Enhanced collisionless heating in a nonuniform plasma at the bounce resonance condition

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The importance of accounting for a nonuniform density profile for modeling of collisionless electron heating in a bounded low-pressure plasma is demonstrated. A drastic enhancement of the power transfer into an inductive plasma under the condition of a bounce resonance is observed if the nonuniformity of the plasma density profile is accounted for. This enhanced plasma heating is attributed to the increase of the number of resonant electrons, for which the bounce frequency of electrons confined inside the plasma potential is equal to the rf field frequency. © *2005 American Institute of Physics*. [DOI: 10.1063/1.1986163]

At low pressures (millitorr region), inductive plasmas exhibit a number of peculiar physical effects typical for warm plasmas, such as a resonant wave-particle interaction and an anomalous skin penetration.[1–3] An interesting effect that can lead to enhanced heating for bounded low-pressure plasmas is a bounce resonance between the frequency ω of the driving rf field and the frequency Ω_b of the bounce motion of the plasma electrons confined into the potential well by an electrostatic ambipolar potential $\phi(x)$ and the sheath electric fields near the plasma edges.[4–8] Most earlier theoretical and numerical studies on this subject assumed for simplicity a uniform plasma density and the absence of an electrostatic potential. As a result, the electrons bounced inside a potential that is flat inside the plasma and infinite at the walls.[7,9,10] Although these suppositions can result in a qualitative description of the plasma behavior under nonresonant conditions, the plasma parameters under resonant conditions can be greatly altered by accounting for the presence of the electrostatic potential, which always exists in real-life bounded plasmas.

Note that though we consider inductively coupled plasmas, the formalism developed in this Letter, can be applied to other problems for the description of the wave-particle interaction in nonuniform plasmas, i.e., in semiconductor physics, laser-plasma interaction, collective phenomena in intensive beams, rf heating of plasmas in discharges and tokamaks, and so forth.

It is a well-known result of the quasilinear theory that the electron heating of low-collisional, warm plasmas essentially depends on the resonant electrons or electrons with velocities equal to the phase velocities of the plane waves constituting the rf field $\omega=\mathbf{v}\cdot\mathbf{k}$.[5,6] For bounded plasmas the k spectrum is discrete, and the above condition transforms into the requirement that the rf field frequency must be equal to or be an integer (n) multiple of the bounce electron frequency $\omega=n\Omega_b$. If the electron mean free path is much larger than the discharge gap L, the resonant electrons (with $\Omega_b=\omega/n$) accumulate velocity changes in successive interactions with the rf electric field, which lead to a very effective electron heating.[4,11,12] The electron bounce frequency is very sensitive to the actual shape of the electrostatic potential, especially for low-energy electrons. Accounting for the electrostatic potential can bring the plasma electrons into a resonant region, even if they were not there in the absence of the potential. The increase of the number of the resonant electrons results into a drastic enhancement of the plasma heating.

In this Letter, we present the results of a full, self-consistent numerical modeling of the low-pressure plasma on the specific example of an inductively coupled (ICP) discharge and demonstrate the pronounced influence of the electrostatic potential on the plasma parameters at the bounce resonance condition. However, the phenomenon described above is of importance for a wide range of problems related to penetration of electromagnetic waves into bounded low-pressure warm plasmas, and the developed formalism can be applied to other cases.[13]

Our model assumes a one-dimensional, slab geometry, ICP discharge of a plasma bounded on both sides by parallel walls with a gap length L. The surface currents, produced by an external rf source, flow in opposite directions. The induced rf electric field E_y is directed along the walls. The static electric field $E_x=-d\phi/dx$, directed towards the discharge walls, keeps electrons confined and the plasma quasineutral, i.e., $n_e(x)=n_i(x)$. In the present treatment of plasmas with density $n_e\sim 10^8-10^{12}$ cm^{-3}, the sheath width is neglected, because it is of the order of a few hundreds of microns, which is small compared with the discharge dimension L. Furthermore, it is assumed that the plasma electrons experience specular reflection either at the discharge walls $x_w=0$, L by the sheath electric field, if the electron energy $\varepsilon_x=mv_x^2/2-e\phi(x)$ is larger than $-e\phi(x_w)$, or at the turning points $x_\pm(\varepsilon_x)$, where $-e\phi(x_\pm)=\varepsilon_x$, by the static electric field in the plasma. The above one-dimensional (1D) scheme is a good approximation for a cylindrical ICP discharge, if the rf

