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Fast Kinetic Modelling of Inductively Coupled Low-Pressure Discharges

Collaboration:

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Motivation

- To develop the fast self-consistent kinetic model of low-collisional ICP (the alternative to PIC-MCC),

I.D. Kaganovich and O.V. Polomarov, Phys. Rev. E 68, 026411 (2003).

- Generalize the “non-local approach” to include kinetic and resonant effects.

I.D. Kaganovich, O.V. Polomarov and C.E. Theodosiou, Phys. Plasmas 11, 2399 (2004).

- To study the effects of resonances, plasma non-uniformity and self-consistency in non-magnetized and magnetized ICP plasmas.

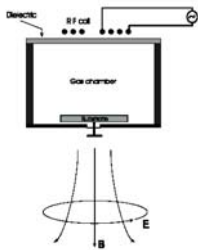
O.V. Polomarov, C.E. Theodosiou, and I.D. Kaganovich, accepted, PoP, August (2005).

O. V. Polomarov, C. E. Theodosiou, and I. D. Kaganovich, accepted, PoP, September, (2005).

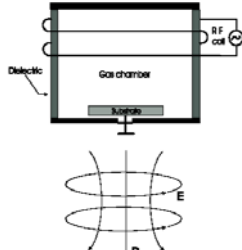
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ICP – the discharge supported by an electromagnetic wave

"Coil on top"



"Cylindrical"



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The Low Pressure Inductively Coupled Discharges Produce Quiet, Stable Plasmas



Discharge frequency $f=0.45-29$ MHz

Argon gas pressures of 0.3-10 mTorr

Plasma density $n = 10^9 - 10^{13}$ cm⁻³

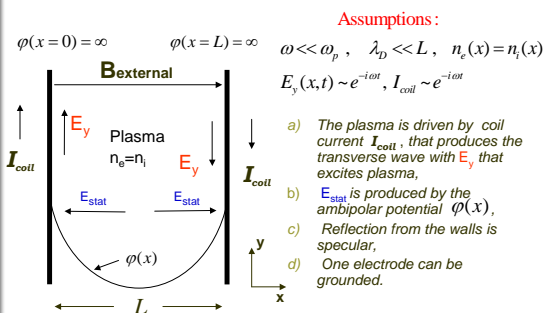
Electron temperature $T_e = \text{few eV}$

RF power dissipation in the plasma 6-400 W.

Plasma size $L \sim 10$ cm

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1-D Plasma Slab Of Width L



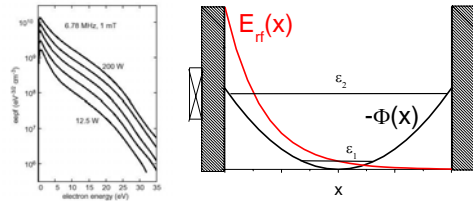
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Characteristic features of low-pressure, inductive plasma

- Non-Maxwellian Electron Energy Distribution Function (EEDF)
- Collisionless heating
- Anomalous skin effect

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Electron Distribution Function is not Maxwellian

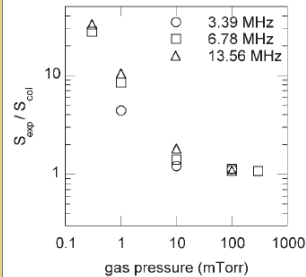


Measured EDF shows departure from a Maxwellian:
 a) Depopulated by high-energy electrons - inelastic collisions.
 b) Enriched by low-energy electrons - cold electrons are trapped in a small rf electric field.

