Quasilinear theory of collisionless electron heating in radio frequency gas discharges

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On the basis of quasilinear kinetic theory, the electron heating of rf discharges is treated for characteristic scale lengths of the heating field much shorter than the electron mean free path. The analysis considers plasmas bounded by walls providing specular reflection. The expressions for the coefficients of electron diffusion in energy space are obtained and analyzed for inductively and capacitively coupled plasmas. Accounting for the oscillatory spatial structure of the penetrating electric field in the regime of anomalous skin effect leads to a decrease of the diffusion coefficient for high, and an increase for low, energy electrons. It is shown that the presence of a second boundary in the case of collisionless plasma slabs results in a similar effect. The formation of energy distribution functions in various regimes of collisionless heating is discussed. The diffusion coefficients are presented taking into account ambipolar electric fields. © *1997 American Institute of Physics.* [S1070-664X(97)02807-3]

I. INTRODUCTION

A modern trend in plasma technology aims at decreasing the gas pressures down to the milliTorr range. For these low pressures it is easier to maintain uniform plasmas with well-controlled parameters. Due to the large value of the mean free path (MFP) the main mechanism of electron heating turns out to be a collisionless one rather than the conventional Joule heating dominant for higher pressures.

The problem of collisionless plasma heating appeared about half a century ago in connection with research on thermonuclear fusion. In spite of the fact that the cross section of Coulomb scattering of charged particles decreases with increasing temperature, anomalously high plasma diffusion was observed in experiments with magnetic confinement. Plasma heating by direct current and external high frequency electric fields was also unexpectedly intensive. Later this effect was explained as originating from plasma instabilities resulting in the excitation of wave fields with small space scales. Interaction of charged particles with these fields leads to collisionless plasma heating. In order to explain the mechanisms of anomalous transport phenomena, the quasilinear and nonlinear theory of plasma wave turbulence was created (see, e.g., Refs. 1 and 2). The simplest examples for the influence of collective effects on the motion of charged particles are Landau damping of Langmuir waves and anomalous skin effects for transverse waves (see, e.g., Ref. 3). The resonance particles moving with a velocity close to the wave phase velocity intensively interact with wave fields and receive or lose energy. The result of particle⇔wave interaction depends on the shape of the particle distribution function in velocity space. For Maxwellian distributions the wave amplitude decreases, while the particle energy increases. For particle beams the situation can be the inverse one, resulting in the excitation of small scale waves. In this case the plasma exhibits high levels of turbulent noise and, as a consequence, the effective frequency of electron scattering becomes larger than the usual particle–particle collision frequency.

The quasilinear theory was initially developed for weak wave turbulence. But it can also be applied to stable plasmas, when small scale fields with a wide spectrum of wave numbers are excited by an external source. This situation is realized in the cases of Debye screening of longitudinal electric fields or cases of anomalous skin effects for transversal fields. A local spectrum of small scale Langmuir waves appears also in the region of plasma resonances for inhomogeneous plasmas interacting with electric fields capacitively coupled with plasmas (CCPs).

The problem of the plateau formation in the electron energy distribution function (EEDF) in the quasilinear theory was investigated first in Refs. 4 and 5. In Ref. 6 it is shown that collisionless heating due to local plasma oscillations results in plateau formation in the region of high electron energy. The effect of collisions on electron heating by local plasma oscillations are investigated in Ref. 7. Collisions smooth out the quasilinear plateau in the EEDFs[2] and result in an increased number of high energy electrons. A bi-Maxwellian EEDF with the temperature of fast electrons defined by the intensity of a strong rf electric field was obtained in Ref. 7. Problems of collisionless heating in gas discharges plasmas are addressed in Refs. 8–14. The model of oscillating plasma sheaths was proposed and explored to obtain the collisionless power dissipation of CCPs in Ref. 8. In Ref. 9, by using a kinetic equation for electrons of a plasma slab with oscillating rigid walls, the electron diffusion coefficient in energy space is derived under the assumption that the wall velocity is much slower than the particle velocity. Particle velocity kicks in longitudinal (normal to the walls) as well as in transverse directions are taken into account and comparisons with a one-particle approach are performed. In Ref. 10 diffusion coefficients obtained from a kinetic treatment of plasma heating in regimes of anomalous skin effect (of the

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same form as presented in Ref. 9) are used for calculating EEDFs, accounting for electron energy losses by elastic collisions. This treatment yields a power law energy dependence in the EEDFs. Also on the basis of a oscillating sheath approximation,[11] EEDFs with power law dependence are presented and used for self-consistent modeling of rf gas discharges. A quasilinear approach to collisionless electron heating in the regions of plasma resonance in low pressure discharges, sustained by electromagnetic surface waves, is developed in Refs. 12 and 13. In Ref. 14 it is demonstrated that—in view of experimental situations—self-consistent modeling has to account for nonlocal effects[15,16] as well as for collisionless heating. Some recent developments on stochastic electron heating in bounded rf plasmas in a model of oscillating rigid walls are presented in Refs. 17 and 18.

In view of the history of the successful use of the quasilinear approach as a well-established method, particularly in hot plasma physics, this paper aims at systematically applying this method also to the study of collisionless heating in various low temperature plasmas and to summarize existing results using a common basis, as well as to gain extended new results. The paper intends to demonstrate that the quasilinear theory is a powerful and easily applicable method for investigations of these type of problems.

This paper is organized as follows: In Sec. II, the derivation of the diffusion coefficient in energy space for uniform boundless plasmas, heated by localized high frequency field is presented. In Sec. III the collisionless heating in inductively coupled plasmas (ICPs) is analyzed for semi-infinite and slab geometry. Both cases can be reduced to the boundless problem with specular reflecting walls. The profile of the electric field under the condition of the anomalous skin effect is found in analytical form and the electric field is shown to oscillate spatially with zero average value. As a result the diffusion coefficient of fast electrons is suppressed, but in the low energy region it is enhanced in comparison to the case of an exponential profile. For ICPs in slab geometry (of width L) it is demonstrated that the second boundary plays a similar role. In this case the corresponding Fourier spectrum of the electric field is discrete ($k_n=\pi n/L$) in contrast to a continuous one for semi-infinite geometry. In Sec. IV the quasilinear theory is applied for capacitively coupled plasmas (CCPs). It is shown that the diffusion coefficient has a clear resonant structure with a main peak at $v=\omega L/\pi$. An analysis of the solutions of the kinetic equation for the EEDF in different regimes of plasma heating is presented. In Sec. V the influence of the ambipolar electric field on the efficiency of collisionless heating is discussed. The general expression for the diffusion coefficient in energy space is obtained for cases both of ICPs and CCPs. The presence of ambipolar electric fields can strongly modify the condition for particle⇔wave resonances. As a consequence the form of diffusion coefficient in energy space is changed, especially for low energy electrons. In a parabolic potential $\Phi=ax^2/2$ the bounce frequency of trapped electrons ($\sqrt{a/m}$) is the same for all energies. So the resonance condition ($\omega=\sqrt{a/m}$) can be fulfilled for one value of a only, and there is no collisionless heating for other values of a. In the general case of a nonparabolic potential the bounce frequency is energy dependent. Section VI contains the conclusion and outlook.