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field penetration depth or skin depth δ into the plasma is less than the plasma cylinder radius R.[14]

A self-consistent study of the discharge properties involves calculation of the electron energy distribution function (EEDF) $f_0(\varepsilon)$, the rf electric field $E_y(x)$, and the ambipolar potential $\phi(x)$. The detailed description of the mathematical formalism is given in Refs. 13 and 15.

The EEDF $f_0(\varepsilon)$ is obtained from the temporal-spatial-averaged Boltzmann equation,

$$-\frac{d}{d\varepsilon}(D_\varepsilon+\overline{D_{\mathrm{ee}}})\frac{df_0}{d\varepsilon}-\frac{d}{d\varepsilon}[\overline{V_{\mathrm{ee}}}+\overline{V_{\mathrm{el}}}]f_0=St_{\mathrm{inel}}. \tag{1}$$

Here, the upper bar denotes spatial averaging according to Ref. 15, St_{inel} is the inelastic collision integral, and the coefficients V_{el}, D_{ee}, and V_{ee} stem from the elastic and electron-electron collision integrals, respectively, and are given in Ref. 16.

The energy diffusion coefficient, responsible for electron heating, is given by

$$D_\varepsilon(\varepsilon)=\frac{\pi}{4}\sum_{n=-\infty}^{\infty}\int_0^{\varepsilon}d\varepsilon_x \times|\Delta v_y(\varepsilon_x)|^2\frac{\varepsilon-\varepsilon_x}{\Omega_b(\varepsilon_x)}\frac{\nu}{[\Omega_b(\varepsilon_x)n-\omega]^2+\nu^2}, \tag{2}$$

where ν is the collision frequency and $\Omega_b(\varepsilon_x)=2\pi/T_b(\varepsilon_x)$, where $T_b(\varepsilon_x)=2\int_{x_-}^{x^+}dx/\sqrt{2[\varepsilon_x+e\varphi(x)/m]}$ is the half of the bounce period of electron motion in the potential well, and $\Delta v_y(\varepsilon_x)=e/m\int_0^{T_b(\varepsilon_x)}E_y[x(t)]e^{i\omega t}dt$ is the velocity kick acquired by an electron with energy ε_x during one bounce period.[15]

Electric field is obtained from a single scalar equation,

$$\frac{d^2E_y}{dx^2}+\frac{\omega^2}{c^2}E_y=-\frac{4\pi i\omega}{c^2}[j(x)+I\delta(x)-I\delta(x-L)], \tag{3}$$

where I is the surface (coil) current. $j(x)$ is the plasma electron current density calculated from the anisotropic part f_1 of the EEDF, obtained from the linearized Boltzmann equation.[15]

The electrostatic potential $\phi(x)$ is obtained using the quasineutrality condition $n_e(x)=n_i(x)$, where $n_e(x)=\int_{\varphi(x)}^{\infty}f_0(\varepsilon)\sqrt{\varepsilon-\varphi(x)}d\varepsilon$ is the electron-density profile and $n_i(x)$ is the ion density profile, obtained from a set of the fluid conservation equations for ion density and momentum.[17]

The total power P, deposited into the plasma per unit square of a side surface, is related to the electron energy diffusion coefficient $D_\varepsilon(\varepsilon)$ as[15]

$$P=-\sqrt{2m}\int_0^{\infty}d\varepsilon D_\varepsilon(\varepsilon)\frac{df_0(\varepsilon)}{d\varepsilon}. \tag{4}$$

