Transverse conductivity in a braided magnetic field

I. Kaganovich[a] and V. Rozhansky
St. Petersburg State Technical University, St. Petersburg 195251 Russia

(Received 22 October 1997; accepted 26 May 1998)

The cross-field current is calculated as a function of the applied electric field, density, and temperature gradients in a given perturbed magnetic field. The current is calculated both for the cases of regular and stochastic magnetic fields. The analysis is focused on the impact of local ambipolarity. It is demonstrated that the local ambipolarity requirement results in the significant suppression of the cross-field radial current with respect to the current of test particles. © *1998 American Institute of Physics.* [S1070-664X(98)03511-3]

I. INTRODUCTION

The issue of transverse conductivity is very important, since it determines the radial potential profile in various magnetic field configurations addressed in the context of fusion research. Usually of main interest is the self-consistent radial ambipolar electric field, which corresponds to the condition of zero radial current averaged over the flux surface. The interest is further enhanced by the large body of evidence obtained on many tokamaks that the formation of transport barriers is consistent with the paradigm that increased shear in the $\mathbf{E}\times\mathbf{B}$ flow leads to a suppression of plasma turbulence, thereby improving the confinement. Magnetic field perturbations produce additional current perpendicular to flux surfaces, and thus contribute to the zero current condition. Hence the value of the ambipolar electric field is changed by magnetic field perturbations. Consequently, to calculate the ambipolar electric field in the presence of magnetic field perturbations, it is necessary to solve two independent problems. First one has to calculate the radial current in the absence of magnetic field perturbations as a function of electric field, density, and temperature gradients. The second step is to obtain a similar expression for the current associated with magnetic field fluctuations. Then the resulting electric field can be determined from the zero current condition. Therefore, the problem of determination of the cross-field conductivity, as well as the response of the radial current to density and temperature gradients, arises.

Many authors have addressed the first part of the problem. In Refs. 1, 2, and references therein, the emergence of the electric field profile has been described in terms of the momentum balance between the damping provided by neoclassical parallel viscosity and anomalous transport of the parallel momentum caused by turbulence. As a result, the perpendicular ion conductivity in a tokamak has been calculated. The theoretical predictions were verified by several biasing experiments [Continuous Current Tokamak at the University of California, Los Angeles (CCT),[3] Tuman-3,[4] and Toroidal Experiment for Technically Oriented Research (TEXTOR)], where the effective cross-field conductivity has been measured. In the absence of biasing, when the radial current has to be zero, the negative neoclassical electric field is predicted, provided toroidal rotation is damped by the anomalous viscosity (in the absence of an externally generated toroidal rotation, e.g., by an unbalanced neutral beam injection).

However, on some tokamaks [TM-4, Ref. 5, the Texas Experimental Tokamak (TEXT) Ref. 6, Tore Supra, Ref. 7] electric fields less negative than the neoclassical, or even positive fields, were observed for special regimes. A tentative explanation is connected with an intrinsic magnetic field stochasticity, which can create an additional current of electrons, thus reducing the negative neoclassical electric field. On TEXT and Tore Supra, an ergodic magnetic limiter created the extrinsic stochasticity. As a result, a strong reduction of the negative electric field was observed (Ref. 8). This effect could be interpreted in terms of a modified transverse conductivity associated with perturbations of the magnetic field.

The problem of calculation of the current perpendicular to the flux surfaces in the braided magnetic field also has some history. However, the particles that contribute to the current were considered to be test particles. For collisionless plasma, the issue of cross-field current in a stochastic magnetic field has been analyzed in Refs. 9–11. According to their approach, the current was assumed to be carried by test electrons. In contrast, in reality, the motion of electrons in a braided magnetic field is accompanied by the emerging of potential, density, and temperature perturbations, which arise to provide local quasineutrality. These perturbations strongly affect the motion of charged particles, generate the local perturbed currents, and, as a result, change significantly the physical pattern and the averaged cross-field current. Therefore, the restrictions imposed by the ambipolarity constraints, which were ignored previously, are very important. It should be noted that the calculation of the transverse conductivity and cross-field current should not be mixed with the problem of a kinetic dynamo (Refs. 12, 13), where the current along the flux surfaces is analyzed. This interesting and important problem is not considered here; still the ambipolar constraints should also play a significant role in this case.[14]

In the present paper the cross-field current and the cross-field conductivity of a plasma in a braided magnetic field (as

[a]Present address: University of Antwerp, Universiteitsplein 1, B-2610, Belgium; electronic mail: kaganovi@plasma4.chee.uh.edu

well as the response to density and temperature gradients) with a given level of magnetic field perturbations is addressed. In contrast to Refs. 9–11, the ambipolarity constraint is employed locally, accounting for the fact that the current is divergence-free throughout the plasma column. In other words, the parallel electron current, arising due to the electron flux along a braided magnetic field line, is short-circuited by the ion flux across the field. Since electrons are much more mobile than ions along a magnetic field, a significant perturbed electric field has to arise, thereby suppressing the electron current. Hence, the value of the transverse current obtained in Refs. 9 and 10 can be only treated as the upper estimate. It is demonstrated that the properly calculated radial current is significantly smaller.

Similar self-consistent constraints were also analyzed in Refs. 15–18. In contrast to our approach, a special type of intrinsic drift-Alfvén turbulence, which is responsible for fluctuations of the magnetic field, has been considered. In Refs. 15 and 16 the radial current has been studied. It was shown that for the case of two-dimensional (2-D) turbulence the self-consistency constraint results in complete inhibition of the radial current. In Ref. 18 it was shown that heat transport is also suppressed for a peaked spectrum shifted from local resonance modes, which is typical for the edge region of reversed-field pinch (RFP). Qualitatively, the conclusion is similar to ours: the transverse current is strongly reduced due to the ambipolarity constraint.

In the present paper we consider the more general three-dimensional (3-D) case with a finite k_z component of the magnetic field perturbation spectrum. This situation principally differs from the 2-D case. In particular, the cross-field current averaged over the flux surface is not zero in the 3-D case. On the other hand, we restricted ourselves to the more modest problem when the externally created magnetic field perturbations can be considered as the given ones.

In the case considered, the resulting net current averaged over the flux surface can flow only due to the finite local effective cross-field conductivity of ions. The ion viscosity and inertia terms associated with small-scale plasma perturbations cause this effective local cross-field conductivity. The considerable enhancement of the transverse current (with respect to the current driven by the averaged viscosity and inertia) occurs even in a regularly perturbed magnetic field. It becomes especially large in the case of a stochastic magnetic field, when the field lines come very closely to each other. In this case the average current depends logarithmically on the ratio of the parallel conductivity to perpendicular local conductivity. In both cases the resulting averaged cross-field current is shown to be proportional to the squared magnetic field perturbations. Both collisional and collisionless cases are considered. The results obtained can be employed for the determination of the self-consistent electric field in an ergodic divertor. This can be done by putting to zero the sum of the ion perpendicular current caused by neoclassical viscosity and the perpendicular current generated by an imposed stochastic magnetic field. In spite of the fact that the analysis quantified below corresponds to the calculation of cross-field conductivity in plasma with a given perturbed magnetic field, it also has a significant reference to tokamaks and RFPs with a large degree of intrinsic stochasticity.

The paper is organized as follows: in Sec. II the model is described with the assumptions made, in Sec. III we deal with the collisional case so that fluid equations are solved, and in Sec. IV the more complicated collisionless case is studied. In both cases, we start the analysis from the nonstochastic magnetic field, and then investigate the case of the stochastic magnetic field.

