

Nonlinear plasma waves excitation by intense ion beams in background plasma

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Plasma neutralization of an intense ion pulse is of interest for many applications, including plasma lenses, heavy ion fusion, cosmic ray propagation, etc. An analytical electron fluid model has been developed to describe the plasma response to a propagating ion beam. The model predicts very good charge neutralization during quasi-steady-state propagation, provided the beam pulse duration τ_b is much longer than the electron plasma period $2\pi/\omega_p$, where $\omega_p = (4\pi e^2 n_p / m)^{1/2}$ is the electron plasma frequency, and n_p is the background plasma density. In the opposite limit, the beam pulse excites large-amplitude plasma waves. If the beam density is larger than the background plasma density, the plasma waves break. Theoretical predictions are compared with the results of calculations utilizing a particle-in-cell (PIC) code. The cold electron fluid results agree well with the PIC simulations for ion beam propagation through a background plasma. The reduced fluid description derived in this paper can provide an important benchmark for numerical codes and yield scaling relations for different beam and plasma parameters. The visualization of numerical simulation data shows complex collective phenomena during beam entry and exit from the plasma.

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I. INTRODUCTION

Ion beam pulses are frequently used in many applications, including heavy ion inertial fusion,^{1,2} positron beams for electron-positrons colliders,³ high-density laser-produced proton beams for the fast ignition of inertial confinement fusion targets,⁴ etc. In these applications unneutralized ballistic focusing is difficult due to the large repulsive space-charge force of the beam ions. To neutralize the ion beam charge, the ion beam can be transported through a background plasma. The plasma electrons tend to effectively neutralize the ion beam charge, and the background plasma can provide an ideal medium for ion beam focusing. There are many critical parameters for ion beam transport in a background plasma, including beam current, type of ion species, radial and longitudinal profiles of the beam density and plasma density, stripping and ionization cross sections of the beam ions and gas atoms, etc. This necessitates an extensive study for a wide range of parameters to determine the conditions for optimum beam propagation. To complement the numerical simulation studies, a number of reduced models have been developed.⁵⁻⁹ Based on well-verified assumptions, reduced models can yield robust analytical and numerical descriptions, and provide important scaling laws for the degrees of charge and current neutralization. Typically, the ion beam pulse propagation duration through the background plasma is long compared with the electron response time, which is determined by the electron plasma frequency, $\omega_p = (4\pi n_p e^2 / m_e)^{1/2}$, where n_p is the background plasma density. Therefore, after the beam passes through a short transition region, the plasma disturbances are stationary in the beam frame. The goal of this

study is to develop a reduced nonlinear model, which describes the plasma disturbance excited by the intense ion beam pulse.

If the beam density is small compared to the background plasma density, a linear perturbation theory can describe the plasma perturbations well.⁵ Here, we focus on the general case where the plasma density has an arbitrary value compared to the beam density. Reference 6 studied the transport of a long ion beam pulse utilizing the assumptions of complete current and charge neutralization. In the present study, we determine the degree of current and charge neutralization from a nonlinear analysis. Nonlinear fluid models have been developed for studies of the plasma response to a propagating laser pulse.⁷ Considerable simplification of these models has been made by assuming that the plasma parameters depend on z and t exclusively through the combination $\zeta = z - ct$. Therefore, the solutions are time stationary in the laser pulse frame. Here, z is the coordinate along the laser pulse propagation direction in the laboratory frame, t is the time, and c is the speed of light *in vacuo*. The reduced models are also based on the fact that the laser pulse moves with the speed of light. Plasma perturbations do not propagate faster than the speed of light. Therefore, different cross sections of the perturbed plasma perpendicular to the propagation direction do not communicate nonlocally with one another. As a result, a considerable reduction in the amount of necessary computation can be achieved, as will be explained in more detail below.