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Electron Heating Is Collisionless

The ratio of the total measured power S_{exp} to collisional S_{col} power dissipation at 100W.



$$S_{col} = \langle j_{col} E \rangle$$

$$j_{col} = \frac{e^2 n E}{m(-i\omega + \nu)}$$

ν is the collision frequency
 ω is the rf field frequency

The enhanced power dissipation is due to resonant wave-particle interaction

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Collisionless Heating is Analogous to Landau Damping

Collisionless (Landau) plasma wave damping is larger than collisional damping for $v \ll \omega$

Analogy: a particle sits on the crest of the wave

Inhomogeneous rf field $\Rightarrow E(k) \Rightarrow \omega \Rightarrow \omega - vk$

$$j_{col} = \frac{e^2 n E}{m(-i\omega + \nu)} \Rightarrow j(k) = \frac{e^2 n E(k)}{m} \int d\nu \frac{f(\nu)}{-i(\omega - \nu k) + \nu}$$

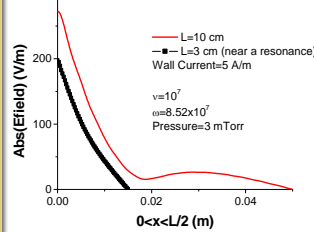
Due to resonance $\omega = vk$

$$j(k) > j_{col}$$

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Typical profile of rf electric field shows anomalous skin effect

Rf electric field for different lengths for full self consistent simulation



$$\frac{d^2 E_y}{dx^2} = -\frac{4\pi i \omega}{c^2} j(x)$$

Normal skin effect:

$$j(x) = \sigma E \text{ for}$$

$$l_{mfp} < \delta$$

Anomalous skin effect:

$$j(x) = \int \sigma(x, y) E(y) dy$$

$$\text{for } l_{mfp} > \delta$$

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Overview of Kinetic Model for Discharge Simulations

- Calculate nonlocal conductivity in non-uniform plasma.
- Find a non-Maxwellian electron energy distribution function driven by collisionless heating of resonant electrons.
- What to expect: self-consistent system for kinetic treatment of collisionless and nonlocal phenomena in inductive discharge.

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The Model Consists Of 3 Blocks:

- The **rf electric field** using Maxwell's equations and the non-local conductivity operator,
- The **electron distribution function (EEDF)** using the averaged over fast electron motions kinetic equation,
- The **electrostatic potential** using quasi-neutrality condition and the fluid equations for ion density and ion momentum.

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Sketch of the non-local approach

Electron Velocity Distribution Function for $\frac{v^*}{\omega} \ll 1$, $\frac{v^*}{\Omega_e} \ll 1$ and $\frac{v^*}{v} \ll 1$

is represented as $f(\vec{x}, \vec{v}, t) = f_0(\varepsilon) + f_1(\vec{x}, \vec{v}, t)$,

where $\varepsilon = \frac{mv^2}{2} - e\phi(x)$, v - elastic collision frequency, v^* - inelastic collision frequency.

$\Omega_b = \frac{\pi}{L} \sqrt{\frac{2T_e}{m_e}}$ - electron bounce frequency.

The Boltzmann equations splits in 2 equations:

1) for the main isotropic part, or EEDF $f_0(\varepsilon)$, which gives density: $\int f_0 d\vec{v} = n_e(\vec{x})$

2) for the small anisotropic part $f_1(\vec{x}, \vec{v}, t)$, which gives

the non-local conductivity operator: $e \int \vec{v} f_1 d\vec{v} = \vec{j}(\vec{x})$.

Averaging used: over the rf period, over fast bouncing, over all velocities with given total energy ε .

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1) The transverse RF Electric Field

$$\frac{d^2 E_y}{dx^2} + \frac{\omega^2}{c^2} E_y = -\frac{4\pi i \omega}{c^2} [j(x) + I \delta(x) - \delta_{anti} I \delta(x-L)]$$

Using spectral method

$$E_y = \sum_{s=0}^{\infty} \Xi_s \cos(k_s x) \quad , \quad \text{where: } k_s = (2s+1)\pi / L \quad ,$$

it gives:

$$\left(-k_s^2 + \frac{\omega^2}{c^2} \right) \Xi_s = -\frac{4\pi i \omega}{c^2} \left[j_s + \frac{I(1 + \delta_{anti})}{L} \right]$$

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1a) Current Density

The non-local conductivity operator:

$$j_s = \frac{e^2}{m} \frac{1}{(2s+1)\Omega_{bT}} \sum_{l=0}^{\infty} \Xi_l Z_{s,l}^{gen} \left(\frac{\omega + iv}{(2s+1)\Omega_{bT}} \right)$$

