepsilon}\overline{\mathcal{D}_\varepsilon}\frac{dF}{d\varepsilon} + \overline{S^*(F)} = 0 \tag{78}$$

The bars indicate averaging over the slab width L. The diffusion coefficient averaged over the angles in velocity space is:

$$\overline{\mathcal{D}_\varepsilon} = \frac{\pi e^2 L}{2} \int_0^\varepsilon \frac{d\varepsilon_x}{m} \sum_n E_n^2(\varepsilon_x) \delta[\Omega^*(\varepsilon_x) - \pi n] \tag{79}$$

with

$$E_n(\varepsilon_x) = \frac{1}{L} \int_{x_-(\varepsilon_x)}^{x_+(\varepsilon_x)} E_x(x) \sin \Omega(x, \varepsilon_x)\, dx \tag{80}$$

Thus (78) with the energy diffusion coefficient (79) appears to be a generalized form of the kinetic equation accounting for collisionless heating. The coefficients E_n are generalized Fourier coefficients (80)*.

Resonances occur, if the time of particle motion from one turning point to another (bounce time) is equal to $nT/2$, where $T = 2\pi/\omega$. This is in accordance with the resonance condition $\Omega^*(\varepsilon_x) = \pi n$, as follows from (79). If the value of $\Omega^*(\varepsilon_x)$ does not exceed π, there are no resonances at all and no collisionless heating occurs. It should be noted that, if the ambipolar potential is approximated by rigid walls, $\Omega^*(\varepsilon_x)$ is proportional to $\pi/\sqrt{\varepsilon_x}$ and the resonance conditions can always be fulfilled. In the case of a parabolic potential, which is realized for low energy electrons trapped near the discharge center and is defined by the potential profile $\Phi(x) = \Phi_0(x/l)^2/2$, the dimensionless period $\Omega^* = \sqrt{(\omega^2 m l^2)/\Phi_0}$ does not depend on the electron energy ε_x and the resonance conditions can be fulfilled in this case for one value of Φ_0/l^2 only. All plasma electrons are in or out of wave–particle resonance. Thus the "degeneracy" could be removed by taking into account the deviation of the ambipolar potential from the parabolic one.

To demonstrate the influence of the ambipolar potential on the energy diffusion coefficient an example for its calculation is given. Usually at the discharge center both the density and the potential profile are parabolic. At the periphery a more rapid decrease of density results in a more rapid increase of ambipolar potential. Therefore the latter is modeled in the form

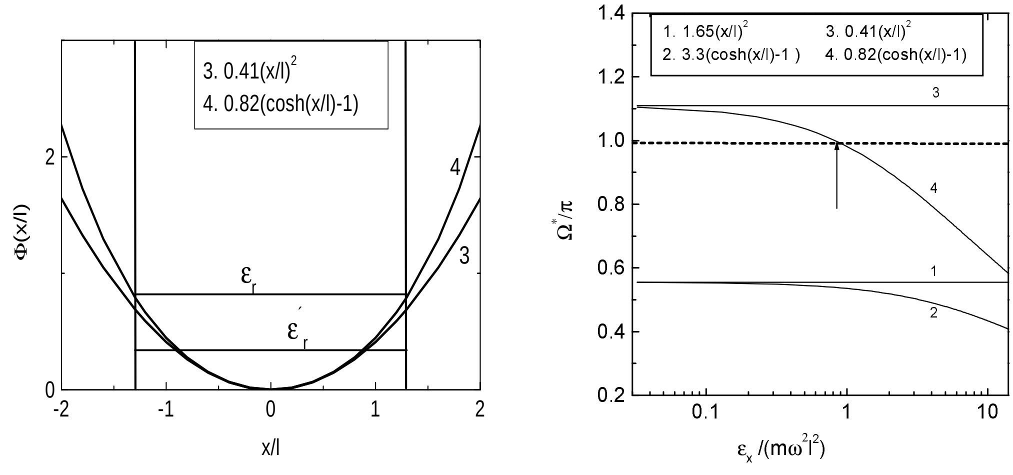
$$\Phi(x) = \Phi_0(\cosh(x/l) - 1), \tag{81}$$

which is parabolic at the center $|x/l| \lesssim 1$ and higher than parabolic at the periphery (see Fig. 7a). The plot of the dimensionless period

$$\Omega^*(\varepsilon) = \int_{x_-}^{x_+} \frac{\omega dx}{v_x} \tag{82}$$

is shown in Fig. 7b for electrons moving in ambipolar potential (81). In contrast to the case of no ambipolar potential the function $\Omega^*(\varepsilon)$ is finite at small ε. For a given potential (81) e.g. this situation occurs, if $\Phi_0/m\omega^2 l^2 > 1$. For smaller Φ_0 the bounce time is larger and a first resonance $\Omega^* = \pi$ appears. For example for $\Phi_0 = 0.82$ resonance corresponds to a energy $\tilde{\varepsilon} = 0.82 m\omega^2 l^2$. With a given resonance energy the energy diffusion coefficient (76) can be calculated, its plot is shown in Fig. 8.