In the present study, a similar procedure has been developed for an ion beam pulse propagating through a background plasma. The ion beams under consideration are typically not relativistic. As a result, different cross sections perpendicular to the propagation direction do communicate

nonlocally with one another via the Poisson equation: the electrostatic potential is a nonlocal function of the space charge over the entire beam pulse. However, for long beams with beam half-length much longer than the beam radius, $l_b \gg r_b$, only neighboring slices interact with one another. The assumption of long beams ($l_b \gg r_b$) allows one to efficiently reduce the dimensionality of the equations. In recent calculations,^{8,9} we studied the nonlinear quasiequilibrium properties of an intense, long ion beam pulse propagating through a cold, background plasma, assuming that the beam pulse duration τ_b is much longer than the inverse electron plasma frequency, i.e., $\omega_p \tau_b \gg 1$. In the present study, we extend the previous results to general values of the parameter $\omega_p \tau_b$. The key assumption in this paper is that the electron thermal velocity can be neglected, because it is much smaller than ion beam velocity. This assumption allows us to use a fluid approximation and obtain an analytical solution for the self-electric and self-magnetic fields of the ion beam pulse.

II. BASIC EQUATIONS FOR ION BEAM PULSE PROPAGATION IN BACKGROUND PLASMA

In many applications, the background plasma electrons are cold—the electron thermal velocity is small compared with the directed beam velocity.¹⁰ Particle-in-cell simulations show that in most cases the electron flow is laminar and does not form multistreaming. For purposes of the plasma wave excitation study, we assume that the ion beam motion is unperturbed. Thus, the cold-fluid equations are used for the electron description, and thermal effects are neglected in the present study. The electron fluid equations, together with Maxwell's equations, comprise a complete system of equations describing the electron response to the propagating ion beam pulse. The electron cold-fluid equations consist of the continuity equation

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{V}_e) = 0, \quad (1)$$

and the force balance equation

$$\frac{\partial \mathbf{p}_e}{\partial t} + (\mathbf{V}_e \cdot \nabla) \mathbf{p}_e = -e \left(\mathbf{E} + \frac{1}{c} \mathbf{V}_e \times \mathbf{B} \right), \quad (2)$$

where $-e$ is the electron charge, $\mathbf{V}_e$ is the electron flow velocity, $\mathbf{p}_e = \gamma_e m_e \mathbf{V}_e$ is the average electron momentum, m_e is the electron rest mass, and $\gamma_e = (1 - \mathbf{V}_e^2/c^2)^{-1/2}$ is the relativistic mass factor. Maxwell's equations for the self-generated electric and magnetic fields, $\mathbf{E}$ and $\mathbf{B}$, are given by

$$\nabla \times \mathbf{B} = \frac{4\pi e}{c} (Z_b n_b \mathbf{V}_b - n_e \mathbf{V}_e) + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}, \quad (3)$$

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \quad (4)$$

where $\mathbf{V}_b = \text{const}$ is the ion beam flow velocity, n_e and n_b are the number densities of the plasma electrons and beam ions, respectively (far away from the beam $n_e \rightarrow n_p$, where n_p is background plasma density), and Z_b is the ion beam charge state. The plasma ions are pushed away from the ion beam pulse by the radial electric field. However, if the ion beam

pulse duration is sufficiently short ($2l_b < r_b \sqrt{M/m_e}$, where M is the plasma ion mass), the plasma ions do not have time to move outside the beam radius, and they can be assumed to remain stationary with $V_i = 0$ and $n_i = n_p$.⁸

Additional simplification is achieved by applying the conservation law of generalized vorticity Ω ,⁸ where

$$\Omega \equiv \nabla \times \mathbf{p}_e - \frac{e}{c} \mathbf{B}. \quad (5)$$

In fluid mechanics, for incompressible flow, the vorticity $\nabla \times \mathbf{V}$ is conserved along the path of a fluid element. Moreover, if all the streamlines originate from the region where the vorticity is equal to zero, then the vorticity is equal to zero everywhere, and the flow is eddy free. In the background plasma without the beam pulse, $\mathbf{p}_e = \mathbf{B} = 0$ and $\Omega = 0$. Therefore, it follows from the conservation of generalized vorticity, similar to fluid mechanics, that Ω is equal to zero in the beam pulse as well, i.e.,