II. MODEL

The magnetic field perturbations, potential, plasma density n, electron, T_e, and ion, T_i, temperatures are assumed to be constant. We start with the case of the uniform density and temperatures to focus attention on the effects caused by the electric field perpendicular to the flux surface. The general formulas taking account of density and temperature gradients, will be derived at the end of each section. The spatial scales of magnetic perturbations are assumed to be significantly smaller than the small radius of plasma devices. Therefore, for simplicity, slab geometry is chosen with plasma parameters depending on the x coordinate, which is normal to the flux surfaces. The unperturbed magnetic field is given by $\mathbf{B}=B_z\mathbf{e}_z+B_y(x)\mathbf{e}_y=B[\mathbf{e}_z+\Theta(x)\mathbf{e}_y]$, $\Theta(x)=B_y/B_z\ll 1$. Here the y axis corresponds to the poloidal direction in a tokamak, the z axis—to the toroidal direction, and the x axis—to the radial direction. The unperturbed potential is denoted as $\varphi_0(x)$, and the corresponding electric field is $E_0=-d\varphi_0/dx$. Toroidal effects are neglected in this model. A strong anomalous viscosity, diffusion, and heat conductivity, originating due to the small-scale electrostatic turbulence, are included into the model. It is assumed that this electrostatic turbulence is independent of the magnetic one, and can be described by corresponding diffusion and viscosity coefficients.

In contrast to Refs. 15 and 16, where the perturbed magnetic field was generated by turbulent plasma currents, so that currents were strictly related to magnetic perturbations, Ampère's law, $\mathbf{j}=(c/4\pi)\boldsymbol{\nabla}\times\mathbf{B}'$, we consider the case when magnetic field perturbations $\mathbf{B}'$ are created by the external time-independent currents in the coils. Hence, in the plasma we have $\boldsymbol{\nabla}\times\mathbf{B}'=0$. The perturbed magnetic field is taken in the form $\mathbf{B}'=\Sigma_{\mathbf{k}}\mathbf{B}'_{0\mathbf{k}}\exp(i\mathbf{kr})$, where $|k_z|\ll|k_x|$, $|k_y|$. Then, from the Maxwellian equations it follows that $\mathbf{k}^2=0$. Therefore, $k_x=\pm i\sqrt{k_y^2+k_z^2}$, i.e., the perturbations of the magnetic field reach a maximum at the separatrix and decrease toward the core. This situation corresponds to the case of an ergodic divertor. The magnetic field perturbations are assumed to be sufficiently small, so that the following conditions are fulfilled: $|k_xb_x/k_\parallel|\cong|k_yb_y/k_\parallel|\ll 1$, $|k_x|L\gg 1$, where $b_{x,y}=B'_{x,y}/B$, $L=(d\ln\varphi_0/dx)^{-1}$, $k_\parallel$ is the parallel wave vector, $k_\parallel(x)=[k_yB_y(x)+k_zB_z]/B$. Therefore, we can apply the quasilinear theory based on the small parameter $|k_xb_x/k_\parallel|\cong|k_yb_y/k_\parallel|\ll 1$. The latter condition means that the displacement of the point at magnetic field line in y or x directions is smaller than the transverse spatial scale of the perturbations provided the point pass the distance $k_\parallel^{-1}$ along the magnetic field.

Finally, we see that the following basic assumptions are made, which are typical for the case of an ergodic divertor

(i) Stationary small-scale perturbations of magnetic field of low amplitude are considered.

(ii) A magnetic field is created externally by currents in the coils.

(iii) Anomalous transport caused by intrinsic electrostatic turbulence exists.

There are several important parameters in the problem considered. One can introduce three scales along the magnetic field. In the absence of resonance (when $k_\parallel$ remains finite everywhere) the longitudinal scale is simply $k_\parallel^{-1}$. If, for a given wave number, there exists the resonance surface where $k_\parallel \to 0$, then the contribution to the plasma perturbations is large near such a flux surface. In this case we introduce a typical longitudinal scale corresponding to $k_\parallel^{-1}$: the value $k_{\parallel\sigma}=\sqrt{|\sigma_{\perp \mathbf{k}} k_\perp^2/\sigma_\parallel|}$, where the parallel, $\sigma_\parallel$, and the perpendicular, $\sigma_\perp$, conductivities are denoted below. Similarly, one can introduce $k_{\parallel D}$ and $k_{\parallel T}$-the inverse scales associated with diffusion and heat conductivity (see below). When the magnetic field becomes stochastic, the typical scale along $\mathbf{B}$ is given by the Kolmogorov length of magnetic field line divergence L_k. The corresponding parallel wave vector is defined as $k_{\parallel k} \sim L_k^{-1}$. In our analysis we have to choose the largest of all the parallel wave vectors or the smallest of the corresponding typical longitudinal scales. The result depends also on the ratio of the mean-free path to the smallest of these longitudinal scales.

In the present paper we shall consider both collisional and collisionless cases, stochastic and nonstochastic magnetic fields.

III. TRANSVERSE CONDUCTIVITY IN THE COLLISIONAL CASE $\lambda_{\mathrm{mfp}} k_\parallel \ll 1$

We start from the most transparent collisional case when the mean-free path $\lambda_{\mathrm{mfp}} \ll k_\parallel^{-1}$. The collisional case, being rarely found in practical applications, is a good example for the demonstration of the considered effects. To find the averaged cross-field current $\langle j_x \rangle$, we have to find at first the perturbed parallel current $j_\parallel^1$. Under validity conditions of quasilinear theory, the averaged cross-field current is then given by the averaged projection of parallel current on the x axis, $j_x = \langle j_\parallel^1 b_x \rangle$. We follow here the method proposed in Ref. 19 for the calculation of electron heat conductivity.

A. Cross-field current driven by transverse electric field

To find the perturbations of parallel current, we have to solve the fluid equations of Ref. 20:

$$\nabla \cdot \mathbf{j} = 0, \tag{1}$$

$$\nabla \cdot \mathbf{\Gamma}_i = 0, \tag{2}$$

$$m_i n (u \nabla) u = -\nabla p + \frac{1}{c}[\mathbf{j} \times B] - \nabla \cdot \boldsymbol{\pi}, \tag{3}$$

$$j_\parallel = \sigma_\parallel \mathbf{b}\left(-\nabla \varphi + \frac{T_e}{e}\nabla \ln n + \frac{1.71}{e}\nabla T_e\right), \tag{4}$$

$$\frac{3}{2} n(\mathbf{u}_e \cdot \nabla) T_e + n T_e (\nabla \cdot \mathbf{u}_e) = -\nabla \cdot \mathbf{q}_e, \tag{5}$$

$$\mathbf{q}_e = -\chi_{e\parallel} \nabla_\parallel T_e - \chi_{e\perp} \nabla_\perp T_e - \frac{0.71}{e} T_e \mathbf{j}_\parallel, \tag{6}$$

$$\frac{3}{2} n(\mathbf{u} \cdot \nabla) T_i + n T_i (\nabla \cdot \mathbf{u}) = -\nabla \cdot \mathbf{q}_i, \tag{7}$$

$$\mathbf{q}_i = -\chi_\parallel \nabla_\parallel T_i - \chi_{i\perp} \nabla_\perp T_i, \tag{8}$$

where $\mathbf{j}$ is the current, $\mathbf{u}$ is the plasma velocity, $\mathbf{u}_e$ is the electron velocity, $\sigma_\parallel$ is Spitzer–Harm parallel conductivity, $\mathbf{b} = \mathbf{B}/|\mathbf{B}|$, $\boldsymbol{\pi}$ is the ion viscosity tensor, and $\chi_{e,i}$ is the heat conductivity. Perpendicular anomalous heat ion and electron conductivities can be estimated as $\chi_{e,i\perp} \approx nD$. We have neglected the divergence-free heat fluxes proportional to $\mathbf{B} \times \nabla Te,i$, since they do not contribute to the transport equations.

The ion viscosity can be caused by the ion–ion collisions (see, for example, Ref. 20) and/or can be anomalous, i.e., caused by an electrostatic turbulence, which can exist in the plasma. For the time being, we consider only the case when the zeroth-order temperatures and density perturbations are absent. The external magnetic field perturbations $\mathbf{b}$ cause the perturbation of electric potential φ^1, ion velocity u^1, and current $\mathbf{j}$. Note that in the test particle approach, (Refs. 9 and 10) the potential perturbations are neglected so that the calculated current is not divergence-free, as it should be according to Eq. (1) (ambipolarity constraint).