II. KINETIC THEORY OF COLLISIONLESS ELECTRON HEATING

A. Separation of the space and times scales in the kinetic description

The kinetic equation for the electron velocity distribution function $\mathscr{F}(\mathbf{r},\mathbf{v},t)$ is

$$\frac{\partial\mathscr{F}}{\partial t}+\mathbf{v}\frac{\partial\mathscr{F}}{\partial\mathbf{r}}+\frac{e}{m}\left(\mathscr{E}+\frac{1}{c}\mathbf{v}\times\mathscr{B}\right)\frac{\partial\mathscr{F}}{\partial v}=S(\mathscr{F}) \tag{1}$$

where e and m are the electron charge and mass, respectively, $S(F)$ is the collisional integral, $\mathscr{E}(\mathbf{r},t)$ is the electric and $\mathscr{B}(\mathbf{r},t)$ the magnetic field.

In low pressure discharges the space scale of the electric heating becomes small in comparison to the MFP λ. Under these conditions it is possible to simplify the kinetic description of the plasma by separation of the space scales. Thus the two fields can be separated into two parts

$$\mathscr{E}=\mathbf{E}+\tilde{\mathbf{E}},\quad \mathscr{B}=\mathbf{B}+\tilde{\mathbf{B}}, \tag{2}$$

where $\mathbf{E}$ and $\mathbf{B}$ have large spatial scales compared to λ, while $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{B}}$ have small spatial scales $1/k$ compared to λ (i.e., $k\lambda\gg 1$). An analogous separation can be performed for the distribution function

$$\mathscr{F}(\mathbf{r},\mathbf{v},t)=F(\mathbf{r},\mathbf{v},t)+\tilde{F}(\mathbf{r},\mathbf{v},t), \tag{3}$$

where F is averaged over a scale in the order of λ and $\tilde{F}$ describes the deviations of the distribution function on scales smaller than λ. Also the usual quasilinear approximation $\tilde{F}\ll F$ is used.[1,2] As a result the kinetic equation (1) can be separated into two parts

$$\frac{\partial\tilde{F}}{\partial t}+v\frac{\partial\tilde{F}}{\partial r}+\frac{e}{m}\left(\mathbf{E}+\frac{1}{c}\mathbf{v}\times\mathbf{B}\right)\frac{\partial\tilde{F}}{\partial v}=-\frac{e}{m}\left(\tilde{\mathbf{E}}+\frac{1}{c}\mathbf{v}\times\tilde{\mathbf{B}}\right)\frac{\partial F}{\partial v}, \tag{4}$$

$$\frac{\partial F}{\partial t}+\mathbf{v}\frac{\partial F}{\partial\mathbf{r}}+\frac{e}{m}\left(\mathbf{E}+\frac{1}{c}\mathbf{v}\times\mathbf{B}\right)\frac{\partial F}{\partial\mathbf{v}}=-\frac{e}{m}\overline{\left(\tilde{\mathbf{E}}+\frac{1}{c}\mathbf{v}\times\tilde{\mathbf{B}}\right)\frac{\partial\tilde{F}}{\partial v}}+S(F). \tag{5}$$

In (4) the collisionless term is omitted, since it is small compared to the second term on the left-hand side: $k\lambda\gg 1$. The bar in (5) denotes spatial averaging on scales smaller than λ.

B. The form of quasilinear integral for weakly inhomogeneous boundless plasmas

First the case of weakly inhomogeneous boundless plasmas, when $\mathbf{E},\mathbf{B}=0$ and $\tilde{\mathbf{E}}$, $\tilde{\mathbf{B}}$ are only rf fields, is considered

$$\tilde{\mathbf{E}}(\mathbf{r},t)=\tfrac{1}{2}\{\mathbf{E}(\mathbf{r})\exp(-i\omega t)+\mathbf{E}^*(\mathbf{r})\exp(i\omega t)\}, \tag{6}$$

where * denotes the complex conjugation.

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Under conditions, when the frequency of ineleastic collisions ν^* is small compared to the rf $\omega(\nu^* \ll \omega)$, the distribution function does not depend on time, i.e., $F=F(\mathbf{r},\mathbf{v})$, and (4), (5) can be simplified to

$$\frac{\partial \widetilde{F}}{\partial t}+\mathbf{v}\,\frac{\partial \widetilde{F}}{\partial \mathbf{r}}=-\frac{e}{m}\left(\widetilde{\mathbf{E}}+\frac{1}{c}\,\mathbf{v}\times\widetilde{\mathbf{B}}\right)\frac{\partial F}{\partial v}, \tag{7}$$

$$\mathbf{v}\,\frac{\partial F}{\partial \mathbf{r}}=S_{\mathrm{ql}}(F)+S(F). \tag{8}$$

The first term on the right-hand side of (8) is well-known in the theory of weak turbulent plasmas as the quasilinear collision integral, describing interaction of electrons with waves,

$$S_{\mathrm{ql}}(F)=-\frac{e}{m}\overline{\left\langle\left(\widetilde{\mathbf{E}}+\frac{1}{c}\,\mathbf{v}\times\widetilde{\mathbf{B}}\right)\frac{\partial \widetilde{F}}{\partial \mathbf{v}}\right\rangle}, \tag{9}$$

where the $\langle\cdots\rangle$ brackets indicate temporal averaging over the wave period $2\pi/\omega$.

Equation (7) can be solved by the Fourier method giving

$$\widetilde{F}(\mathbf{k})=-\frac{ie}{m}\cdot\frac{\mathbf{E}(\mathbf{k})+\frac{1}{c}\,\mathbf{v}\times\mathbf{B}(\mathbf{k})}{\omega-\mathbf{kv}}\cdot\frac{\partial F}{\partial \mathbf{v}}, \tag{10}$$

where $\widetilde{F}(\mathbf{k})$, $\mathbf{E}(\mathbf{k})$ and $\mathbf{B}(\mathbf{k})$ are the Fourier transformations of the functions $\widetilde{F}(\mathbf{r})$, $\mathbf{E}(\mathbf{r})$ and $\mathbf{B}(\mathbf{r})$, respectively.

