COLLISIONLESS ELECTRON HEATING IN RF GAS DISCHARGES: II. THE ROLE OF COLLISIONS AND NON-LINEAR EFFECTS

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INTRODUCTION

A new trend in applications of low pressure discharges is to decrease the pressure below 10 mTorr. For these low pressures the mean free path of electrons (λ) is large (comparable with discharge slab) and collisionless heating dominates Ohmic one. Being initially proposed for plasma heating in [1], it was first explored in gas discharge plasma for a capacitively coupled plasma (CCP) [2-4], and is now widely discussed for application to inductively coupled plasmas (ICP) [5,6]. The classification of various scenaria of collisionless heating has been done in [7]. The similar problem of cyclotron heating were extensively analyzed for magnetic traps (see for example review[8]).

Here we concentrate on the effects of stochastization due to collisions and non-linear effects and investigate the interaction between them. For a single wave the collisionless heating corresponds to Landau damping. The role of collisions and non-linear effects has been investigated in detail for a single finite-amplitude wave [9,10]. Recently it has been shown that collisions play a key role in Landau damping and have to be accounted for[11]. In a gas discharge plasma the rf electric field is strongly inhomogeneous. The corresponding Fourier spectrum contains many wave vectors. Our aim is to study the electron heating resulting from the interaction with the whole spectrum of waves.

In common models the collisionless heating does not depend on the collision frequency. We demonstrate in a number of examples that collisions play an important role and that collisionless heating depends on collision frequency for some cases.

It is known that in low-pressure discharges electron heating may occur without collisions [1,12]. In absence of collisions the conditions for such a collisionless heating arise if there exists some other phase randomization mechanism. This stochastic heating can only occur if an electron “forgets” the field phase between subsequent interactions during its dynamic motion. The sequence of non-correlated interactions with the field results in diffusive electron motion along the energy axis toward high energies, i.e. in the electron heating. In a bounded plasma the situation may be quite different. Due to the presence of plasma boundaries (potential barriers) that specularly reflect the electrons, the phases of subsequent electron interactions with the field may be strongly correlated. Also, the picture essentially depends on the direction of the rf field with respect to the boundaries.

The CCP is sustained by longitudinal — i.e. perpendicular to the boundary — electric field. The ICP is sustained by the electric field along the plasma boundary. It results in velocity kicks along the plasma boundary. If the influence of the rf magnetic field is taken into the account, the kicks are transverse to the boundary [13]. Thus, in general, a variety of heating scenaria may arise.

We consider different mechanisms of the electron heating using a simple model. Let L denote the gap length and $\delta \ll L$ the layer thickness where electrons interact with the localized rf fields. For a CCP such a model corresponds to a strongly asymmetric discharge with a much larger current density at the powered electrode then at the grounded electrode. δ is the width of the sheath. In the ICP, δ is the skin depth. In addition to the rf fields, a static space-charge field is present, which confines the majority of plasma electrons. We shall approximate its influence in the model by rigid reflecting walls and neglect the ambipolar potential in the bulk. The influence of the bulk potential on the electron collisionless heating was analyzed in [14]. In the plasma bulk electrons experience isotropic collisions, and there shall be no collisions in the δ layer ($\delta \ll \lambda$).

The electron motion is governed by three frequencies: the frequency of the rf field ω, the collision frequency ν and the bounce frequency $\Omega(v_x) = v_x/2L$.

The electron heating is adequately described in terms of the diffusion coefficients in energy space D_ε or velocity space D_v [15]:

$$D_\varepsilon = \left\langle \frac{(\Delta\varepsilon)^2}{2\Delta t} \right\rangle, \; D_v = \left\langle \frac{(\Delta v)^2}{2\Delta t} \right\rangle, \; D_\varepsilon = 2m\varepsilon D_v. \tag{1}$$