1. Nonresonant modes $k_\parallel(x) \neq 0$

We start the analysis from the simplest case of nonresonant modes. Such a magnetic field perturbation can be created by slightly rippled coils. In the absence of the transverse current, from the continuity, Eq. (1), $(ik_\parallel j_\parallel = 0)$, it immediately follows that the parallel net current should be zero ($\mathbf{j}_\parallel = -\sigma_\parallel [b_x (d\varphi_0/dx) + ik_\parallel \varphi^1] = 0$). In other words, in contrast to the test particle case, the potential perturbations emerge and cancel the part of the parallel current associated with radial electric field projection on the magnetic field line. The solution of the linearized equations (1) and (4), corresponds to the constant potential along the perturbed magnetic field line. Therefore, the potential perturbations of the first order are

$$\varphi_0^1 = -\frac{b_x}{ik_\parallel}\frac{d\varphi_0}{dx}. \tag{9}$$

In Refs. 9 and 10 these potential perturbations were neglected. In Fig. 1 the sketch of two possibilities with and without potential perturbations, Eq. (9), is shown.

The nonzero parallel current arises if the transverse current is taken into account. The latter can be caused by collisions with neutrals or driven by ion viscosity and inertia. The electron finite Larmor radius effects considered in Ref. 21 (for the case of test electrons) are negligible with respect to the ion perpendicular currents. The corresponding mechanism of the cross-field conductivity consists of the following. The perturbed electric field, Eq. (9), causes $\mathbf{E} \times \mathbf{B}$ drifts along the equipotentials. Moreover, the perturbed fields produce

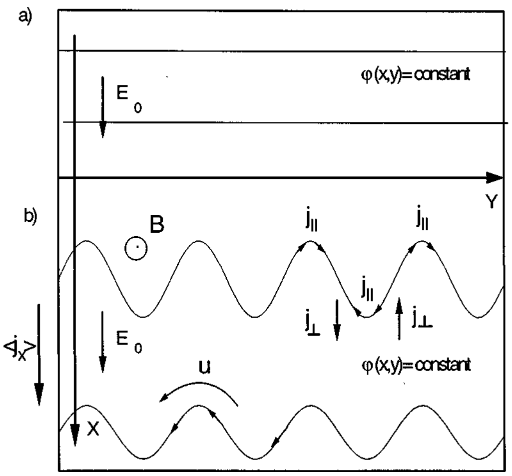

FIG. 1. Scheme of equipotentials (a) without an account of the local ambipolarity constraint; (b) with an account of potential perturbation, Eq. (7). Cross-field and parallel currents are also shown.

strongly inhomogeneous drifts. For the potential perturbations $\varphi_{\mathbf{k}}=\varphi_{0\mathbf{k}}\exp(i\mathbf{kr})$, the drift velocity is $\mathbf{u}_\perp=-(ci/B)\varphi_{\mathbf{k}}\mathbf{k}\times\mathbf{e}_z$. These small-scale drifts result in the ion viscosity and inertia forces. From the momentum balance equation (3), it follows that these forces generate the perpendicular ion current,

$$\mathbf{j}_\perp=-c[(m_i n(\mathbf{u}_\perp\nabla)\mathbf{u}+\nabla\cdot\boldsymbol{\pi})\times\mathbf{B}]/B^2 \tag{10}$$

[see Fig. 1(b)]. The parallel current, which arises to close the circuit, can be calculated from the continuity equation (1): $ik_\parallel j_\parallel+\nabla_\perp\cdot j_\perp=0$. Accounting for perpendicular currents, the potential perturbations differ from Eq. (9). They can be found from Eq. (1). Since we are interested mainly in divergence of the perpendicular current, it is convenient to introduce the effective cross-field conductivity as

$$\sigma_{\perp k}=-\nabla\cdot\mathbf{j}_{\perp k}/\Delta_\perp\varphi_k. \tag{11}$$

In our case of externally created magnetic field perturbations $(k_x=\pm i\sqrt{k_y^2+k_z^2};\ |k_z|\ll|k_x|,\ |k_y|)$ $\Delta_\perp\varphi_{\mathbf{k}}=-k_\perp^2\varphi_{\mathbf{k}}\equiv-(k_x^2+k_y^2)\varphi_{\mathbf{k}}=k_z^2\varphi_{\mathbf{k}}$. Substituting the expression for the drift velocity as a function of potential into Eq. (10) for current, we have, for the effective cross-field conductivity,

$$\begin{aligned}\sigma_{\perp\mathbf{k}}&=\sigma_{\perp\mathbf{k},i}+\sigma_{\perp\mathbf{k},v}\\&=im_ic^2(v_0k_y)n/B^2+\eta_1(\mathbf{k}_\perp^2+4k_z^2)c^2/B^2.\end{aligned} \tag{12}$$

The first term here is the contribution from the inertia force, where $v_0=cE_0/B$ is the velocity of unperturbed $\mathbf{E}\times\mathbf{B}$ drift along the flux surface (in the y direction). Note that the inertia term is the result of a combination of both unperturbed drift (plasma rotation) and small-scale drifts. The contribution to the effective perpendicular conductivity from the viscosity is calculated by means of substituting the expression for the viscosity tensor given in Ref. 22, where η_1 is the ion transverse viscosity coefficient. The contribution of other components to Eq. (1) results in the divergence-free terms. In the real turbulent plasma, the contribution from the anomalous viscosity is much larger than from the classical viscosity. It can be taken into account by replacement of the classical viscosity coefficient η_1 with the coefficient of anomalous viscosity, which should be of the order of m_inD, where D is the anomalous diffusion coefficient. However, for typical plasma parameters the viscosity contribution often remains small with respect to the inertia term, even taking into account the anomalous effects. In other words, the inequality $|\sigma_{\perp\mathbf{k},v}/\sigma_{\perp\mathbf{k},i}|\sim Dk_z^2/|v_0k_y|\ll1$ is usually satisfied.

It is worthwhile emphasizing that the finite cross-field conductivity exists only for the magnetic field perturbations with finite k_z. In the 2-D case, when $k_z=0$, the divergence of the perpendicular currents, caused by inertia and viscosity, vanish, the magnetic field line remains equipotential, and no parallel currents arise. The absence of the net radial current for two-dimensional magnetic turbulence, due to the self-consistence character of current and magnetic fluctuations coupled by Ampère's law, was pointed out in Refs. 15 and 16. However, if the 3-D structure of magnetic field perturbations is accounted for, the net radial current is not zero and, as we shall see, can be large.

In the presence of the transverse ion currents, the parallel current arises. Substituting the expression for parallel current: $j_\parallel=-\sigma_\parallel[b_x(d\varphi_0/dx)+ik_\parallel\varphi^1]=0$ into the condition $ik_\parallel j_\parallel+\nabla_\perp\cdot j_\perp=0$ and using Eq. (11), we find the solution of the linearized equations (1) for the potential,

$$\varphi_{\mathbf{k}}^1=\left(1-\frac{\sigma_{\perp\mathbf{k}}k_\perp^2}{\sigma_{\perp\mathbf{k}}k_\perp^2+\sigma_\parallel k_\parallel^2}\right)\frac{i}{k_\parallel}\frac{d\varphi_0}{dx}b_{x\mathbf{k}}. \tag{13}$$

Accounting for the transverse conductivity, the magnetic field lines are not equipotential, due to the second term on the right-hand side (RHS) of Eq. (13), which depends on $\sigma_{\perp\mathbf{k}}$, and $\sigma_{\perp\mathbf{k}}k_\perp^2\ll\sigma_\parallel k_\parallel^2$. In spite of the fact that the deviation from the equipotential case, Eq. (9), is small, this deviation determines the parallel current,

$$j_\parallel^1=\sum_{\mathbf{k}}j_{\parallel,\mathbf{k}}^1=\sum_{\mathbf{k}}\frac{\sigma_\parallel\sigma_{\perp\mathbf{k}}k_\perp^2}{\sigma_{\perp\mathbf{k}}k_\perp^2+\sigma_\parallel k_\parallel^2}b_{x\mathbf{k}}E_0. \tag{14}$$