$$\nabla \times \mathbf{p}_e = \frac{e}{c} \mathbf{B}. \quad (6)$$

Note that Eq. (6) is a consequence of the fluid description and is not an *a priori* assumption. Substituting Eq. (6) into Eq. (2) yields

$$\frac{\partial \mathbf{p}_e}{\partial t} + \nabla K_e = -e \mathbf{E}, \quad (7)$$

where $K_e = (\gamma_e - 1) m_e c^2$ is the electron kinetic energy. Note that the inertia terms in Eq. (7) are comparable in size to the Lorentz force term and cannot be omitted. Estimating the self-magnetic field from Eq. (6), we conclude that the electron gyroradius is of order of the beam radius. This is a consequence of the fact that the electrons originate from the region of zero magnetic field in front of the beam. If most electrons are dragged along with the beam and originate from the region of large magnetic field, the situation may be different.^{11,12}

III. APPROXIMATE SYSTEM OF EQUATIONS FOR LONG CHARGE BUNCHES ($l_b \gg r_b$)

In this section, we assume a long ion beam pulse ($l_b \gg r_b$), but relax the assumption of a dense plasma used in Refs. 8 and 9, i.e., the condition $V_b/\omega_p \ll l_b$. The typical longitudinal scale of electron density perturbations is V_b/ω_p . If $V_b/\omega_p \ll r_b$, the main spatial variations are in the longitudinal direction and the electron dynamics can be described by a one-dimensional model. In the opposite case, when $V_b/\omega_p \gg r_b$, the main spatial variations are in the transverse direction, and the longitudinal derivatives can be neglected in comparison with the radial derivatives in Poisson's equation. The criterion $V_b/\omega_p \gg r_b$ can be expressed as a condition on the total ion beam current, $I_b < 17\beta^3 n_b/n_p kA$, and this condition pertains to ion beams that aren't extremely intense. We also assume an axisymmetric beam pulse ($\partial/\partial\theta = 0$). The dependence on z and t is assumed to be exclusively through $\zeta = z - V_b t$. Therefore, solutions are time stationary in the beam frame. This gives

$$(V_{ez} - V_b) \frac{\partial n_e}{\partial \zeta} + \frac{1}{r^n} \frac{\partial}{\partial r} (r^n n_e V_{er}) = 0, \quad (8)$$

$$\frac{1}{r^n} \frac{\partial}{\partial r} (r^n E_r) = 4\pi e (Z_b n_b + n_p - n_e), \quad (9)$$

$$(V_{ez} - V_b) \frac{\partial}{\partial \zeta} p_{er} + \frac{\partial K_e}{\partial r} = -e E_r. \quad (10)$$

Here, $n=0$ for slab geometry ($r \rightarrow x$) and $n=1$ for a cylindrical beam. Single-charged plasma ions are assumed with $Z_b=1$. As a result, the two-dimensional problem is reduced to a one-dimensional problem, with time derivatives being replaced by $(V_{ez} - V_b) \partial / \partial \zeta$. It follows from Eq. (6) and the assumption of a long charge bunch that the azimuthal self-magnetic field for $\partial / \partial \theta = 0$ is determined in terms of the longitudinal flow velocity, which gives

$$B_\theta = -\frac{c}{e} \frac{\partial p_{ez}}{\partial r}, \quad (11)$$

$$-\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} p_{ez} = \frac{4\pi e}{c} (Z_b n_b V_b - n_e V_{ez}). \quad (12)$$

Equation (12) determines the longitudinal electron flow velocity and can be used to calculate the degree of current neutralization of the beam. In Eq. (12), we have neglected the displacement current.¹³ The electric field along the beam propagation direction can be determined from Eq. (7), which gives

$$(V_{ez} - V_b) \frac{\partial}{\partial \zeta} p_{ez} + \frac{\partial K_e}{\partial \zeta} = -e E_z. \quad (13)$$

It follows from Eqs. (10) and (13) that $E_z / E_r \sim r_b / l_b \ll 1$. Therefore, the longitudinal electric field is small compared with the radial electric field, and E_z has been neglected in comparison with E_r in Poisson's equation (9).

