An implicit method for M3D and prospect for large scale hybrid simulations

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Outline

- An implicit method for M3D (resistive MHD)
- Prospect for large scale hybrid simulations

An implicit method for M3D

- In the current version of M3D, only compressional Alfven waves are advanced implicitly. Thus, the time step size is limited by CFL condition due to shear Alfven waves.
- We have recently developed an implicit method which is valid for the full resistive MHD equations.
- The new implicit method has been validated for linear regime. Work is in progress to demonstrate its use for nonlinear regime and to extend it to two fluid model.

Resistive MHD equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0;$$

$$\rho \frac{d\mathbf{v}}{dt} = \mathbf{J} \times \mathbf{B} - \nabla p + \mu \nabla^2 \mathbf{v}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E};$$

$$\frac{dp}{dt} = -\gamma p \nabla \cdot \mathbf{v} + \rho \nabla \cdot \kappa \cdot \nabla \frac{p}{\rho}$$

$$\mathbf{J} = \nabla \times \mathbf{B};$$

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{J}$$

$$\nabla \cdot \mathbf{B} = 0.$$

In (R, Z, φ) coordinates

2.1 Decompose v into (u, χ, v_φ), we have

$$\mathbf{v} = R^2 \epsilon \nabla u \times \nabla \varphi + \nabla_\perp \chi + v_\varphi \hat{\varphi}$$

2.2 Decompose B into (ψ, I), we have

$$\mathbf{B} = \nabla \psi \times \nabla \varphi + \frac{1}{R} \nabla_\perp F + R_0 I \nabla \varphi$$

2.3 F equation

$$\nabla_\perp^2 F = -\frac{a}{R} \tilde{I}', \ \ \text{with} F \equiv \frac{\partial f}{\partial \varphi}, I = \frac{R}{R_0} B_\varphi = 1 + \epsilon \tilde{I}.$$

