Collective Instabilities and Beam-Plasma Interactions for an Intense Ion Beam Propagating through Background Plasma *

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Abstract

This paper develops a formulary that summarizes the growth rates (e-folding lengths) for a wide range of collective beam-plasma instabilities for an intense ion beam propagating through a background plasma that provides complete charge and current neutralization. The instabilities considered here include: the electron-ion two-stream instability; the multispecies electromagnetic Weibel instability; and several beam-plasma instabilities that include the effects of an applied solenoidal magnetic field on the electron dynamics.

INTRODUCTION

There is growing interest in collective instabilities and beam-plasma interaction processes for an intense charged particle beam propagating through neutralizing background plasma, with applications ranging from the focusing of intense ion charge bunches to a small spot size, to ion-beam-driven high energy density physics and heavy ion fusion [1, 2]. Recent theoretical investigations [3, 4, 5, 6, 7, 8] have included advanced analytical and numerical studies of collective interactions and instabilities, including: the electron-ion two-stream instability [3, 4, 5] between the beam ions ($j = b$) and the plasma electrons ($j = e$), and the plasma ions ($j = i$) and plasma electrons ($j = e$); the effects of a velocity tilt on reducing two-stream instability growth rates [6]; the multispecies electromagnetic Weibel instability [3, 4, 5]; and the effects of a solenoidal magnetic field on several beam-plasma instabilities[7, 8].

In the present paper, a formulary is developed that summarizes the growth rates (e-folding lengths) for a wide range of collective beam-plasma instabilities for an intense ion beam propagating through a background plasma. The plasma is assumed to provide complete charge and current neutralization with $\sum_{j=b,e,i} n_j e_j = 0 = \sum_{j=b,e,i} n_j e_j \beta_j c$, where n_j and e_j are the number density and charge, respectively, of species j, and $\beta_j c = V_{zj}$ is the average axial velocity of species j in the z-direction. The analysis generally allows for a uniform solenoidal magnetic field $B\hat{\mathbf{e}}_z$ in the direction of beam propagation. It is assumed that the applied solenoidal field is weak enough that the applied magnetic field influences only the electron dynamics. In this case, an important dimensionless parameter that determines the stability behavior is defined by [7, 8]

$$\alpha \equiv \frac{\beta_b^2 \omega_{pe}^2}{\omega_{ce}^2} = 10^{-11} \beta_b^2 \frac{(n_0/cm^{-3})}{(B/kG)^2} \left(1 + Z_b \frac{n_b}{n_0}\right), \quad (1)$$

where $\omega_{pe} = (4\pi n_0 e^2/m_e)^{1/2}$ is the electron plasma frequency, and $\omega_{ce} = eB/m_e c$ is the electron cyclotron frequency. Here, n_0 is the number density of neutralizing plasma electrons, m_e is the electron mass, n_b is the ion beam number density, and A_s and Z_s, for $s = (i, b)$, denote the beam ion and background ion atomic mass number and charge state, respectively. For weakly relativistic ions, we express the parameter α in terms of the accelerating energy U using the relation

$$\beta_b^2 = 1.1 \times 10^{-3} \frac{(U/MeV)}{A_b}, \quad (2)$$

and Eq. (1) becomes

$$\alpha \equiv \frac{\beta_b^2 \omega_{pe}^2}{\omega_{ce}^2} = 1.1 \times 10^{-14} \left(\frac{U/MeV}{A_b}\right) \times \frac{(n_0/cm^{-3})}{(B/kG)^2} \left(1 + Z_b \frac{n_b}{n_0}\right). \quad (3)$$

In the subsequent analysis, we consider the two limiting cases corresponding to weak magnetic field ($\alpha \gg 1$), or strong magnetic field ($\alpha \ll 1$).

WEAK MAGNETIC FIELD ($\alpha \gg 1$)

The case of weak magnetic field ($\alpha \gg 1$) corresponds to the circumstances where the solenoidal magnetic field has a negligible effect on the electron dynamics. (Of course this also includes the case where $B = 0$.)