By inserting (10) into (9) the quasilinear integral for the one-dimensional geometry can be written in the form

$$S_{\mathrm{ql}}=\delta(x-x_0)\,\frac{\partial}{\partial v_i}\,D_{ij}(\mathbf{v})\,\frac{\partial F}{\partial v_j}, \tag{11}$$

where the δ function appears as a result of space averaging and reflects the fact of localization of the heating electric field in the region $x\approx x_0$ with a characteristic scale $\widetilde{\lambda}$ much smaller than λ and

$$D_{ij}(\mathbf{v})=-\frac{e^2\pi}{m^2}\int dk\left[\mathbf{E}(k)+\frac{1}{c}\,\mathbf{v}\times\mathbf{B}(k)\right]_i \times\left[\mathbf{E}(k)+\frac{1}{c}\,\mathbf{v}\times\mathbf{B}(k)\right]_j \mathrm{Im}\left(\frac{1}{\omega-kv_x}\right) \tag{12}$$

is the tensor of the diffusion coefficient in velocity space. By excluding the magnetic field from (12) with help of the Maxwell equation

$$\widetilde{\mathbf{B}}(\mathbf{k})=\frac{c}{\omega}\,\mathbf{k}\times\widetilde{\mathbf{E}}(\mathbf{k}) \tag{13}$$

it can be shown that the tensor (12) has only one component D_{xx}. This stems from the fact that the electrons receive impulse from the wave impulse directed along $\mathbf{k}$ $(k,\,0,\,0)$ independent of the type of heating field (transverse or longitudinal).

$$D_{xx}(\mathbf{v})=\frac{\pi e^2}{m^2 v_x^2}\int dk|\mathbf{vE}(k)|^2\delta(\omega-kv_x). \tag{14}$$

The relation

$$\mathrm{Im}\left(\frac{1}{\omega-kv_x+io}\right)=-\pi\delta(\omega-kv_x), \tag{15}$$

corresponding to the phase resonance between travelling wave and moving electron, was used to obtain expression (14).

It should be stressed that D_{xx} is a nonlocal function of the heating electric field due to the integral representation

$$\mathbf{E}(k)=\int\frac{dx}{2\pi}\exp(-ikx)\widetilde{\mathbf{E}}(x). \tag{16}$$

This type of nonlocality is also discussed in Ref. 19

The simple physical meaning of expression (14) shall be illustrated by the example of plasma heating due to the localized field of a longitudinal wave $\widetilde{\mathbf{E}}$ $(E_x,0,0)$. In this case one obtains from (14)

$$D_{xx}=\frac{\pi^2e^2}{m^2}\,\frac{1}{|v_x|}\left|E\left(k=\frac{\omega}{v_x}\right)\right|^2. \tag{17}$$

Expression (17) can be presented in the form

$$D_{xx}=\frac{|v_x|}{4}\,(\Delta v)^2, \tag{18}$$

where Δv is the amplitude of the velocity kick after interaction with the wave field

$$\Delta v=\int_{-\infty}^{+\infty}e\widetilde{E}_x(k=v_xt)\exp(-i\omega t)dt =\frac{2\pi e}{v_x}\,E\left(k=\frac{\omega}{v_x}\right). \tag{19}$$

C. Kinetic equations for the EEDF

In gas discharge plasmas the anisotropy of the distribution function in velocity space is small due to the elastic collision frequency ν being large compared to the inelastic one (ν^*), i.e., $\nu\gg\nu^*$. In this case the conventional two-term approach is applicable and for the spherically symmetric part of the EEDF F_0 one has the following equation for one-dimensional geometry:

$$\frac{v^2}{3\nu(v)}\,\frac{\partial^2F_0}{\partial x^2}-S_{\mathrm{ql}}^0(F_0)-S^*(F_0)=0, \tag{20}$$

where

$$S_{\mathrm{ql}}^0(F_0)=\frac{1}{v^2}\,\frac{\partial}{\partial v}\left(v^2\mathscr{D}(v)\,\frac{\partial F_0}{\partial v}\right)\delta(x-x_0) \tag{21}$$

with

$$\mathscr{D}(v)=\frac{1}{v^2}\,\frac{\pi^2e^2}{m^2}\int_{-\infty}^{+\infty}dk\int_0^{\pi}\sin\vartheta\,d\vartheta \times\int_0^{2\pi}\frac{d\varphi}{4\pi}\,|\mathbf{vE}(k)|^2\delta(\omega-kv_x) \tag{22}$$

is the averaged (over the velocity angles) quasilinear collision integral S_{ql} (11) and $S^*(F_0)$ is the inelastic collision integral.

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To obtain the EEDF (20) should be integrated over a small vicinity of the point $x=x_0$. One gets as a result

$$-\frac{v^2}{3\nu(v)}\frac{\partial F_0}{\partial x}\bigg|_{x=x_0+0}=\frac{1}{2}\frac{1}{v^2}\frac{\partial}{\partial v}\left(v^2\mathscr{D}(v)\frac{\partial F_0}{\partial v}\bigg|_{x=x_0+0}\right). \tag{23}$$

In obtaining (23) symmetry of plasma heating and EEDF with respect to the space position $x=x_0$ is assumed. Relation (23) should be considered as boundary condition for the equation

$$\frac{v^2}{3\nu(v)}\frac{\partial^2 F_0}{\partial x^2}-S^*(F_0)=0 \tag{24}$$

describing the space evaluation of the EEDF outside the region of rf power input.

By representing the collisional integral $S^*(F_0)$ in the form

$$S^*(F_0)=-\nu^*(v)F_0(v,x), \tag{25}$$

one obtains from (24)

$$F_0(v,x)=F_0(v)\exp\left(-\frac{x}{\lambda_\epsilon}\right), \tag{26}$$

where $\lambda_\epsilon=v/\sqrt{3\nu(v)\nu^*(v)}$ is the energy relaxation length. Substituting (26) into the boundary condition (23) now results in the equation for the EEDF:

$$v\sqrt{\frac{\nu^*(v)}{3\nu(v)}}F_0(v)=\frac{1}{2}\frac{1}{v^2}\frac{\partial}{\partial v}\left(v^2\mathscr{D}(v)\frac{\partial F_0}{\partial v}\right). \tag{27}$$

Equation (27) has previously been analyzed for the case of localized longitudinal rf fields in the ionosphere[7] and SW produced plasmas.[12] The character of the solutions of (27) essentially depends on the diffusion coefficient $\mathscr{D}(v)$. For its determination the space spectra of the heating EM fields have to be known.

