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6 authors, including:

J. P. Sheehan
University of Wisconsin–Madison

Igor Kaganovich
Princeton University

Y. Raitses
Princeton University

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Effects of emitted electron temperature on the plasma sheath

J. P. Sheehan,[1,a)] I. D. Kaganovich,[2] H. Wang,[2] D. Sydorenko,[3] Y. Raitses,[2] and N. Hershkowitz[4]
[1]Department of Aerospace Engineering, University of Michigan, Ann Arbor, Michigan 48109, USA
[2]Princeton Plasma Physics Laboratory, Princeton, New Jersey 08543, USA
[3]Physics Department, University of Alberta, Edmonton, Alberta T6G 2E9, Canada
[4]Department of Engineering Physics, University of Wisconsin–Madison, Madison, Wisconsin 53706, USA

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It has long been known that electron emission from a surface significantly affects the sheath surrounding that surface. Typical fluid theory of a planar sheath with emitted electrons assumes that the plasma electrons follow the Boltzmann relation and the emitted electrons are emitted with zero energy and predicts a potential drop of $1.03T_e/e$ across the sheath in the floating condition. By considering the modified velocity distribution function caused by plasma electrons lost to the wall and the half-Maxwellian distribution of the emitted electrons, it is shown that ratio of plasma electron temperature to emitted electron temperature significantly affects the sheath potential when the plasma electron temperature is within an order of magnitude of the emitted electron temperature. When the plasma electron temperature equals the emitted electron temperature the emissive sheath potential goes to zero. One dimensional particle-in-cell simulations corroborate the predictions made by this theory. The effects of the addition of a monoenergetic electron beam to the Maxwellian plasma electrons were explored, showing that the emissive sheath potential is close to the beam energy only when the emitted electron flux is less than the beam flux. © *2014 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4882260]

I. INTRODUCTION

The problem of sheath formation near a collecting boundary is one of the oldest problems in plasma physics and has been rigorously studied.[1–4] In laboratory plasmas, however, most surfaces emit electrons due to secondary electron emission, making the question of how these emitted electrons affect the sheath structure quite important. Emissive probes make use of electron emission to measure the plasma potential and the details of the emissive sheath are critical to its operation.[5,6] Secondary electron emission in tokamak divertors can have a profound effect on heat lost and plasma confinement. Additionally, performance of plasma devices, such as Hall thrusters, is greatly impacted by secondary electron emission from the plasma facing surfaces. In a recently reported Letter,[7] we proposed a kinetic theory of emissive sheaths that takes into account modifications to the Electron Velocity Distribution Function (EVDF) by the electrons lost to the wall and the temperature of the electrons emitted from the surface. This article expands on that theory and explores it in greater depth.

The first fluid theory of a collisionless, emissive sheath was developed by Hobbs and Wesson.[8] Their model was of a one dimensional, planar, floating, electron emitting surface facing a plasma with cold ions (ion temperature $T_i = 0\,\mathrm{eV}$) and Maxwellian electrons with temperature T_{ep}. The plasma electrons were assumed to follow the Boltzmann relation in the sheath, meaning that the flux of electrons to the surface was small compared to the flux of electrons into the sheath or, equivalently, that the EVDF was not modified by the electrons lost to the surface. The emitted electrons were assumed to be emitted with zero energy at the surface and the ions were assumed to be much heavier than the electrons. The three fluxes—plasma electron, plasma ion, and emitted electron—were described with fluid equations based on the conservation laws.

The case of a floating surface was considered, so the currents balanced leaving no net current to the surface. By solving Poisson's equation

$$\frac{d^2\phi}{dx^2} = \frac{e}{\epsilon_0}(n_{ep} + n_{ee} - n_i), \tag{1}$$

where ϕ is potential referenced to the sheath edge (the position at which Bohm's criterion is fulfilled), x is position, e is the elementary charge, ϵ_0 is the permittivity of free space, n_{ep} is the plasma electron density, n_{ee} is the emitted electron density, and n_i is the plasma ion density, and Bohm's criterion[9] modified to account for the emitted electrons[3]

$$\left(\frac{dn_{ep}}{d\phi} + \frac{dn_{ee}}{d\phi} - \frac{dn_i}{d\phi}\right)\bigg|_{\phi=0} \geq 0, \tag{2}$$

they calculated the sheath potential ϕ_w (the potential difference between the sheath edge and the electron emitting wall) and ion energy at the sheath edge E_0 for a given level of emission. This emission level was based on the parameter γ, the ratio of emitted electron flux to collected electron flux. As the emission level increases, the sheath potential shrinks due to space-charge effects and at some critical level of emission γ_c the sheath potential saturates at $\phi_w = -1.03T_{ep}/e$. The emitted electrons modify Bohm's

a)Electronic mail: sheehanj@umich.edu

criterion, as expected,[10] so the electrons reach the sheath edge with energy $E_0 = 0.58T_{ep}$. This analysis of emissive sheaths was generalized by Ye and Takamura to consider the sheath potential of non-floating surfaces.[11]