We see that the averaged perpendicular current appears in the second order on magnetic fluctuations $\sigma_{\perp\mathbf{k}}k_\perp^2\ll\sigma_\parallel k_\parallel^2$. Substituting the expression for $j_\parallel$ into the expression for the net current $j_x=\langle j_\parallel^1b_x\rangle=1/2\ \mathrm{Re}\ \Sigma_{\mathbf{k}}j_\parallel^1b_{x\mathbf{k}}$, we find

$$j_x=\frac{\sigma_\parallel E_0}{2}\times\sum_{\mathbf{k}}\frac{|\sigma_{\perp\mathbf{k},i}k_\perp^2|^2+\sigma_{\perp\mathbf{k},v}k_\perp^2(\sigma_\parallel k_\parallel^2+\sigma_{\perp\mathbf{k},v}k_\perp^2)}{|\sigma_{\perp\mathbf{k},i}k_\perp^2|^2+(\sigma_\parallel k_\parallel^2+\sigma_{\perp\mathbf{k},v}k_\perp^2)^2}|b_{x\mathbf{k}}|^2. \tag{15}$$

In the absence of the resonance surfaces, where $k_\parallel(x)\to0$, estimating $|k_\parallel(x)|\sim|k_y|\Theta$, accounting for $\sigma_{\perp\mathbf{k},v}k_\perp^2\ll|\sigma_{\perp\mathbf{k},i}k_\perp^2|\ll\sigma_\parallel k_\parallel^2$, we have

$$\begin{aligned}j_x&=\frac{E_0}{2}\sum_{\mathbf{k}}\sigma_{\perp\mathbf{k},v}\left(\frac{k_\perp|b_{x\mathbf{k}}|}{k_\parallel}\right)^2\\&=E_0\sum_{\mathbf{k}}\frac{3\eta_1k_z^2c^2}{2B^2}\left(\frac{k_\perp|b_{x\mathbf{k}}|}{k_\parallel}\right)^2.\end{aligned} \tag{16}$$

The last factor in Eq. (16) has to be smaller than unity, since the quasilinear approach is valid if the magnetic field is rather small, so that $|k_\perp b_{x\mathbf{k}}/k_\parallel| \ll 1$. Hence, the resulting transverse conductivity $\Xi_{\perp b} \equiv j_x/E_0$ is determined by the perpendicular viscosity (and does not depend on ion inertia). It can reach the value of $\sigma_{\perp \mathbf{k}v}$ by the order of magnitude.

Let us estimate the perpendicular conductivity calculated above. To do this we compare the result with the cross-field conductivity $\Xi_\perp$ in unperturbed magnetic field, which is caused by the perpendicular viscosity, given in Ref. 22,

$$\Xi_\perp = m_i D n c^2/B^2 L_v^2, \quad L_v^{-2} = \frac{\partial^2 \varphi_0}{\varphi_0 \partial x^2}. \tag{17}$$

In both cases, we chose $\eta_1 = n m_i D$ for anomalous viscosity coefficient. Since the ratio $\sigma_{\perp \mathbf{k}v}/\Xi_\perp \sim k_\perp^2 L_v^2 \gg 1$, the conductivity associated with magnetic field perturbations dominates. In a tokamak the perpendicular conductivity is caused by the parallel neoclassical viscosity and by the radial transport of the toroidal momentum, as is shown in Refs. 1 and 2. It can be one to two orders of magnitude larger than given by Eq. (17). Nevertheless, due to the large factor $k_\perp^2 L_v^2$, the transverse conductivity, Eq. (16), may be significant, even in the devices for controlled fusion (in the edge region).

It is worthwhile to note that the ambipolar diffusion (with diffusion coefficient D) plays an important role in the process. In the absence of averaged density and temperature gradients, neglecting diffusion, we have

$$\frac{\partial n_e}{\partial t} - i k_\parallel j_\parallel = \frac{\partial n_i}{\partial t} + \nabla_\perp \cdot \mathrm{j}_\perp = 0.$$

If the diffusion were not taken into account, these density perturbations would grow until the particle fluxes driven by the perturbed gradients compensate the initial ones. Therefore, the resulting transverse current would be zero. In the presence of large anomalous ambipolar diffusion $e^2 D n_0 (T_e + T_i) \gg |\sigma_\perp|$ (which is typical for real experiments), the density perturbations are washed out. Since the anomalous diffusion fluxes are automatically ambipolar they do not influence the current directly. We shall discuss this problem in more detail in Sec. III B.

2. Resonant modes $k_\parallel(x) \to 0$

In the presence of the resonance surfaces the impact of the magnetic field perturbations is most pronounced near these surfaces. The simplified equation (16) has a singularity at the resonance surfaces where $k_\parallel \to 0$, so that the sum over $k_\perp$ becomes infinite, since the corresponding perpendicular current $j_x \sim k_\parallel^{-4}$. To calculate the contribution from resonances, we have to explore Eq. (15). This yields the typical values of $k_\parallel \sim k_{\parallel\sigma} - \sqrt{|\sigma_{\perp \mathbf{k} i} k_\perp^2|/\sigma_\parallel}$ (the resonance width), which give the main contribution to the sum. Note that now the effect is determined by the local conductivity driven by inertia. When $k_{\parallel\sigma} > \delta k_y \Theta$ (where δk_y is the difference between two neighboring k_y), many modes contribute to the resonance and the sum in Eq. (15) can be replaced by the integral. This criterion is similar to overlapping criterion (Ref. 23) for a stochastic magnetic field. However, the important difference consists in the fact that resonance width here is determined by perpendicular conductivity and not by nonlinear effects, as for the stochastic magnetic field case. Replacing the sum in Eq. (15) by the integral, we find

$$j_x = \frac{\sigma_\parallel E_0 D_B}{L_\sigma}, \tag{18}$$

where $L_\sigma = k_{\parallel\sigma}^{-1}$, $D_B = \int \{ |b_{x\mathbf{k}}|^2 / [1 + (k_\parallel / k_{\parallel\sigma})^4] \} \times (dk_y / 2 k_{\parallel\sigma} \delta k_y)$. The last integral can be easily calculated, provided the amplitude of magnetic fluctuations $b_{x\mathbf{k}}$ changes insignificantly at the resonance width. If $k_{\parallel\sigma} > \Delta k_y \Theta$, where Δk_y is the width of the spectrum $b_{x\mathbf{k}}$, we have $D_B = \pi\sqrt{2} |b_{x\mathbf{k}}|^2 / (4 \Theta\, \delta k_y)$. Here $b_{x\mathbf{k}}$ correspond to resonance modes $k_\parallel(x) = 0$. The resulting current depends on the ion local cross-field conductivity $\sigma_{\perp\mathbf{k}}$ only as a square root. From Eq. (18) it follows that the conductivity can be significantly enhanced, even for a *nonstochastic* magnetic field. The enhanced conductivity, Eq. (18), $\Xi_{\perp b} \equiv j_x/E_0$, can be rewritten in the form

$$\Xi_{\perp b} \sim |\sigma_{\perp\mathbf{k}}| \frac{k_{\parallel\sigma}}{\Theta\, \delta k_y} \left(\frac{k_\perp |b_{x\mathbf{k}}|}{k_{\parallel\sigma}} \right)^2. \tag{19}$$

In the case $k_{\parallel\sigma} > \delta k_y \Theta$, many modes contribute to the sum, Eq. (15), and the current, Eq. (18), becomes a continuous function of x, and peaks of the current j_x near the resonance surfaces are smoothed out. It means that the continuous enhanced conductivity can exist even for a nonstochastic magnetic field (when there is no overlapping of magnetic islands). However, the validity of Eq. (19) is restricted by the severe condition $k_\perp |b_{x\mathbf{k}}| / k_{\parallel\sigma} < 1$. Moreover, the situation $k_{\parallel\sigma} < \delta k_y \Theta$ is more typical. Let us consider a numerical example. For the typical tokamak parameters (Tore Supra, Ref. 7, major radius $R \sim 238$ cm, minor radius $a \sim 80$ cm, $B = 3$ T) we take at the edge region $T_e \sim 20$ eV, $n \sim 10^{13}$ cm^{-3}, $k_y = m/a \sim 0.6$ cm^{-1} (m is the typical poloidal number for magnetic field perturbations). The characteristic widths of resonances in Eq. (14) and (15) are very small: $k_{\parallel\sigma} = \sqrt{|\sigma_{\perp\mathbf{k}} k_\perp^2 / \sigma_\parallel|} \sim 2 \times 10^{-6}$ cm$^{-1} \ll \delta k_y \Theta \sim 1.310^{-3}$ cm^{-1}, hence the contribution to the cross-field conductivity, Eq. (19), is less significant than conductivity, Eq. (16).