In contrast to the boundless plasmas considered above, gas discharge plasmas are always bounded. Thus in the next section, a procedure of reducing the bounded problem to a boundless one will be developed.

III. SPACE SPECTRA OF HEATING ELECTRIC FIELDS AND DIFFUSION COEFFICIENTS FOR INDUCTIVELY COUPLED PLASMAS

A. The semi-infinite plasma

In order to use the results of boundless plasmas, specular reflection of the electrons from the discharge wall $(x=0)$ is assumed

$$\tilde{F}(v_x,v_y,v_z,x<0)=\tilde{F}(-v_x,x_y,v_z,x>0). \tag{28}$$

Thus Eq. (7) can be continued into the complete x range $(-\infty<x<\infty)$ by the following *ansatz*:

$$\begin{aligned}\tilde{B}_y(x<0)&=-\tilde{B}_y(x>0),\\ \tilde{E}_y(x<0)&=\tilde{E}_y(x>0),\\ \tilde{E}_x(x<0)&=-\tilde{E}_x(x>0),\end{aligned} \tag{29}$$

and the problem reduces to the case of boundless plasmas.

Now the possible regimes of shielding the transverse EM field by the semispace plasma are addressed. In the case of weak collisions $(\omega>\nu)$ the field penetrates into the overdense plasma $[\omega\ll\omega_p=(4\pi e^2n/m)^{1/2}]$ over the depth $\delta_0=c/\omega_p$. For such a regime the frequency of the electric field should be sufficiently high ($\omega>v_T/\delta_0$, with v_T being the thermal electron velocity). A regime of collisionless plasma heating is possible, if the mean electron energy is not too low so that the skin length δ_0 does not exceed the MFP λ, i.e., if $v_T>\nu\cdot\delta_0$. In this case one should use the following form for the electric field, continued into the complete x range to calculate the diffusion coefficient for semi-infinite plasmas:

$$E_y(x)=E_0\exp\left(-\frac{|x|}{\delta_0}\right), \tag{30}$$

with the Fourier spectrum given by

$$E(k)=\frac{E_0\delta_0}{\pi(1+k^2\delta_0^2)}. \tag{31}$$

It should be noted that accounting for low collisionality results in weak spatial field oscillations and a small EM power flux into the plasma. Substitution of (31) into (22) yields the expression for the diffusion coefficient $\mathscr{D}(v)$

$$\mathscr{D}(v)=\omega\delta_0v_E^2g\left(\frac{v}{\omega\delta_0}\right), \tag{32}$$

where $v_E=eE_0/m\omega$ and

$$g(\tilde{v})=\frac{1}{4\tilde{v}^3}[(2+\tilde{v}^2)\ln(1+\tilde{v}^2)-2\tilde{v}^2] \tag{33}$$

which should be used in the kinetic equation (27) for obtaining the EEDF $F_0(v)$.

In the region of low electron energy $(v/\omega\delta_0\ll1\rightarrow\tilde{v}\ll1)$ one has

$$g(\tilde{v})\approx\frac{1}{24}\tilde{v}^3. \tag{34a}$$

In the limit of high electron energy $(v/\omega\delta\gg1\rightarrow\tilde{v}\gg1)$ one obtains

$$g(\tilde{v})\approx\frac{\ln(\tilde{v})}{\tilde{v}}. \tag{34b}$$

For lower frequencies $\omega^*<\omega<v_{T_e}/\delta_0$ the case of anomalous skin effect[3] is realized $(\omega^*=\nu^3c^2/\omega_p^2v_{T_e})$, independent of the relation between ω and ν. In this situation the depth of the electric field penetration becomes less than the MFP λ and electrons are heated in a collisionless manner.

If one uses a description of the electric field in the form

$$E_y(x)=E_0\exp\left(-\frac{|x|}{\lambda_{sk}}\right), \tag{35}$$

where $\lambda_{sk}=(c\mathscr{Z})/(i\omega)$ is chosen from the condition of a correct representation for the power deposition into the plasma with the surface impedance $\mathscr{Z}=2/3\delta(1+1/3\sqrt{3}i)$, the diffusion coefficient $\mathscr{D}(v)$ is

$$\mathscr{D}(v)=\omega\delta_0v_E^2g\left(\frac{v}{\omega\delta_0}\right). \tag{36}$$

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The function $g(\tilde{v})$ has the following approximation in the region of high electron energy ($\tilde{v}=v/\omega\delta_0 \gg 1$):

$$g(\tilde{v}) \approx \frac{4}{27}\frac{\ln(\tilde{v})}{\tilde{v}} \tag{37a}$$

and in that of low electron energy ($\tilde{v} \ll 1$)

$$g(\tilde{v}) = \frac{9}{128}\tilde{v}^3. \tag{37b}$$

The exact Fourier transformation of the electric field obtained from the kinetic approach[3] yields

$$E(k) = \frac{E_0}{\pi L_{\mathrm{sk}}\left(k^2 - i\dfrac{b}{|k|}\right)}, \tag{38}$$

where

$$b = \frac{4\pi\omega e^2}{c^2 m}\int v_y \frac{\partial F}{\partial v_y}\,\delta(v_x)\,d\mathbf{v}. \tag{39}$$

In the case of a Maxwellian EEDF one has:

$$b = \frac{2e^2 n\omega}{c^2}\sqrt{\frac{2\pi}{mT}}, \tag{40}$$

where n is the electron density and

$$L_{\mathrm{sk}} = \frac{2(\sqrt{3}+i)}{3^{3/2}b^{1/3}}. \tag{41}$$

The space dependence of the electric field penetrating into the plasma over the region $0<x<v_T/\omega$ has the form

$$E_y(x) = \frac{E_0}{\pi L_{\mathrm{sk}} b^{1/3}}\left\{\frac{\pi}{3}\left[(\sqrt{3}i+1)\exp\left(i\frac{\sqrt{3}}{2}-\frac{1}{2}\right)xb^{1/3} + \exp(-xb^{1/3})\right] - 2i\fint_0^\infty \frac{\xi\exp(-xb^{1/3}\xi)}{1-\xi^6}\,d\xi\right\}. \tag{42}$$

The last term on the right-hand side of (42) represents the contribution of the branch point $k=0$ and the exponential terms are a result of the poles in the integration when the inverse Fourier transformation of expression (38) is performed. A plot of $E_y(x)$ is presented in Fig. 1. The electric field profile has a oscillatory structure which reflects the existence of power flux into the plasma due to collisionless dissipation. It should be noted that at large distances from the plasma surface ($x>v_{T_e}/\omega$) the influence of electron thermal motion is not important and the electric field penetration becomes purely exponential with a scale length $\delta_0 = c/\omega_p$. But this effect of changing the regime of field penetration can be neglected because at this distance $E_y(x)$ has decreased to a small fraction of its value at the surface if the frequency ω is in the range $\omega<(v_T/c)\omega_p$ where the anomalous skin effect is applicable.