Attempts to formulate a kinetic theory of emissive sheaths have been published, but neglect one or more important aspects of the problem.[12,13] Some authors did not consider the electrons from the plasma that are not confined by the sheath and lost to the wall.[14,15] Others did not account for the modification of the Bohm criterion by the emitted electrons.[16] Many papers that consider electron emitting sheaths only address these issues qualitatively, as focuses of these works were other than emissive sheath kinetic theory.[17–19]

A kinetic theory of emissive sheaths was proposed by Schwager which were verified by Particle In Cell (PIC) simulations.[20,21] In order to better compare the theory to the simulation, the boundary condition between the sheath and the plasma was that the ions and electrons had equal fluxes into the sheath. This caused a "source sheath" to form, which was a double layer that accelerated the ions and reduced the electron density so the ion and electron densities were equal at the sheath edge. It is known that this is not an accurate model of a real plasma sheath, but it was chosen to better match the PIC simulations.

Schwager considered Maxwellian distributions of the plasma electrons, plasma ions, and emitted electrons and the modification to the plasma EVDF caused by losses to the surface. For $\tau \equiv T_i/T_{ep} = 0.1$, $\Theta \equiv T_{ep}/T_{ee} = 10^4$, and $\mu \equiv m_i/m_e = 10^4$ (where m_i is the ion mass and m_e is the electron mass) the source sheath potential was calculated to be $-0.9T_{ep}/e$ while the sheath potential was $-0.58T_{ep}/e$. These results are substantially different from the fluid theory result, a difference which may be due to the non-physical boundary condition of this formulation. Schwager explored the effect of ion mass on the sheath potential for a few values of τ but did not investigate the effect of Θ on the sheath potential.

II. DERIVATION OF PLANAR EMISSIVE SHEATH POTENTIAL

This theory of emissive sheaths assumes a collisionless plasma sheath adjacent to a floating surface emits electrons such that the electric field at the surface is zero. The following normalized values were used:

$$\Phi \equiv -\frac{e\phi}{T_{ep}} \qquad \mathcal{E}_0 \equiv \frac{E_0}{T_{ep}}.$$

Here $\mathcal{E}_0$ is the ion energy normalized to the plasma electron temperature.

The plasma electrons were assumed to be Maxwellian with temperature T_{ep}. After they enter the sheath most are reflected back out, but some are energetic enough to reach the surface or wall, where they are lost. It was assumed that no electrons reflected off of the surface. The electrons lost to the wall modify the plasma electron density in the sheath, which can be calculated by integrating over a Maxwellian velocity distribution function that is missing the tail where $v > \sqrt{\frac{2T_{ep}}{m_e}(\Phi_w - \Phi)}$:

$$\frac{n_{ep}(\Phi)}{n_{ep}(0)} = \exp(-\Phi)\left(\frac{1+\mathrm{erf}\left(\sqrt{\Phi_w - \Phi}\right)}{1+\mathrm{erf}\left(\sqrt{\Phi_w}\right)}\right), \qquad (3)$$

where $\Phi = 0$ at the sheath edge and $n_{ep}(0)$ is the plasma electron density at the sheath edge.[22]

The emitted electrons were assumed to have a half-Maxwellian distribution with temperature T_{ee}. This is an accurate assumption for thermionic emission which emits electrons with such a distribution with a temperature equal to that of the surface[23] but is not a good assumption for secondary electron emission which has a significantly more complex distribution.[24] The electrons emitted from the surface accelerate through the sheath into the plasma. By integrating over the half-Maxwellian distribution, the emitted electron density is:

$$\frac{n_{ee}(\Phi)}{n_{ee}(\Phi_w)} = \exp[\Theta(\Phi_w - \Phi)]\mathrm{erfc}\left(\sqrt{\Theta(\Phi_w - \Phi)}\right), \qquad (4)$$

where Θ is the ratio of plasma electron temperature to emitted electron temperature. Charge neutrality at the sheath edge dictates that

$$n_{ep}(0) + n_{ee}(0) = n_0, \qquad (5)$$

where n_0 is the ion density at the sheath edge.