The generalized dispersion function:

$$Z_{s,l}^{gen}(\zeta) = \sqrt{\frac{2}{m}} \frac{(2s+1)\pi \Omega_{bT}}{L} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\Gamma(\varepsilon)}{n\Omega_b(\varepsilon) - (2s+1)\Omega_{bT}\zeta} \frac{G_{s,n}(\varepsilon) G_{l,n}(\varepsilon)}{\Omega_b(\varepsilon)} d\varepsilon \quad ,$$

$$\text{where } G_{l,n}(\varepsilon) = \frac{1}{T} \int_0^T \cos[k_l x(\tau)] \cos\left(\frac{\pi n \tau}{T}\right) d\tau$$

can be effectively computed using Fast Fourier Transform (FFT).

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2) Kinetic Equation Is Averaged over Fast Electron Bouncing in Potential Well

$$-\frac{d}{d\varepsilon} (D_e + \overline{D_{ee}}) \frac{df_0}{d\varepsilon} - \frac{d}{d\varepsilon} \overline{v_{ee}} f_0 = \sum_k \left[v_k^*(u + \varepsilon_k) \frac{\sqrt{(u + \varepsilon_k)}}{\sqrt{u}} f_0(\varepsilon + \varepsilon_k) - \overline{v_k} f_0 \right] \quad ,$$

D_{ee} , $\overline{v_{ee}}$ are from the electron-electron collision integral, v^* is inelastic collision frequency and upper bar denotes space averaging with constant total energy.

$$D_e = \frac{\pi e^2}{4m^2} \sum_{n=-\infty}^{\infty} \int_0^{\varepsilon} d\varepsilon_x |E_{yn}(\varepsilon_x)|^2 \frac{\varepsilon - \varepsilon_x}{\Omega_b(\varepsilon_x) [\Omega_b(\varepsilon_x) n - \omega]^2 + \nu^2}$$

Energy diffusion D_e coefficient is a function of the rf electric field E_y and the plasma potential $\phi(x)$.

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3) The ambipolar potential

The quasineutrality condition

$$n_{ion}(x) = n_{electron}(x) = \int_{\phi(x)}^{\infty} f_0(\varepsilon) \sqrt{\varepsilon - \phi(x)} d\varepsilon \quad ,$$

gives the electrostatic potential:

$$\frac{d\phi(x)}{dx} = -T_e^{scr}(x) \frac{d \ln[n_{ion}(x)]}{dx} \quad ,$$

where T_e^{scr} is the electron "screening temperature".

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3a) The ion density profile

The ion density profile $n_{ion}(x)$ is given by:

$$\frac{d(n_{ion} u)}{dx} = R_{ion}$$

$$u_+ \frac{du_+}{dx} = -\frac{e T_e^{scr}(x)}{M} \frac{d \ln n_{ion}}{dx} - v_{ion}(u_+) u_+ - \frac{R_{ion}}{n_{ion}} u_+$$

$$\text{where } R_{ion}(x) = N_{gas} \sqrt{\frac{2e}{m}} \int_{\phi(x)}^{\infty} \sigma_{ionic}(u) (\varepsilon - \phi(x)) f_0(\varepsilon) d\varepsilon$$

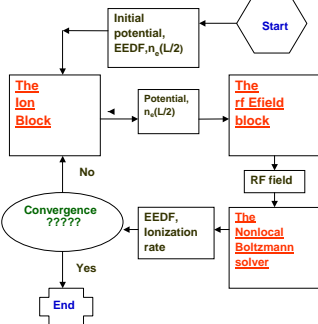
is the ionization rate, and $v_{ion}(u)$

is the ion-neutral and ion charge transfer frequency.