By substituting expression (38) into (22) one obtains

$$\mathscr{D}(v) = \omega\delta_0 v_E^2 g\left(\frac{v}{\omega\delta_0}\right), \tag{43}$$

where

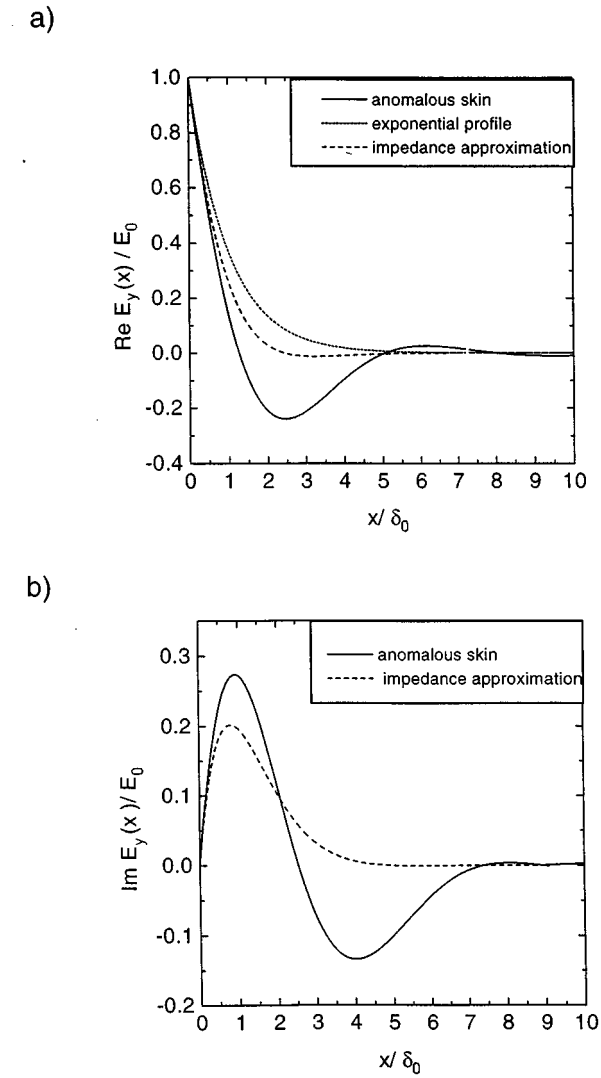

FIG. 1. Plot of the electric field at anomalous skin effect as a function of normalized coordinates x/δ_0. The solid curve corresponds to the exact profile of the electric field (42), the dashed one to the exponential profile (30) and the small dashes represent the impedance approximation (35): (a) Real, (b) imaginary part of the electric field intensity.

$$g(\tilde{v}) = \frac{9}{64\tilde{v}}\left\{\sqrt{3}\left[\arctan\left(\frac{2\tilde{v}^2-1}{\sqrt{3}}\right)+\frac{\pi}{6}\right] - \frac{1}{\tilde{v}^2}\ln(1+\tilde{v}^6) - \ln\left(\frac{1+\tilde{v}^2}{\sqrt{1-\tilde{v}^2+\tilde{v}^4}}\right)\right\}. \tag{44}$$

In the region of large values of the argument $\tilde{v} \gg 1$ the function $g(\tilde{v})$ decays and can be approximated by

$$g(\tilde{v}) \approx \frac{9\pi}{32\sqrt{3}}\frac{1}{\tilde{v}}. \tag{45a}$$

For small values of $\tilde{v} \ll 1$ (44) goes to zero by the law

$$g(\tilde{v}) \approx \frac{9}{128}\tilde{v}^3. \tag{45b}$$

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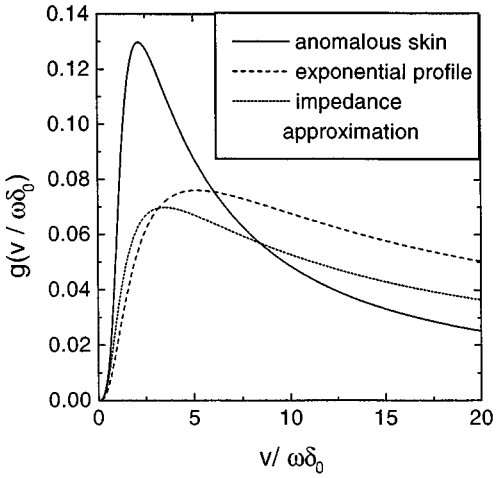

FIG. 2. The normalized diffusion coefficient $g=\mathscr{D}/(\delta_0/2)\omega v_E^2$ as a function of the dimensionless velocity $\tilde{v}=v/\omega\delta_0$ for various models of electric field profile. The solid curve corresponds to the exact profile of the electric field (42), the dashed one to the exponential profile (30) and the small dashes represent the impedance approximation (35).

$g(\tilde{v})$ has a maximum in the region $\tilde{v}\approx 1$ (see Fig. 2).

Diminishing of the diffusion coefficient (43) in regions of a high electron velocity $v>\omega\delta_0$ has a transparent physical meaning. To the electron with such a high velocity the electric field appears as a stationary one. So the increase of the electron energy by interacting with the localized electric field should be equal to zero and

$$\int_0^\infty E(x)dx=0. \tag{46}$$

Note that it follows from (46) that the Fourier component $E(k=0)$ should be equal to zero, which is in agreement with (35).

In the region of high electron energy the diffusion coefficients $\mathscr{D}(v)$ (32), (45) decays much less than in the case when the electric field is described exactly (38). This is a consequence of the fact that in the former two cases the long wave part of the Fourier spectrum of the electric field does not vanish and $E(k=0)\neq 0$. In Fig. 2 plot of $g(\tilde{v})$ for different cases of electric field representation is shown.