They determine the microscopic characteristics of the electron ensemble such as the electron distribution function (EDF) $f(\varepsilon)$ and the power deposition rate into a unit volume of plasma P, which can be expressed in terms of D_ε and $f(\varepsilon)$ [7]. The principal part of the EDF $f(\varepsilon)$ satisfies the stationary kinetic equation:

$$\frac{1}{\sqrt{\varepsilon}} \frac{d}{d\varepsilon} \sqrt{\varepsilon} D_\varepsilon \frac{df(\varepsilon)}{d\varepsilon} + St^* f(\varepsilon) = 0, \tag{2}$$

where $St^* f(\varepsilon)$ is the inelastic collision integral. The macroscopic quantities such as the rate of energy input deposition into the unit volume of plasma, P, can be expressed in terms of D_ε and $f(\varepsilon)$:

$$P = \frac{4\sqrt{2}\pi}{m^{3/2}} \int_0^\infty f(\varepsilon) \frac{d}{d\varepsilon} (\sqrt{\varepsilon} D_\varepsilon(\varepsilon)) d\varepsilon. \tag{3}$$

Thus, the energy diffusion coefficient contains all information about the electron heating. We have performed a Monte Carlo simulation at various ω, δ, L, ν and compare its results ($f(v)$, D_ε or D_v) with the quasi-linear theory [14] and analyse non-linear effects.

THE COLLISIONLESS HEATING IN THE CASE $\lambda < L$

If $\lambda < L$, there are many collisions in the gap and subsequent interactions with the electric field (δ-layer) are random. The energy diffusion coefficient can be evaluated as the product of the square of the step of random-walk in energy and the frequency of such steps Ω, which is the average frequency of electron-field interactions [7]. Since the process of velocity isotropisation is faster then the energy loss mechanism in gas discharges, the EDF is isotropic [16]. Thus, the energy diffusion coefficient should be averaged over velocity directions:

$$D_\varepsilon = \frac{1}{2} \left\langle \left\langle \Omega (\Delta\varepsilon)^2 \right\rangle \right\rangle , \tag{4}$$

where $\langle\langle ... \rangle\rangle$ means averaging in time and in direction of velocity, $\Delta\varepsilon = m\boldsymbol{v} \cdot \Delta\boldsymbol{v}$ is the change in absolute value of energy and $\Delta\boldsymbol{v}$ is the kick in velocity.

For example in a CCP $\Delta\varepsilon = mv_x \Delta v_x(t)$ with $\Delta v_x = \Delta v_{x0} \sin\omega t$. Averaging in time and velocity direction gives [3,4]:

$$D_\varepsilon = m^2 (\Delta v_{x0})^2 v^3 / 32L. \tag{5}$$

The expression for the energy diffusion coefficient does not explicitly depend on ν. The role of collisions is only stochastization of subsequent interactions with the δ-layer and isotropisation of the EDF. More precisely the disappearance of ν from the expression for D_ε is the result of averaging over all electrons in a volume. The main contribution to D_ε corresponds to the particles at a distance of the order of one mean free path $\lambda = v/\nu$ from the δ-layer. They have the largest possible electron–field interaction frequency which is of the order of ν. The interaction frequency averaged over all particles is of the order $\nu\lambda/L = \Omega$ which does not depend on collision frequency.

THE COLLISIONLESS HEATING IN THE CASE $\lambda >> L$

If $\lambda >> L$, collisions in the gap are rare and subsequent interactions with the electric field are not random (Fig. 1).

Most interactions with the electric field in the δ-layer result in oscillations of the velocity around a constant average value. Only due to collisions there is decorrelation and alternation of the average value and as a result diffusion in velocity space. The main contribution to this diffusion is due to resonances $\omega/\Omega \approx 2\pi n$: the electrons interact with the rf field practically always at the same phase, so kicks are summed up. Due to small divergence from resonance conditions the phase is changing. This results in large oscillations in velocity (Fig. 1). The quasi-linear theory gives the energy diffusion coefficient for this case [14]:

$$D_\varepsilon = \frac{m^2}{8L^2} \sum_n \int v_x^2 \left(\boldsymbol{v} \cdot \Delta_0 \boldsymbol{v}\right)^2 \Delta \left(\omega - \frac{\pi n |v_x|}{L} \right) \frac{d\boldsymbol{\Omega}}{4\pi} \tag{6}$$

$$\Delta(\omega^*) = \frac{\nu}{\nu^2 + \omega^{*2}}, \ \lim_{\nu \to 0} \Delta(\omega^*) = \pi\delta(\omega^*), \ \omega^* = \omega - \frac{\pi n |v_x|}{L} \tag{7}$$

where $\boldsymbol{\Omega}$ is the solid angle in velocity space and δ is the Dirac function, $\Delta_0\boldsymbol{v}$ is the amplitude of velocity kick.

