Effects of Collisions and Particle Trapping on Collisionless Heating

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(Received 19 August 1998)

Collisionless power dissipation has been calculated analytically taking into account particle trapping in the wave and electron collisions with neutrals. The approximation of analytical calculations for a decrement of nonlinear Landau damping gives, within an error less than 5%, $\gamma_{nl} = \gamma_l \tanh(2\nu\tau_r)$, where γ_l is the linear Landau damping, ν is the total collision frequency, and τ_r is a bounce time of trapped electrons. The theory is applied to the calculation of collisionless heating in a bounded low-pressure glow discharge plasma. It is shown that the difference with previously published results of linear theory on collisionless power dissipation can be as large as 3 orders of magnitude. [S0031-9007(98)08192-7]

PACS numbers: 52.35.Mw, 52.65.Ff, 52.80.Pi

Collisionless power dissipation is of fundamental interest in plasma physics. Principal examples are Landau damping of longitudinal waves [1] and anomalous skin effect of transversal waves [2]. In addition to purely theoretical interests, collisionless power dissipation plays an important role in many applications. Examples are supplementary plasma heating in fusion devices [3], sustenance of radio-frequency (rf) gas discharges at low pressures [4], etc.

The linear theory of collisionless damping breaks down for times longer than the bounce time of trapped resonance electrons $\tau_r \equiv (m/e\Phi_0 k^2)^{1/2}$, where k is the wave number and Φ_0 is the amplitude of the electric field potential. For finite perturbations, when $\gamma_l \tau_r < 1$, where γ_l is linear Landau damping [1], the problem is essentially nonlinear. It is generally believed that in this regime of nonlinear Landau damping, the initial decay of the wave amplitude will soon turn into nonlinear oscillations and eventually approach a Bernstein-Green-Kruskal (BGK) steady state [5] with a lower value of wave amplitude [6]. Recently, this picture has been confirmed by long-time numerical calculations [7]. Results of simulations have shown that claims of other papers [8,9] that the wave amplitude will eventually decay to zero, are not conclusive.

In a practical plasma, electron collisions with neutral atoms, electrons, and ions have to be taken into account [10]. Although the collision frequency is small, collisions are the only remaining mechanism providing wave damping in the nonlinear regime. The decrement of longitudinal waves (nonlinear Landau damping γ_{nl}) was calculated in [11] under the conditions $\gamma_l \tau_r \ll 1$ accounting for rare Coulomb collisions. Unlike the linear decrement, the nonlinear decrement depends on the amplitude of the wave and collision frequency.

In the present article a partially ionized plasma is considered, where electrons collide mainly with neutral atoms. These conditions are met for a gas discharge plasma, in which Landau damping has frequently been measured [12]. It is assumed that the differential cross section of electron-atom scattering has no singularity at small angles in the range of energies up to about 30 eV [13], so that small angle scattering does not contribute to the total cross section. This makes it possible to calculate the decrement of nonlinear Landau damping analytically for any collision frequency. An approximation to the analytical calculation for the decrement of nonlinear Landau damping gives, within an error of less than 5%, $\gamma_{nl} = \gamma_l \tanh(2\nu\tau_r)$, where ν is the total collision frequency. The derived formula gives the nonlinear wave damping ($\gamma_l \tau_r \ll 1$) for any value of collision frequency. In the limit $\nu\tau_r \gg 1$, $\tanh(\nu\tau_r) \to 1$, and the obtained result coincides with the linear theory of Landau. In the opposite case $\nu\tau_r \ll 1$, $\tanh(\nu\tau_r) \to 0$, and my result corresponds to the O'Neil theory [6]. So, this result is a natural generalization of both theories, and gives the wave damping for any value of collision frequency. The theory is applied to the calculation of collisionless heating in a bounded plasma. In the traditional theory, collisionless heating is constant when the collision frequency tends to zero. In contrast to this, nonlinear effects cause the collisionless dissipation to tend to zero as ν approaches zero and to vanish in the limit $\nu = 0$.