The ions were assumed to be cold ($T_i = 0$) and have a velocity at the sheath edge directed toward the emitting surface with energy E_0. The ions are, therefore, described by the fluid equations which have been often used.

$$\frac{n_i(\Phi)}{n_0} = \sqrt{\frac{1}{1+\frac{\Phi}{\mathcal{E}_0}}}. \qquad (6)$$

This equation can be derived from the continuity equation.

The total electron density is graphed in Fig. 1 where $\Phi_w = 1$ is assumed. By taking kinetic effects into account, the electron density near the surface is reduced. The ion density profile is the same for each condition since the ions were assumed to be cold and follow the fluid equations regardless of the behavior of the electrons. A reduction of the electron

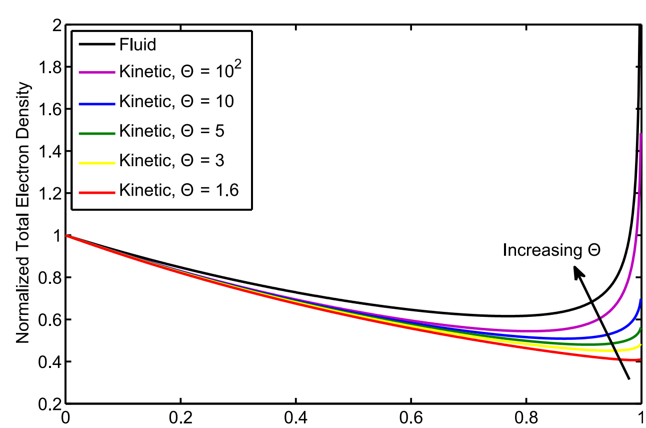

FIG. 1. Total electron density normalized to density at the sheath edge as a function of potential, assuming $\Phi_w = 1$.

density near the surface results in a reduced net space-charge in the sheath, as compared to the fluid theory.

The boundary condition at the sheath edge is that the potential is zero by definition and the electric field is zero. The potential of the emissive surface is Φ_w and when the emissive sheath is marginally space-charge limited, the electric field is zero. At greater emission currents a non-monotonic structured called a virtual cathode forms where a potential barrier forms between the wall and the potential minimum. This potential barrier reduces the emitted electron current to that of the space-charge limited case and is typically small compared to the sheath potential.[25] It does not affect the region of interest between the sheath edge and the potential minimum where the electric field equals zero. This case is of particular interest due to its use in determining the plasma potential with emissive probes.[26]

By integrating Poisson's equation over potential, the differential equation can be reduced to an integral equation

$$\int_0^{\Phi_w} \left[n_{ep}(\Phi) + n_{ee}(\Phi) - n_i(\Phi)\right] d\Phi = 0. \tag{7}$$

Assuming the ion flux is small compared to the emitted electron flux (terms on the order of $\sqrt{m_i/m_e}$ are neglected), the densities $n_{ep}(0)$ and $n_{ee}(\Phi_w)$ are functions of n_0 and Φ_w only. The electron densities can then be written as a function of Φ for given n_0 and Φ_w, and by inserting those expressions into Eq. (7) and integrated directly the following equation can be produced:

$$2\mathcal{E}_0\left(\sqrt{1+\frac{\Phi_w}{\mathcal{E}_0}} - 1\right) = \frac{1 + \mathrm{erf}(\sqrt{\Phi_w}) - \exp(-\Phi_w)\left[1 + \frac{1}{\sqrt{\Theta}}(1 - \exp(\Theta\Phi_w)\mathrm{erfc}(\sqrt{\Theta\Phi_w}))\right]}{1 + \mathrm{erf}(\sqrt{\Phi_w}) + \sqrt{\Theta}\exp[(\Theta-1)\Phi_w]\mathrm{erfc}(\sqrt{\Theta\Phi_w})}. \tag{8}$$

Bohm's criterion (Eq. (2)) can be solved to find the expression for the ion energy at the sheath edge.

$$\mathcal{E}_0 = -\left(\frac{2}{n_0}\frac{d(n_{ep}+n_{ee})}{d\Phi}\bigg|_{\Phi=0}\right)^{-1}. \tag{9}$$

These two equations can be solved to calculate the sheath potential Φ_w and the ion energy at the sheath edge $\mathcal{E}_0$ for a given value of Θ.