To check the theoretical predictions, we have utilized a two-dimensional (2D) electromagnetic particle-in-cell (PIC) code, called EDPIC.¹⁴ The code uses a leapfrog, finite-difference scheme¹⁵ to solve Maxwell's equations (3) and (4) on a planar, two-dimensional rectangular grid in the laboratory frame. EDPIC uses a moving-window approach. The window of the simulations is shifted after a few time steps so that the window is moving with the beam on average. The current deposition scheme is designed to conserve charge exactly,¹⁶ so there is no need to solve Poisson's equation. Since the plasma ahead of the pulse is electrically neutral, the boundary conditions for the fields on the front boundary of the simulation box are trivial ($\mathbf{E} = \mathbf{B} = 0$). The boundaries are located at $3r_b$ and $1.5l_b$ in the radial and axial directions, respectively. As can be seen from Fig. 1, the plasma and field perturbations do not reach the boundaries (as the boundaries are located far away from the beam at distances larger than the beam radius and the skin depth). In the present simulations, the dynamics of the (stationary) background ions is neglected, and the background plasma electrons are initially cold. The beam ions are represented by a stationary (in the moving frame) current density on the simulation grid. To

advance the electrons, we use the time-centered, leapfrog scheme first introduced in Ref. 17. Various simulations obtained using the EDPIC particle-in-cell code are presented in Refs. 8, 18, and 19. The fluid simulations utilize the MacCormack finite-difference scheme²⁰ to solve the system of equations (8)–(12) on a two-dimensional rectangular grid.

In Fig. 1, we present a detailed comparison of the fluid and PIC results in slab geometry. The fluid simulation results agree well with the results of the two-dimensional electro-magnetic PIC simulations. Figure 1 shows similar beams with a constant ratio of beam length to radius ($l_b / r_b = 10$) and different beam densities. The background plasma density is maintained at twice the beam density. If the plasma density is sufficiently small that the plasma period is long compared with the beam pulse duration, i.e., $2\pi / \omega_p > 2l_b / V_b$, the plasma does not have time to respond to the beam pulse and a plasma wake appears behind the beam pulse, as evident in Fig. 1(a). As the beam and background plasma density increase, the plasma period shortens and the number of plasma oscillations increases during the beam pulse. Despite the fact that $n_b = 0.5n_p$, the plasma oscillations excited by the beam pulse are quite large—the electron density perturbation can be as large as $3n_p$ [see Fig. 1(b)].

Note that the longitudinal scale of the plasma oscillations is of order V_b / ω_p and can be smaller than the beam radius ($V_b / \omega_p < r_b$), as in Fig. 1(d). As a result, the assumption of mainly radial dynamics fails in this case, and the agreement between the fluid and particle-in-cell simulations is not as good as in the case where $V_b / \omega_p > r_b$. However, if the plasma oscillations are not large amplitude, i.e., the perturbations in the electron density are small compared with the background density, which corresponds to $Z_b n_b \ll n_p$, the plasma oscillations can be described by linear theory. Interestingly, the reduced system of equations (8)–(12) recovers the results of the full system of equations in the linear approximation. Indeed, in the linear case, when $Z_b n_b \ll n_p$, the equation for the plasma oscillations is⁵

$$V_b^2 \frac{\partial^2}{\partial \zeta^2} n_e + \omega_p^2 (n_e - Z_b n_b - n_p) = 0. \quad (14)$$

Equation (14) is readily derived from the linearized version of Eqs. (1) and (2) and Poisson's equation, and is not restricted by any requirements on the beam radius. It can also be derived from the reduced system of equations (8)–(12), if the nonlinear terms are neglected. Because the reduced model consisting of the system of equations (8)–(12) gives the same results in the limit $V_b / \omega_p \gg r_b$, and in the linear case $Z_b n_b \ll n_p$ for any values of V_b / ω_p and r_b , it also applies reasonably well in the intermediate case where $V_b / \omega_p \sim r_b$, as can be seen in Fig. 1(d).