The received result is of general character and can be applied to any bounded plasma (in metals, semiconductors, etc.).

The cause of collisionless damping is the interaction of resonant electrons with the wave. The average scattering angle $\overline{\theta}$ is considered to be $\overline{\theta} \gg \Delta u/v_{ph}$, where $\Delta u = (e\Phi_0/m)^{1/2}$, $v_{ph} = \omega/k$ is the phase velocity, and $\Delta u/v_{ph} \ll 1$. This allows one to assume that, after scattering in elastic collisions, resonant electrons immediately leave the resonance region. To obtain the damping coefficient, the rate of increase of kinetic energy of resonant electrons has to be calculated.

Exact solution of the nonlinear Landau problem.— We consider a stationary wave in a coordinate system moving with the wave's phase velocity. We examine the stationary electron distribution function (EDF) at times larger than the collision time. The EDF is close to isotropic everywhere at velocities far from the resonance velocity v_{ph}. For the resonance region where strong interaction with the wave occurs, one has to solve the kinetic equation taking the collisional integral into account,

$$\frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} - eE(t,x)\frac{\partial f}{m\partial v_x} = \int (f' - f) v \, d\sigma \,, \tag{1}$$

where $d\sigma$ is a differential cross section of elastic electron-atom collisions, and focusing the EDF mainly in the velocity range: $|v - v_{ph}| \leq 2 * \Delta u$. Outside the resonance region, the EDF is close to isotropic $f_0(w)$, where $w = 0.5m(v_x^2 + v_y^2 + v_z^2)$ is kinetic energy. The collision integral influx term gives $\int f' v \, d\sigma = \nu f_0$, where ν is total collision frequency (not just the transport frequency as in the BGK integral). If f^1 is the difference between f and $f_0(w)$, Eq. (1) takes the form

$$\frac{\partial f^1}{\partial t} + v_x \frac{\partial f^1}{\partial x} - eE(t,x)\frac{\partial f^1}{m\partial v_x} - eE(t,x) v_x \frac{\partial f_0(w)}{\partial w} = -\nu f^1. \tag{2}$$

Note that in Eq. (2) the nonlinear term involving the product $eE(t,x)\frac{\partial f^1}{m\partial v_x}$ is included, in contrast to linear theory. The solution of Eq. (2) is

$$f^1 = \left[-\int_{-\infty}^{t} eE(\tau, x(\tau)) e^{-\nu(t-\tau)} \, d\tau \right] v_x \frac{\partial f_0(w)}{\partial w}, \tag{3}$$

where $x(\tau)$ is the electron trajectory in the wave. Inserting Eq. (3) into Eq. (1) and averaging over time and velocity, one can find for the slow evolution of the main part of the EDF $f_0(w)$:

$$\frac{\partial f_0}{\partial t} + \frac{\partial}{\sqrt{w}\,\partial w}\left[\sqrt{w}\, D(w) \frac{\partial f_0}{\partial w}\right] = St^*(f_0), \tag{4}$$

where $St^*(f_0)$ is a collisional integral accounting for energy losses in elastic and inelastic collisions, and $D(w)$ is the energy diffusion coefficient:

$$D(w) = \int D_v m^2 v_x^2 \frac{d\cos\alpha \, d\beta}{4\pi}, \tag{5}$$

which is the average, over velocity angle ($\cos\alpha = v_x/v; \cos\beta = v_y/\sqrt{v_y^2 + v_z^2}$) of the diffusion coefficient in velocity space:

$$D_v = e^2 \left\langle E(\tau, x(\tau)) \int_{-\infty}^{t} E(\tau, x(\tau)) e^{-\nu(t-\tau)} \, d\tau \right\rangle, \tag{6}$$

where angular brackets $\langle\rangle$ denote averaging over time t. According to the equation of electron motion in the wave $eE(\tau, x(\tau)) = d[v_x(t) - v_x(\tau)]/d\tau$. Substituting this expression for the electric field and integrating by parts, the diffusion coefficient Eq. (6) takes the form

$$D_v = \frac{\nu}{2}\left\langle \int_0^{\infty} [v_x(t) - v_x(t - \tau)]^2 \nu e^{-\nu\tau} d\tau \right\rangle. \tag{7}$$

The expression for D_v of Eq. (7) has a very transparent physical meaning. It is the product of the squared velocity step $[v_x(t) - v_x(t - \tau)]$ by the frequency of this step ν, averaged over the probability to make the step, or to remain in the resonance region without collisions for a time $\tau - \nu e^{-\nu\tau}$.