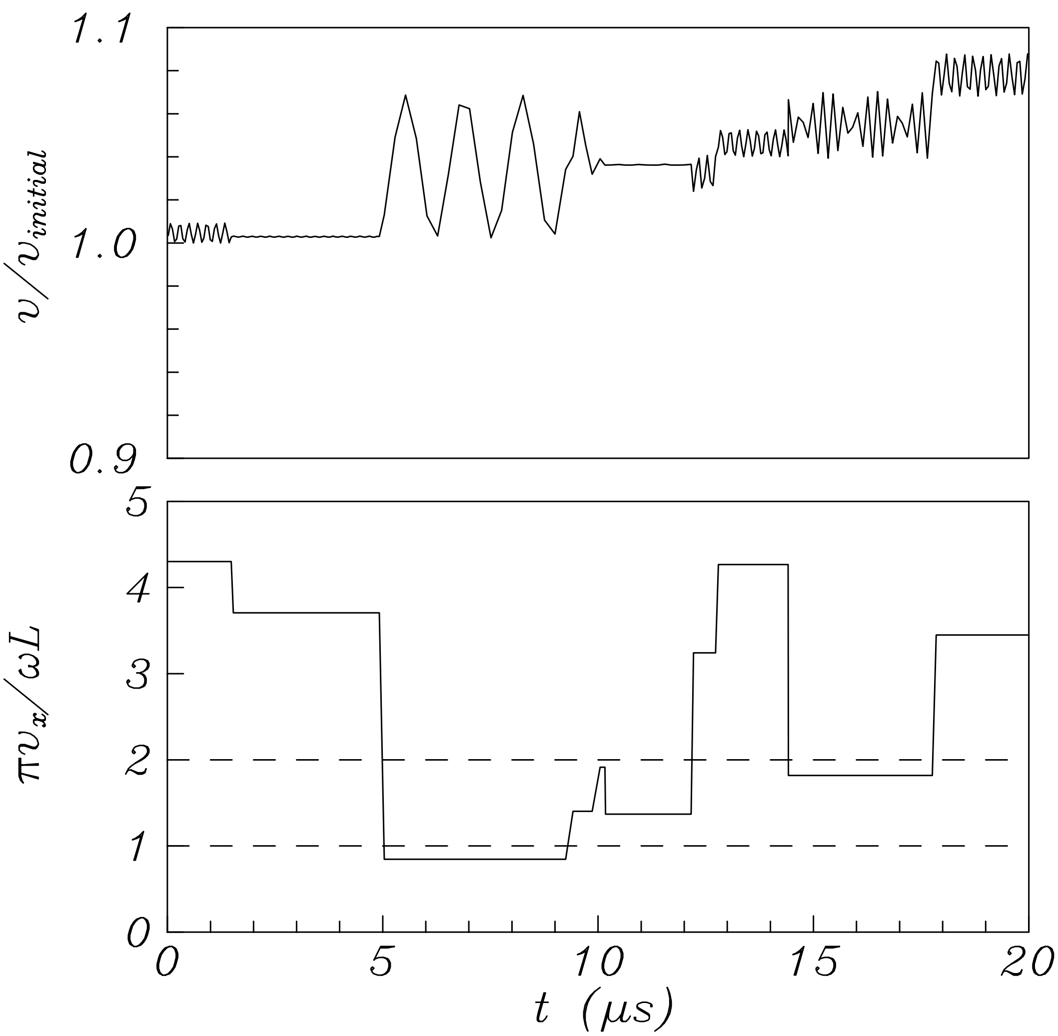

Figure 1. Time evolution of the the velocity $v(t)$ and its x-projection $v_x(t)$ for a model of ICP, velocity kicks are in y-direction: $\omega = 3 \cdot 10^7 s^{-1}$, $\delta = 1cm$, $\nu = 5 \cdot 10^5 s^{-1}$, $L = 3.14cm$, $E_0 = 10V/m$, $v_{initial} = 1.5 \cdot 10^6 m/s$. Dashed lines correspond to first and second resonance.