III. ANALYSIS OF EMISSIVE SHEATH

A. Single component, Maxwellian bulk plasma

The sheath potential as a function of plasma to emitted electron temperature ratio is graphed in Fig. 2. The dashed lines indicate the solutions as $\Theta \rightarrow \infty$. The fluid theory result for cold emitted electrons ($\Theta \rightarrow \infty$) is $\phi_w = -1.03T_{ep}/e$, the result first derived by Hobbs and Wesson. If the effect of the electrons lost to the wall on the distribution function are considered, the sheath potential drops to $\phi_w = -0.91T_{ep}/e$, only a 10% reduction from the fluid theory result.

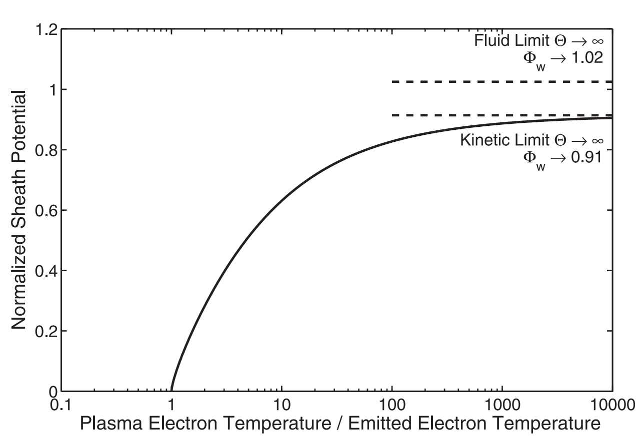

FIG. 2. The sheath potential normalized to plasma electron temperature as a function of Θ. Reprinted with permission ©American Physical Society 2013.

The emitted electron temperature has an even greater effect on the emissive sheath potential than that of the electrons lost to the wall on the distribution function. A typical thermionically heated emissive probe emits electrons with a temperature of ~0.2 eV, while the plasma electron temperature can be as low as 1 eV or less in some low-temperature laboratory experiments.[27] For these parameters, $\Theta = 5$ and $\phi_w = -0.51T_{ep}/e$. Including the effects of the plasma electrons lost to the wall and non-zero emitted electron temperature yields a sheath potential half that predicted by the widely used fluid theory. These results are consistent with the electron density profiles as shown in Fig. 1. Less net space-charge means a lower electric field magnitude in the sheath, causing the reduced electric fields and sheath potential.

As the emitted electron temperature approaches the plasma electron temperature ($\Theta \rightarrow 1$) the sheath potential goes to zero. This is expected because at $\Theta = 1$ the electrons lost to the wall would be replaced by electrons emitted from the wall at the same temperature and it is a result that has been observed in PIC simulations.[28] For the electrons, it would be as if the surface was not there. If the ion flux was taken into account, the sheath potential would be nonzero, but small, because the ion flux is smaller than the electron fluxes by an order of $\sqrt{m_e/m_i}$.

The emitted electrons modify not only the sheath potential but also the ion energy at the sheath edge. Figure 3 shows E_0 as a function of Θ using the kinetic theory described above. In the widely used fluid theory $E_0 = 0.58T_{ep}$, while this new kinetic theory predicts that $E_0 \rightarrow 0.53T_{ep}$ as $\Theta \rightarrow \infty$. As $\Theta \rightarrow 1$, $E_0 \rightarrow 0.5T_{ep}$, which is expected since the EVDF would be a pure Maxwellian at the sheath edge, assuming Bohm's criterion. The maximum of E_0 is $0.57T_{ep}$ which occurs at $\Theta = 2.55$.

The emitted electrons and plasma electrons together make up the total electron distribution function ($f_{et}(v, \Phi)$) which is expressed as

$$f_{et}(v,\Phi) = A\begin{cases} \exp\left(-\dfrac{m_e v^2}{2T_{ep}} - \Phi\right), & \text{for } v \geq \sqrt{\dfrac{2T_{ep}}{m_e}(\Phi_w - \Phi)} \\ \Theta\exp\left(\dfrac{-m_e v^2}{2T_{ee}} + \Theta(\Phi_w - \Phi) - \Phi_w\right), & \text{for } v < \sqrt{\dfrac{2T_{ep}}{m_e}(\Phi_w - \Phi)} \end{cases}. \tag{10}$$

$$A = n_0\sqrt{\frac{2m_e}{\pi T_{ep}}}\left(1 + \operatorname{erf}\left(\sqrt{\Phi_w}\right) + \sqrt{\Theta}\exp((\Theta-1)\Phi_w)\operatorname{erfc}\left(\sqrt{\Theta\Phi_w}\right)\right)^{-1}. \tag{11}$$

Here A is a normalizing factor such that the integral over all of velocity space at the sheath edge yields n_0, the density at the sheath edge. The velocity $v_c = \sqrt{\frac{2T_{ep}}{m_e}(\Phi_w - \Phi)}$ is the velocity that separates the plasma electrons ($v \geq v_c$) from the emitted electrons ($v < v_c$). The plasma electrons that would have been at $v < v_c$ were absorbed by the surface and emitted electrons with $v > v_c$ are not possible because they have at least as much energy as gained by accelerating through the sheath. Because in a collisionless sheath the Debye length is much longer than the collisional mean free path, collisions are not able to rethermalize the distribution function.