The characteristic e-folding length L_{e-f} of an instability with maximum temporal growth rate $(Im\omega)_{max}$ is

$$L_{e-f} = \frac{V_g}{(Im\omega)_{max}}, \quad (4)$$

where V_g is the group velocity of the perturbation with the most unstable wavenumber. The group velocity is different for different instabilities [3, 4, 5]. For example for the two-stream instability between the beam ions ($j = b$) and the background electrons ($j = e$), $V_g \simeq (2/3)V_b$, whereas for the two-stream instability between the background ions ($j = i$) and the background electrons ($j = e$), $V_g \simeq (1/3)V_e \simeq (1/3)Z_b(n_b/n_0)/[1 + Z_b(n_b/n_0)]V_b$. On the other hand, for the multispecies Weibel instability, the group velocity is, $V_g \approx V_b$. Therefore, for simplicity, we estimate an upper limit L_p on the e-folding length by redefining it as

$$L_p \equiv \frac{V_b}{(Im\omega)_{max}} > L_{e-f}. \quad (5)$$

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Below we summarize the expressions for L_p for various beam-plasma instabilities [3, 4, 5, 6, 7, 8].

a. Multispecies Weibel instability: For the case of weak magnetic field with $\alpha \gg 1$, the e-folding length for the multispecies Weibel instability is given by [3, 4, 5]

$$L_W = \frac{c}{\omega_{pb}} \left[\left(1 - \frac{\beta_e}{\beta_b}\right)^2 + \left(\frac{\beta_e}{\beta_b}\right)^2 \left(\frac{\omega_{pi}}{\omega_{pb}}\right)^2 \right]^{-1/2}, \quad (6)$$

or equivalently,

$$L_W = 2.26 \times 10^7 \times \left[\frac{A_b}{Z_b^2 (n_b/cm^{-3})} \left(1 + Z_i \frac{n_b}{n_0} \frac{A_b}{A_i}\right)^{-1} \right]^{1/2} \times \left(1 + Z_b \frac{n_b}{n_0}\right) cm. \quad (7)$$

b. Two-stream instability between the beam ions and the background plasma electrons: For the two-stream instability between the beam ions and the background plasma electrons [3, 4, 5], we obtain

$$L^b_{TS} = \frac{2(2)^{1/3}}{(3)^{1/2}} \beta_b \frac{c}{\omega_{ps}} \left(\frac{\omega_{ps}}{\omega_{pe}}\right)^{1/3}, \quad (8)$$

or equivalently, for $s = b$,

$$L^b_{TS} = 0.94 \times 10^7 \frac{\beta_b}{(n_0/cm^{-3})^{1/2}} \times \left\{ \frac{A_b}{Z_b^2 (n_b/n_0)[1 + Z_b(n_b/n_0)]^{1/2}} \right\}^{1/3} cm. \quad (9)$$

c. Two-stream instability between the background ions and the background electrons: On the other hand, for the two-stream instability between the background ions and the background electrons [3, 4, 5], we obtain

$$L^i_{TS} = \frac{2(2)^{1/3}}{(3)^{1/2}} \beta_b \frac{c}{\omega_{ps}} \left(\frac{\omega_{ps}}{\omega_{pe}}\right)^{1/3}, \quad (10)$$

or equivalently, for $s = i$,

$$L^i_{TS} = \left(\frac{A_i Z_b^2}{A_b Z_i} \frac{n_b}{n_0}\right)^{1/3} L^b_p = 0.94 \times 10^7 \times \frac{\beta_b}{(n_0/cm^{-3})^{1/2}} \left\{ \frac{A_i}{Z_i [1 + Z_b(n_b/n_0)]^{1/2}} \right\}^{1/3} cm. \quad (11)$$

STRONG MAGNETIC FIELD ($\alpha \ll 1$)

