Analytical and numerical studies of heavy ion beam transport in the fusion chamber

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Abstract
The propagation of a high-current finite-length ion charge bunch through a background plasma is of interest for many applications, including heavy ion fusion, plasma lenses, cosmic ray propagation, and so forth. Charge neutralization has been studied both analytically and numerically during ion beam entry, propagation, and exit from the plasma. A suite of codes has been developed for calculating the degree of charge and current neutralization of the ion beam pulse by the background plasma. The code suite consists of two different codes: a fully electromagnetic, relativistic particle-in-cell code, and a relativistic Darwin model for long beams. As a result of a number of simplifications, the second code is hundreds of times faster than the first one and can be used for most cases of practical interest, while the first code provides important benchmarking for the second. An analytical theory has been developed using the assumption of long charge bunches and conservation of generalized vorticity. The model predicts nearly complete charge neutralization during quasi-steady-state propagation provided the beam pulse duration \( \tau_b \) is much longer than the inverse electron plasma frequency \( \omega_p^{-1} \), where \( \omega_p = (4\pi n_p e^2/m_e)^{1/2} \) and \( n_p \) is the background plasma density. In the opposite limit, the beam head excites large-amplitude plasma waves. Similarly, the beam current is well neutralized provided \( \omega_p \tau_b \gg 1 \) and the beam radius is much larger than plasma skin depth \( \delta_p = c/\omega_p \). Equivalently, the condition for current neutralization can be expressed in terms of the beam current as \( I_b \gg 4.25Z_b \beta_c(n_b/n_p)kA \), where \( n_b \) is the beam density, \( Z_b \) is the ion charge, and \( \beta_c = \beta_c c \) is the beam velocity; and the condition for charge neutralization can be expressed as \( I_b \gg 4.25\beta_c(n_b/n_p)(r_b/r_l)^2kA \), where \( l_b \) and \( r_b \) are the beam length and radius, respectively. For long charge bunches, the analytical results agree well with the results of numerical simulations. The visualization of the data obtained in the numerical simulations shows complex collective phenomena during beam entry into and exit from the plasma.

Keywords: Ion beam; Plasma neutralization; Plasma waves; Self-fields

1. INTRODUCTION
Heavy ion beams are envisioned as one of the principal drivers for inertial confinement fusion (Bangerter, 2001). In heavy ion fusion design schemes, the ion beams are focused onto an indirect-drive target, to produce X-ray radiation, which compresses the deuterium-tritium pellet and initiates the fusion process (Meier, 2001). To protect the chamber walls and focusing magnets from neutron radiation, liquid Flibe jets create a region into which the heavy ion targets are injected (Peterson, 2001). These jets evaporate gas with a saturated vapor pressure of few milliTorr. As a result of electron stripping in ion–atom collisions, the charge state of the beam ions increases up to 5–8 during propagation over about 3 m in the chamber (Olson, 2001b). The space-charge potential of a typical beam with parameters at the chamber entrance corresponding to 4 kA current, 10 ns pulse duration, and 0.28c velocity, where c is the speed of light, is approximately 1.7 MV. Due to the stripping of electrons from the beam ions, the space-charge potential increases even further. Such high space-charge potentials inhibit beam focusing, and therefore ballistic focusing relies on various neutralization schemes to reduce the space-charge potential to acceptable levels. During ion beam propagation in the chamber, electrons are drawn into the beam by the positive ion charge, and the electrons provide some degree of charge neutralization. For effective neutralization, there should be enough electron production mechanisms to assure the generation of total electron charge equal to the ion beam pulse charge. Sources of electrons include: emission of electrons by the chamber walls (Bugaev et al., 1975), extraction of electrons from a preformed plasma plug (Efthimion & Davidson, 2001), and photoionization of the chamber gas by soft X rays emitted by the target (Sharp et al., 2001).

Neutralization of the beam charge and current in a plasma is also an important issue for many other applications. For example, high energy physics applications involve the transport of positive charges in plasma, for example, positrons for electron–positrons colliders (Rajagopalan et al., 1995),
and high-density laser-produced proton beams for the fast ignition of inertial confinement fusion targets (Roth et al., 2001). The recent resurgence of interest in charged particle beam transport in background plasma is brought about by the recognition that plasmas can be used as magnetic lenses. Applications of the plasma lens concept, ranging from heavy ion fusion to high-energy lepton colliders are discussed by Rajagopalan et al. (1995) and Tauschwitz et al. (1996).

