New CDX-U Equilibrium, M3D Results & Error Field Calculations

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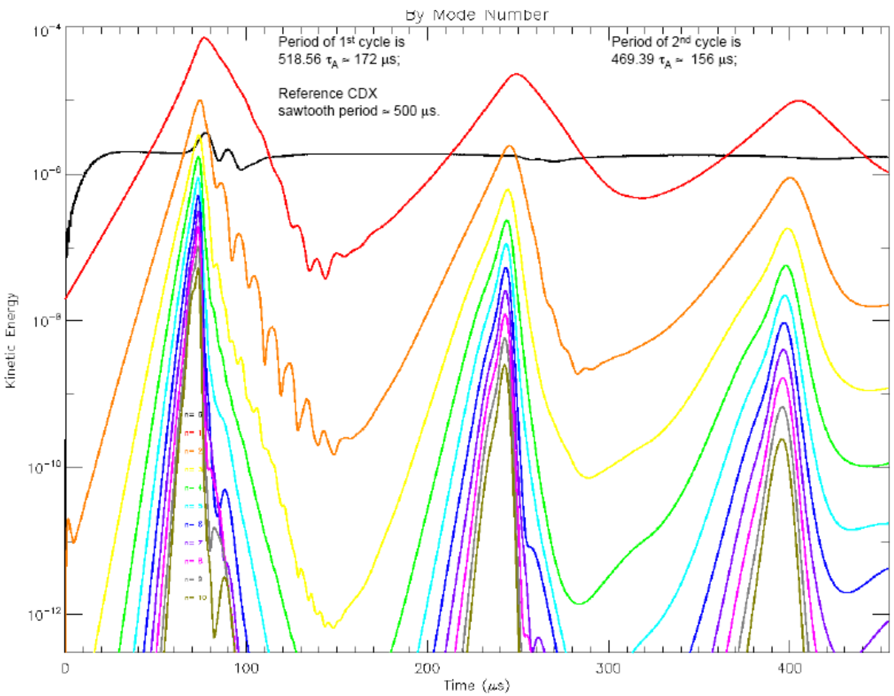
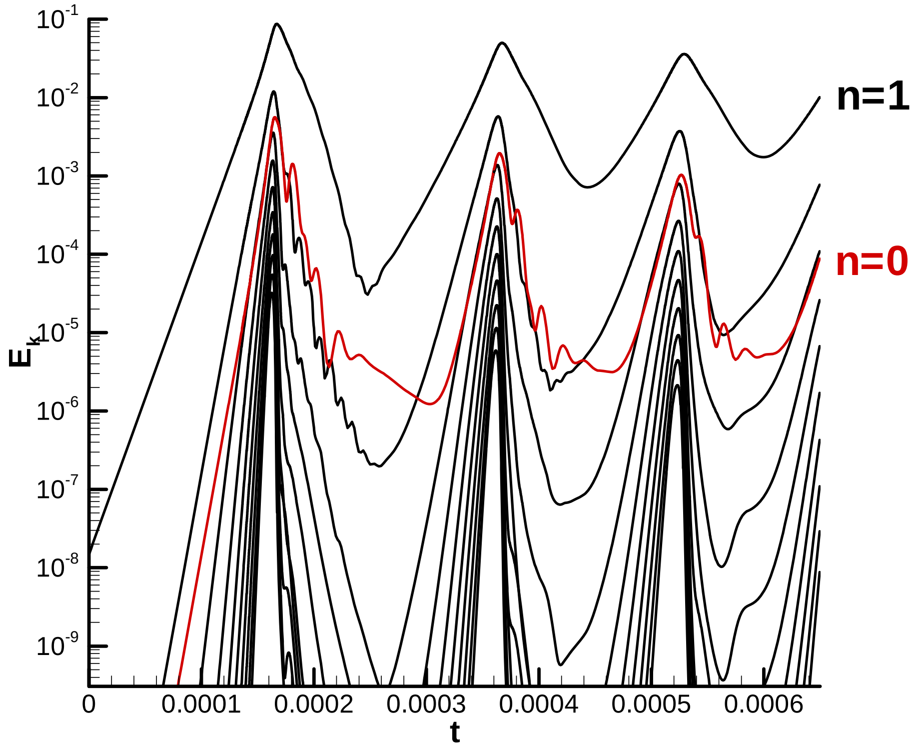
CEMM Meeting
Orlando
November 11, 2007

I. CDX-U Sawtooth

Previous Nonlinear M3D-NIMROD Comparison



M3D Kinetic energy in first 10 modes



NIMROD Kinetic energy in first 10 modes

Good agreement with each other; period **not** in agreement with experiment.