Figure 1(c) shows clearly the limitations of the fluid approach. Pairs of fluid electron trajectories intersect at the points $x = \pm 0.19c / \omega_p$ and $y = -1.8c / \omega_p$. Correspondingly, the fluid equations have a singularity at this point, and the electron density tends to infinity [see Fig. 1(c), bottom frame]. Particle-in-cell simulations do not show any singularity in the electron density, as evident in Fig. 1(c) (top left and bottom frames). Therefore, the presence of the singular-

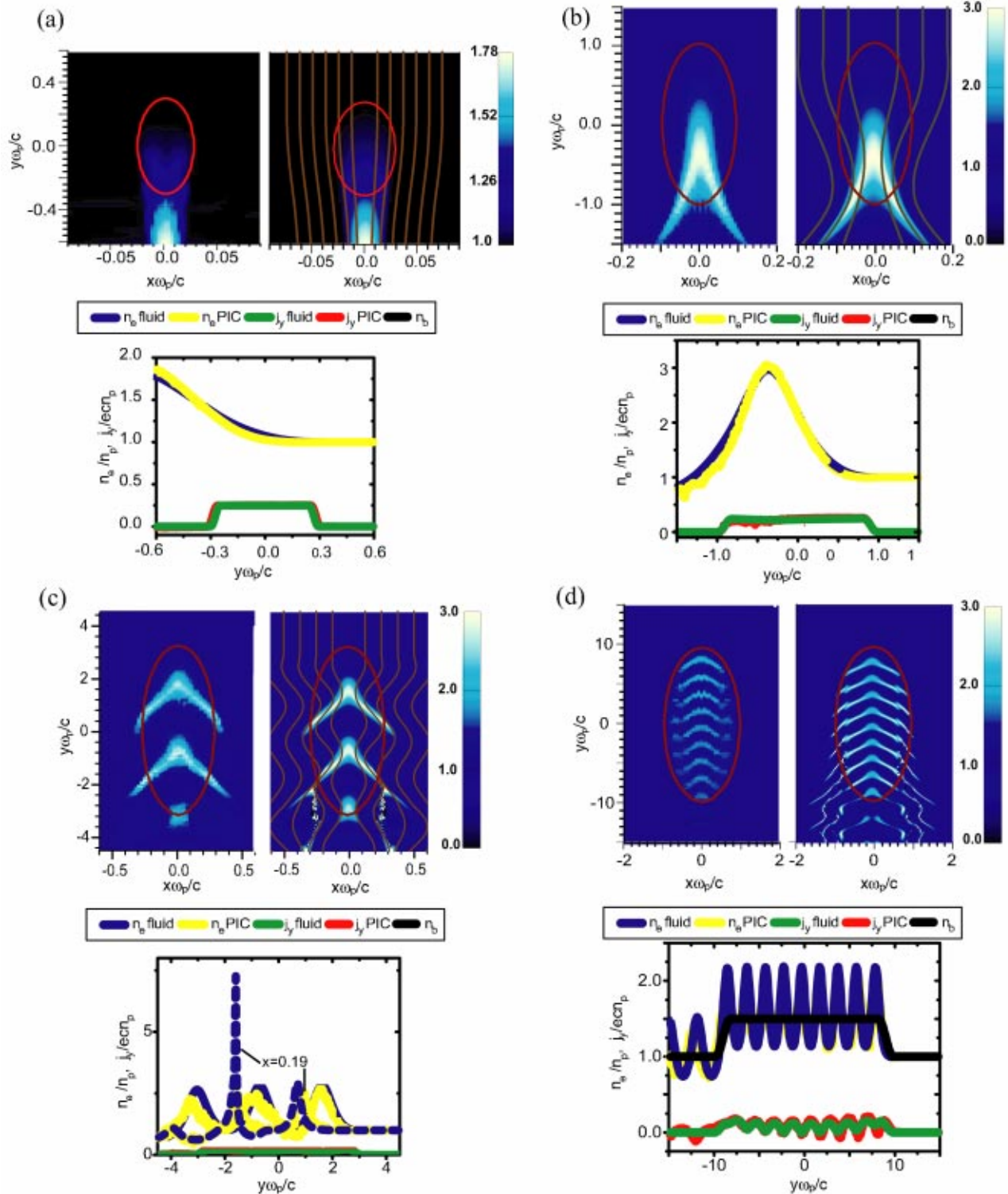


FIG. 1. (Color online). Neutralization of an ion beam pulse during steady-state propagation of the beam pulse through a cold, uniform, background plasma in planar geometry. The figure shows comparisons between the PIC simulations and the fluid description. The beam propagates in the y direction. The beam density has a flat-top density profile, and the red lines show the beam pulse edge. Shown in the figure are color plots of the normalized electron density (n_e/n_p) for particle-in-cell simulations (top left) and the fluid model consisting of Eqs. (8)–(12) (top right) in $(x\omega_p/c, y\omega_p/c)$ space. The lower figure shows the normalized electron density (n_e/n_p), and the normalized longitudinal