The evolution of electron velocity is governed by the Hamiltonian

$$H(v_x, x) = \frac{m}{2}(v_x - v_{ph})^2 - e\Phi_0 \cos kx \,. \tag{8}$$

Following O'Neil [6] we normalize velocity with Δu, time with τ_r, and introduce the phase $\phi = kx$, and dimensionless parameter χ instead of total energy $H/e\Phi_0 \equiv \frac{2}{\chi^2} - 1$. The solution of equation of motion is to be found in terms of elliptic functions (dn and cn) [6]:

$$\chi < 1\colon v_x - v_{ph} = \frac{2}{\chi}\,\mathrm{dn}[(t + t_0)/\chi \mid \chi], \tag{9}$$

$$\chi > 1\colon v_x - v_{ph} = \frac{2}{\chi}\,\mathrm{cn}[t + t_0 \mid 1/\chi]. \tag{10}$$

It is convenient to replace variables from v_x and ϕ to action I and angle Θ: $I \equiv \oint u \, d\phi$, $\Theta \equiv \int \frac{d\phi}{u} / \oint \frac{d\phi}{u}$. Θ has a simple interpretation: $\dot\Theta = \frac{\partial H}{\partial I} \equiv \frac{2\pi}{T}$, where T is the period of $v_x(t)$ oscillations in nonlinear resonance. The phase volume is conserved after this variable transformation: $dv_x d\phi = dI d\Theta$. Thus averaging over velocity angle dv_x/v and initial phase $d\phi$ in Eq. (5) is equivalent to integration over $dId\Theta$. Substituting the Fourier series expansion (9),(10) for $v_x(t)$ and changing the integration from I to χ, $\frac{\partial I}{\partial \chi} = \oint \frac{\partial u}{\partial \chi} d\phi$ we find

$$D_v = \frac{\pi k e \Phi_0^2 v_{ph}^2 \Pi(\tilde\nu)}{2m^2 v^3}, \tag{11}$$

where the function $\Pi(\tilde\nu)$ is a function of the dimensionless collision frequency $\tilde\nu = \nu * \tau_r$ and is defined as

$$\Pi(\tilde\nu) \equiv 128\tilde\nu \sum_{n=1} \int_0^1 \left[\left(\frac{q^n}{1 + q^{2n}}\right)^2 \frac{1}{1 + \left(\frac{\tilde\nu}{n\pi/K\chi}\right)^2} \frac{1}{K\chi^4} + \left(\frac{q^{n-\frac{1}{2}}}{1 + q^{2n-1}}\right)^2 \frac{1}{1 + \left(\frac{\tilde\nu}{(2n-1)\pi/2K}\right)^2} \frac{\chi}{K} \right] d\chi \approx \tanh(2\tilde\nu), \tag{12}$$

where $q = \exp(-\frac{\pi K'}{K})$, $K' = K(\sqrt{1 - \chi^2})$, $K(\chi) = F(\chi, \pi/2)$. Surprisingly, the complex function $\Pi(\tilde\nu)$ can be very well approximated simply by $\tanh(2\tilde\nu)$. The $\tanh(2\tilde\nu)$ approximation is valid to within error less than 5%. Function $\Pi(\tilde\nu)$ is plotted in Fig. 1.