When $v \ll \omega L/\pi$ there are many resonances enabling the sum to be changed into an integral and the energy diffusion coefficient (6) coincides with (4) [14].

It should be noted that this result is only valid with accounting for collisions. The collisions play two important roles:

1. They stochastizate the phase of electron motion with respect to the phase of the electric field and

2. they transfer electrons from non–resonance velocity to resonance and vice versa.

This complex process of electron motion can be considered as diffusion with a diffusion coefficient (6). The identity of diffusion coefficient (6) with (4) is not general, since accounting for nonlinear effects will change the results as shown below.

We also note importance of three dimensional effects. In the one dimensional case (e.g. in magnetic traps[8]), the situation is quit different: A single collision is not able to move the electrons from a non-resonant velocity to a resonant one and vice versa, since a considerable change of kinetic energy is required. In the three dimensional case, only the projection of the velocity perpendicular to the wall has to be changed. As a result the diffusion coefficient is much smaller in one dimensional case compared to the three dimensional one. More precisely, in one dimensional case the averaging over region of many resonances is to be performed not over D_ε, but over $1/D_\varepsilon$. This is due to modification of EDF near the resonances, where plateaus are formed $\frac{df(\varepsilon)}{d\varepsilon} \approx 0$. Indeed if we define the averaged diffusion coefficient as

$$< D_\varepsilon >= \frac{\Gamma}{(f_1 - f_2)/(\varepsilon_2 - \varepsilon_1)} \tag{8}$$

where Γ is flux in energy space, and the region $\varepsilon_2 - \varepsilon_1$ includes many resonances. With the use of relation $\Gamma = D_\varepsilon \frac{df}{d\varepsilon}$ we find that[8]:

$$< D_\varepsilon >= \frac{1}{\int_{\varepsilon_1}^{\varepsilon_2} \frac{d\varepsilon}{D_\varepsilon}} \tag{9}$$

As a result the regions of resonances, where D_ε is large do not contribute to average diffusion coefficient (9) and main contribution is due to non-resonant regions with small D_ε. So $< D_\varepsilon >$ is small and proportional to collision frequency. Physically this means that electrons diffuse very fast across the resonance and get stuck in the region between them. In three dimensional case the scattering of electrons allows them to escape from this non-resonant region and participate in farther diffusion.

If $v > \omega L/\pi$ the sum in (6) cannot be transformed into an integral. The main contribution is due to first resonance ($n = 1$) and the diffusion coefficient according to (6) is small compared to that from (4) for $v > 3\, v_1$, where $v_1 = L\omega/\pi$ corresponds to the velocity of first resonance. This is due to the fact that most electron field interactions result in no heating and compensate each other (Fig. 1). The numerical example is shown in Fig. 2 for an ICP-type field $E_y = E_0 \exp{(-x/\delta)}$ (without account for the influence of the rf magnetic field). In the Monte Carlo simulation the diffusion coefficients were calculated according to their definition (1) as the ratio of the square of averaged change of energy or absolute value of velocity over some time interval $\Delta t >> 1/\nu$, $1/\Omega$.

From Fig. 2 one can see the remakable influence of a finite gap on the diffusion coefficient. The diffusion coefficients are characterized by a sharp rise at the velocity of