There have been a number of numerical simulations schemes that study chamber transport of a heavy ion beam pulse. These simulations employ electromagnetic particle-in-cell codes. For example, the BPIC code has been used in the studies reported by Sharp et al. (2001) and Vay and Deutsch (2001), while Rose et al. (2001), Ottinger et al. (2001), and Welch et al. (2002) have utilized the LSP code. These simulations are typically numerically intensive and require up to several days of computational time. In addition, particle-in-cell codes have a considerable numerical noise in the electron density and the electric field, which may result in an artificial electron heating.

There are many critical parameters for ion beam transport in the chamber, including beam current, type of ion species, radial and longitudinal profiles of the beam density, chamber gas density, stripping and ionization cross sections, and so forth. This necessitates an extensive study for a wide range of parameters to determine conditions for optimum beam propagation. To complement comprehensive numerical simulations, a number of reduced models have been developed. Based on well-verified assumptions, reduced models can yield robust analytical and numerical descriptions and provide important scaling laws for the degrees of charge and current neutralization. Such general treatments also have relevance for other applications, which use positively charged beams, for example, plasma lenses in high energy physics, and the propagation of cosmic rays in astrophysics. Depending on the assumptions and simplifications, a suite of numerical codes has been developed and the codes benchmarked against one another. This suite is the subject of this article.

The code suite consists of two different codes: a fully electromagnetic, relativistic, particle-in-cell (PIC) code, and a nonrelativistic Darwin model for long beam pulses. The two-dimensional electromagnetic PIC code uses a leap-frog, finite-difference scheme to solve Maxwell’s equations on a two-dimensional rectangular grid in the frame moving with the beam. The current deposition scheme is designed to conserve charge exactly, so there is no need to solve Poisson’s equation. The other code uses the approximation of a very long charge bunch, that is, the beam length is much longer than the beam radius, and therefore the beam can be described by a number of weakly interacting slices. The electron motion is described in the quasi-stationary approximation, assuming that the ion beam evolves on a time scale much longer than the electron plasma period. The electric field is determined from Poisson’s equation, separately for each beam slice. As a result of the simplification, the second code is hundreds of times faster than the first (PIC) code. The second code can be used for most cases, while the first code provides benchmarking for the second.

The electron response frequency is of order of the electron plasma frequency, \( \omega_p = (4\pi n_p e^2/m_e)^{1/2} \), where \( n_p \) is the background plasma density. For heavy ion fusion applications, the ion pulse propagation time through the chamber is much longer than the inverse electron plasma frequency \( \omega_p^{-1} \). Therefore, a beam–plasma quasi-steady state forms during beam propagation. The initial step of the study is to describe the steady-state propagation (in the beam frame) of an ion beam pulse through a background plasma.

The case where the beam propagates through a cold plasma, with plasma density large compared with the beam density, can be studied by use of linear perturbation theory (Chen et al., 1985). Here, we focus on the nonlinear case where the plasma density has an arbitrary value compared with the beam density, and, correspondingly, the degrees of current and charge neutralization are arbitrary. The transport of stripped, pinched ion beams has also been discussed by Hahn and Lee (1996), where the assumptions of current and charge neutrality were made to determine self-consistent solutions for the electric and magnetic fields. M. Rosenbluth, E.P. Lee, and R. Briggs (pers. comm.), have considered the equilibrium of an isolated, charge-neutralized, self-pinched ion beam pulse in the absence of background plasma. In contrast, we consider here the case where “fresh” plasma is always available in front of the beam, and there are no electrons comoving with the beam (Kaganovich et al., 2001).