B. Particle in cell simulations

The predictions of the planar kinetic theory were compared against the one dimensional planar Electrostatic Direct Implicit Particle In Cell (EDIPIC) code.[28–31] The simulated system length was 5 mm, the ions were singly ionized argon, the plasma electron temperature was 1 eV, and the ion temperature was 0.025 eV. There were no collisions in these simulations so as to be consistent with the collisionless kinetic theory. At the source ($x = 0$ mm) a constant flux of $7.12 \times 10^{17}\,\mathrm{m^{-2}s^{-1}}$ electron-ion pairs were injected into the system and any electrons escaping to the source boundary were reinjected with a Maxwellian distribution. The electric field at this boundary was fixed at zero. At the emitting surface ($x = 5$ mm), the electric potential was fixed at zero and electrons were emitted with a flux of $3.7 \times 10^{19}\,\mathrm{m^{-2}s^{-1}}$, which was determined to be sufficient to allow the space-charge limited sheath to form. The emitted electron temperature was varied from 0.2 eV to 0.01 eV.

The steady state potential profiles of these simulations are shown in Fig. 4. The structure is a source sheath in first 1 or 2 mm to accelerate the ions to fulfill Bohm's criterion. The potential then plateaus before dropping into the typical sheath structure. The plateau was more pronounced for longer systems lengths (15 mm), but ion acoustic instabilities in the longer systems prevented a steady state solution from being obtained. These instabilities may play an important role in the formation of the emissive sheath[7] but are beyond the scope of this paper. Near the emissive boundary, one can observe that the virtual cathode forms in all cases.

In the longer system length simulations that reached a steady state it was observed that the source sheath did not accelerate the ions to marginally fulfill Bohm's criterion ($E_0 = \frac{1}{2}T_{ep}$) but rather the ions reached an energy of $0.71T_{ep}$ at the sheath edge. This was due to the thermalization and reflection boundary condition at the source. While the kinetic theory presented above was formulated for the marginal

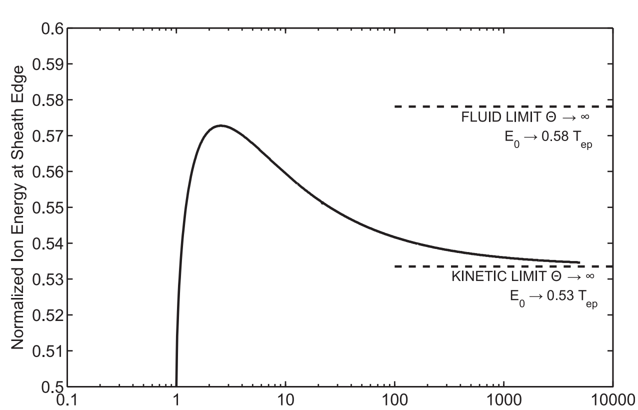

FIG. 3. The ion energy normalized to plasma electron temperature at the sheath edge as a function of Θ.

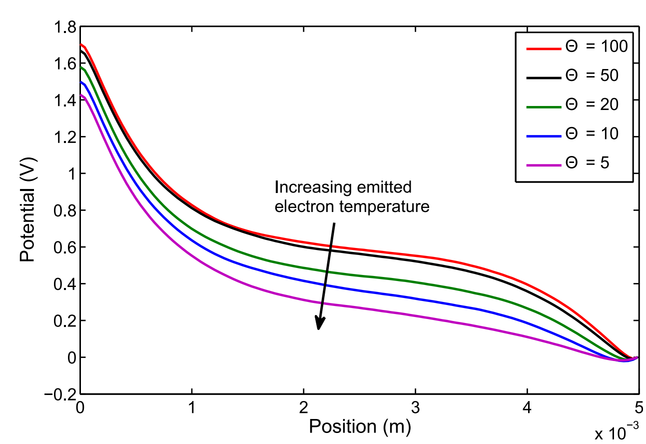

FIG. 4. PIC simulation results of electric potential as a function of position for a variety of plasma electron temperature to emitted electron temperature ratios.

solution to Bohm's criterion, it can easily be rewritten for supersonic ion flow at the sheath edge. This modification was made in order to make a meaningful comparison between the PIC simulations and the kinetic theory.