Refinements to Physics Model Required

Next study to include these refinements:

- Apply ohmic heating instead of volumetric heat source, with self-consistent evolving resistivity profile.
- Apply loop voltage rather than volumetric current source to better model the inductive discharge.
- Choose a more realistic perpendicular thermal conductivity profile, consistent with quasi-equilibrium state.
- Include additional two-fluid terms as necessary/feasible.
- Begin with an analytically specified CDX-like equilibrium.

Specification of Analytic Equilibrium

Quantity	Value
Major radius R_0	0.341 m
Minor radius *a*	0.247 m (aspect ratio = 1.38)
Ellipticity κ	1.35
Triangularity δ	0.25
Central temperature (T_e=T_i)	100 eV
Normalized central pressure $\mu_0 p_0$	2.5×10^{-4} (implies $n_0 = 1.8 \times 10^{-19}$ m^{-3})
α Parameter in pressure equation*	0.1
Vacuum value g_0 of R·B_T	0.042 T·m
Effective ion charge Z_{EFF}	2.0
Loop voltage V_L	3.1741 V (implies $q_0 \approx 0.82$)

$$*p(\psi) = p_0\left[\alpha\tilde{\psi} + (1-\alpha)\tilde{\psi}^2\right], \text{ where } \tilde{\psi} \equiv \frac{\psi - \psi_{\text{limiter}}}{\psi_{\text{axis}} - \psi_{\text{limiter}}}.$$

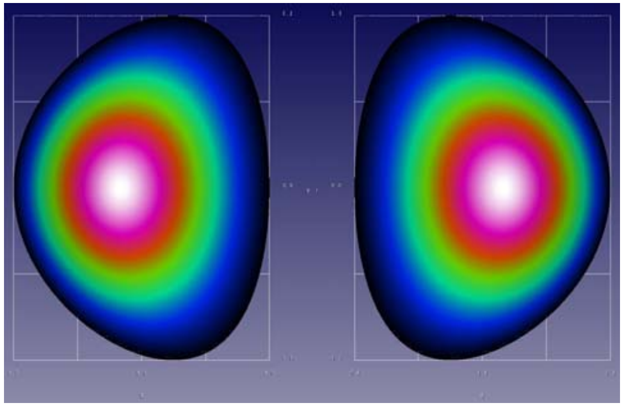
Use equilibrium code to solve Grad-Shafranov equation, with profile of heat conduction coefficient χ computed self-consistently to keep temperature constant given profile, energy supplied by applied V_L.