B. The slab geometry

Now the case of a uniform plasma slab (of width L) bounded by two plane walls with specular reflection of electrons is considered. For the case $L\gg\lambda$ collisionless heating of overdense plasmas occurs only in the nearest vicinity of the walls, while between then the usual Joule heating takes place. By using the method of continuation considered in the previous section the following solution for the EEDF is obtained:

$$F_0(v,x)=F_0(v)\cdot\frac{\cosh\left(\dfrac{x-(L/2)}{\lambda_\epsilon}\right)}{\cosh\left(\dfrac{L}{2\lambda_\epsilon}\right)}, \tag{47}$$

generalizing the result (26) for the case of plasma slabs with symmetrical heating on both walls. The kinetic equation for the EEDF $F_0(v)$ instead of (27) now has the form

$$v\sqrt{\frac{\nu^*}{3\nu}}\,F_0(v)=\frac{\coth(L/2\lambda_\epsilon)}{2v^2}\frac{\partial}{\partial v}\left(v^2\mathscr{D}(v)\frac{\partial F_0(v)}{\partial v}\right). \tag{48}$$

The diffusion coefficient $\mathscr{D}(v)$ in (48) is defined by (22). The limit $L\gg\lambda_\epsilon$ corresponds to the case of semi-infinite plasmas and (48) changes to (27). In the opposite limit of thin slabs ($L\ll 2\lambda_\epsilon$), on the right-hand side of (48) a large parameter $2\lambda_\epsilon/L\gg 1$ arises corresponding to more effective heating. In this case (48) takes the form

$$\nu^* F_0(v)=\frac{1}{v^2}\frac{\partial}{\partial v}\left(v^2\mathscr{D}(v)\frac{\partial F_0(v)}{\partial v}\right). \tag{49}$$

Equation (49) expresses the fact that losses of electron energy by inelastic collisions in the plasma volume balances the collisionless heating. It should be noted that in this limit the EEDF is spatially uniform and is equal to $F_0(\epsilon)$ in spite of the localization of energy input regions near the walls. This limit corresponds to the so-called ''nonlocal'' heating regime of plasmas with space dimensions less than the length of electron energy relaxation λ_ϵ.[15,16]

If the slab width does not exceed the MFP ($L<\lambda<\lambda_\epsilon$), the kinetic equation for the EEDF has a form identical to (49) but with the diffusion coefficient

$$\mathscr{D}(v)=\frac{1}{v^2}\frac{2\pi e^2}{m^2}\sum_{n=-\infty}^{\infty}\int_0^\pi \sin\vartheta\, d\vartheta\cdot\int_0^{2\pi}\frac{d\varphi}{4\pi}\cdot v_y|E_n|^2 \cdot\delta\left(\omega-\frac{\pi n}{L}v_x\right), \tag{50}$$

where

$$E_n=\frac{1}{L}\int_0^L E_y(x)\cos\left(\frac{\pi}{L}nx\right)dx \tag{51}$$

is the Fourier transformation of the periodically continued—in accordance with (29)—electric field $E_y(x)$ (with period L).

It can be shown that the spectrum E_n is always proportional to $1/n^2$, independent of the form of dependence of $E_y(x)$. This behavior is connected to the symmetric continuation of $E_y(x)$ and originates from the jump of the magnetic field component $B_y=-ic/\omega\partial E_y/\partial x$ and is easy to check by twice partially integrating (51). For such a spectrum the sum in (50) can be changed to an integral for low energy electrons ($v<\omega L/\pi$) and the diffusion coefficient takes the form of (22) with the function $E(k)$ corresponding to the Fourier spectrum of the electric field

$$E_y(x)=\begin{cases}E_y(|x|), & -L<x<L,\\ 0 & |x|>L.\end{cases} \tag{52}$$

In the region of high electron energy $v\geqslant\omega L/\pi$) the sum in (50) cannot be transformed into an integral. To demonstrate this the simple case of an exponential profile of the penetrating electric field with skin depth $\delta_0\leqslant L$ is considered. The spectrum E_n for this case is

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$$E_n = \frac{E_0 \dfrac{\delta_0}{L}}{1+\left(\dfrac{n}{n^*}\right)^2} \tag{53}$$

(with $n^* = L/\pi\delta_0$), and the diffusion coefficient (50)

$$\mathscr{D}(v) = \omega \frac{\delta_0}{2L} v_E^2 g\left(\frac{v}{\omega\delta_0}\right) \tag{54}$$

is obtained, where

$$g(\tilde{v}) = \frac{1}{2\tilde{v}} \sum_{n>\frac{n^*}{\tilde{v}}}^{+\infty} \frac{1-\left(\dfrac{n^*}{n\tilde{v}}\right)^2}{\left[1+\left(\dfrac{n}{n^*}\right)^2\right]^2 \cdot n}. \tag{55}$$

For small $\tilde{v}$ ($\tilde{v} < n^*$) the main contribution to the sum in (55) stems from large n so that the sum can be represented by an integral and the result coincides with the result (34) for the case of semi-infinite plasmas.

For large $\tilde{v}$ ($\tilde{v} > n^*$) the sum in (55) yields a smaller diffusion coefficient compared with that in the case of semi-infinite plasmas. This is connected to the effective cut-off at small $k \leqslant \pi/L$ for the slab geometry.

IV. CAPACITIVELY COUPLED PLASMAS

CCPs have a more complicated structure than ICPs due to the presence of large oscillating sheaths. The space scale length of the heating field and of the ambipolar field can be comparable in CCPs (in contrast to ICPs). For such a situation the approximation of the sheath potential as a moving rigid wall has repeatedly been used (see, e.g. Ref. 19). In this model the sheath velocity is $v_{\rm sh} = v_E \cos(\omega t + \varphi)$, where $v_E = eE_0/m\omega$ and E_0 is the amplitude of the sheath electric field strength. The electric field in the plasma bulk is neglected.

It is possible to transfer the problem into a noninertial system moving with the sheath velocity $v_{\rm sh}$. In this system there is a stationary specularly reflecting boundary and an alternating electric field connected to the inertial force

$$E_x(t) = -\frac{m}{e}\frac{du_{\rm sh}}{dt} = E_0 \sin(\omega t + \varphi). \tag{56}$$

Now the quasilinear approach is easily applied and with the help of the continuation method described above (28), (29) it yields the following expression for the diffusion coefficient:

$$\mathscr{D}(v) = \frac{1}{v^2}\frac{\pi e^2}{2m^2} \sum_{n=-\infty}^{\infty} \int_0^{\pi} \sin\vartheta \, d\vartheta \int_0^{2\pi} \frac{d\varphi}{4\pi} v_x^2 |E_n|^2 \cdot \delta\left(\omega - \frac{\pi n}{L} v_x\right), \tag{57}$$

where

$$E_n = \frac{1}{L}\int_0^L E_x(x) \sin\left(\frac{\pi n x}{L}\right) dx. \tag{58}$$

In the case considered here $E_x(x) = E_0 =$ constant and the diffusion coefficient is