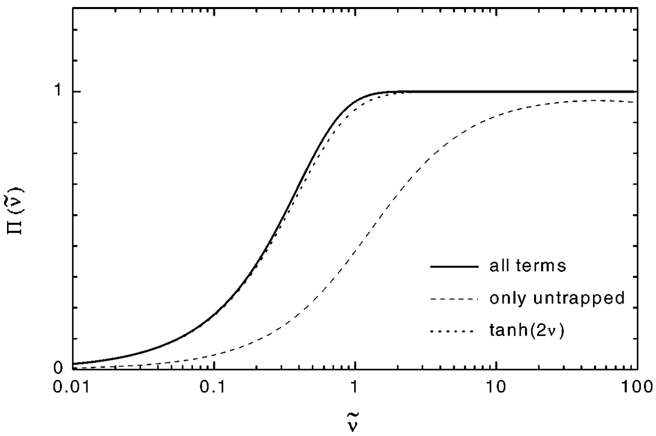

FIG. 1. Dimensionless function $\Pi(\nu)$.

Decrement of nonlinear wave.—At $\tilde{\nu} \gg 1 \Pi(\tilde{\nu}) \to 1$, Eq. (11) corresponds to the quasilinear theory, and the total power dissipation gives linear Landau decrement. We can then deduce that nonlinear damping is related to linear damping by

$$\gamma_{nl} \cong \gamma_l \tanh(2\nu\tau_r)\,. \qquad (13)$$

As can be seen from Fig. 1, the main contribution at $\tilde{\nu} \gg 1$ is due to untrapped electrons (not trapped in the wave); see the first term on right-hand side (rhs) of Eq. (12). For $\tilde{\nu} < 1$, $\Pi(\tilde{\nu})$ is less than unity, and, correspondingly, the power dissipation and nonlinear decrement of the wave decrease. For $\tilde{\nu} \ll 1 \Pi(\tilde{\nu}) \cong 2\tilde{\nu}$ is proportional to the collision frequency, similar to the result of [11]. Note that in contrast to [11], where only the limit of rare collisions was considered, Eq. (13) is valid for arbitrary values of $\tilde{\nu}$. At small $\tilde{\nu}$ the main contribution to the power dissipation is due to trapped-in-the-wave electrons [second term on rhs of Eq. (12)], the contribution of untrapped electrons is only about 25% compared with that of trapped electrons. Note, that the obtained result is also very different from that of [11], where nonlinear Landau damping with account for Coulomb collisions was explored. The main contribution to wave damping in that case is due to narrow boundary layer around separatrix.

We have considered electron heating by a monochromatic longitudinal wave. The theory can be applied to the calculation of collisionless heating in any bounded plasma for an arbitrary electric field. As an example, we have chosen the anomalous skin effect in a bounded plasma.

Influence of nonlinear effects on diffusion coefficient in velocity space in bounded plasma.—Let us consider transverse electric fields $E_y(x)e^{-i\omega t}$ corresponding to inductively coupled plasma (ICP) in slab geometry, with sharp boundaries at $x = 0$ and $x = L$. The diffusion coefficient in velocity space was derived [14] by using a quasilinear theory in:

$$D(v) = \frac{\pi e^2 L}{2v^2 m^2} \sum_{n=-\infty}^{\infty} \int_0^{\pi} \frac{d\cos\alpha\, d\beta}{4\pi} (v_y)^2 |E_n|^2 \times \Delta\left(\omega - \frac{\pi n}{L} v_x\right), \qquad (14)$$

where $\Delta(\omega - \frac{\pi n v_x}{L}) = \frac{\nu}{\nu^2 + [\omega - (\pi n |v_x|/L)]^2}$, and $E_n = \frac{1}{L}\int_0^L E_y(x)\cos(\frac{\pi}{L}nx)\,dx$ is the Fourier transform of the electric field $E_y(x)$. In the case $\nu \ll \omega, v/L$, the term $\Delta(\omega - \frac{\pi n v_x}{L}) \to \pi\delta(\omega - \frac{\pi n |v_x|}{L})$, where $\delta()$ is the delta function. As a result, the diffusion coefficient does not depend on collision frequency. Equation (14) shows that only resonant particles ($v_x = \omega L/\pi n$) contribute to collisionless heating, similar to the case of longitudinal waves, for which only resonant particles ($v_x = \omega k$) contribute to heating.