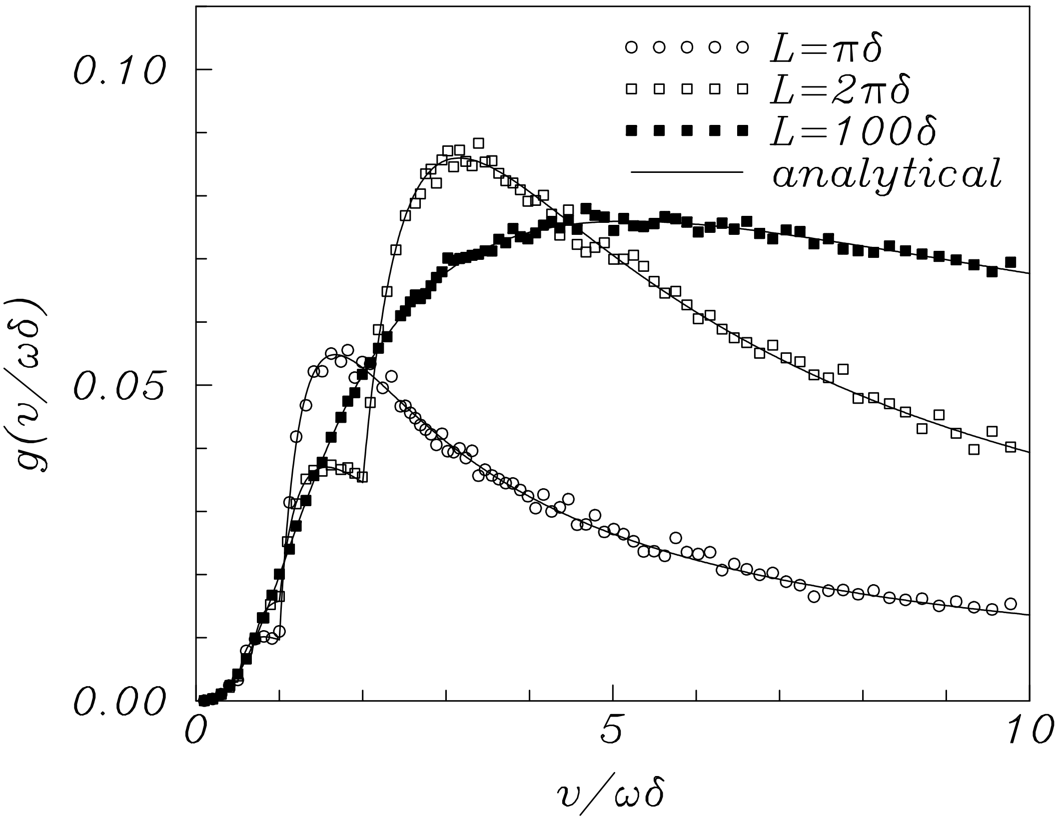

Figure 2. Influence of the second boundary on collisionless heating. Dimensionless diffusion coefficient for this field $g = D_v / \frac{e^2 E_0^2 \delta}{2m^2 \omega L}$ as a function of the normalized velocity for different slab widths L. Solid curves correspond to the analytical formulae (6), symbols to Monte Carlo simulations.

first resonance ($v_1/\omega\delta = 1$). A strong suppression of diffusion compared to the "infinite" gap situation occurs for $v > 3v_1$. The vanishing of correlation effects for $v < \omega L$ (all curves coincide) was proposed first in [1], but the conclusion that it can occur without collisions is not correct. The reduction of collisionless heating due to the influence of the second boundary was observed in numerical simulations [5].

The resonance effect also considerably influences the EDF. Since the flux in energy space $D_\varepsilon df/d\varepsilon$ does not change as rapidly as the diffusion coefficient at the first resonance, the derivative $df/d\varepsilon$ decreases above the resonance. This leads to the formation of a sort of plateau on the EDF for $\varepsilon > \frac{1}{2}mv_1^2$. Fig. 3 depicts the EDF resulting from the simulation of a CCP with electron generation (injection) at 0.5eV and loss at 20eV. The flux is constant between these energies. For a collision frequency of $\nu = 5 \cdot 10^7 s^{-1}$ the mean free path is (MFP) $\lambda \sim 4cm < L$ and electrons return with random phase. This corresponds to the diffusion coefficient (5), which gives a smooth growth of D_ε. For $\nu = 5 \cdot 10^6 s^{-1}$, $\lambda \sim 40cm >> L = 5cm$ and electrons return with correlated phase. The use of (6) gives

$$D_\varepsilon = \frac{(m \Delta v_{x0} v_1^2)^2}{8vL} \tag{10}$$

for $v > v_1$, $v_1 = \omega L/\pi$. Comparing (5) and (10) shows that, the diffusion coefficient of (5) four times smaller than that of (10). The deviation of simulation results from the analytical formula (10) is due to a large value of the velocity kick $\Delta v_{x0}/v \sim 0.5$ so that the sharp rise of D_ε is smoothed out. The first resonance appears at $\varepsilon \simeq 5eV$, above this energy the sort of plateau is formed.