current ($j_y/en_p c$) in the beam cross section at $x=0$ (lowest curves). The brown contours in the upper figure show the electron trajectories in the beam frame. The beam velocity is $V_b=0.5c$, the beam density is $n_b=0.5n_p$, and the ion beam charge state is $Z_b=1$. The beam dimensions correspond to $l_b/r_b=10$ and (a) $l_b=0.3c/\omega_p$; (b) $l_b=1.0c/\omega_p$; (c) $l_b=3.0c/\omega_p$; and (d) $l_b=10c/\omega_p$. The dashed lines in the lower part (c) show wave breaking at $x=0.19c/\omega_p$.

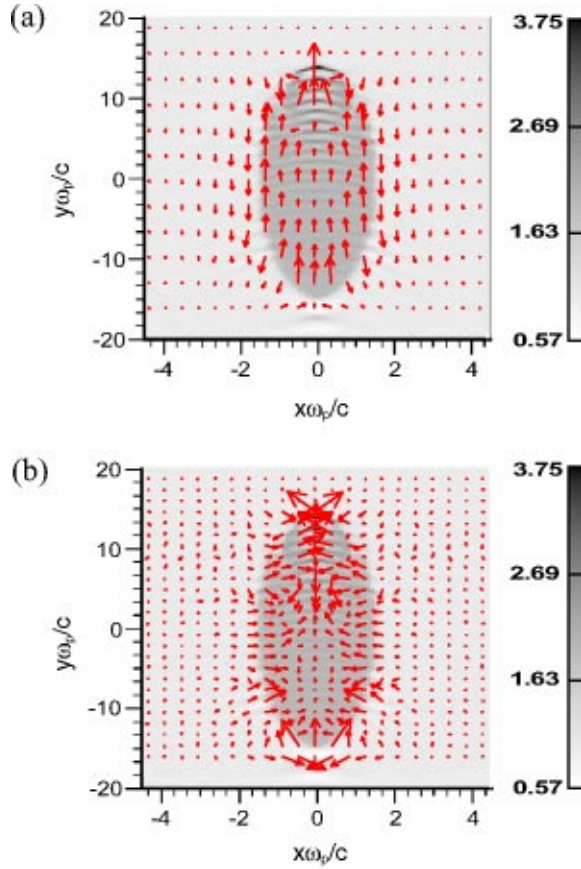


FIG. 2. (Color online). The excitation of plasma waves by the beam head is calculated in two-dimensional slab geometry using the EDPIC code (Ref. 14) for the following beam parameters: $V_b = 0.5c$, $Z_b = 1$, $n_b = n_p$, $l_b/r_b = 10$, and $l_b = 15c/\omega_p$. Shown in the figure are electron charge density contour plots in $(x\omega_p/c, y\omega_p/c)$ space and the vector fields for (a) the total current, and (b) minus the electric field, $-E$.

ity in the fluid solution for the electron density points to a failure of the fluid model. Specifically, the basic assumption of the fluid approach, that there is a uniquely defined, single-valued electron flow velocity at every point, breaks down.