$$
\begin{aligned}
\frac{\partial}{\partial t}\Delta^\dagger U &= \epsilon R \nabla_\perp U \times \nabla_\perp (\Delta^\dagger U)\cdot\hat{\varphi} - \nabla_\perp \chi \cdot \nabla_\perp(\Delta^\dagger U) - \Delta^\dagger U(2\epsilon\frac{\partial U}{\partial Z} + \Delta^\dagger \chi) \\
&-\frac{v_\varphi}{R}\frac{\partial}{\partial\varphi}\Delta^\dagger U - \nabla_\perp(\frac{v_\varphi}{R})\cdot\nabla_\perp(\frac{\partial U}{\partial \varphi}) \\
&+2R_0\frac{v_\varphi}{R}\frac{\partial}{\partial Z}\frac{v_\varphi}{R} + \frac{R_0}{R}\nabla_\perp(\frac{v_\varphi}{R})\times\nabla_\perp(\frac{\partial\chi}{\partial\varphi})\cdot\hat{\varphi} \\
&+R_0[\mathbf{B}\cdot\nabla(\frac{C}{d}) + \mathbf{J}\cdot\nabla(\frac{I}{d})] \\
&+\frac{2}{d}\frac{\partial p}{\partial Z} + R\nabla_\perp\frac{1}{d}\times\nabla_\perp p\cdot\hat{\varphi} \\
&-R_0\nabla\varphi\cdot\nabla\times(\mu\frac{R^2}{d}\nabla^2\mathbf{v}) \qquad (14)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial C_a}{\partial t} &= \epsilon R[\nabla_\perp(\Delta^* U)\times\nabla_\perp\psi + \nabla_\perp U\times\nabla_\perp C_a + 2\nabla_\perp(\frac{\partial U}{\partial R})\times\nabla_\perp(\frac{\partial\psi}{\partial R}) + 2\nabla_\perp(\frac{\partial U}{\partial Z})\times\nabla_\perp(\frac{\partial\psi}{\partial Z})]\cdot\hat{\varphi} \\
&\epsilon R[\nabla_\perp(\Delta^* U)\cdot\nabla_\perp F + \nabla_\perp U\cdot\nabla_\perp(\Delta^* F) + 2\nabla_\perp(\frac{\partial U}{\partial R})\cdot\nabla_\perp(\frac{\partial F}{\partial R}) + 2\nabla_\perp(\frac{\partial U}{\partial Z})\cdot\nabla_\perp(\frac{\partial F}{\partial Z})] \\
&-\nabla_\perp(\Delta^*\chi)\cdot\nabla_\perp\psi - \nabla_\perp\chi\cdot\nabla_\perp C_a - 2\nabla_\perp(\frac{\partial\chi}{\partial R})\cdot\nabla_\perp(\frac{\partial\psi}{\partial R}) - 2\nabla_\perp(\frac{\partial\chi}{\partial Z})\cdot\nabla_\perp(\frac{\partial\psi}{\partial Z}) \\
&[\nabla_\perp(\Delta^*\chi)\times\nabla_\perp F + \nabla_\perp\chi\times\nabla_\perp(\Delta^* F) + 2\nabla_\perp(\frac{\partial\chi}{\partial R})\times\nabla_\perp(\frac{\partial F}{\partial R}) + 2\nabla_\perp(\frac{\partial\chi}{\partial Z})\times\nabla_\perp(\frac{\partial F}{\partial Z})]\cdot\hat{\varphi} \\
&+\frac{\partial}{\partial\varphi}\Delta^*\Phi \\
&+\eta(\Delta^* C_a + \frac{1}{R}\frac{\partial}{\partial Z}\Delta^* F) \qquad (13)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \tilde{I}}{\partial t} &= \epsilon R \nabla_\perp U \times \nabla_\perp \tilde{I} \cdot \hat{\varphi} - \nabla_\perp \chi \cdot \nabla_\perp \tilde{I} - \frac{v_\varphi}{R}\frac{\partial \tilde{I}}{\partial \varphi} \\
&+ R\nabla_\perp\left(\frac{v_\varphi}{R}\right) \times \nabla_\perp \psi \cdot \hat{\varphi} + R\nabla_\perp F \cdot \nabla_\perp\left(\frac{v_\varphi}{R}\right) \\
&-\left(\frac{1}{\epsilon} + \tilde{I}\right)\Delta^* \chi + \eta\left[\Delta^* \tilde{I} - \frac{1}{R}\nabla_\perp^2 F' + \frac{2}{R^2}\left(\frac{\partial F'}{\partial R} + \frac{\partial \psi'}{\partial Z}\right)\right] \\
&+ \nabla_\perp \eta \cdot \left[\nabla_\perp \tilde{I} - \frac{1}{R}\nabla_\perp F' - \nabla_\perp \psi' \times \nabla \varphi\right]
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
\frac{\partial}{\partial t}\left(\frac{\partial \chi}{\partial R}\right) &= -\epsilon R \frac{\partial}{\partial z}\left(\frac{\partial U}{\partial t}\right) - \mathbf{v}_\perp \cdot \nabla_\perp\left(\frac{\partial \chi}{\partial R} + \epsilon R \frac{\partial U}{\partial z}\right) \\
&- \epsilon v_\varphi \frac{\partial U'}{\partial z} - \frac{v_\varphi}{R}\frac{\partial \chi'}{\partial R} + \frac{v_\varphi^2}{R} - \frac{R^2}{d}\frac{\partial p}{\partial R} \\
&+ \frac{1}{d}\left(\frac{1}{\epsilon} + \tilde{I}\right)\left[\frac{1}{R}\left(\frac{\partial F'}{\partial R} + \frac{\partial \psi'}{\partial z}\right) - \frac{\partial \tilde{I}}{\partial R}\right] + \frac{C}{d}\left(\frac{\partial F}{\partial z} - \frac{\partial \psi}{\partial R}\right) + \hat{R}\cdot\mu\frac{R^2}{d}\nabla^2 \mathbf{v}
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
\frac{\partial}{\partial t}\left(\frac{\partial \chi}{\partial Z}\right) &= \epsilon R \frac{\partial}{\partial R}\left(\frac{\partial U}{\partial t}\right) - \mathbf{v}_\perp \cdot \nabla_\perp\left(\frac{\partial \chi}{\partial z} - \epsilon R \frac{\partial U}{\partial R}\right) \\
&+ \epsilon v_\varphi \frac{\partial U'}{\partial R} - \frac{v_\varphi}{R}\frac{\partial \chi'}{\partial z} - \frac{R^2}{d}\frac{\partial p}{\partial z} \\
&+ \frac{1}{d}\left(\frac{1}{\epsilon} + \tilde{I}\right)\left[\frac{1}{R}\left(\frac{\partial F'}{\partial z} - \frac{\partial \psi'}{\partial R}\right) - \frac{\partial \tilde{I}}{\partial z}\right] - \frac{C}{d}\left(\frac{\partial F}{\partial R} + \frac{\partial \psi}{\partial z}\right) + \hat{z}\cdot\mu\frac{R^2}{d}\nabla^2 \mathbf{v}
\end{aligned} \tag{17}
$$