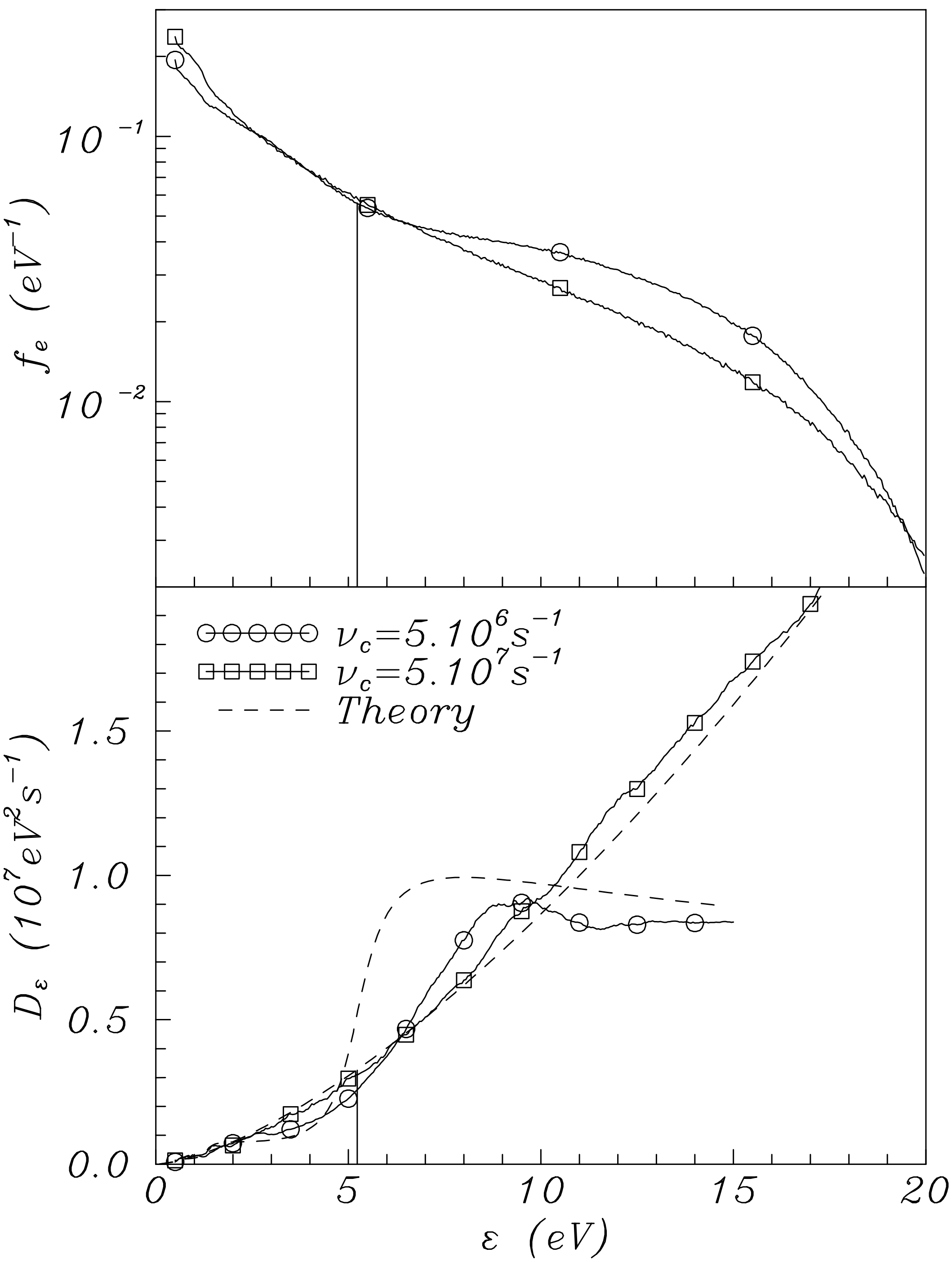

Figure 3. The EDFs (upper part) and energy diffusion coefficients (lower part) obtained in a Monte–Carlo simulation of a CCP with two different collision frequencies. $f = 13.56$MHz, sheath length 0.6cm and slab width $L = 5cm$. The first resonance is marked by a vertical line.

It should be mentioned that this mechanism can be responsible for the formation of a bi-Maxwellian distribution in capacitively coupled discharges as found in experiments [17]. The resonance effects and hence the plateau vanish if $\lambda < 2L$ (Fig. 3).

INFLUENCE OF NONLINEAR EFFECTS ON COLLISIONLESS HEATING.

Nonlinear effects are introduced in the case $\Delta \boldsymbol{v} \parallel \boldsymbol{x}$ by the fact that the bounce frequency itself depends on v_x. The kicks change the bounce frequency in contrast to the case $\Delta \boldsymbol{v} \perp \boldsymbol{x}$. Thus, electrons move out of the resonance. The amplitude of the velocity oscillation (see Fig. 1) in the resonances are limited by a resonance width $\Delta_{rw} v = \Delta v \Omega / \theta$ [15], where θ is characteristic frequency of non-linear oscillations:

$$\theta = \Delta_{rw} v \frac{\omega d\Omega}{\Omega dv} = \sqrt{\Delta v \omega \frac{d\Omega}{dv}} \tag{11}$$

If the resonance width $\Delta_{rw} v$ is larger then the distance between resonances $\delta v = \pi v^2 / \omega L$, diffusion in velocity space can occur even without collisions [3,4].

Otherwise, collisions are necessary for diffusion. The non-linear effects should be accounted for by limiting the amplitude of velocity oscillation in resonances. This can be done by using a modified resonant function $\Delta(\omega^*)$, approximated as:

