I. WEIBEL INSTABILITY IN UNIFORM BEAM.

Analysis of exact dispersion relation for fillamentation gives a low frequency mode with frequency (increament)

$$\omega^2 = -k_\perp^2 \frac{(V_b - V_e)^2 \omega_{pb}^2 + V_e^2 \omega_{pi}^2}{k_\perp^2\ c^2 + \omega_{pe}^2}. \tag{1}$$

If the return current is absent, i.e. $V_{\parallel e} = 0$

$$\omega^2 = -\frac{k_\perp^2 V_b^2 \omega_{pb}^2}{k_\perp^2 c^2 + \omega_{pe}^2} \tag{2}$$

which describes attraction of parallel current filaments in the ion beam ($\omega = iV_b\omega_{pb}/c$ for $k_\perp^2 c^2 >> \omega_{pe}^2$). If the width of the mode is longer than the skin depth ($k_\perp^2 c^2 << \omega_{pe}^2$), the magnetic field of the mode is screened by the electron return current. The reduction of increment for long wave length is accounted by denominator.

A. Explanation of dispersion relationship for $V_{\parallel e} \neq 0$.

During fillmentation, $\omega << \omega_{pe}$, so the quasineutrality has to be maintained, electrons should be in a radial balance (otherwise their density perturbations are too big), then the radial force on electrons is nearly zero $-V_e B_\perp/c + E_\perp = 0$

Perturbation of the beam ion density is then

$$-i\omega\delta n_b = -ik_\perp v_{\perp b} n_b, \tag{3}$$

where

$$-i\omega v_{\perp b} = -\frac{e_b}{m_b c} V_{\parallel b} B_\perp + \frac{e_b}{m_b} E_\perp = -\frac{e_b}{m_b c}(V_\parallel - V_e) B_\perp \tag{4}$$

and

$$\delta n_b = -\frac{ik_\perp}{\omega^2} \frac{e_b n_b}{m_b c}(V_{\parallel b} - V_e) B_\perp. \tag{5}$$

Similarly for ions

$$\delta n_i = -\frac{ik_\perp}{\omega^2} \frac{e_i n_i}{m_i c}(V_{\parallel i} - V_e) B_\perp. \tag{6}$$

Electron density can be determined from the quasineutrality condition

$$e\delta n_e = e_i \delta n_i + e_b \delta n_b. \tag{7}$$

Finally, substituting it to Maxwell's equations (in the last term only electron acceleration in the inductive field has to be accounted for):

$$ik_\perp B_\perp = \frac{4\pi}{c}\left(e_b V_{\parallel b}\delta n_b - eV_{\parallel e}\delta n_e + e_i V_{\parallel i}\delta n_i\right) - \left(\frac{\omega_{pe}^2}{ic\omega} + \frac{i\omega}{c}\right)E_\parallel, \tag{8}$$

$$E_\parallel = -\frac{\omega}{ck_\perp}B_\perp, \tag{9}$$

and we have

$$-k_\perp^2 c^2 = k_\perp^2\left[\frac{\omega_{pb}^2(V_{\parallel b} - V_{\parallel e})^2}{\omega^2} + \frac{\omega_{pi}^2(V_{\parallel i} - V_e)^2}{\omega^2}\right] + \left(\omega_{pe}^2 - \omega^2\right), \tag{10}$$

or

$$-\omega^2 = \frac{k_\perp^2\left[\omega_{pb}^2(V_{\parallel b} - V_{\parallel e})^2 + \omega_{pi}^2(V_{\parallel i} - V_e)^2\right]}{k_\perp^2 c^2 + \omega_{pe}^2}. \tag{11}$$

There additional large terms in instability are associated with perturbation of the ion background plasma density by the radial electric field and subsequent filling by electrons, where ions left. Perturbations of the electron density produces additional current perturbations and enhancement of the magnetic field perturbations, which amplifies instability, especially for the case of light ions in the background plasma.