*Expression (79) for the diffusion coefficient in energy space can also be derived by introducing a new variable—phase $\Omega = \int \frac{dx}{v_x}$ instead of coordinates[32]—and then performing a Fourier transformation on this variable.



a) b)

Figure 7. (a) Plot of the ambipolar potential $\Phi(x) = \Phi_0(\cosh(x/l) - 1)$ with $\Phi_0 = 0.82 m\omega^2 l^2$. $x_\pm(\tilde{\varepsilon}_r) = \pm 1.3l$ are the coordinates of the turning points. Model with no ambipolar field corresponds to the slab width L, so that the discharge area available for resonance particles is the same as before: $L = 2x_+(\tilde{\varepsilon}_r)$. Resonance occurs at the energy $\varepsilon = 0.82 m\omega^2 l^2$. with and $\varepsilon_r = 2m\omega^2(x_\pm(\varepsilon))^2/\pi^2$ without accounting for the ambipolar potential. (b) Plot of the phase Ω^*/π as function of the normalised energy $\tilde{\varepsilon} = \varepsilon/(m\omega^2 l^2)$. For normalized potentials $\tilde{\Phi} = \Phi/(m\omega^2 l^2)$ 1. $\tilde{\Phi}(x) = 1.65(x/l)^2$ 2. $\tilde{\Phi}(x) = 3.3(\cosh(x/l) - 1)$, 3. $\tilde{\Phi}(x) = 0.41(x/l)^2$ 4. $\tilde{\Phi}(x) = 0.82(\cosh(x/l) - 1)$

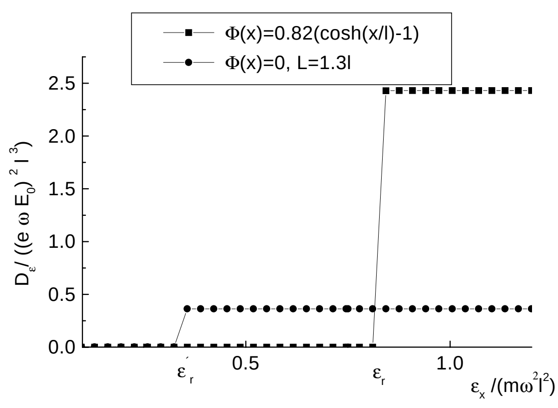


Figure 8. c) Plot of the dimensionless diffusion coefficient $g(\tilde{\varepsilon}) = \mathcal{D}_\varepsilon(\tilde{\varepsilon})/e^2 E_0^2 \omega^2 l^3$ as a function of the dimensionless energy $\tilde{\varepsilon}$ (see Fig. 7b). Solid curve: with ambipolar normalised potential $\tilde{\Phi}(x) = 0.2(\cosh x - 1)$, $\tilde{\Phi}_0 > 1$ corresponds to the absence of resonance particles and $g = 0$. Small dashes: no ambipolar field, but smaller slab width L, so that the discharge area available for resonance particles is the same as before: $L = 2x_+(\tilde{\varepsilon}_r)$.

In Fig. 8 also the energy diffusion coefficient is presented without accounting for the ambipolar potential for a slab width the same as distance between turning points for resonance particles remains the same (i.e. $L = 2x_+(\varepsilon_r)$) (see Fig. 8) †.

The corresponding energy diffusion coefficient is considerably smaller than that with account of ambipolar field.

Indeed, for a purely parabolic profile, the function Ω^* does not depend on ε and $d\Omega^*/d\varepsilon = 0$, so for a resonance in this situation $\mathcal{D}_\varepsilon \to \infty$. As a consequence the smaller the nonlinearity of the ambipolar potential (the difference to the parabolic profile) the larger is the diffusion coefficient and, correspondingly, heating.