3. Stochastic magnetic field

When the magnetic island width exceeds the distance between the neighboring islands, the magnetic field becomes stochastic. The stochasticity of magnetic field lines in this case are characterized by the Kolmogorov's length of exponential divergence $L_\mathrm{K} = (\delta k_y \Theta / k_y^2 \Theta^2 b^2)^{1/3}$ of two neighboring magnetic lines from being initially at distance $\delta_0 : \delta = \delta_0 \exp(-z/L_\mathrm{K})$. The overlapping criterion of magnetic islands (Chirikov criterion) is equivalent to the condition that Kolmogorov's length is smaller than the parallel correlation length $(\Theta\, \delta k_y)^{-1}$, as is shown in Ref. 23. In the stochastic magnetic field, the local transverse transport is strongly amplified because the magnetic field lines approach each other very closely for distances larger than L_K along magnetic field. This effect strongly effects the final expression for the average current perpendicular to the flux surface.

Let us consider the flux tube of initial size δ_0. Due to stochastic instability, the average distance between neighbor-

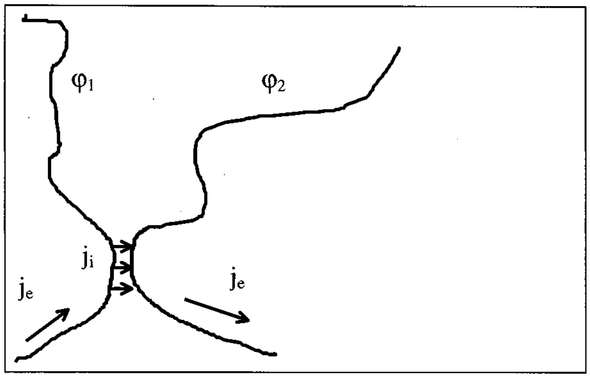

FIG. 2. The scheme of equipotentials for stochastic magnetic lines.

ing lines is increasing exponentially. But the cross section of a flux tube conserves. Therefore, since the distance between the neighboring magnetic field lines exponentially diverges in one direction, in the other direction it decreases exponentially: $\delta=\delta_0\exp(-z/L_{\mathrm{K}})$; see Ref. 24. So, if the distance between the two magnetic field lines at $z=0$ is δ_0, it becomes equal to $\delta_0\exp(-z/L_{\mathrm{K}})$ for $z>L_{\mathrm{K}}$. Due to rather small cross-field conductivity, the equipotentials almost coincide with the magnetic field lines, Eq. (13). Hence, an initial potential drop $\delta_0 E_0$ between the two magnetic field lines becomes applied to the very small distance δ at $z>L_{\mathrm{K}}$. As a result, a large electric field of the order of $E_0\exp(z/L_{\mathrm{K}})$ arises. At such places, where magnetic field lines approach each other closely, the parallel current is short-circuited by the ion transverse current, in spite of the low value of the cross-field conductivity; see Fig. 2.

This effect is less important in the regime of high collisionality when $L_{\mathrm{K}}>L_\sigma$, since for these lengths of the order L_σ that give the main contribution to the sum, Eq. (15), the exponential divergence of neighboring lines is small. In contrast to the previously analyzed situations, in the more frequently met case when $L_{\mathrm{K}}<L_\sigma$, the quasilinear theory is not valid and the fractal structure of the magnetic field has to be taken into account. The estimate of conductivity in this case can be obtained by the method proposed in Ref. 19 by applying the variation principal. From the other side, we can obtain the same results by taking into account the enhancement of the transverse current by the substitution $k_\perp^2\Rightarrow k_\perp^2\exp(2/|k_\parallel L_{\mathrm{K}}|)$ into Eq. (15), accounting for an exponential decrease of the distance between equipotentials for large wavelengths $k_\parallel^{-1}$. This procedure gives a correct result in the case of test particle diffusion in the collisional case: see Refs. 19 and 23. The result is also given by Eq. (18), where the length L_σ is replaced by the factor $\tilde{L}_{\mathrm{K}}\sim L_{\mathrm{K}}\ln(L_\sigma/L_{\mathrm{K}})$:

$$j_x=\frac{\sigma_\parallel E_0 D_B}{\tilde{L}_{\mathrm{K}}}. \tag{20}$$

For the stochastic magnetic field lines, the value of D_B is the diffusion coefficient of the magnetic field lines. Finally, the resulting transverse current almost everywhere is given by the projection of the parallel current flowing along the magnetic field lines, and only in some ''narrow'' places the current flows directly perpendicular to the magnetic field.

B. The net cross-field current driven by transverse electric field, density, and temperature gradients

In the presence of density and temperature gradients additional transverse currents appear. The averaged cross-field current is then given by the averaged projection of parallel current on the x axis $j_x=\langle j_\parallel^1 b_x\rangle$. Hence, to calculate the parallel current $\mathbf{j}_\parallel=\sigma_\parallel\mathbf{b}[-\boldsymbol{\nabla}\varphi+(T_e/e)\boldsymbol{\nabla}\ln n+(1.71/e)\boldsymbol{\nabla}T_e]$ one has to find density, temperature, and potential perturbations. The perturbations of potential, density, and temperature are coupled.[7] To find them one has to solve the linearized set of fluid equations (1)–(7). Let us introduce the variables $\varphi_0^1=-(b_x/ik_\parallel)(d\varphi_0/dx)$, $n_0^1=-(b_x/ik_\parallel)\times(dn_0/dx)$, $T_{e0}^1=(b_x/ik_\parallel)(dT_{e0}/dx)$, $T_{i0}^1=-(b_x/ik_\parallel)\times(dT_{i0}/dx)$. These perturbations correspond to the constant value of density, temperature, or potential along the perturbed magnetic field. First we consider case of nonresonant modes.

1. Nonresonant modes $k_\parallel(x)\neq 0$

Due to large parallel electron conductivity, for nonresonance modes the following inequality is satisfied: $k_\parallel\gg\sqrt{e^2Dn_0T_ek_\perp^2/\sigma_{e\parallel}}$. Thus, it is easy to see from Eq. (1) that parallel current is much less than the contribution from each of the terms, and the potential perturbation is given by

$$\varphi^1\approx\varphi_0^1-\frac{T_e}{en_0}(n_0^1-n^1)-\frac{1.71}{e}(T_{e0}^1-T_e^1). \tag{21}$$

Similarly, due to the large parallel electron heat conductivity, the electron temperature is nearly constant along magnetic lines, and perturbed temperature is $T_e^1\cong T_{e0}^1$. The parallel current can be estimated from Eq. (1): $ik_\parallel j_\parallel=-ik_\perp j_\perp=-k_\perp^2\sigma_{i\perp}[\varphi^1+(T_{i0}/en_0)n^1+T_i^1/e]$. After substituting the potential perturbation in the form of Eq. (21), we have $j_\parallel=-(k_\perp^2\sigma_{i\perp}/ik_\parallel)[\varphi_0^1-(T_{e0}/en_0)(n_0^1-n^1)+(T_{i0}/en_0)n^1+T_i^1/e]$. If $n^1\approx n_0^1$ and $T_i^1\approx T_{i0}$, then $j_\parallel=-(k_\perp^2\sigma_{i\perp}/ik_\parallel)(\varphi_0^1+(T_{i0}/en_0)n_0^1+T_{i0}^1/e)=-(k_\perp^2\sigma_{i\perp}b_x/k_\parallel^2)(d\varphi_0/dx+T_{i0}dn_0/en_0dx+dT_{i0}/edx)$, and zero net current is given by modified Eq. (16) with substitution $E_0\to-(d\varphi_0/dx+T_{i0}dn_0/en_0dx+dT_{i0}/edx)$:

$$j_x=-\frac{1}{2}\left(\frac{d\varphi_0}{dx}+\frac{T_{i0}dn_0}{en_0dx}+\frac{dT_{i0}}{edx}\right)\sum_{\mathbf{k}}\sigma_{\perp\mathbf{k},v}\left(\frac{k_\perp b_{x\mathbf{k}}}{k_\parallel^2}\right)^2.$$

We see that in the absence of ion temperature gradient the parallel current becomes zero, when the radial electric field corresponds to the Boltzmann distribution of ions. This result contrasts with the wide spread opinion that in the presence of magnetic turbulence the radial electric field corresponds to the Boltzmann distribution for electrons.