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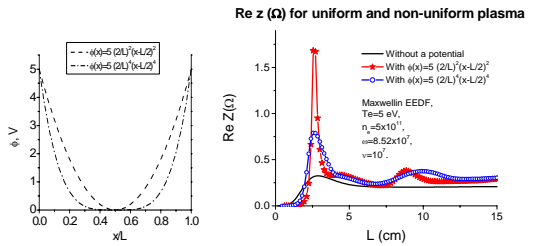
Simulation flow chart.

Each block depends on the results of other blocks – iterations.



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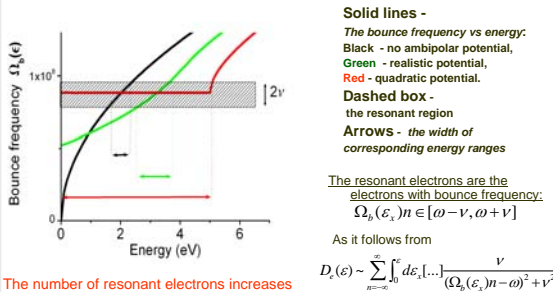
The role of plasma non-uniformity at the bounce resonance : $\omega = \Omega_b(\epsilon_x)$



O. V. Polomarov, C. E. Theodosiou, and I. D. Kaganovich, "Enhanced collisionless heating in non-uniform plasma at the bounce resonance condition." Accepted PoP August (2005).

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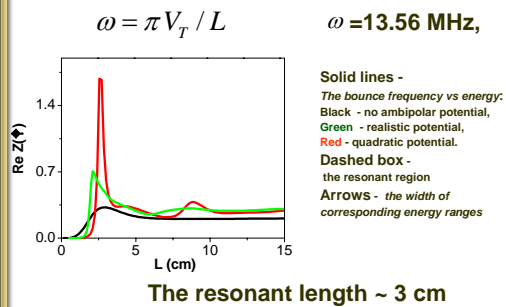
Influence of ambipolar potential on the bounce frequency and the corresponding enlargement of the number of resonant electrons



The number of resonant electrons increases if the ambipolar potential is taken into account. More resonant electrons-larger dissipated power.

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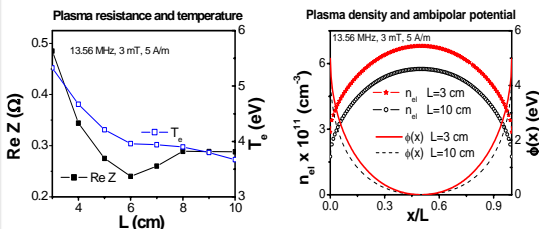
The corresponding surface resistance for Maxwellian EEDF.



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Plasma resistance, T_e , density and ambipolar potential profiles

Full self-consistent simulation Resonant length ~ 3 cm



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1-D simulation of magnetized ICP discharges

- The dc magnetic field is applied perpendicularly to boundaries. where E^\pm is left and right polarized electric fields, respectively.
- $$\frac{d^2 E^\pm}{dx^2} + \frac{\omega^2}{c^2} E^\pm = -\frac{4\pi i \omega}{c^2} (j^\pm(x) + I \delta(x) - \delta_{ant} I \delta(x-L))$$
- $$D_\epsilon = \frac{\pi e^2}{8m^2} \sum_{n=-\infty}^{\infty} \int_0^\epsilon d\epsilon_x |E_n^+(\epsilon_x)|^2 \frac{\epsilon - \epsilon_x}{\Omega_b(\epsilon_x) [\Omega_b(\epsilon_x)n - (\omega + \Omega_c)]^2 + \nu^2} + \frac{\pi e^2}{8m^2} \sum_{n=-\infty}^{\infty} \int_0^\epsilon d\epsilon_x |E_n^-(\epsilon_x)|^2 \frac{\epsilon - \epsilon_x}{\Omega_b(\epsilon_x) [\Omega_b(\epsilon_x)n - (\omega - \Omega_c)]^2 + \nu^2}$$
 where Ω_c is the electron-cyclotron frequency.