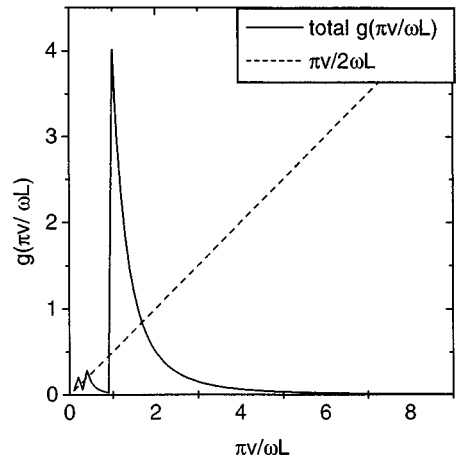

FIG. 3. The normalized diffusion coefficient $g = \mathscr{D}/(\delta_0/2)\omega v_E^2$ as a function of the dimensionless velocity $\tilde{v} = \pi v/\omega L$ for CCPs. The dashed curve represents $g = \tilde{v}/2$.

$$\mathscr{D}(v) = \frac{\omega}{2\pi} v_E^2 g\left(\frac{\pi v}{\omega L}\right) \tag{59}$$

with

$$g^*(\tilde{v}) = \frac{4}{\tilde{v}^3} \sum_{\substack{n=1,3,5,\ldots \\ n>1/\tilde{v}}} n^{-5}. \tag{60}$$

For small $\tilde{v}$ ($v < \omega L/\pi$) the sum in (60) can be replaced by an integral and $g(\tilde{v})$ is approximated by

$$g(\tilde{v}) = \frac{\tilde{v}}{2} \tag{61}$$

so that

$$\mathscr{D}(v) = \frac{v v_E^2}{4L}. \tag{62}$$

Thus in the low energy region the expression for the diffusion coefficient (62) is identical to the result obtained in Ref. 11 with an one-particle approach.

In the high velocity region ($v > \omega L/\pi$) the main contribution in the sum of (60) is due to the first resonance and one obtains

$$g(\tilde{v}) = \frac{4}{\tilde{v}^3} \tag{63}$$

and correspondingly the diffusion coefficient decays $\sim v^{-3}$

$$\mathscr{D}(v) = \frac{2\omega^4 L^3 v_E^2}{\pi^4}\frac{1}{v^3}. \tag{64}$$

From Fig. 3 the large deviation of the quasilinear diffusion coefficient (solid line) from the one obtained from the moving rigid wall approximation (dashed line) in the region of high electron velocity is evident. The difference in the nature of the heating field for CCPs and ICPs rests in the structure of the diffusion coefficient near resonances. In the case of CCPs every resonance gives a well-defined peak near $v_n = \pi\omega/Ln$ (see Fig. 3). With increasing v, an additional resonance v_n yields at first a considerable contribution, fol-

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lowed, however, by a rapid decrease $\sim v^{-3}$. As a consequence at $v > \pi\omega/3L$ (i.e., $n \geqslant 3$) the diffusion coefficient is practically determined by the lowest resonance ($n=1$) and its curve resembles a sequence of peaks (see Fig. 3). For ICPs the peak structure is smoothed out, since every new resonance results only in a small contribution $\sim(v-v_n)^2$ and the decrease of the resonance contributions is also slower than for CCPs ($\sim v^{-1}$). Therefore, only the peak of the first resonance is pronounced in the net diffusion coefficient.

V. THE DIFFUSION COEFFICIENTS FOR INHOMOGENEOUS PLASMAS ACCOUNTING FOR STATIONARY AMBIPOLAR ELECTRIC FIELDS

In inhomogeneous plasmas the stationary ambipolar electric field appears which tends to trap electrons in the plasma volume. As a consequence, low energy electrons cannot reach the periphery of the discharge where the rf field is large. Therefore, taking into account the ambipolar potential leads to a decrease in the efficiency of heating of low energy electrons. The diffusion coefficient for these electrons is small and the effective temperature of the low energy part of the EEDF can be considerably smaller than that of the higher energy part. For instance, in CCPs EEDFs resembling bi-Maxwellian ones have been observed, with a temperature of low energy electrons ($\epsilon < 2$ eV) ten times smaller than that of high energy electrons ($\epsilon > 2$ eV).[20]

In this section the case of a collisionless slab for CCPs ($\lambda \gg L$) will be considered. The kinetic equation (4) for the fast varying part of the distribution function $\widetilde{F}(x,v)$ with the ambipolar potential $\Phi(x)$ takes the form

$$-i\omega\widetilde{F}+v_x\frac{\partial\widetilde{F}}{\partial x}-\frac{e}{m}\frac{\partial\Phi}{\partial x}\frac{\partial\widetilde{F}}{\partial v_x}=-\frac{e}{m}\widetilde{E}_x\frac{\partial F}{\partial v_x}. \tag{65}$$

The solution of (65) is given by (see, e.g. Ref. 21)

$$\widetilde{F}(x,\epsilon_x)=e\frac{\partial F(\epsilon)}{\partial\epsilon}\exp(i\Omega)\times\left[-\frac{\int_{x_-}^{x_+}\sin(\Omega^*-\Omega(x'))E_0(x')dx'}{\sin\Omega^*}+\int_{x_-}^{x}\exp(-\Omega^*(x'))E_0(x')dx'\right], \tag{66}$$

where

$$\Omega(x,\epsilon_x)=\text{sign } v_x\int_{x_-(\epsilon_x)}^{x}\frac{\omega\, dx'}{\sqrt{\frac{2}{m}(\epsilon_x-e\Phi(x'))}}, \tag{67}$$

$\Omega^*=\Omega(x_+,\epsilon_x)$, $\epsilon_x=e\Phi(x)+1/2mv_x^2$ and the turning points $x_\pm(\epsilon_x)$ are defined by the relation

$$e\Phi(x_\pm)=\epsilon_x. \tag{68}$$

The kinetic equation for the main part of the distribution function $F=F(\epsilon)$—with $\epsilon=e\Phi+1/2mv^2$ being the total energy—reads

$$\frac{d}{d\epsilon}\overline{\mathscr{D}_\epsilon}\frac{dF}{d\epsilon}+\overline{S^*(F)}=0. \tag{69}$$

The bars indicate averaging over the slab width L. The diffusion coefficient averaged over the angles in velocity space is

$$\overline{\mathscr{D}_\epsilon}=\frac{\pi e^2L}{2}\int_0^\epsilon\frac{d\epsilon_x}{m}\sum_n E_n^2(\epsilon_x)\delta[\Omega^*(\epsilon_x)-\pi n] \tag{70}$$

with

$$E_n(\epsilon_x)=\frac{1}{L}\int_{x_-(\epsilon_x)}^{x_+(\epsilon_x)}E_x(x)\sin\Omega(x,\epsilon_x)dx. \tag{71}$$