The ion density and temperature fluctuations are to be found from continuity equations (2) and (7). It is easy to see that divergences of ion fluxes are $ik_\parallel\Gamma_{i\parallel}=(k_\parallel^2\sigma_{i\parallel}/e^2)\{[(T_e+T_i)/n_0](n^1-n_0^1)+(T_e^1-T_{e0}^1)+(T_i^1-T_{i0}^1)\}$, where we have introduced effective ion parallel ''conductivity:'' $\sigma_\parallel=e^2n/m_ii(k_yv_0+k_xu_0)$, $u_0=-Ddn_0/n_0dx$ is diffusion velocity in the x direction. In the perpendicular ion flux, we can leave only the ambipolar diffusion flux and $E\times B$ drift, since the diamagnetic fluxes are divergence-free, while ion inertia fluxes are assumed to be small compared to the diffusive ones ($eDn \gg \sigma_{i\perp}T_{e,i}$). Hence, $\boldsymbol{\nabla}_\perp\cdot\boldsymbol{\Gamma}_{i\perp}=k_\perp^2 Dn^1 - ik_y[(en_0/T_e)v_{e0}^{\mathrm{B}}(\varphi^1-\varphi_0^1)-v_0(n^1-n_0^1)]$, where $v_{e0}^{\mathrm{B}} = cT_e d\ \ln n_0/eBdx$ is the drift associated with the electron Boltzmann electric field $\varphi_0^{\mathrm{B}}=(T_e/e)\ln n_0$.

For heat fluxes, we have $ik_\parallel q_\parallel = k_\parallel^2\chi_\parallel(T_i^1-T_{0i}^1) - (0.71/e)T_e ik_\parallel j_\parallel$, and $\boldsymbol{\nabla}_\perp\cdot\mathbf{q}_{i\perp}=k_\perp^2\chi_\perp T_i^1-\frac{3}{2}ik_y n_0 \times[(ev_0^T(\varphi^1-\varphi_0^1)-v_0(T_i^1-T_{i0}^1)]$, where $v_0^T=cdT_i/eBdx$. For typical edge tokamak conditions, discussed above, parallel ion transport is large: $e^2k_\perp^2 Dn_0 \ll k_\parallel^2|\sigma_{i\parallel}|(T_e+T_i)$ and $k_\perp^2\chi_\perp \ll k_\parallel^2\chi_\parallel$, so $n^1\approx n_0^1$ and $T_i^1\approx T_{0i}^1$; ion density and temperature are nearly constant along magnetic lines.

In the opposite case, when

$$e^2k_\perp^2 Dn_0 \gg |k_\parallel^2\sigma_{i\parallel}(T_e+T_i)|;|k_y e^2 n_0(v_{e0}^{\mathrm{B}}-v_0)|, \tag{22}$$

$$k_\perp^2\chi_\perp \gg k_\parallel^2\chi_{i\parallel};|k_y n_0 v_0|, \tag{23}$$

density and ion temperature perturbations are small, $n^1 \ll n_0^1$, $T_i^1 \ll T_{i0}$. The parallel current is then $j_\parallel = (k_\perp^2\sigma_{i\perp}/ik_\parallel)(\varphi_0^1-T_{e0}n_0^1/en_0) = -(k_\perp^2\sigma_{i\perp}b_x/k_\parallel^2)(d\varphi_0/dx - T_e dn_0/en_0 dx)$. So, only for rather exotic parameters, the ambipolar condition (zero net current) is achieved for the radial electric field, which corresponds to the electron Boltzmann distribution.

Another mechanism of cross-field conductivity associated with large parallel ion flux exists.[25] The density gradients produce plasma flow along the curvilinear magnetic field. As a result, in the second order on magnetic perturbations, cross-field current appears. In the Chew–Goldberger–Low approximation, the ion viscosity is given by

$$\boldsymbol{\pi}=(p_{i\parallel}-p_{i\perp})[(\mathbf{B}/B)(\mathbf{B}/B)-\mathsf{I}/3],$$

where, in accordance with the Braginskii expression (Ref. 20),

$$p_{i\parallel}-p_{i\perp}=-3\,\eta_0(ik_\parallel u_{\parallel\mathbf{k}}-\tfrac{1}{3}\boldsymbol{\nabla}\cdot\mathbf{u}),\qquad \eta_0=0.96nT_i/\nu_{ii}.$$

The x component of the flux-averaged current is determined by the perpendicular component and by the projection of the parallel current $\langle j_x\rangle=\langle j_{\perp x}\rangle+\langle j_\parallel\cdot b_x\rangle$. From the momentum balance equation it follows that the first term on the RHS of the previous equation is defined by curvilinear viscosity,

$$\langle j_{\perp x}\rangle=\left\langle \frac{c}{B}\,(p_{i\perp}-p_{i\parallel})(\mathbf{b}\boldsymbol{\nabla})b_y\right\rangle.$$

The projection of the parallel current is calculated from the current continuity equation,

$$\langle j_\parallel b_x\rangle=\sum_{\mathbf{k}}\left\langle \frac{c}{B}\,(p_{i\perp}-p_{i\parallel})\,\frac{k_z}{3k_\parallel}\,(\mathbf{b}\boldsymbol{\nabla})b_y\right\rangle.$$

The detailed calculations can be found in Ref. 25.

2. Case of resonant modes $k_\parallel\rightarrow 0$ and stochastic magnetic field

As we have seen in previous section, the main contribution to the current is due to resonance modes $k_\parallel \sim\sqrt{|\sigma_\perp/\sigma_{e\parallel}|k_\perp^2}$. For this very small $k_\parallel$, ion density and temperature perturbations are also small, according to conditions (22) and (23). In the presence of large anomalous ambipolar diffusion, the electron temperature fluctuations can be neglected with respect to the potential ones for these resonance modes, since electron temperature fluctuations are washed out by much stronger cross-field transport $\chi_{e\perp}\sim nD \gg|\sigma_{i\perp}|T_{e,i}/e^2$. As a result, we have the same final expressions, as before, but with a replacement $-\boldsymbol{\nabla}\varphi \Rightarrow -\boldsymbol{\nabla}\varphi+(T_e/e)\nabla\ \ln n+(1.71/e)\boldsymbol{\nabla}T_e$. For example, Eq. (20) modifies to

$$j_x=\frac{\sigma_\parallel D_B}{\widetilde{L}_k}\left(-\frac{d\varphi_0}{dx}+\frac{T_{0e}}{e}\frac{d\ \ln n_0}{dx}+\frac{1.71}{e}\frac{dT_{0e}}{dx}\right). \tag{24}$$

The combination in the brackets is rather natural in this expression, however, it differs from the similar result of Ref. 9 for the collisionless case, where it was $j_x\sim[-\partial\varphi_0/\partial x +(T_e/e)(\partial\ \ln n_0/\partial x)+(0.5/e)(\partial T_{0e}/\partial x)]$. Also important is the presence of correlation length $\widetilde{L}_k$.