The dependence of plasma heating on resonant electrons is especially pronounced for $\nu\ll\omega$, Ω_b,[13] because in this case the last factor on the right-hand side of Eq. (2) tends to the Dirac delta function. As a result, the electron heating does not depend on the collision frequency and accounts explicitly for the bounce resonance,

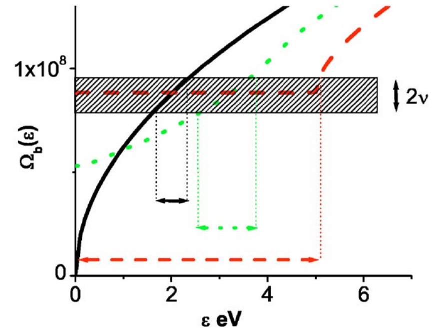

FIG. 1. (Color online). The electron bounce frequency $\Omega_b(\varepsilon_x)$, as a function of the electron energy $\varepsilon_x=mv_x^2/2-e\phi(x)$ for different potential wells, consisting of the reflecting walls and different ambipolar potentials $\phi(x)$. The solid line corresponds to a uniform plasma with $\phi(x)=0$, dashed line—quadratic potential $\phi(x)=5\times(2x/L-1)^2$ eV and dotted line—the realistic potential obtained from the ion fluid model with $T_e=5$ eV. The discharge length is $L=3$ cm. The box of height 2ν shows the resonance region, corresponding to $\omega=8.52\times10^7$ s^{-1}. The arrows show electron energies in the resonance region.

$$\Omega_b(\varepsilon_x)n=\omega. \tag{5}$$

However, if nonlinear effects are taken into account the collisionless heating may depend explicitly on the collision frequency.[18] Note that if $L\to\infty$, the summation in Eq. (2) can be replaced by integration over the corresponding wave vectors k, and the bounce resonance condition $\Omega_b(\varepsilon_x)n=\omega$ transforms into the wave-particle resonance condition $k_xv_x=\omega$ for a continuous-wave spectrum. The presence of ambipolar potential can greatly affect the electron heating due to the fact that the number of resonant electrons is generally larger for a nonuniform plasma than for a uniform plasma.[13] Equation (2) shows that for $\nu\ll\omega$ (when the effect of a bounce resonance is important) only resonant electrons, i.e., electrons in the energy range corresponding to $|\Omega_b(\varepsilon_x)n-\omega|<\nu$ or $\Omega_b(\varepsilon_x)n\in[\omega-\nu,\omega+\nu]$, give essential contributions to the energy diffusion coefficient. As is evident from Fig. 1, which shows the dependence of the electron bounce frequency $\Omega_b(\varepsilon_x)$ on the electron energy ε_x for different potential wells, the number of resonant electrons increases if the ambipolar potential is taken into account. In the limit of a parabolic potential, the bounce frequency is the same for all electrons and *all electrons can be resonant simultaneously.* The realistic potential is close to parabolic in the discharge center and changes faster at the plasma periphery. As a result, the number of resonant electrons in nonuniform plasma is much larger than in uniform plasma, see Fig. 1.

To explicitly show the importance of accounting for ambipolar potential on the calculation of plasma heating, we performed numerical simulations of the plasma resistance for uniform and nonuniform plasmas (*with* and *without* ambipolar potential) for a given Maxwellian EEDF. Specifically, we obtained results for the plasma surface resistance, or the real part of the surface impedance $Z=4\pi/c\times E_0/B_0$, as a function of the plasma length. E_0 and B_0 are the electric and magnetic fields at the wall.[3] The plasma surface resistance is related to the power deposition as $P=I^2\,\mathrm{Re}\,Z$, where I is the effective amplitude of the driving current. From Fig. 2 it is evident that the presence of ambipolar potential significantly