Figure 2 shows the total current and electric field in the case of a long beam pulse. Because the beam pulse length, $2l_b/V_b$, is much longer than the plasma period, $2\pi/\omega_p$ ($l_b\omega_p/\pi V_b = 9.6$), the ion beam charge is well neutralized. However, the ion density rises steeply at the head of the beam pulse on a time scale comparable with the plasma period. This sudden increase in ion density drives many plasma oscillations during the beam pulse. Because the ion beam charge density is well neutralized in an average sense, the total current (the sum of the ion beam current and the electron current) is divergence free ($\nabla \cdot \mathbf{j} = 0$). This means that the ion beam current should be short-circuited outside the beam pulse by the electron return current, which is illustrated in Fig. 2(a). In Fig. 2(b), we note that the longitudinal electric field is small compared with the radial electric field ($E_z/E_r \sim r_b/l_b$). Short wavelength plasma oscillations at the beam pulse head, however, can produce larger longitudinal electric fields.

The fluid approach may not be applicable for a tenuous background plasma. In Fig. 1(c), the intersecting fluid elec-

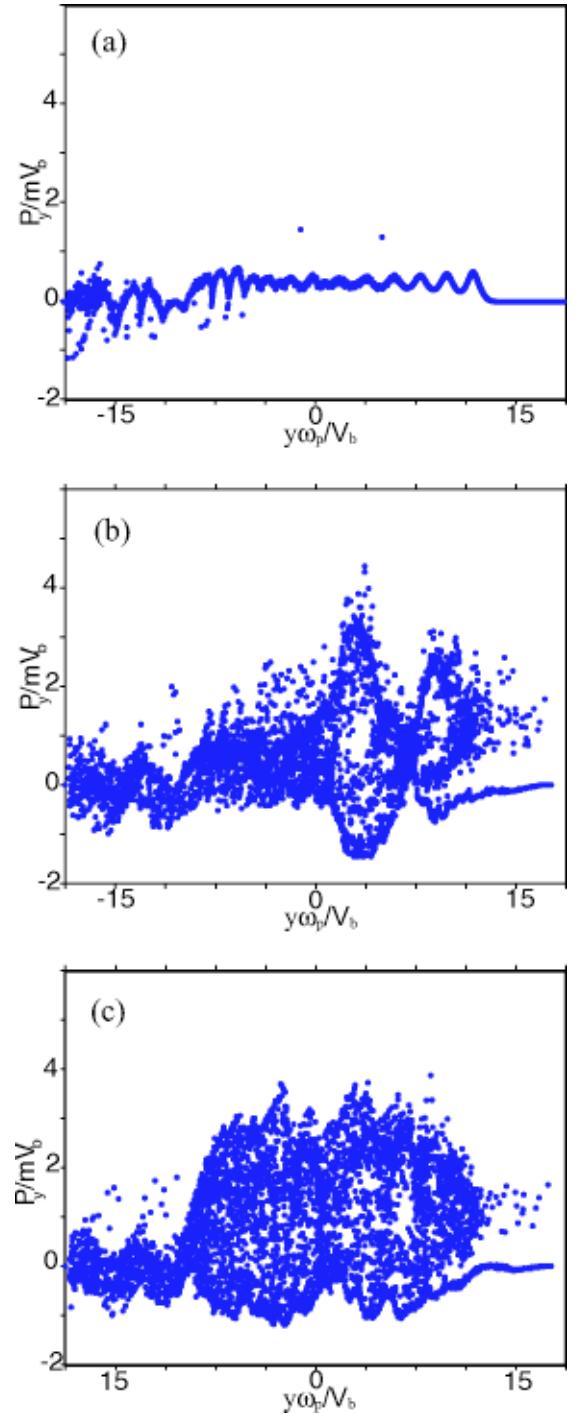


FIG. 3. (Color online). Electron phase space for 1D PIC simulations of the beam entering the plasma at $t = 0$. Here, $V_b = 0.5c$, $Z_b = 1$, $l_b = 7.5c/\omega_p$, and (a) $n_b = n_p$; (b) $n_b = 2n_p$; and (c) $n_b = 2n_p$. The steady state establishes after the beam enters the plasma in case (a). Under these circumstances, the plasma phase space also shows applicability of the fluid approximation. However, in case (b) the plasma wave breaks and the electrons are heated by wave-particle interactions. In this situation, the cold-fluid approximation is not applicable. Moreover, the phase space slowly evolves with time as shown. The times after entering the plasma correspond to (b) $t = 113/\omega_p$, and (c) $t = 245/\omega_p$.