$$\Delta(\omega^*) = \frac{\nu}{\nu^2 + \theta^2 + \omega^{*2}}, \ \lim_{\nu \to 0} \Delta(\omega^*) = \frac{\pi \nu}{\theta} \delta(\omega^*) \tag{12}$$

A more detailed calculation is performed in[8]. As a result D_ε is proportional to ν (at $\nu < \theta$) and tends to zero with $\nu \to 0$. This is in contrast to the case of transversal kicks, where the non-linear effect is absent, and D_ε remains a constant when $\nu \to 0$.

The non-linear effect is important for the capacitive discharge and inductive discharge with account of magnetic field, where the kicks are along x. Fig. 4 is a plot of power dissipation (3) as function of the collision frequency for Maxwell distribution function with electron temperature 3 eV.

If the gap is small ($L = \pi \delta$), the diffusion decreases when $\lambda > 2L$ ($\nu < \Omega \approx 5 \cdot 10^7 s^{-1}$). There is no decrease in the case of the large gap, because the corresponding velocity of first resonance $v_1 = \omega L / \pi$ is larger than thermal velocity and decrease of diffusion coefficient is not pronounced. Accounting for B_{rf} leads to a decrease of the diffusion coefficient at $\nu < \theta$ from (11) due to the introduction of the non-linear effect as described above. As a result the curves for power dissipation with and without account for induced magnetic field diverge below $\nu = \theta$. Increasing L or diminishing E_0 decreases θ and hence the critical collision frequency for divergence (see Fig. 4). The typical numbers for θ are : $\theta = 2.3 \cdot 10^7$ in case of $L = \pi \delta$, $E_0 = 5V/m$, and $\theta = 3.2 \cdot 10^6$ in case of $E_0 = 0.1V/m$, $\theta = 2.1 \cdot 10^6$ in case of $L = 25\delta$, $E_0 = 5V/m$.