The potential at the sheath edge was taken to be the potential at which the ion energy was 0.71 eV. The emissive sheath potential was the difference between the potential at the sheath edge and that at the minimum of the virtual cathode. The potential drop between the emissive surface and the virtual cathode minimum serves to limit the flux of electrons out of the sheath and was not considered in the theory. The emissive sheath potential as predicted by the kinetic theory (Eqs. (3)–(7)) and calculated from the PIC simulation results is shown in Fig. 5. One can see the excellent agreement between the simulation and theory, corroborating the predictions of the theory.

C. Maxwellian plasma with electron beam

The model presented in this paper can be extended to account for EVDFs other than Maxwellian distributions. As long as the EVDF is known, a system of equations can be generated to solve for the Φ_w and $\mathcal{E}_0$. Consider, as an example, a Maxwellian plasma that contains an electron beam with energy E_b directed towards the emissive surface. At low beam flux relative to the flux of plasma electrons, all of the beam electrons will be collected by the surface when their energies are higher than the sheath potential. At higher fluxes, however, the beam electrons can dominate the formation of the sheath potential[32,33] and some beam electrons can be reflected off of the sheath. In consideration of this, we define the density fraction β as the density ratio of the beam electrons incident on the sheath edge to the total density of electrons incident on the sheath edge:

$$\beta = \frac{\int_0^\infty f_b(v,0)dv}{\int_0^\infty (f_m(v,0)+f_b(v,0))dv}, \tag{12}$$

where $f_m(v, \Phi)$ is a Maxwellian velocity distribution function and $f_b(v, \Phi)$ is a beam distribution function centered on energy E_b with a temperature much less than that of the plasma electrons ($T_b \ll T_{ep}$). With this definition, the sheath potential and reflected electrons do not affect the beam density fraction, which is dictated by the plasma itself. Similarly, the flux ratio γ is defined as the ratio of the incident flux of beam electrons to the incident flux of Maxwellian electrons

$$\gamma = \frac{\beta}{1-\beta}\sqrt{\frac{\pi E_b}{T_{ep}}}. \tag{13}$$

Solutions to the emissive sheath potential in the presence of an electron beam were generated numerically. The plasma electron density (now consisting both of Maxwellian and beam electrons) as a function of sheath potential was calculated by integrating the distribution function, accounting for the electrons lost to the surface.

$$\frac{n_{ep}(\Phi)}{n_{ep}(0)} = \int_{v_{\min}(\Phi)}^{\infty} (f_m(v,\Phi)+f_b(v,\Phi))dv. \tag{14}$$

Here, $v_{\min}$ (Φ) is the minimum electron velocity at some sheath potential. Higher energy electrons would have been collected by the surface and not reflected

$$v_{\min}(\Phi) = -\sqrt{\frac{2T_{ep}}{m_e}(\Phi_w-\Phi)}. \tag{15}$$

The equation for emitted electron density (Eq. (4)) and ion density (Eq. (6)) in the sheath remain valid regardless of the plasma EVDF. The normalized densities of the plasma electrons, emitted electrons, ions can be calculated by assuming that the plasma is quasineutral at the sheath edge and the surface is floating in the same way as was done for the case of a Maxwellian plasma. By solving Poisson's equation (Eq. (7)) and Bohm's criterion (Eq. (9)), the sheath potential and initial ion energy can be calculated. Note that Bohm's criterion is modified by the presence of the beam electrons.

Results are shown in Fig. 6 for the example of an electron beam with $E_b/T_{ep} = 10$ and $T_b/T_{ep} = 0.01$ at a variety of beam

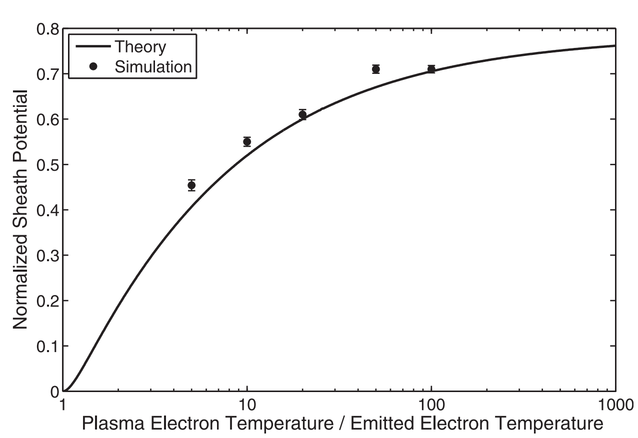

FIG. 5. A comparison of the emissive sheath as predicted by the kinetic theory (solid line) to that simulated using EDIPIC (points).