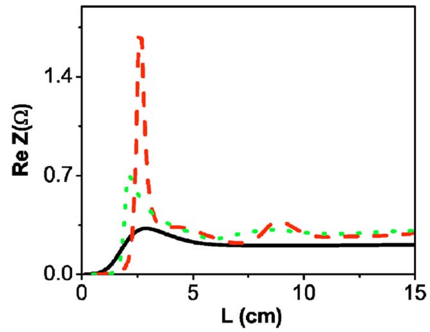

FIG. 2. (Color online). The plasma resistance Re Z as a function of the discharge gap L for a uniform plasma (without any ambipolar potential—the solid line) and nonuniform plasma (quadratic potential—the dashed line, and the realistic potential obtained from the ion fluid model—the dotted line) with a given Maxwellian EEDF. The plasma parameters are electron temperature $T_e=5$ eV, peak electron density at the discharge center $n_e=5\times10^{11}$ cm^{-3}, rf field frequency $\omega/2\pi=13.56$ MHz, and the electron collision frequency $\nu=10^7$ s^{-1}.

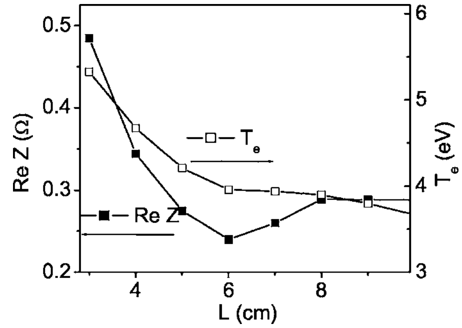

FIG. 3. Self-consistent simulations of the plasma surface resistance Re Z and the electron temperature T_e (defined as 2/3 of the average electron energy) at the discharge center as functions of the discharge gap. The discharge parameters are the coil current $I=5$ A/cm, the rf field frequency $\omega/2\pi=13.56$ MHz, and argon pressure $P=3$ mTorr.

enhances plasma resistance under the bounce resonance condition ($L\approx2.6$ cm), comparing to the case of a uniform plasma. The most profound change in resistance is observed for a quadratic potential, because in this case all trapped electrons have the same bounce frequency, and, thus, all of them are resonant. The maximum of plasma surface resistance in Fig. 2 occurs due to the first bounce resonance $n=1$ in Eq. (5). At larger L a smaller maximum exists due to the second resonance $n=2$ in Eq. (5). The obtained results explicitly show that neglecting the ambipolar potential, as is often done for simplicity, can lead to large discrepancies, especially for conditions close to the bounce resonance.

The bounce frequency increases with decreasing of the gap size for both uniform and nonuniform plasmas, but in the nonuniform plasma the bounce frequency for low-energy electrons does not tend to zero, as shown in Figs. 1 and 4(c). As a result, $\omega<\Omega_b$ can be satisfied for all electrons, which leads to complete disappearance of the collisionless heating for small gaps in the nonuniform plasma [see Figs. 2 and 4(a)] in contrast to the limit of uniform plasma.

The aforementioned phenomena have been observed in a fully self-consistent simulation of the EEDF, rf electric field, and ambipolar potential for a given coil current which have been performed for 13.56-MHz rf driving frequency. Figure 3 shows the dependence of the plasma surface resistance on the discharge dimension. The simulations have been performed for discharge gaps in the range of 3–10 cm (discharge cannot be sustained for $L<3$ cm). The calculated plasma surface resistance has a sharp maximum for the resonant condition $\omega=\Omega_b(\varepsilon_x)$, which corresponds to the discharge gap of 3 cm. Note that the plasma density is not a constant as in Fig. 2; it is approximately proportional to the plasma surface resistance, as more power ($P=I^2$ Re Z) is deposited for larger Re Z. Additional simulations have been performed for the twice lower discharge frequency −6.78 MHz. Figure 4(a) shows the result of self-consistent simulation of the plasma surface resistance for two coil currents, 1 and 5 A/cm. For lower discharge frequencies, the first bounce resonance corresponds to larger L. Correspondingly the maximum of plasma surface resistance shifts toward larger L, compare Figs. 4(a) and 3. However, the positions of the surface resistance maxima are different for different coil currents. This is due to the different plasma density and correspondingly skin depth in the two cases. The larger coil current transfers a larger power into the plasma, which results in a higher plasma density.[19] The higher plasma density, in turn, leads to the smaller skin depth. Further, it follows from Eqs. (4) and (2) that the electron heating is maximal if two conditions are met: the electron velocity kick $\Delta v_y(\varepsilon_x)$ is large for electron energies corresponding to the first bounce resonance $\omega=\Omega_b(\varepsilon_x)$, and the fraction of the