The results presented in Fig. 4 correspond to two values of electric fields. For small E_0 kicks are small ($\Delta v << v_1$) and D is proportional to ν. For larger value of kicks (Δv comparable with v_1) the dependence of D on ν is even more complex. For example D can have a minimum at some ν and then increase with decreasing ν. The reason for this effect is that many subsequent interactions with the δ-layer can result in a change of v. We discuss these effects elsewhere.

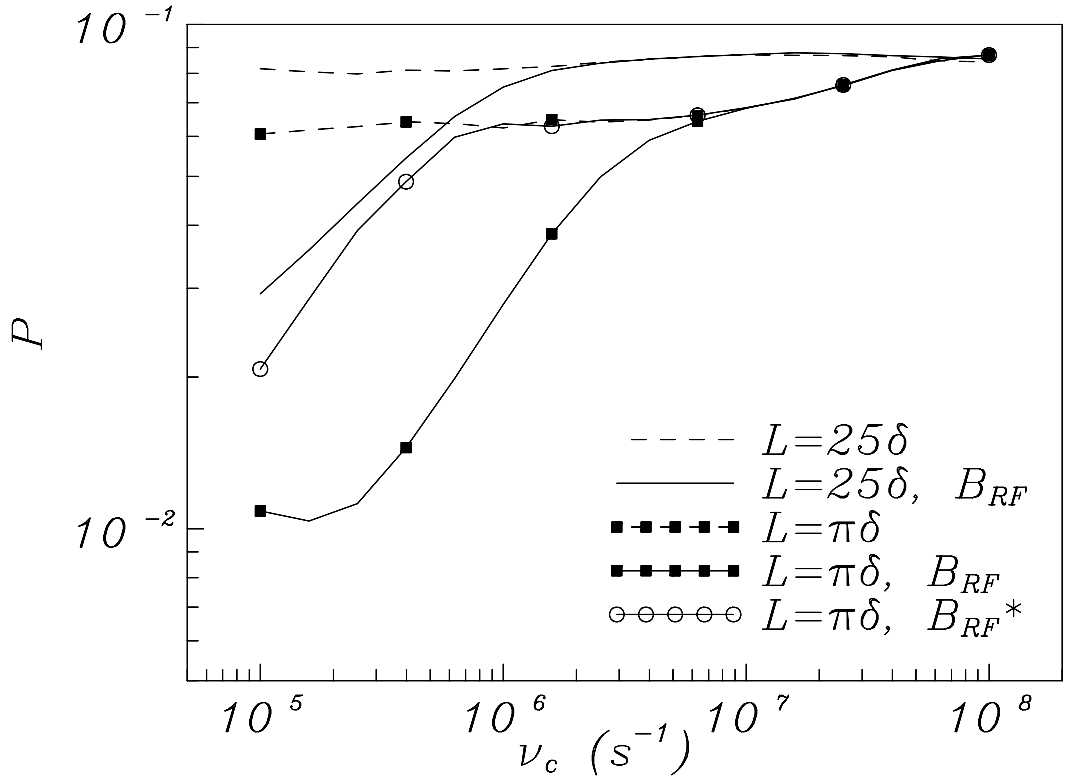

Figure 4. Dimensionless power dissipation, normalized on $\frac{ne^2E_0^2\delta}{m\omega L}$ for the same conditions as in Fig. 2 as function of the collision frequency for two slab widths L with and without account for B_{rf}, $\omega = 6 \cdot 10^7 s^{-1}$, $\delta = 1cm$, $E_0 = 5V/m$ except (*): $E_0 = 0.1V/m$.

CONCLUSIONS

1. It is shown that when $\lambda > L$, only resonance particles ($\omega L/\pi v_x = n$) contribute to the heating and as result for large velocities, where the fraction of resonance particles is small, collisionless heating is suppressed.

2. A plateau in the distribution function in the region of first resonance can be observed.

3. At smaller collision frequency the nonlinear effects should be accounted for. If kicks are perpendicular to the discharge boundaries a considerable suppression of collisionless heating appears due to nonlinear effects. In this case collisionless heating is proportional to collision frequency ($D \sim \nu$).

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