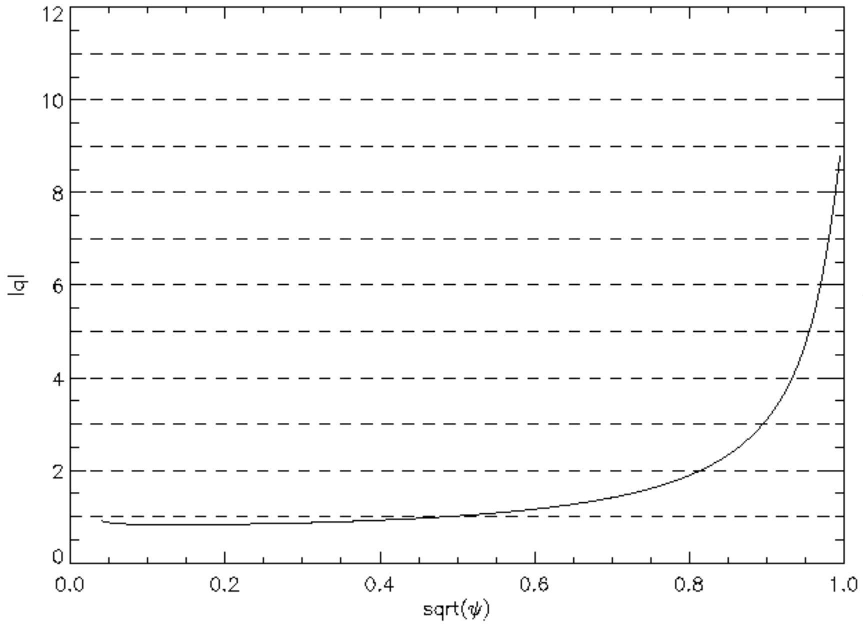
Form of New Equilibrium

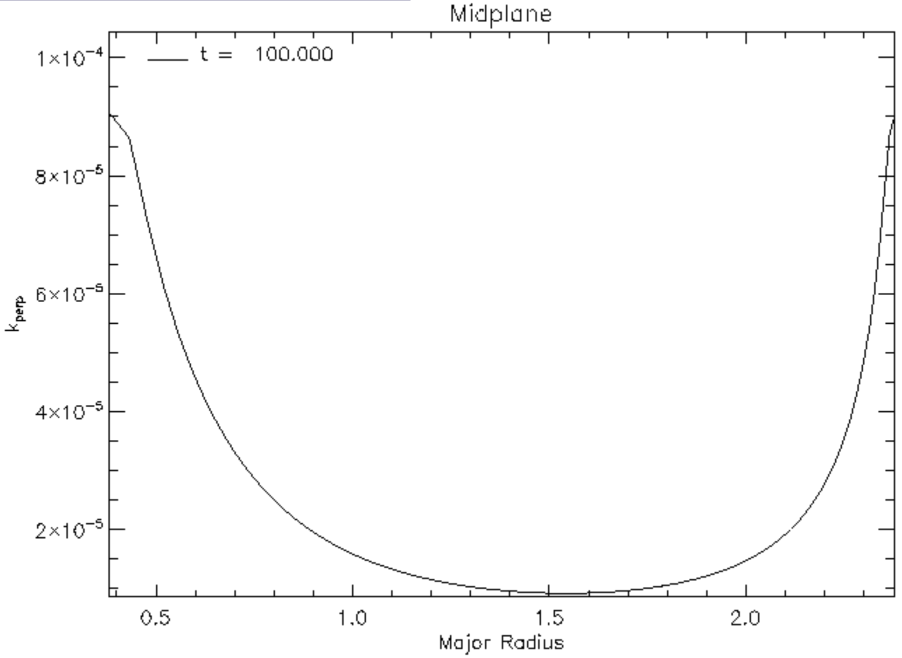
$$R(\theta) = R_0 + a\cos\left[\theta + \delta\sin(\theta)\right]$$
$$z(\theta) = a\kappa\sin(\theta)$$

$$T(\psi) = T_0\tilde{\psi},$$

$$n(\psi) = \frac{p}{2k_B T} = \frac{p_0}{2k_B T_0}\left[\alpha + (1-\alpha)\tilde{\psi}\right]$$