For the case of sufficiently strong solenoidal magnetic field that $\alpha \ll 1$ [7, 8], it follows from Eq. (3 that

$$\frac{\beta_b^2 \omega_{pe}^2}{\omega_{ce}^2} = 1.1 \times 10^{-14} \left(\frac{U}{MeV}\right) \times \frac{(n_0/cm^{-3})}{(B/kG)^2} \left(1 + Z_b \frac{n_b}{n_0}\right) \ll 1. \quad (12)$$

a. Multispecies Weibel instability: In the case where $\alpha \ll 1$, the e-folding length for the multispecies Weibel instability is [8]

$$L_W = \frac{c}{\omega_{pb}} \left[1 + \left(\frac{\omega_{pb}}{\omega_{pi}}\right)^2 \right]^{1/2}, \quad (13)$$

or equivalently,

$$L_W = 2.26 \times 10^7 \times \left[\frac{A_b}{Z_b^2 (n_b/cm^{-3})} \left(1 + Z_b \frac{n_b}{n_0} \frac{Z_b}{Z_i} \frac{A_i}{A_b}\right) \right]^{1/2} cm. \quad (14)$$

b. Lower-hybrid instability: For the lower-hybrid instability [8], we obtain for $\alpha \ll 1$

$$L_{LH} = \beta_b \frac{c}{\omega_{pb}} \left(\frac{\omega_{pb}}{\omega_{pi}}\right)^{1/3} \sqrt{1 + \left(\frac{\omega_{pe}}{\omega_{ce}}\right)^2}, \quad (15)$$

or equivalently,

$$L_{LH} = 2.26 \times 10^7 \beta_b \times \left[\frac{A_b}{Z_b^2 (n_b/cm^{-3})} \left(Z_b \frac{n_b}{n_0} \frac{Z_b}{Z_i} \frac{A_i}{A_b}\right)^{1/3} \right]^{1/2} \times \left[1 + 10^{-11} \frac{(n_0/cm^{-3})}{(B/kG)^2} \left(1 + Z_b \frac{n_b}{n_0}\right) \right]^{1/2} cm. \quad (16)$$

c. Modified two-stream instability: For the modified two-stream instability [8] between the beam ions ($s = b$), or the background ions ($s = i$), and the background electrons we obtain for $\alpha \ll 1$

$$L^s_{MTS} = \frac{2(2)^{1/3}}{3^{1/2}} \beta_b \frac{c}{\omega_{ps}} \left(\frac{\omega_{ps}}{\omega_{pe}}\right)^{1/3} \sqrt{1 + 2\left(\frac{\omega_{pe}}{\omega_{ce}}\right)^2}, \quad (17)$$

or equivalently,

$$L^b_{MTS} = 0.94 \times 10^7 \frac{\beta_b}{(n_0/cm^{-3})^{1/2}} \times \left\{ \frac{A_b}{Z_b^2 (n_b/n_0)[1 + Z_b(n_b/n_0)]^{1/2}} \right\}^{1/3} \times \left[1 + 2 \times 10^{-11} \frac{(n_0/cm^{-3})}{(B/kG)^2} \left(1 + Z_b \frac{n_b}{n_0}\right) \right]^{1/2} cm, \quad (18)$$

and

$$L^i_{MTS} = \left(\frac{A_i Z_b^2}{A_b Z_i} \frac{n_b}{n_0}\right)^{1/3} L^b_p = 0.94 \times 10^7 \frac{\beta_b}{(n_0/cm^{-3})^{1/2}} \left\{ \frac{A_i}{Z_i [1 + Z_b(n_b/n_0)]^{1/2}} \right\}^{1/3} \times \left[1 + 2 \times 10^{-11} \frac{(n_0/cm^{-3})}{(B/kG)^2} \left(1 + Z_b \frac{n_b}{n_0}\right) \right]^{1/2} cm. \quad (19)$$

d. Upper-hybrid instability: For the upper-hybrid instability [8] between the beam ions ($s = b$), or the background ions ($s = i$), and the background electrons, we obtain for $\alpha \ll 1$