2. Basic Equations for Description of Ion Beam Pulse Propagation in a Background Plasma

In most applications, the background plasma electrons are cold—the electron thermal velocity is small compared with the beam velocity. Particle-in-cell simulations show that in most cases, the electron flow is laminar and does not form multistreaming. Thus, the electron fluid equations can be used for the electron description, and thermal effects are neglected in the present study. The electron fluid equations together with Maxwell’s equations comprise a complete system of equations describing the electron response to a propagating ion beam pulse. The electron cold-fluid equations consist of the continuity equation,

\[
\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e V_e) = 0, \tag{1}
\]

and the force balance equation,

\[
\frac{\partial p_e}{\partial t} + (V_e \cdot \nabla) p_e = -e \left( E + \frac{1}{c} V_e \times B \right), \tag{2}
\]

where \(-e\) is the electron charge, \( V_e \) is the electron flow velocity, \( p_e = \gamma_e m_e V_e \) is the average electron momentum,
m_e is the electron rest mass, and γ_e is the relativistic mass factor. Maxwell’s equations for the self-generated electric and magnetic fields, E and B, are given by

\[ \nabla \times B = \frac{4\pi e}{c} (Z_b n_b V_b - n_e V_e) + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}, \]

\[ \nabla \times E = -\frac{1}{c} \frac{\partial B}{\partial t}, \]

where \( V_b \) is the ion beam flow velocity, \( n_e \) and \( n_b \) are the number densities of the plasma electrons and beam ions, respectively (far away from the beam \( n_e \rightarrow n_{b0} \)), and \( Z_b \) is the ion beam charge state. The plasma ions are assumed to remain stationary with \( V_i = 0 \). The assumption of immobile plasma ions is valid for sufficiently short ion pulses with \( 2l_b < r_b \sqrt{M/m_e} \) (Kaganovich et al., 2001). Here, \( r_b \) and \( 2l_b \) are the ion beam radius and length, respectively, and \( M \) is the plasma ion mass.

Considerable simplification can be achieved by applying the conservation of generalized vorticity \( \Omega \) (Kaganovich et al., 2001). If \( \Omega \) is initially equal to zero ahead of the beam, and all streamlines inside of the beam originate from the region ahead of the beam, then \( \Omega \) remains equal to zero everywhere, that is,

\[ \Omega = \nabla \times \mathbf{p} - \frac{e}{c} \mathbf{B} = 0. \]

Substituting Eq. (5) into Eq. (2) yields

\[ \frac{\partial \mathbf{p}_e}{\partial t} + \nabla K_e = -e\mathbf{E}. \]

where \( K_e = (\gamma_e - 1) m_e c^2 \) is the electron kinetic energy. Note that the inertia terms in Eq. (6) are comparable in size to the Lorentz force term and cannot be omitted. Estimating the self-magnetic field from Eq. (5), we conclude that the electron gyroradius is of the order of the beam radius. This is a consequence of the fact that the electrons originate from the region of zero magnetic field in front of the beam. If most electrons are dragged along with the beam and originate from the region of large magnetic field, the situation may be different (Kaganovich et al., 2001).

3. APPROXIMATE SYSTEM OF EQUATIONS FOR LONG DENSE CHARGE BUNCHES

\( (V_b/\omega_p, r_b \ll l_b) \)

In this section, an approximate set of equations is derived for a long \( (r_b \ll l_b) \), cylindrically symmetric ion charge bunch satisfying

\[ V_b/\omega_p \ll l_b. \]

For long bunches \( (r_b \ll l_b) \), radial derivatives are much larger than longitudinal derivatives. Therefore, it follows from Eq. (5) for cylindrically symmetric beams that the azimuthal self-magnetic field is determined in terms of the longitudinal flow velocity, which gives

\[ B = \frac{c}{e} \frac{\partial p_{ec}}{\partial r}. \]

The displacement current, the last term on the right-hand side of Eq. (3), can be neglected under the condition in Eq. (7). Thus, Eq. (3) simplifies to become

\[ -\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial p_{ec}}{\partial r} \right) = \frac{4\pi e}{c} (Z_b n_b V_{bc} - n_e V_e). \]

Equation (9) has been also derived by Welch et al. (2002). Equation (9) describes the degree of current neutralization of the beam. Under the condition in Eq. (7), the degree of charge neutralization is very close to unity (Kaganovich et al., 2001), and the quasi-neutrality condition holds with

\[ n_e = Z_b n_b + n_p, \]

where \( n_p \) is the background plasma ion density.