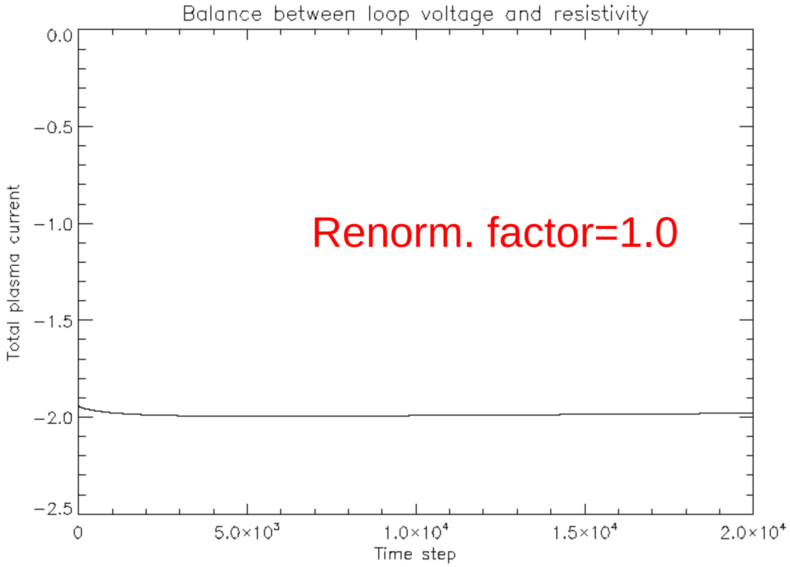


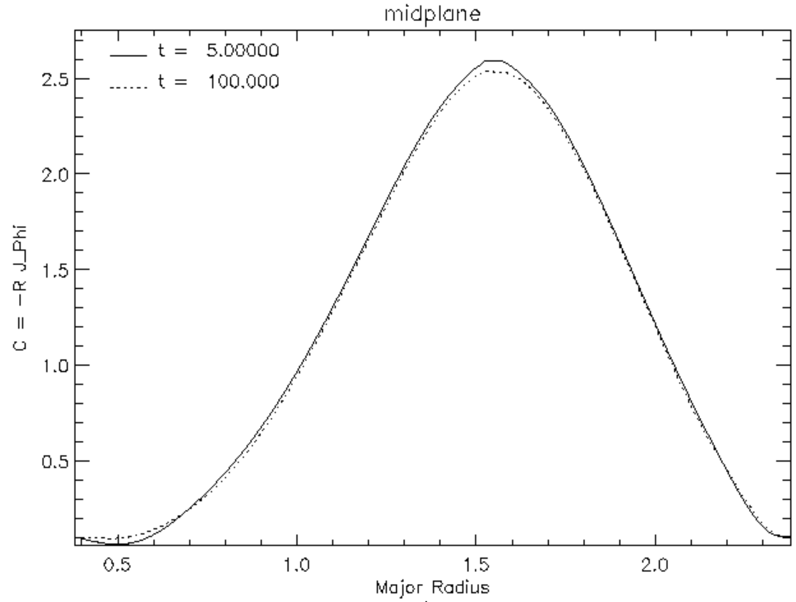
$q_{min} = 0.8203$

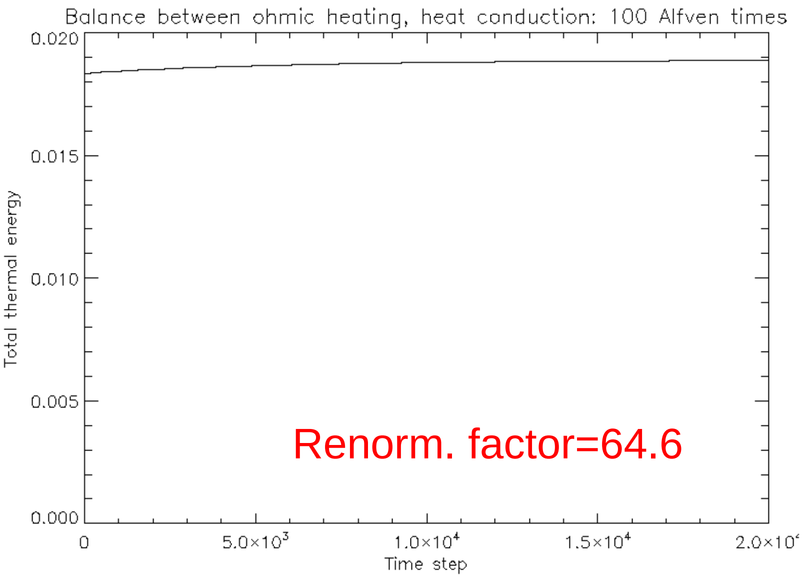
Minimum value: 9.21×10^{-6}

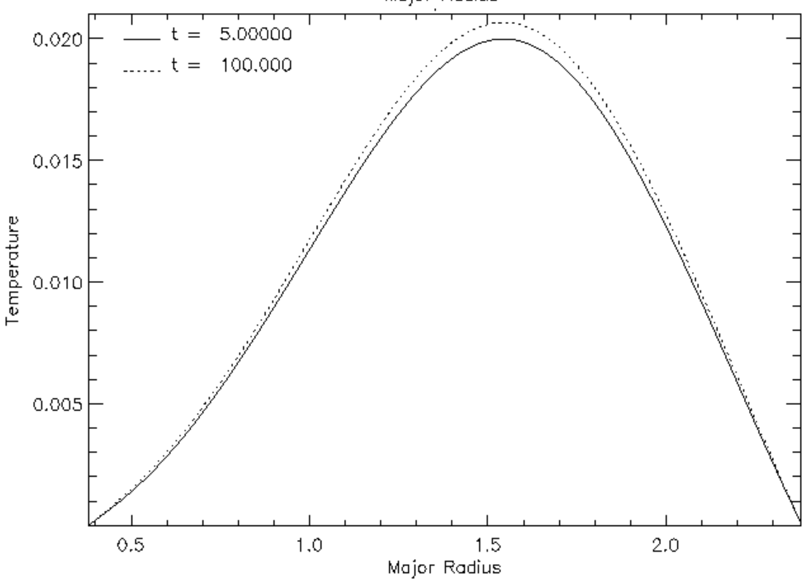
Old case: pkkk $\equiv 9.09 \times 10^{-4}$