$$
\begin{aligned}
\frac{\partial p}{\partial t} &= \epsilon R \nabla_\perp U \times \nabla_\perp p \cdot \hat{\varphi} - \nabla_\perp \chi \cdot \nabla_\perp p - \frac{v_\varphi}{R}\frac{\partial p}{\partial \varphi} \\
&\quad -\gamma p[\Delta^\dagger \chi + 2\epsilon \frac{\partial U}{\partial Z} + \frac{1}{R}\frac{\partial v_\varphi}{\partial \varphi}] \\
&\quad +d\nabla \cdot \kappa \cdot \nabla \left(\frac{p}{d}\right)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial v_\varphi}{\partial t} &= \epsilon R \nabla_\perp U \times \nabla_\perp v_\varphi \cdot \hat{\varphi} - \nabla_\perp \chi \cdot \nabla_\perp v_\varphi - \frac{v_\varphi}{R}[\epsilon R \frac{\partial U}{\partial Z} + \frac{\partial \chi}{\partial R} + \frac{\partial v_\varphi}{\partial \varphi}] \\
&\quad +\frac{1}{d}[\nabla_\perp \tilde{I} \cdot \nabla_\perp F - \frac{1}{R}(\nabla_\perp F' \cdot \nabla_\perp F + \nabla_\perp \psi' \cdot \nabla_\perp \psi) \\
&\quad +\nabla_\perp \tilde{I} \times \nabla_\perp \psi \cdot \hat{\varphi} + \frac{1}{R}\nabla_\perp \psi' \times \nabla_\perp F \cdot \hat{\varphi} + \frac{1}{R}\nabla_\perp F' \times \nabla_\perp \psi \cdot \hat{\varphi}] \\
&\quad -\epsilon \frac{R}{d}\frac{\partial p}{\partial \varphi} + \hat{\varphi} \cdot (\mu \frac{R^2}{d}\nabla^2 \mathbf{v}) \qquad (15)
\end{aligned}
$$

Implicit operator for shear Alfven waves

$$\frac{dw}{dt} = R_0 B \cdot \nabla (c/d) + other\ terms$$

$$\frac{dc}{dt} = \epsilon R^2 B \cdot \nabla w + other\ terms$$

Thus implicit equation for w can be written as

$$w^{n+1} - (\Delta t)^2 B \cdot \nabla (R^2/d) B \cdot \nabla w^{n+1} = RHS$$

Implicit operator for V_ϕ

$$V_\phi^{n+1} - (\Delta t)^2 (1/d) R B_p \cdot \nabla R B_p \cdot \nabla V_\phi^{n+1} = RHS$$

Prospect for large scale hybrid simulations

- At present, the domain decomposition for particle is 2D.
- Previous scaling studies for hybrid runs showed good parallel efficiency up to 1024 processors.
- We plan to upgrade the particle domain decomposition to 3D in order to use > 1024 processors effectively.
- We expects better parallel efficiency for hybrid runs as compared to that of MHD runs.