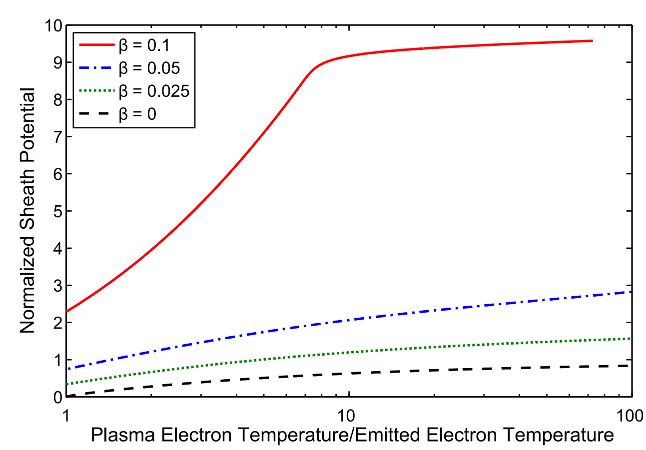

FIG. 6. Emissive sheath potentials normalized to the Maxwellian plasma component electron temperature versus plasma electron temperature to emitted electron temperature ratio for a variety of beam fractions when $E_b/T_{ep} = 10$.

fractions. When the beam fraction and flux ratio are low ($\gamma < 1$), the beam modifies the sheath potential, increasing it for all values of the temperature ratio to reduce the plasma electron flux to the surface which was increased by the beam electrons. The trend of the sheath potential decreasing as the plasma electron temperature approaches the emitted electron temperature remains, although the sheath potential does not go to zero when the two temperatures are equal.

When $\gamma > 1$ and the emitted electron temperature is very small ($T_{ee} \ll T_{ep}$) the beam electrons dominate the sheath formation, resulting in an emissive sheath potential close to the beam energy. This result has been observed for collecting Langmuir sheaths[32,33] and predicted for emissive sheaths using fluid theory.[34] As the emitted electron temperature is increased, the sheath potential drops slightly at first, though still remaining close to the beam energy. The increased emitted electron temperature increases the emitted electron flux. Once the emitted electron flux from the surface is larger than the beam electron flux to the surface the sheath potential must decrease substantially to collected enough Maxwellian electrons to maintain current balance. Higher fluxes of beam electron will allow the sheath potential to stay near the beam energy for higher emitted electron temperatures, as is shown in Fig. 7.

Since the knee in Fig. 7 occurs where the beam electron flux equals the emitted electron flux, a condition can be written for when the sheath potential will be near the beam energy:

$$\gamma(1-\beta)\sqrt{\Theta} > \frac{n_{ee}(\Phi_w)}{n_{ep}(0)}. \quad (16)$$

The density ratio on the right hand side is a weak function of Φ_w, going like $\sqrt{1/\Phi_w}$, and $1-\beta$ is typically near unity. This condition shows that as the beam flux increases the knee moves to a lower value of Θ, as observed in Fig. 7. A precise calculation of the densities requires solving Poisson's equation and Bohm's criterion.

IV. DISCUSSION

The results from this model of the emissive sheath differ significantly from Schwager's results.[21] For a large plasma electron temperature to emitted electron temperature ratio and $T_i = 0.1T_{ep}$, Schwager calculated that the ion energy at the sheath edge is $0.9T_{ep}$ and the space-charge limited sheath potential is $0.6T_{ep}/e$. Our analysis concludes that the ion energy at the sheath edge is $0.53T_{ep}$ and the sheath potential is $0.91T_{ep}/e$. The difference in the ion energy is due to the ion acceleration mechanism. In the PIC formulation, which Schwager adopted for her analytic model, a source sheath accelerates the ions to beyond marginal fulfillment of Bohm's criterion ($\sim 0.5T_{ep}$) while we assumed a presheath marginally fulfilled Bohm's criterion at the sheath edge. Ion energy in excess of $0.5T_{ep}$ causes the emissive sheath potential to be reduced as can be seen in Fig. 5 where an ion energy of $0.71T_{ep}$ was enforced to compare the analytic theory to the PIC simulations. A larger ion energy at the sheath edge increases the normalized ion density for a given potential (see Eq. (6)) so the sheath potential must be reduced to increase the emitted electron density (Eq. (4)). In experiments, presheaths marginally fulfill Bohm's criterion, so the analysis in this paper is appropriate for application to emissive probes and electron emitting boundaries.