For a flat-top ion beam density profile with constant velocity \( V_b \), Eq. (9) has the solution in the nonrelativistic limit

\[ V_{ec}(r) = \frac{Z_b n_b V_{bc}}{Z_b n_b + n_p} \left\{ 1 - \frac{W}{1 + W} \frac{I_0(r/\delta_{p0})}{I_0(r_b/\delta_{p0})} \right\}, \]

\[ r < r_b, \]

\[ 1 + \frac{W}{1 + W} \frac{K_0(r/\delta_{p0})}{K_0(r_b/\delta_{p0})}, \]

\[ r > r_b, \]

where

\[ W = \frac{\delta_{p0} I_0(r_b/\delta_{p0})}{\delta_{p0} I_0(r_b/\delta_{p0})} \]

\[ \frac{K_0(r_b/\delta_{p0})}{K_0(r_b/\delta_{p0})}. \]

Here, \( I_0(x) \) and \( K_0(x) \) are the modified Bessel functions, \( \delta_{p0} = c/(4\pi n_p e^2/m_e)^{1/2} \), and \( \delta_{p0} = c/(4\pi e^2 (n_p + Z_b n_b)/m_e)^{1/2} \).

The fractional degree of local current neutralization \( f_e(r) \) is defined by Davidson and Qin (2001)

\[ f_e(r) = \frac{n_e V_{ec}(r)}{Z_b n_b V_{bc}}. \]

Substituting Eqs. (10) and (11) into Eq. (13) gives the degree of current neutralization at the beam center and at the beam edge. We obtain

\[ f_e(0) = 1 - \frac{W}{1 + W} \frac{1}{I_0(r_b/\delta_{p0})}, \]

and

\[ f_e(r_b) = \frac{1}{1 + W}. \]
Note that in the limit \( r_b \gg \delta_p, f_e(0) \to 1 \), and \( f_e(r_b) \to \delta_p/(\delta_p + \delta_{pb}) \), and the current flows at the edge of the beam over distances of the order of \( \delta_{pb} \). Moreover, for more realistic ion beam density profiles, the edge profile falls off over a finite width \( \delta r_b \). Equation (15) is valid only if \( \delta r_b \ll \delta_{pb} \), and in the opposite limit with \( \delta r_b \gg \delta_{pb} \), we obtain \( f_e(r_b) \to 1 \).

The self-magnetic field can be calculated making use of Eq. (8) and Eq. (11). The degree of magnetic neutralization \( f_m(r) \) inside the beam is determined by the degree of current neutralization defined by

\[
f_m(r) = \frac{I_e(r)}{I_b(r)}.
\]

where \( I_e(r) \) is the electron current, \( I_b(r) = -e \int_0^r n_e V_e e 2\pi r \, dr \), and \( I_b(r) \) is the ion beam current, \( I_b(r) = Z_b e \int_0^r n_b V_b e 2\pi r \, dr \), both within radius \( r \). Substituting Eqs. (8) and (11) into Eq. (16) gives

\[
f_m(r) = 1 - \frac{2\delta_{pb}}{r} \left[ \frac{I_e(r)}{I_b(r)} \right],
\]

where \( I_e = (m_e c^3/e) \beta_b \approx 17 \beta_b kA \) is the (nonrelativistic) Alfven current for electrons with velocity \( \beta_b c \) and \( f_p = n_p/(Z_b n_b) \) is the normalized plasma density. Therefore, the normalized net current \( I_{net}/I_b \) is a function of \( I_b/I_A \) and \( f_p \), which can be expressed as

\[
\frac{I_{net}}{I_b} = \left[ \frac{I_b}{I_e} \left( 1 + f_p \right) \right]^{1/2} = \frac{W}{1 + W} \frac{I_1(r_b/\delta_{pb})}{I_0(r_b/\delta_{pb})}.
\]

The degree of net current neutralization is illustrated in Figure 1 as a function of the normalized plasma density for different values of \( I_b/I_A \). For \( f_p > 0.5 \), Eq. (19) can be approximated within 5% accuracy as

\[
\frac{I_{net}}{I_b} = \frac{\Lambda(f_p)}{[4I_b(f_p + 1)/I_A + \Lambda^2(f_p)]^{1/2}},
\]

where \( \Lambda(f_p) = 2\sqrt{f_p/(\sqrt{f_p + 1})} \).