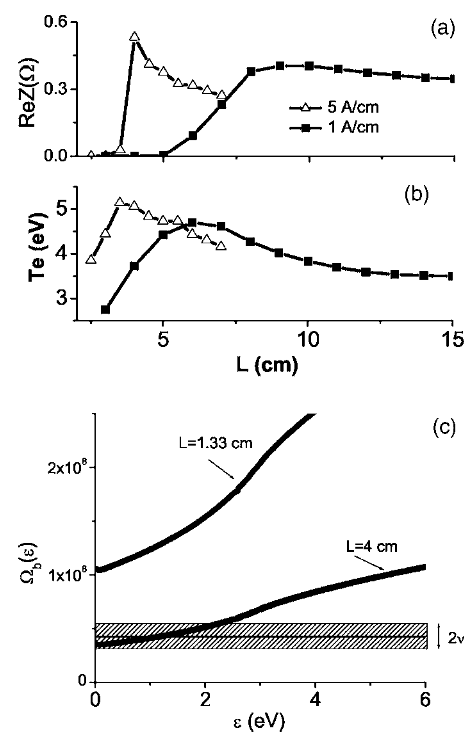

FIG. 4. Self-consistent simulations for coil currents $I=1$ A/cm and $I=5$ A/cm and the given discharge parameters $P=3$ mTorr and $\omega/2\pi=6.78$ MHz. Shown are (a) the plasma surface resistance, (b) the electron temperature in the discharge center vs the discharge gap, and (c) the electron bounce frequency.

resonant electrons satisfying the bounce resonance condition is not small. The velocity kick amplitude Δv_y is maximal if the transit time through the skin layer δ / v_x is approximately equal to $1/\omega$, i.e., $\omega \approx v_x / \delta$ (transit resonance). Combining the transit resonance condition with the first bounce resonance condition $\omega \approx v_x \pi / L$ estimated in uniform plasmas yields $L \approx \pi\delta$. As it is shown in Figs. 1 and 4(c), the fraction of the resonant electrons is not small if $\omega\delta \leq V_T$, where V_T is the thermal velocity. Thus, the resulting rate of the electron heating depends on both transit and bounce resonances and is maximal when both resonances are satisfied simultaneously, which occur at $L \approx \pi\delta$. Correspondingly, for larger discharge currents, the skin layer length is smaller and the position of the surface resistance maximum shifts into the region of smaller discharge gaps, as evident from Fig. 4(a). Similar results have been obtained in the numerical simulations in Ref. 20 (see Fig. 2 of that paper).

Figure 4(b) shows the electron temperature versus the discharge gap. Note, that our calculations show that the electron temperature grows with the increase of the discharge gap for small L. It differs from the predictions of the global model[21–23] based on the Maxwellian EEDF and particle balance $\nu_{\mathrm{ion}}[T_e] = \nu_{\mathrm{loss}}[T_e]$, where ν_{ion} is the ionization frequency and ν_{loss} is the loss frequency. The difference is due to non-Maxwellian shape of the EEDF for the conditions of Fig. 4.

In conclusion, enhanced electron heating and larger plasma densities (for a given current in the coil) can be achieved if low-pressure ICP discharges are operated under the bounce resonance conditions. Self-consistent simulations of the discharge plasma surface resistance and the electron energy distribution function demonstrate the significance of explicit accounting for the nonuniform plasma density profile and the correct form of ambipolar electrostatic potential. The formalism developed in this Letter can be applied to many different problems for the description of wave-particle interaction in nonuniform plasmas.

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