IV. STUDY OF THE MORE COMPLICATED COLLISIONLESS CASE

A. Transverse current in the collisionless case $\lambda_{\mathrm{mfp}}k_\parallel \gg 1$

At first we neglect density and temperature gradients. To describe the electron motion under these conditions, we use the drift electron kinetic equation,

$$(v_\parallel\mathbf{b}+v_0\mathbf{e}_y)\boldsymbol{\nabla}f-D\Delta_\perp f+\frac{e}{m_e}\,\mathbf{b}\boldsymbol{\nabla}\varphi\,\frac{\partial f}{\partial v_\parallel}=\mathrm{St}(f\,), \tag{25}$$

where $\mathrm{St}(f\,)$ is the electron collision integral. We also introduce the anomalous diffusion into Eq. (25) through anomalous diffusion coefficient D.

The potential perturbations are given by Eq. (13), but with different electron parallel conductivity $\sigma_{\parallel\mathbf{k}}$. For ions, analogously to the collisional case, the cross-field conductivity is mainly determined by the inertia: $\sigma_{\perp\mathbf{k}} = im_i c^2(v_0k_y)n/B^2$ both for collisionless and collisional cases. To find $\sigma_{\parallel\mathbf{k}}$ one has to solve Eq. (25). Separating the isotropic and the anisotropic parts of the distribution function, from Eq. (25) we find the value of the parallel conductivity given in Ref. 26 (the electron collision integral is taken in the τ approximation):

$$\sigma_{\parallel\mathbf{k}}=\frac{4\pi e^2}{3T_e}\int dv \times\frac{v^4f^{(\mathrm{M})}(v)}{[iv_0k_y+\nu_{\mathrm{eff}}(\mathbf{k},v)+v_{ei}+(k_\parallel v)^2/(iv_0k_y+\nu_{\mathrm{eff}}(\mathbf{k},v))]}, \tag{26}$$

where $f^{(\mathrm{M})}$ is the Maxwellian distribution, ν_{ei} is the electron–ion collision frequency, ν_{eff} is the frequency of the electron phase decorrelation. The first term in the denominator corresponds to additional momentum loss frequency iv_0k_y, the second term represents phase decorrelation; the last term arises due to spatial nonlocality. The spatial nonlocality results in decreasing of conductivity in comparison with the collisional case.

For example, in the absence of collisions and of any mechanism of electrons escape from the magnetic field line, the real part of conductivity $\sigma_{\parallel\mathbf{k}}$ becomes zero. This situation corresponds to the Boltzmann equilibrium for electrons. The frequency of the electron phase decorrelation is governed by the stochastic escape of electrons from the magnetic field line caused by the transverse anomalous diffusion: $\nu_{\mathrm{eff}}(\mathbf{k},v_\parallel)\cong\max\{Dk_\perp^2;[1/3(\partial k_\parallel/\partial x)^2Dv_\parallel^2]^{1/3}\}$; see, for example, Ref. 27. The last term is associated with amplification of phase decorrelation, due to the combined effect of shear of $k_\parallel$ and electron motion along magnetic line. It should be noted that, in spite of the fact that the frequencies k_yv_0, ν_{eff}, ν_{ei} are small with respect to $k_\parallel v$, they cannot be omitted in Eq. (26), otherwise the parallel conductivity becomes zero, since electrons with positive and negative parallel velocities are affected equally by an electric field.

Employing the electron kinetic equation and Eqs. (1), (2), for the Maxwellian distribution function, we find

$$j_x=\frac{1}{2}E_0\ \mathrm{Re}\left(\sum_k |b_{x\mathbf{k}}|^2\sigma_{\parallel\mathbf{k}}I_\sigma(k_\parallel)\right), \tag{27}$$

$$I_\sigma(k_\parallel)=\frac{\sigma_{\perp\mathbf{k}}k_\perp^2}{\sigma_{\perp\mathbf{k}}k_\perp^2+\sigma_{\parallel\mathbf{k}}k_\parallel^2}. \tag{28}$$

Similar to the collisional case, the resulting cross-field current exists even for a nonstochastic magnetic field; the decorrelation between the phases of the parallel current and the perturbation of the magnetic field is caused by the anomalous electron diffusion or by the electron–ion collisions. The resulting $\Xi_{\perp b}$ depends on the local ion perpendicular conductivity. In Ref. 28 it was incorrectly stated that electron radial flux could appear in the presence of magnetic islands only due to collisions. It is clear that without any mechanism of transverse escape, electrons are bounded to the magnetic field line and cannot move in the radial direction, since the magnetic field lines are regular and do not diverge in the radial direction, on average. It can be shown formally. Equation (25) without drifts and diffusion reads as

$$v_\parallel\mathbf{b}\nabla f+\frac{e}{m_e}\mathbf{b}\nabla\varphi\frac{\partial f}{dv_\parallel}=\mathrm{St}(f).$$

Integrating this equation over parallel velocity, one can see that electron current $j_\parallel=\int v_\parallel f d\mathbf{v}=0$, since there are no losses from the magnetic field line and $\int\mathrm{St}(f)d\mathbf{v}=0$. In Ref. 28 the collision integral was taken in the form $\mathrm{St}(f)=-\nu f$, introducing artificial sink. The radial electron shift can arise only for nonstationary magnetic islands, when magnetic islands appear and disappear randomly, resulting in effective radial electron diffusion.

As we have discussed in Sec. III A, for the typical tokamak parameters, the characteristic widths of resonances in Eqs. (27) and (28) are very small: $k_{\parallel\sigma}=\sqrt{|\sigma_{\perp\mathbf{k}}k_\perp^2/\sigma_\parallel|}\sim 3\times10^{-5}\ \mathrm{cm}^{-1}$, $\nu_{\mathrm{eff}}/v\sim5\times10^{-4}\ \mathrm{cm}^{-1}\ll\delta k_y\Theta\sim10^{-2}\ \mathrm{cm}^{-1}$. It means that j_x has strong narrow peaks at resonance surfaces $\sim k_\parallel^{-4}(x)$ at $k_\parallel(x)\to0$.

These peaks are smoothed if the magnetic field is stochastic and the magnetic islands are overlapped. Similar to the collisional case, the transverse current for a stochastic magnetic field is considerably enhanced, and to estimate the current we should perform the substitution $k_\perp^2\Rightarrow k_\perp^2\exp(2/|k_\parallel L_{\mathrm{K}}|)$. The frequency of electron escape from the magnetic field line, $\nu_{\mathrm{eff}}(\mathbf{k},v_\parallel)$, is also considerably increased, since now the effective cross-field electron diffusion is $D_{\mathrm{B}}v_\parallel$, where D_{B} is the diffusion coefficient of the stochastic magnetic field lines. Substituting $D_{\mathrm{B}}v_\parallel$ instead of D, we find $\nu_{\mathrm{eff}}=v_\parallel/L_{\mathrm{K}}$—is determined by the Kolmogorov length. Thus, for typical parameters $\nu_{\mathrm{eff}}\sim10^7-10^8\ \mathrm{s}^{-1}$, which is much larger than $v_0k_y\sim10^6$, $\nu_{ei}\sim10^4$. The electron inertia and collisions can thus be omitted in Eqs. (26) and (28). Since for the stochastic magnetic field $L_{\mathrm{K}}<1/(\Theta\cdot\delta k_y)$ the sum in Eq. (27) can be replaced by the integral. The integration of Eq. (27) yields

$$j_x=e^2i_\sigma D_{\mathrm{st}}nE_0\sqrt{\frac{2}{\pi m_eT_e}},\quad i_\sigma\approx\frac{1.6}{\ln(|\sigma_\parallel L_{\mathrm{K}}^2/\sigma_{\perp\mathbf{k}}k_\perp^2|)}, \tag{29}$$

where $k_\perp$ with logarithmic accuracy is the typical value of k_z. The result of Eq. (29) differs from the current of test electrons found in Refs. 9 and 10 by the factor i_σ, which accounts for the ambipolarity constraint. The ratio of $\sigma_\parallel L_{\mathrm{K}}^{-2}/\sigma_{\perp\mathbf{k}}k_\perp^2$ is very large (can reach 10^6); hence, the value of $\ln(|\sigma_\parallel L_{\mathrm{K}}^{-2}/\sigma_{\perp\mathbf{k}}k_\perp^2|)$ can be of the order of 15. Therefore, the impact of the local ambipolarity constraint is very important.