$$L^s_{UH} = \frac{2(2)^{1/3}}{3^{1/2}} \beta_b \frac{c}{\omega_{ps}} \left(\frac{\omega_{ps}}{\omega_{pe}}\right)^{1/3} \left[1 + \left(\frac{\omega_{ce}}{\omega_{pe}}\right)^2\right]^{1/6}, \quad (20)$$

or equivalently,

$$L^b_{UH} = 0.94 \times 10^7 \frac{\beta_b}{(n_0/cm^{-3})^{1/2}} \quad (21)$$
$$\times \left\{ \frac{A_b}{Z_b^2 (n_b/n_0)[1 + Z_b(n_b/n_0)]^{1/2}} \right\}^{1/3}$$
$$\times \left\{ 1 + 10^{11} \frac{(B/kG)^2}{(n_0/cm^{-3})[1 + Z_b(n_b/n_0)]} \right\}^{1/6} cm,$$

and

$$L^i_{UH} = \left(\frac{A_i Z_b^2}{A_b Z_i} \frac{n_b}{n_0} \right)^{1/3} L^b_p = \quad (22)$$
$$0.94 \times 10^7 \frac{\beta_b}{(n_0/cm^{-3})^{1/2}} \left\{ \frac{A_i}{Z_i [1 + Z_b(n_b/n_0)]^{1/2}} \right\}^{1/3}$$
$$\times \left\{ 1 + 10^{11} \frac{(B/kG)^2}{(n_0/cm^{-3})[1 + Z_b(n_b/n_0)]} \right\}^{1/6} cm.$$

ILLUSTRATIVE EXAMPLES

To illustrate the application of the above formulae, we consider a weakly relativistic ($\beta_b = 0.1$) singly ionized ($Z_b = 1$) Aluminum ion beam ($A_b = 13$) propagating through a background Argon plasma ($Z_b = 1$ and $A_b = 18$) with electron density $n_0 = 10^{12} cm^{-3}$, with ratio of the beam density to the background electron density equal to $n_b/n_0 = 1/6$, and for two different strengths of applied solenoidal magnetic field: (a) $B = 0.1kG$, and (b) $B = 1kG$, corresponding to (a) $\alpha = 11.7 \gg 1$, and (b) $\alpha = 0.117 \ll 1$, respectively. The e-folding lengths for the different instabilities for Case (a) are summarized in Table I, and the e-folding lengths for Case (b) are summarized in Table II.

Table I

Instability Type	**Case (a):** $\mathbf{B = 0.1kG}$ ($\alpha = \mathbf{11.7}$)
Multispecies Weibel instability [Eq. (7)]	$L_W = 220.0cm$
Two-stream instability between the beam ions and the background plasma electrons [Eq. (9)]	$L^b_{TS} = 3.9cm$
Two-stream instability between the background ions and the background electrons [Eq. (11)]	$L^i_{TS} = 2.4cm$

Table II

Instability Type	**Case (b):** $\mathbf{B = 1kG}$ ($\alpha = \mathbf{0.117}$)
Multispecies Weibel instability [Eq. (14)]	$L_W = 221.4cm$
Lower-hybrid instability [Eq. (16)]	$L_{LH} = 55.6cm$
Modified two-stream instability between the beam ions and the background plasma electrons [Eq. (18)]	$L^b_{MTS} = 19.3cm$
Modified two-stream instability between the background ions and the background plasma electrons [Eq. (19)]	$L^i_{MTS} = 11.8cm$
Upper-hybrid instability between the beam ions and the background plasma electrons [Eq. (21)]	$L^b_{UH} = 4.0cm$
Upper-hybrid instability between the background ions and the background plasma electrons [Eq. (22)]	$L^i_{UH} = 2.4cm$

When using the above expressions for the e-folding lengths for instabilities involving the background plasma ions and the background plasma electrons, keep in mind that the group velocity can be smaller than the beam velocity, and therefore, the e-folding length can be smaller by the same factor than the ones quoted in the text.

Acknowledgments

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