If the ion beam density profile has a finite edge thickness \( \delta r_b \), then Eq. (19) is valid only if \( \delta r_b \ll \delta_{pb} \), or equivalently, \( f_p > 0.5(r_b/\delta r_b)^2/\sqrt{I_A/I_b} \).

The electric field is obtained from Eq. (6). Small departures from charge neutrality can be estimated by making use of Poisson’s equation:

\[
\frac{\delta n}{\rho} = \frac{\nabla \cdot \mathbf{E}}{4\pi e}.
\]

Here, \( n_n \) denotes density of background ions. It is convenient to introduce the average degree of charge neutralization \( \langle f \rangle \) over the beam cross section defined by

\[
\langle f \rangle = 1 - \frac{2 \int_0^{r_b} (Z_b n_b + n_p - n_n) r \, dr}{Z_b n_b r_b^2}.
\]

Making use of Poisson’s equation (21), we obtain from Eq. (22)

\[
\langle f \rangle = 1 - \frac{E(r_b)}{2\pi e Z_b n_b r_b}.
\]

In the nonrelativistic case, Eq. (6) gives \( E = -(m_e/c) V_e e \partial V_e / \partial r \), and making use of Eq. (11), the degree of charge neutralization is given by

\[
\langle f \rangle = 1 - \frac{2\beta_b^2}{1 + f_p} \frac{\delta_{pb}^2}{r_b^2 (1 + W^2)} \frac{K_1(r_b/\delta_{pb})}{K_0(r_b/\delta_{pb})}.
\]

Note that in the limit \( r_b \gg \delta_{pb} \), Eq. (24) reduces to

\[
\langle f \rangle = 1 - \frac{2\beta_b^2}{1 + f_p} \frac{\delta_{pb}^2}{r_b^2 (\delta_{pb}^2 + \delta_p^2)}.
\]

It can be readily shown (Kaganovich et al., 2001) that the maximum deviation from quasi-neutrality occurs when \( r_b \sim c/\omega_p \), and the degree of nonquasi-neutrality is bounded by \( (Z_b n_b + n_p - n_n)/(Z_b n_b) < 0.25 \beta_b^2 \). Therefore, for nonrelativistic, long ion pulses, there is almost complete charge neutrality.
neutralization. For heavy ion fusion parameters, $\beta_b < 0.25$ and degree of charge neutralization is more than 98%.

In the nonrelativistic limit, the force acting on the beam ions is (Hahn & Lee, 1996; Kaganovich et al., 2001)

$$ F_r = m_e e (V_{bc} - V_{ec}) \frac{\partial V_{ce}}{\partial r}. $$

(26)

It follows from Eq. (9) that $V_{bc} > V_{ec}$ and $\partial V_{ce}/\partial r < 0$, and therefore the force acting on the beam ions in the presence of a dense plasma is always focusing ($F_r < 0$).

The effective self-electric perveance in the presence of plasma scales as $1 - \langle f \rangle$, where $\langle f \rangle$ is the averaged charge neutralization defined in Eq. (24). Moreover, the total effective perveance including both self-electric and self-magnetic effects scales as (Davidson & Qin, 2001)

$$ \frac{Q_{\text{eff}}}{Q_0} = 1 - (f) - \beta_b^2 [1 - f_m(r_b)], $$

where the magnetic neutralization $f_m(r_b)$ at the beam edge is defined in Eq. (17). Here, the beam perveance $Q_0$ in the absence of plasma is defined by

$$ Q_0 = \frac{2\pi e Z_b n_b r_b^2}{\gamma_b^2 M V_b^2}. $$

(28)