tron trajectories originate from different radial positions ahead of the beam pulse. In the limit $n_b > n_p$, the intersecting electron trajectories can originate from different longitudinal positions ahead of the beam pulse, which corresponds

to plasma wave breaking in one-dimensional geometry⁸ as illustrated in Fig. 3. If the beam density is less than the background plasma density ($n_b < n_p$), the amplitude of the plasma oscillations is finite. In Fig. 3(a), the electron dynamics can be described by the fluid approach: a phase space plot shows that there is a uniquely defined, single-valued electron flow velocity at every point. If $n_b > n_p$, the electron density oscillations may become large, and the intersecting electron trajectories originate from different longitudinal positions. This is evident from the phase space plot shown in Figs. 3(b) and (c). The time evolution in phase space of the electron distribution corresponds to the complex process of electron heating due to plasma wave breaking. The analysis described above shows that for good charge neutralization the ion beam should be neutralized by plasma with density larger than the ion beam density. In the opposite case, the ion beam head may excite large-amplitude plasma waves. This will lead to electron heating and the generation of large, uncontrollable radial electric fields, which may be detrimental to ion beam focusing.

IV. CONCLUSIONS

We have shown that the electron perturbations excited by a long ion beam pulse ($l_b \gg r_b$) can be well described in the fluid approximation by the reduced system of equations (8)–(12). Based on well-verified assumptions, this reduced model can yield robust analytical and numerical descriptions and provide important scaling laws for the degrees of charge and current neutralization. However, the utilization of the cold-fluid model may be limited by the wave-breaking condition (intersection of fluid electron trajectories), especially in the case $n_b > n_p$.

The approach used here can be generalized to the case of a nonuniform, nonstationary, warm electron fluid. This research is now underway. This more general approach will be especially useful for studies of the long-time evolution of ion beam pulses on time scales much longer than the plasma period $2\pi/\omega_p$, because it enables one to eliminate the fast processes (at the plasma frequency) from the governing equations, thereby substantially reducing the computational requirements. For example, simplified beam envelope equations can be developed based on the reduced system of equations.⁹ These envelope equations are known to predict beam focusing with reasonable accuracy.²¹

The analysis described in this paper shows that for good charge neutralization the ion beam should be neutralized by plasma with density larger than the ion beam density. The condition for avoiding large-amplitude plasma wave generation requires that the rise time of the ion beam pulse be much longer than the plasma period $2\pi/\omega_p$. In the opposite case, the ion beam head excites large-amplitude plasma waves. This may lead to electron heating and the generation of large, uncontrollable radial electric fields, which could be detrimental to ion beam focusing. The parameter which controls the degree of current neutralization is the ratio of the skin depth to the beam radius, which is discussed in greater detail elsewhere.^{8,9}

Finally, although steady-state propagation of the ion beam pulse in a background plasma has been well studied, the beam entry and exit from the plasma requires additional research. We have produced movies that illustrate the complex collective phenomena that occur during beam entry into and exit from the plasma.²² Additional discussion of this subject is also provided in Ref. 19.

ACKNOWLEDGMENTS

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- ²²See EPAPS Document No. E-PHPAEN-11-025407 for movies showing highly complex collective behavior of the electron motion during the entry of the intense beam into the background plasma). A direct link to this document may be found in the online article's HTML reference section. The document may also be reached via the EPAPS homepage (<http://www.aip.org/pubservs/epaps.html>) or from <ftp.aip.org> in the directory /epaps. See the EPAPS homepage for more information.