Resonances occur, if the time of particle motion from one turning point to another (bounce time) is equal to $nT/2$, where $T=2\pi/\omega$. This is in accordance with the resonance condition $\Omega^*(\epsilon_x)=\pi n$, as follows from (70). If the value of $\Omega^*(\epsilon_x)$ does not exceed π, there are no resonances at all and no collisionless heating occurs. It should be noted that, if the ambipolar potential is approximated by rigid walls, $\Omega^*(\epsilon_x)$ is proportional to $\pi/\sqrt{\epsilon_x}$ and the resonance conditions can always be fulfilled. In the case of a parabolic potential, which is realized for low energy electrons trapped near the discharge center and is defined by the potential profile $\Phi(x)=ax^2/2$, the dimensionless bounce frequency $\Omega^*=\sqrt{a/m}$ and does not depend on the electron energy ϵ_x the resonance conditions can be fulfilled in this case for one value of a only. All plasma electrons are in or out of wave-particle resonance. Thus the ''degeneracy'' could be removed by taking into account the deviation of the ambipolar potential from the parabolic one.

For ICPs the expression for the diffusion coefficient has the form

$$\overline{\mathscr{D}_\epsilon}=\frac{\pi e^2L}{2}\int_0^\epsilon\frac{d\epsilon_x}{m}\frac{\epsilon-\epsilon_x}{2\epsilon_x}\sum_n E_n^2\delta(\Omega^*-\pi n), \tag{72}$$

where

$$E_n(\epsilon_x)=\int_{x_-(\epsilon_x)}^{x_+(\epsilon_x)}E_y(x)\cos\Omega(x)\sqrt{\frac{\epsilon_x}{\epsilon_x-e\Phi(x)}}\,dx. \tag{73}$$

VI. CONCLUSIONS AND OUTLOOK

The quasilinear approach to the description of collisionless electron heating by given high frequency EM fields has been developed. This approach has been shown to be an effective method of providing a common basis for a variety of conditions in gas discharges and obtaining generalized expressions. It was possible not only to summarize the effects obtained and to compare with results previously obtained by a different approach, but also to obtain new and extended results: The comparison to results from the one-particle approach yields agreement in the diffusion coefficient characterizing heating only for the case of low energy electrons. The expressions derived here are valid also for higher energies. For this region of electron energy a cutoff for the long wave part of the spectrum of heating electric field is required. Accounting for the second boundary of

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bounded plasma automatically provides such a cutoff. The generalized expressions for bounded plasmas given include the effect of ambipolar electric fields, which can be essential for the effectivity of collisionless heating.

It should be noted, if the EM in the plasma has also large space scale parts ($\mathbf{E}\neq 0$, $\mathbf{B}\neq 0$), the complete equation (5) has to be used instead of (8). The influence of static ambipolar fields has been considered in Sec. V. The large space scale and time varying part of the EM give rise to local diffusion coefficients which should be added to nonlocal ones. Modeling of microwave discharges with heating due to two space scales and time varying electric fields has been undertaken in Ref. 14. In general it can be underlined that knowledge of the energy diffusion coefficient gives a good basis for further developments, in particular for calculations of EEDFs needed for specific applications.

Finally it should be noted that the theory used in the present article is *not* a self-consistent quasilinear theory due to neglecting backward influences of the modification of the EEDF on the electric field profile. For the case of surface wave produced plasmas this problem has briefly been discussed in Ref. 13 and is included in numerical kinetic modeling of Ref. 14.

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[1] A. A. Galeev and R. Z. Sagdeev, *Nonlinear Plasma Theory*, Reviews of Plasma Physics (Consultants Bureau, New York, 1976), Vol. 7.
[2] A. A. Vedenov, *Theory of a Weakly Turbulent Plasma*, Reviews of Plasma Physics (Consultants Bureau, New York, 1967), Vol. 3.
[3] A. F. Alexandrov, L. S. Bogdankevich, and A. A. Rukhadze, *Principles of Plasma Electrodynamics*, Springer Series in Electrophysics (Springer, Berlin, 1984), Vol. 9.
[4] A. H. Vedenov, E. P. Velikov, and R. Sagdeev, Nucl. Fusion **1**, 82 (1961).
[5] W. E. Drummond and D. Pines, Nucl. Fusion Suppl. **3**, 1049 (1962).
[6] L. M. Kovrizhnykh and A. Sakharov, Sov. J. Plasma Phys. **5**, 470 (1979).
[7] V. V. Vas'kov, A. B. Gourevick, and Ja. S. Dimant, Sov. Phys. JETP **57**, 310 (1983).
[8] V. A. Godyak, Sov. Phys. Tech. Phys. **16**, 1073 (1972).
[9] Yu. R. Alanakyan, Sov. Phys. Tech. Phys. **24**, 611 (1979).
[10] Yu. R. Alanakyan, Sov. J. Plasma Phys. **5**, 504 (1979).
[11] C. G. Goedde, A. J. Lichtenberg, and M. A. Lieberman, J. Appl. Phys. **64**, 4375 (1988).
[12] Yu. M. Aliev, V. Yu. Bychenkov, A. V. Maximov, and H. Schlüter, Plasma Sources Sci. Technol. **1**, 126 (1992).
[13] Yu. M. Aliev, ''Some aspects of nonlinear theory of ionizing surface plasma waves,'' *Microwave Discharges: Fundamentals and Applications*, edited by C. M. Ferreira and M. Moisan NATO ASI Series B: Physics (Plenum, New York 1993), Vol. 302, pp. 105–115.
[14] Yu. M. Aliev, A. V. Maximov, U. Kortshagen, H. Schlüter, and A. Shivarova, Phys. Rev. E **51**, 6091 (1995).
[15] I. B. Bernstein and T. Holstein, Phys. Rev. **94**, 1475 (1954).
[16] L. D. Tsendin, Plasma Sources Sci. Technol. **4**, 200 (1995).
[17] U. Buddemeier, U. Kortshagen, and I. Pukropski, J. Appl. Phys. Lett. **67** (2), 191 (1995).
[18] I. D. Kaganovich, V. I. Kolobov, and L. D. Tsendin, J. Appl. Phys. Lett. **69**, 3818 (1996).
[19] V. Vahedi, M. A. Lieberman, G. Di Peso, T. D. Rognlien, and D. Hewett, J. Appl. Phys. **78**, 1446 (1995).
[20] V. A. Godyak and R. B. Piejak, Phys. Rev. Lett. **65**, 996 (1990).
[21] M. A. Lieberman, B. E. Meierovich, and L. P. Pitaevski, Sov. Phys. JETP **35**, 904 (1972).

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