O. V. Polomarov, C. E. Theodosiou, and I. D. Kaganovich at all. "Self-consistent kinetic modeling of low-pressure inductively coupled plasmas." submitted to IEEE(2005)

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Resonances in magnetized plasmas

O. V. Polomarov, C. E. Theodosiou, I. D. Kaganovich et al, "Effectiveness of electron-cyclotron and transmission resonance heating in ICP plasmas", accepted, PoP, September, (2005).

● Electron-cyclotron resonance:

$$\Omega_c - \omega \leq k_s V_T$$

● Transmission resonances for $B > B_c$:

$$I) k_s^2 - \frac{\omega_{pe}^2}{c^2} \frac{\omega}{k_s V_T} \operatorname{Re} Z_M \left(\frac{\omega - \Omega_c + i\nu}{k_s V_T} \right) = 0,$$

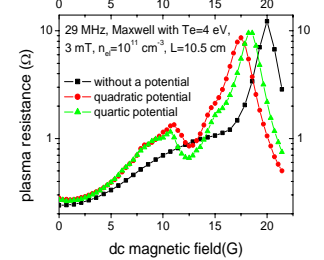
$$II) \Omega_c - \omega \gg k_s V_T$$

Where $\Omega_c = \frac{eB}{mc}$, $k_s = (2s+1) \frac{\pi}{L}$, $V_T = \sqrt{\frac{2T_e}{m}}$, $\omega_{pe} = \sqrt{\frac{4\pi e^2 n_e}{m}}$
and $Z_M(\xi)$ is the "standard" plasma dispersion function.

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Resistance of uniform and non-uniform plasmas for Maxwellian EEDF

Re Z (Ω) for uniform and non-uniform plasma



ECR resonance:

$B \sim 10\text{G}$ Transmission

resonances:

$B \sim 20, 30\text{G}$

$$\text{Power} = \operatorname{Re} Z I^2$$

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Self-consistent simulation of 1-D ICP for fixed coil current

Input parameters of the discharge:

- Coil current, or power
- Pressure
- Frequency and Length

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Self-consistent simulation of the magnetized ICP and comparison with the experiment.

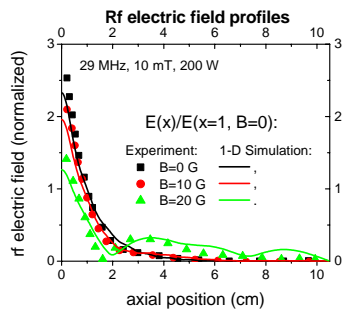
$$\omega = 29 \text{ MHz}, \quad L = 10,5 \text{ cm}$$

Experimental data are taken from:

V. A. Godyak and B. M. Alexandrovich, Phys. Plasmas 11, 3553 (2004).

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Rf electric field



ECR resonance:

$B \sim 10\text{G}$

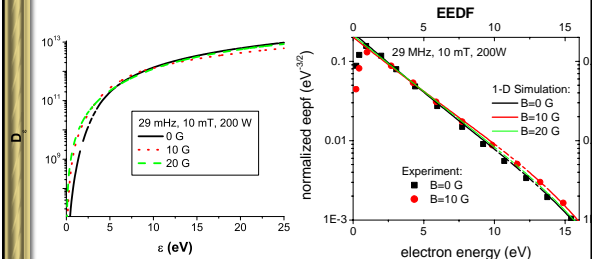
Transmission

resonances:

$B \sim 20\text{G}$

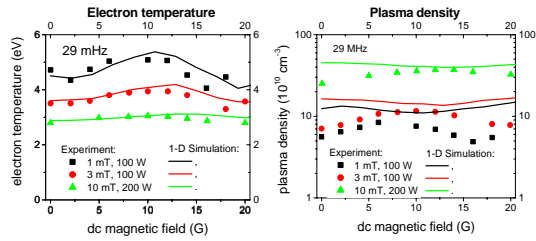
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The energy diffusion coefficient and EEDF



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Electron temperature and density



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Conclusion

- The self-consistent system of equations is derived for description of plasma heating and anomalous skin effect in non-uniform non-magnetized and magnetized plasmas.
- The robust kinetic code was developed for fast modeling of ICP discharges.

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