How to patch active plasma and collisionless sheath: a practical guide

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The purpose of the discussion

- Avoiding complex matching techniques and solving Poisson's equation for whole plasma to calculate sheath properties with a very good accuracy
  - Find a position of the boundary between plasma and sheath and
  - a value of the boundary electric field
- What to expect: robust recipe for sheath calculation

Sheath Calculation

$$-\frac{d^2V}{dx^2} = 4\pi e(n_i - n_e)$$

$$n_i = \frac{\Gamma_s}{c_s\left[1 + 2e(V_s - V)/T_e\right]^{1/2}}$$

$$n_e = n_s \exp\left(-\frac{e(V_s - V)}{T_e}\right)$$

- To solve equations in Collisionless sheath (neglecting ionization) Requires:
  - Position of matching point
  - A value of the electric field

A value of the electric field at the ion sound point

$$\frac{d}{dx}(n_i v_i) = Z n_e$$

$$M\frac{d}{dx}(n_i v_i^2) = -e n_i \frac{dV}{dx}$$

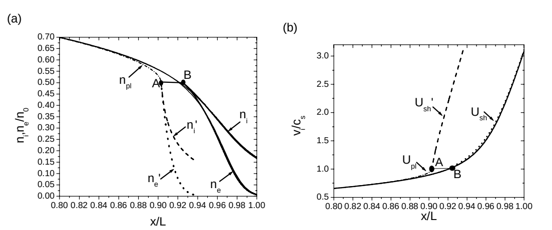
$$n_e = n_0 \exp\left(\frac{eV}{T_e}\right)$$

$$-\frac{d^2V}{dx^2} = 4\pi e(n_i - n_e)$$

- Full discharge calculation with Poisson's Eq.
- E at $U = c_s = \sqrt{T_e/M}$
- Various λ_{D0}/L and wall potentials.
- The reference value $\lambda_{Dr}/L=0.71x10^{-2}$

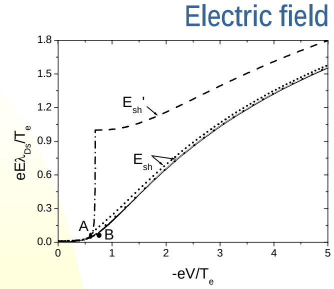
	$3.2\lambda_{Dr}$	λ_{Dr}	$0.32\ \lambda_{Dr}$	$\lambda_{Dr}/10$
$(\lambda_{D0}/L)^{3/5}$	0.102	0.051	0.0257	0.0130
ϕ_w =1	0.102	0.049	0.0240	0.0119
ϕ_w =5	0.112	0.052	0.0243	0.0121
ϕ_w =10	0.117	0.053	0.0244	0.0121

Plasma Density and Ion Velocity Profiles



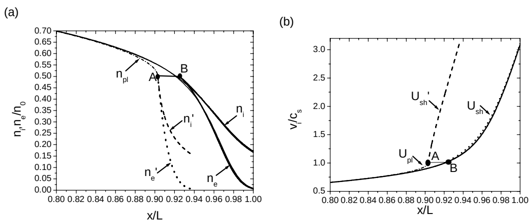
- Fig.1 The the exact solution - solid lines,
- approximate solutions - dashed line n_i and dotted line n_e.
- Approximate solutions shifted from the point A (x=0.907) to the point B (x=0.921) are practically indistinguishable from the exact solution.
- Prime denotes Godyak's solution. The plasma solution - the dash-dotted lines. $\lambda_{D0}/L=0.7071x10^{-2}$, V_w =-5T_e/e.

Electric field Profile



- Fig.2 The electric field as function of the potential. The conditions are the same as in Fig.1. The finely dotted line corresponds to the sheath solution patched at the point where $(\lambda_{D0}/L)^{3/5}$, point A. The coarsely dotted line (practically indistinguishable from the exact solution) patches the sheath solution at the point B.
- The plasma solution - dash-dotted line.
- The dashed line corresponds to the Godyak's sheath solution.

Plasma Density and Ion Velocity Profiles



Transition layer

- At $U = c_s$
- Neither plasma (the electric field can not been determined from the quasineutrality condition)
- no sheath ($n_i \cong n_e$)
- Account for ionization is important

Mathematical Description of Transition layer

$$\Gamma_i \cong n_0 / 2\left[c_s + Z(x - x_s)\right]$$

$$M(v_i^2 - c_S^2)/2 = -e(V - V_s) - Z(x - x_s)$$

$$n_i - n_e = \frac{n_0}{2}\left[\frac{1 + Z/c_s(x - x_s)}{\sqrt{1 + 2\phi - 2Z/c_s(x - x_s)}} - e^{-\phi}\right]$$

$$\frac{d^2\phi}{dx^2} = \lambda_{D0}^{-2}\left[\frac{1}{2}(\phi - \phi_s)^2 + \frac{Z}{c_s}(x - x_s)\right]$$

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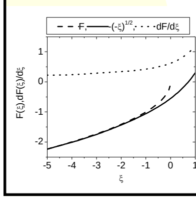
$$\phi - \phi_s = \phi_{tr} F(x / x_{tr})$$



$$\delta\phi_{tr} = \left(\frac{2\lambda_{Ds}Z}{c_s}\right)^{2/5} \qquad \delta x_{tr} = \lambda_{Ds}\left(\frac{c_s}{2\lambda_{Ds}Z}\right)^{1/5}$$

$$dF / d\xi = 0.962$$

$$E_s = 0.962\,\frac{T_e}{e\lambda_{Ds}}\left(\frac{2\lambda_{Ds}Z}{c_s}\right)^{3/5}$$

Conclusion

An approximate procedure to patch sheath and plasma is proposed. The sheath and plasma are patched at the point where the value of the electric field

$$E_s = 0.962\,\frac{T_e}{e\lambda_{Ds}}\left(\frac{2\lambda_{Ds}Z}{c_s}\right)^{3/5}$$

The transition layer is accounted simply by shifting the sheath solution from the patching point of plasma solution ($U = c_s$) by a distance and the potential:

$$\delta x_{tr} = \lambda_{Ds}\left(\frac{c_s}{2\lambda_{Ds}Z}\right)^{1/5} \qquad \delta\phi_{tr} = \left(\frac{2\lambda_{Ds}Z}{c_s}\right)^{2/5}$$

For most practical purposes, the value of $\delta V << T_e / e$ is very small compared with sheath potential and can be neglected.