Substituting Eqs. (17), (24), and (28) into Eq. (27) yields

$$ \frac{Q_{\text{eff}}}{Q_0} = -\gamma_b^2 \beta_b \frac{I_{\text{net}}}{I_b} \left[ 1 - \frac{V_{ce}(r_b)}{V_b} \right], $$

where $V_{ce}/V_b$ is given by Eq. (11). Note that Eq. (29) is similar to Eq. (26). The second term on the right-hand side of Eq. (29) is small except for $f_p \ll 1$ and $I_b \gg I_A/4$. Therefore the self-electric perveance is dominated by the self-magnetic perveance, and the total effective perveance scales as normalized net current defined in Eq. (20) except for the case of very tenuous plasma. To within 5% accuracy, $K_1(x)/K_0(x) \approx 1 + 1/(3x)$ for $x > 0.1$, and $I_0(x)/I_1(x) \approx \sqrt{1 + 4/x^2}$ for arbitrary $x$. Therefore $Q_{\text{eff}}/Q_0(I_b/I_A, f_p)$ in Eq. (30) can be readily calculated as a function of $I_b/I_A$ and the normalized plasma density $f_p$ by making use of the above approximations and Eqs. (12) and (18). The total effective perveance can be expressed as

$$ Q_{\text{eff}} = -\frac{Z_b m_e}{\gamma_b M} \frac{r_b}{\delta_p} \frac{[W + f_p/(1 + f_p)] K_1(r_b/\delta_p)}{(1 + f_p)(1 + W)^2 K_0(r_b/\delta_p)}. $$

(30)

In the nonrelativistic limit, and for $r_b \gg \delta_p$ and $f_p \gg 1$, it follows that $W \rightarrow 1$ and Eq. (30) simplifies to give

$$ Q_{\text{eff}} = -\frac{Z_b m_e}{M} \frac{r_b}{2\delta_p} \frac{1}{(1 + f_p)}. $$

(31)

The effective perveance $Q_{\text{eff}}(I_b/I_A, f_p)$ is illustrated in Figure 2 as a function of the normalized plasma density for different values of $I_b/I_A$. Note that Eq. (31) gives a different sign for the perveance than Olson’s electrostatic result for a plasma plug, $Q_e = Z_b m_e/M$ (Olson, 2001a). Also, the perveance in Eq. (31) is greatly reduced for the case of beam propagation in dense plasma with $n_p \gg Z_b n_b$.

In summary, Eqs. (6)–(21) describe the quasi-steady-state self-consistent electron motion induced by a long, dense ion charge bunch. Examples of calculations and comparisons with the results of electromagnetic particle-in-cell simulations can be found in Kaganovich et al. (2001) and Kaganovich (2002).

5. DISCUSSION

The propagation of a finite-length ion beam pulse through a background plasma has been studied. The analytical solutions for the electric and magnetic fields generated by the ion beam pulse have been determined in the nonlinear case for arbitrary values of beam and plasma densities, under the assumption of a long beam, where the beam length is much longer than the beam radius. Under these conditions, a reduction in the dimensionality of the problem is possible. Assuming an axisymmetric beam pulse, the longitudinal electron flow velocity is determined for one-dimensional variations in the radial direction for each axial slice of the beam [Eq. (9)]. The electric and magnetic fields are then readily calculated from the longitudinal electron flow velocity using Eqs. (6) and (8), respectively.

The approach used here can be generalized to the case of nonuniform, nonstationary plasma density and beam density profiles, and forms the basis for a hybrid semianalytical approach to be used for calculations of beam propagation in
the target chamber. This work is now underway. The analytical formulas derived in this article can provide an important benchmark for numerical codes and provide scaling laws for different beam and plasma parameters.

The charge neutralization depends crucially on the beam length, and is determined by product of beam pulse duration to plasma frequency, \( l_b \omega_p / V_b \). If \( l_b \omega_p / V_b > 1 \), the degree of charge neutralization is very close to unity. Current neutralization is usually weaker than charge neutralization. Therefore, the magnetic pinching force dominates the electric force, and total effective perveance is negative during quasi-steady-state beam propagation through the background plasma. The degree of current neutralization is determined by the ratio of the beam radius to the skin depth, \( r_b / (c/\omega_p) \). The effective perveance of the beam including the effects of background plasma is given by Eq. (30).

In summary, the analytical results agree well with the results of numerical simulations for ion beam charge and current neutralization. The visualization of the data obtained in the numerical simulations shows complex collective phenomena during beam entry into and exit from the plasma, and will be described in future publications. Further visualization is also available on the website http://w3.pppl.gov/~nnp.

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REFERENCES