Nevertheless, the rf electric field is directed along the plasma boundary, and the rf magnetic field results in only the velocity kicks transversal to the boundary [15]. Thus, account for rf magnetic field represents such a nonlinear effect. Nonlinear effects are introduced by the fact that the bounce frequency itself depends on v_x. The velocity kicks change the bounce frequency. Thus, electrons move out of resonance.

This problem becomes similar to the nonlinear Landau damping problem, where nonlinear effects also destroy the resonance condition $\omega = v_x k$. During one bounce forward and back over the gap, resonant electrons get a velocity kick: $\Delta v_y = \int_{-L/v_x}^{L/v_x} \frac{eE_y(x,t)}{m}\,dt = \frac{2eE_nL}{mv_{x,n}}$, $v_{x,n} = \frac{\omega L}{\pi n}$. The rf magnetic field rotates the velocity kick from y to x direction. Because kinetic energy is conserved, $v_x \Delta v_x = v_y \Delta v_y$ and $\Delta v_x = \frac{v_y}{v_x}\frac{2eE_nL}{mv_{x,n}}$. In the resonance region, the evolution of velocity is described by the system

$$\frac{dv_x}{di} = -\Delta v_x \sin\varphi,$$
$$\frac{d\varphi}{di} = -\left(\frac{\omega}{\Omega} - 2\pi n\right) \approx \frac{\omega\, d\Omega}{\Omega^2 dv_x}(v_x - v_{x,n}), \qquad (15)$$
$$\frac{dt}{di} = \frac{1}{\Omega},$$

where $\varphi = \omega t - 2\pi n$, i is a bounce number, and Δv_x is the amplitude of the kick. The system is governed by the Hamiltonian:

$$H(v_x, \varphi) = \frac{\omega\, d\Omega}{2\Omega^2 dv_x}(v_x - v_{x,res})^2 - \Delta v_x \cos\varphi\,. \qquad (16)$$

Introducing the normalized variables for velocity $[v_x] = \sqrt{\frac{\Delta v_x \Omega^2}{\omega(d\Omega/dv_x)}}$ and for time $\tau_{nl}^{-1} = \sqrt{\Delta v_x \omega\, d\Omega/dv_x}$, the dimensionless Hamiltonian of Eq. (16) coincides with the dimensionless Hamiltonian of Eq. (8). Thus, one can use the results of calculation of diffusion coefficient for nonlinear Landau damping Eq. (13), where time τ_r should be replaced by time τ_{nl}. Therefore, the diffusion coefficient accounting for nonlinear effects reads

$$D(v) = \frac{\pi e^2 L}{2v^2 m^2} \sum_{n=-\infty}^{\infty} \int_0^{\pi} \frac{d\cos\alpha\, d\beta}{4\pi} (v_y)^2 |E_n|^2 \times \delta\left(\omega - \frac{\pi n}{L} v_x\right) \tanh(2\nu\tau_{nl})\,. \qquad (17)$$

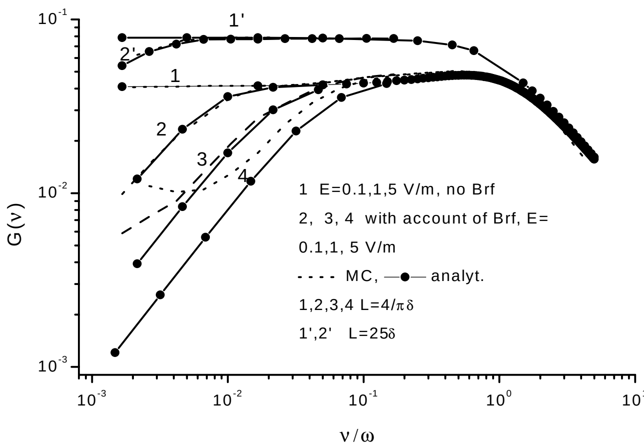

FIG. 2. Influence of the second boundary on collisionless heating. Dimensionless diffusion coefficient in velocity space $G = D_v/(e^2E_0^2\delta/2m^2\omega L)$ for an electric field $E_y = E_0 \exp(-\frac{x}{\delta})$ as a function of $\frac{\nu}{\omega}$ for the different slab widths $L = 4/\pi\delta$, 25δ. Solid curves with circles correspond to the analytical formulas (14),(17), dashed lines are Monte Carlo simulations.