Emissive probes have often been used to measure the plasma potential. The most common technique for doing so is the floating point method where the probe is heated to emit enough electrons for the sheath surrounding the probe to become space-charge limited. This produces a value for the plasma potential that is $\sim 2T_{ep}/e$ below the true plasma potential, one T_{ep}/e from the sheath potential and one T_{ep}/e from the presheath.[6] Caution should be exercised, however, when attempting to use this theory to adjust floating emissive probe measurements of the plasma potential for greater accuracy. Experiments have shown that the emissive sheath potential is larger than expected for intermediate values of the plasma electron temperature to emitted electron temperature ratio.[7,35]

The theory presented in this article treats a planar emissive sheath while emissive probes are typically cylindrical.[5] The effects of cylindrical geometry on the sheath has recently gained attention. Considering radial motion only, the sheath potential surrounding a floating cylindrical electrode is expected to be smaller than that surrounding a planar electrode due to ion convergence.[36] Indeed, when the probe radius equals the Debye length, fluid theory predicts the sheath potential to be less than $0.5T_{ep}/e$. Orbital motion effects have just barely been touched upon, but the initial work indicates that the orbital motion of the emitted electrons can increase the space-charge near the surface which results in a larger sheath potential.[37] Geometric effects can act in opposing ways with ion convergence reducing the sheath potential and orbital motion effects increasing the sheath potential. In short, there are an large number of complicating factors that must be taken into account when determining the emissive sheath potential.

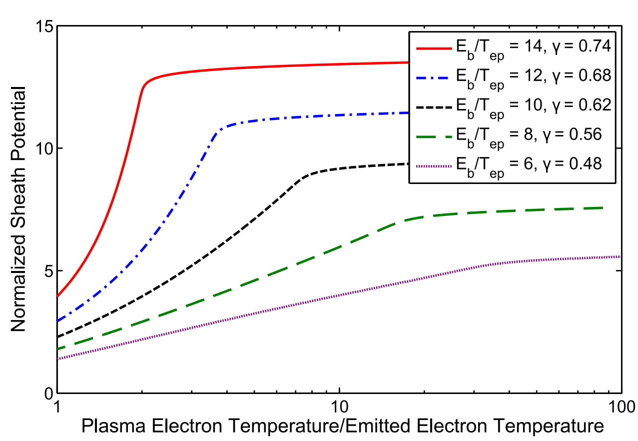

FIG. 7. Emissive sheath potentials normalized to the Maxwellian plasma component electron temperature versus plasma electron temperature to emitted electron temperature ratio for a variety of beam energies when the beam fraction was 0.1.

V. CONCLUSIONS

Using a kinetic model of the emissive sheath, it was shown that the sheath potential is smaller than that predicted by fluid theory when the emitted electron temperature is on the order of the plasma electron temperature, driving it down to $0.51T_{ep}/e$ when $T_{ep} = 5T_{ee}$ and even lower. As the plasma electron temperature approaches the emitted electron temperature the emissive sheath potential goes to zero. The theory was corroborated with a one dimensional EDIPIC simulations of a planar, collisionless plasma. This result affects the interpretation of plasma potential measurements made using emissive probes with the floating potential method.[26] Although the floating potential of an emissive probe is used as an estimate of the plasma potential, it is known to have an error on the order of the electron temperature.[6] The theory presented in this article indicates that the error is smaller than expected and can vary depending on the plasma and emitted electron temperatures. Only if both of these parameters are known the floating potential method can be corrected to give the true plasma potential. Additionally, these results may be important for divertors in fusion plasmas where heat loss is a major concern. Therefore, understanding how temperature effects play a role in the emissive sheath is necessary in order to understand the behavior of a divertor.

For a Maxwellian plasma with a monoenergetic electron beam, it was shown that the beam can significantly affect the sheath potential, demonstrating this theory's applicability to non-Maxwellian EVDFs. If the emitted electron flux is smaller than the flux of beam electrons to the surface and the beam flux into the sheath is larger than the Maxwellian component's flux into the sheath, the emissive sheath potential floats near the beam energy. When the emitted electron flux is larger than the beam electron flux however, the sheath potential is significantly smaller than the beam energy in order to allow contributions from the Maxwellian component of the plasma electron flux to maintain flux balance.

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