From kinetic equation (73) the distribution of current in the discharge can be found. The current in the system is determined by the difference $\Delta\tilde{F} = \tilde{F}(|v_x|) - \tilde{F}(-|v_x|)$. Substituting the solution (75) gives

$$\Delta\tilde{F} = 2ie\frac{\partial F}{\partial\varepsilon}\cdot\left[\int_{x_-}^{x} G(x,x',\varepsilon_x)E(x')\,dx' + \int_{x}^{x_+} G(x',x,\varepsilon_x)E(x')\,dx'\right], \tag{83a}$$

with the kernel

$$G(x,x',\varepsilon_x) = -\frac{\sin\Omega(x')\cdot\sin(\Omega^* - \Omega(x))}{\sin\Omega^*}, \tag{83b}$$

Integrating (83a) over the velocity we get an equation for the current in the discharge:

$$J(x) = \int_{x_-}^{x} \Pi(x,x')E(x')\,dx' + \int_{x}^{x_+} \Pi(x',x)E(x')\,dx' \tag{83c}$$

with

$$\Pi(x,x') = \frac{2\pi i}{m}\int_0^\infty \frac{\partial F}{\partial\varepsilon}\int_0^\varepsilon G(x,x',\varepsilon_x)\,d\varepsilon_x\,d\varepsilon \tag{83d}$$

For small collision frequency the current is shifted by $\pi/2$ in phase compared to the electric field, since it is determined by electron inertia.

For ICPs the diffusion coefficient averaged in velocity space is not influenced by the magnetic field, so it can be omitted for $\overline{\overline{\mathcal{D}_\varepsilon}}$ calculation. For the inductive electric field the current is determined by the sum $\Delta\tilde{F} = \tilde{F}(|v_x|) + \tilde{F}(-|v_x|)$. The function $\Delta\tilde{F}$ obeys to the integral equation (83a) with a kernel[27]

$$G(x,x') = -\frac{\cos(\Omega(x'))\cdot\cos(\Omega^* - \Omega(x))}{\sin(\Omega^*)}\,\frac{v_y}{|v_x|} \tag{84}$$

the expression for the diffusion coefficient has the form:

$$\overline{\mathcal{D}_\varepsilon} = \frac{\pi e^2 L}{2}\int_0^\varepsilon \frac{d\varepsilon_x}{m}\frac{\varepsilon - \varepsilon_x}{2\varepsilon_x}\sum_n E_n^2\delta(\Omega^* - \pi n) \tag{85}$$

where

$$E_n(\varepsilon_x) = \int_{x_-(\varepsilon_x)}^{x_+(\varepsilon_x)} E_y(x)\cos\Omega(x)\sqrt{\frac{\varepsilon_x}{\varepsilon_x - e\Phi(x)}}\,dx \tag{86}$$

†Many resonances exist for small energies, as a result the function $\mathcal{D}_\varepsilon(\varepsilon)$ has the form of a ladder function. The value of steps at higher resonances is small $\propto 1/n^5$ and, thus, this cannot be recognized in Fig. 8. The main contribution is due to the first resonance, which corresponds to a energy $\varepsilon = m/2(\omega L/\pi)^2$.

CONCLUSIONS AND OUTLOOK

The quasilinear approach to the description of collisionless electron heating by given high frequency EM fields has been developed. This approach has been shown to be an effective method of providing a common basis for a variety of conditions in gas discharges and obtaining generalized expressions. It was possible not only to summarize the effects obtained and to compare with results previously obtained by a different approach, but also to obtain new and extended results: The comparison to results from the one–particle approach yields agreement in the diffusion coefficient characterizing heating only for the case of low energy electrons. The expressions derived here are valid also for higher energies. For this region of electron energy a cut–off for the long wave part of the spectrum of heating electric field is required. Accounting for the second boundary of bounded plasma automatically provides such a cut–off. The generalized expressions for bounded plasmas given include the effect of ambipolar electric fields, which can be essential for the effectivity of collisionless heating.

It should be noted, if the EM in the plasma has also large space scale parts ($\boldsymbol{E} \neq 0$, $\boldsymbol{B} \neq 0$), the complete equation (5) has to be used instead of (8). The influence of static ambipolar fields has been considered. The large space scale and time varying part of the EM give rise to local diffusion coefficients which should be added to nonlocal ones. Modeling of microwave discharges with heating due to two space scales and time varying electric fields has been undertaken in[17]. In general it can be underlined that knowledge of the energy diffusion coefficient gives a good basis for further developments, in particular for calculations of EEDFs needed for specific applications.

Finally it should be noted that the theory used in the present article is *not* a self consistent quasilinear theory due to neglecting backward influences of the modification of the EEDF on the electric field profile. For the case of surface wave produced plasmas this problem has briefly been discussed in[16] and included in numerical modeling in[17].

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