B. The net cross-field current driven by transverse electric field, density, and temperature gradients in the collisionless case $\lambda_{\mathrm{mfp}}k_\parallel\gg1$

Analogously to Sec. III B, the ion and electron temperature and density perturbations can be neglected compared to the potential ones for resonance modes. As a result of the solution of the kinetic equation (25) accounting for density and temperature gradients, we have[29]

$$j_x=\sum_k |b_{x\mathbf{k}}|^2G(k_\parallel),$$

$$G(k_\parallel)=I_\sigma(k_\parallel)\frac{2\pi e^2}{3}\int dv\,v^4f(v)\mathrm{Re}$$
$$\times\left[\frac{\left[\frac{\partial\ln n}{\partial x}-\frac{e}{T_e}\frac{\partial\varphi_0}{\partial x}+\left(\frac{mv^2}{2T_e}-\frac{3}{2}\right)\frac{\partial\ln T_e}{\partial x}\right]}{\left(iv_0k_y+\nu_{\mathrm{eff}}(\mathbf{k},v)+\nu_{ei}+\frac{(k_\parallel v)^2}{iv_0k_y+\nu_{\mathrm{eff}}(\mathbf{k},v)}\right)}\right].$$

Similar to Sec. IV A, in the case of a stochastic magnetic field,

$$j_x=i_\sigma eD_{\mathrm{st}}n\sqrt{\frac{2T_e}{\pi m_e}}\left(\frac{\partial\ln n}{\partial x}-\frac{e}{T_e}\frac{\partial\varphi_0}{\partial x}+\frac{1}{2T_e}\frac{\partial T_e}{\partial x}\right). \tag{30}$$

The resulting equation (30) differs from the current of test electrons found in Refs. 9 and 10 by the factor i_σ, which accounts for the ambipolarity constraint. The result obtained in the approximation of the test particles can be reduced by the order of magnitude.

V. CONCLUSIONS

We calculated the effective transverse conductivity with the ambipolarity constraint employed locally. The local self-consistent electric field is taken into account, which results in the reducing of the effective transverse conductivity with respect to the conductivity calculated in the model of the test electrons. For the case of two-dimensional magnetic perturbations, the average current is exactly zero, as pointed out in Refs. 15 and 16, so that one has to develop a three-dimensional model. The resulting average current perpendicular to the flux surface is determined by the local cross-field conductivity of ions, which becomes considerable for the small-scale magnetic field perturbations. Due to this fact, the considerable average perpendicular current exists both in stochastic and regularly perturbed magnetic fields.

ACKNOWLEDGMENTS

The authors would like to thank G. Fiksel, A. Smolyakov, and M. Tendler for valuable comments. Discussions with A. Bogaerts and R. Gijbels are gratefully acknowledged. The work of V.R. was supported by Russian Basic Research Foundation (RBRF) Grant No. N 9602 16919a and of I.K. was supported by the Humboldt Foundation, INTAS, Grant No. N 96-0235, by Grant No. RBRF N 98-02-16000, and partially sponsored by the Belgian Office for Scientific, Technical, and Cultural Affairs W&T collaboration with Central and East Europe, and by the University of Antwerp, Belgium (New Research Initiative ''Mathematical Simulation of Pulsed, Radio-Frequency and Magnetron Discharges'').

[1] V. Rozhansky and M. Tendler, Phys. Fluids B **4**, 1877 (1992).
[2] V. Rozhansky and M. Tendler, in *Reviews of Plasma Physics*, edited by B. B. Kadomtsev (Consultants Bureau, New York, 1996), Vol. 19, p. 147.
[3] R. J. Taylor, M. L. Brown, B. D. Fried, H. Grote, J. R. Liberati, G. J. Morales, P. Pribyl, D. Darrow, and M. Ono, Phys. Rev. Lett. **63**, 2365 (1989).
[4] L. G. Askinazi, V. E. Golant, S. V. Lebedev, V. A. Rozhanskij, and M. Tendler, Nucl. Fusion **32**, 271 (1992).
[5] V. I. Bugarya, A. V. Gorshkov, S. A. Grashin *et al.*, Nucl. Fusion **25**, 1707 (1985).
[6] D. L. Brower, W. A. Peebles, and N. C. Luhman, Nucl. Fusion **27**, 2055 (1987).
[7] Ph. Ghendrih, A. Grosman, and H. Capes, Plasma Phys. Controlled Fusion **38**, 1653 (1996).
[8] X. Z. Yang, B. Z. Zhang, A. J. Wotton, P. M. Schoch, B. Richards, D. Baldwin, D. L. Brower, G. G. Castle, R. D. Hazeltine, J. W. Heard, R. L. Hickok, W. L. Li, H. Li, S. C. McCool, V. J. Simic, Ch. P. Ritz, and C. X. Yu, Phys. Fluids B **3**, 3448 (1991).
[9] R. W. Harvey, M. G. McCoy, J. Y. Hsu, and A. A. Mirin, Phys. Rev. Lett. **47**, 102 (1981).
[10] A. V. Gurevich, K. P. Zybin, and Ya. N. Istomin, Nucl. Fusion **27**, 453 (1987).
[11] T. Kubota and S.-I. Itoh, Phys. Plasmas **2**, 3368 (1995).
[12] A. Gurevich, K. Zybin, A. Luk'anov, and R. Nebel, Sov. Phys. JETP **73**, 976 (1991).
[13] G. Giruzzi and E. Martines, Phys. Plasmas **1**, 2653 (1994).
[14] I. Kaganovich, V. Rozhansky, and M. Tendler, ''Bootstrap current due to shear of stochasticity of magnetic field,'' *International Conference on Plasma Physics ICPP96*, Nagoya, pp. 314–317.
[15] R. E. Waltz, Phys. Fluids **25**, 1269 (1982).
[16] P. W. Terry, P. H. Diamond, and T. S. Hahm, Phys. Rev. Lett. **57**, 1899 (1986).
[17] P. W. Terry and P. H. Diamond, Phys. Fluids B **2**, 1128 (1990).
[18] P. W. Terry, G. Fiksel, A. F. Algmari, M. Cekic, D. J. Den Hartog, P. H. Diamond, S. C. Prager, J. S. Starff, W. Shen, M. Stoeking, and A. S. Ware, Phys. Plasmas **3**, 1999 (1996).
[19] B. B. Kadomtsev and O. P. Pogutse, *Plasma Physics and Controlled Nuclear Fusion Research*, Proceedings, 7th International Conference, Innsbruck, 1978 (International Atomic Energy Agency, Vienna, 1978), Vol. 1, p. 649.
[20] S. I. Braginski, in *Reviews of Plasma Physics*, edited by M. Leontovich (Consultants Bureau, New York, 1963), Vol. 1, p. 193.
[21] J. H. Misguich, M. Vlad, F. Spineanu, and R. Balescu, Comments Plasma Phys. Control. Fusion **17**, 45 (1995).
[22] V. Rozhansky, Contrib. Plasma Phys. **36**, 366 (1996).
[23] J. A. Krommes, C. Oberman, and R. G. Kleva, J. Plasma Phys. **30**, 11 (1983).
[24] A. B. Rochester and M. N. Rosenbluth, Phys. Rev. Lett. **40**, 38 (1978).
[25] I. Kaganovich, V. Rozhansky, and M. Tendler, ''Transverse conductivity in a braided magnetic field,'' in *Proceedings of the 10th International Conference on Plasma Physics*, Joint Conference on Plasma Theory and 10th International Congress on Waves and Instabilities in Plasmas, Foz do Iguacu PR—Brazil (American Institute of Physics, Woodbury, NY, 1995).
[26] Yu. F. Baranov and A. D. Piliya, Sov. Phys. JETP **68**, 289 (1989).
[27] A. A. Galeev and L. M. Zelenii, JETP Lett. **29**, 669 (1979).
[28] J. D. Callen, Phys. Rev. Lett. **39**, 1540 (1977).
[29] I. Kaganovich, V. Rozhansky, and M. Tendler, ''Transverse current in stochastic magnetic field,'' *Plasma Physics and Controlled Fusion*, Kiev (European Physical Society, Petit Lancy, 1996), p. 136.