From Eq. (17) one can see that nonlinear effects are important at small ν. When $\nu \ll \tau_{nl}^{-1}$, the diffusion coefficient is proportional to ν and $D(v) \rightarrow 0$ as $\nu \rightarrow 0$. This is in contrast to the linear theory, when $D(v)$ remains a constant at $\nu \rightarrow 0$.

Figure 2 is a plot of the diffusion coefficient for a fixed velocity ($v = 5\omega\delta$) as a function of ν/ω, for gap lengths $L = 4\delta/\pi$ and 25δ.

For small fields velocity kicks are also small ($\Delta v_x \ll v_x$) and theory agrees well with MC simulation. For larger values of the electric field the kicks are not small ($\Delta v_x \sim v_x$) and the dependence of D on ν is even more complex.

Influence of nonlinear effects on surface impedance in bounded plasmas.—In [16,17] the anomalous skin effect was considered ignoring the induced rf magnetic field. To show the importance of nonlinear effects, the real part of the surface impedance Z with and without taking nonlinear effects into account was calculated. The real part of surface impedance is related to the power absorption P by $\mathrm{Re}(Z) = 2P|Z|^2/E_0^2$. The power deposition into a unit volume of plasma, P, can be expressed in terms of D_v [Eqs. (14) or (17)] and the electron distribution function $f(v)$ [14]:

$$P = 4\pi m \int_0^\infty v^3 D_v(v) \frac{d}{dv} f(v)\, dv\,. \qquad (18)$$

Figure 3 depicts the real part of the surface impedance as a function of ν/ω. The profile of the electric field and the imaginary part of the surface impedance were taken in analytical form from [16]. Figure 3 shows that the value of the real part of the surface impedance decreases considerably at $\nu < \tau_{nl}^{-1}$ due to the influence of nonlinear effects. For typical values of electric field

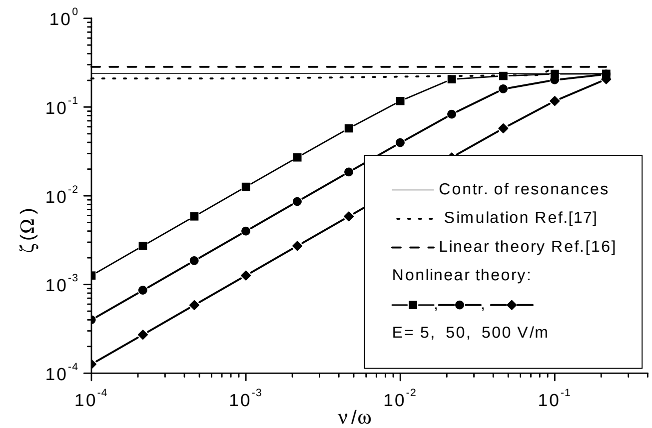

FIG. 3. The real part of the surface impedance in ohms, ζ, as a function of ν/ω. The plasma parameters are density $N = 10^{11}$ cm^{-3}, $T_e = 5$ eV, and $L = 4$ cm.

amplitudes in self-sustained ICP (about several V/cm [4]), the nonlinear effects start to be important for $\nu \sim 0.3\omega$ and the difference with the linear theory can be as large as 3 orders of magnitude.

Helpful discussions with Professor Tsendin and Professor Economou are greatly acknowledged. This work was sponsored in part by the Alexander von Humboldt Foundation, by INTAS Grant No. N96-0235, by the University of Antwerp, Belgium and by the Belgian Office for Scientific Technical and Cultural Affairs W T Collaboration with Central and East Europe.

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