Conservation properties



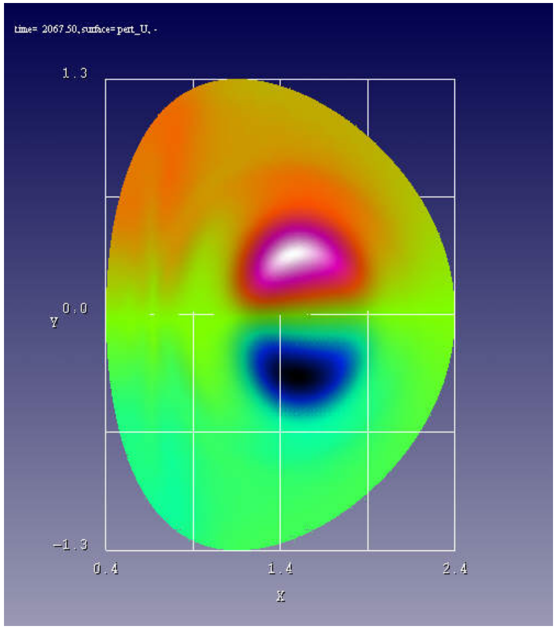




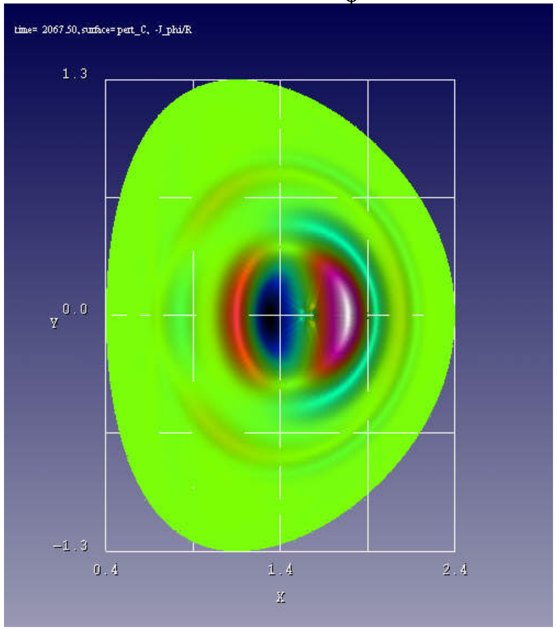


n=1 eigenmode

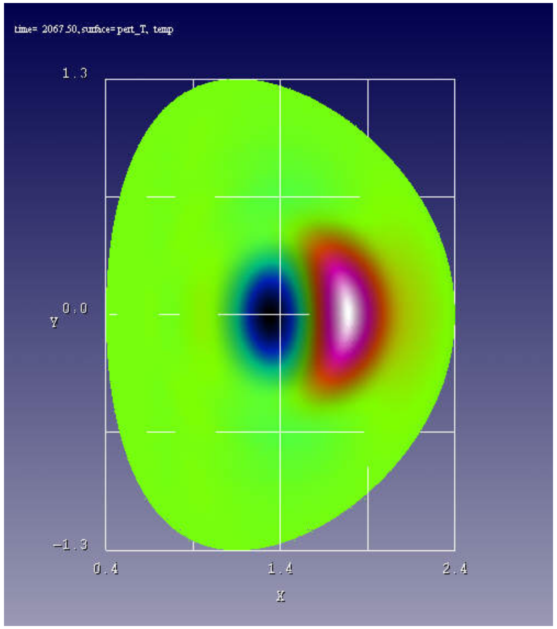
Velocity stream function U



C = $-RJ_\phi$



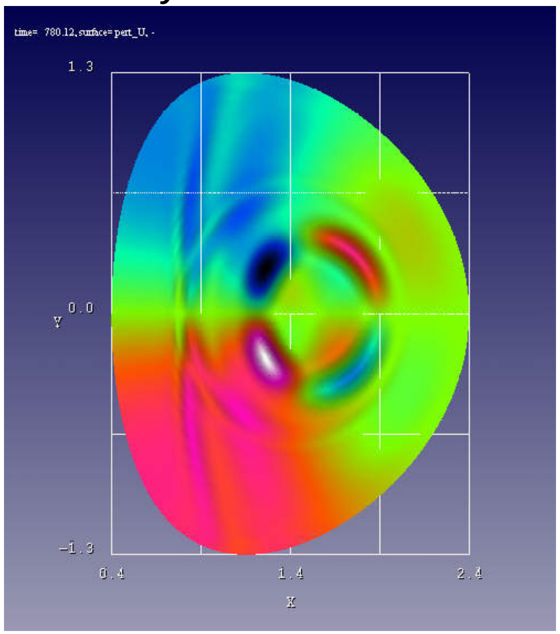
Temperature



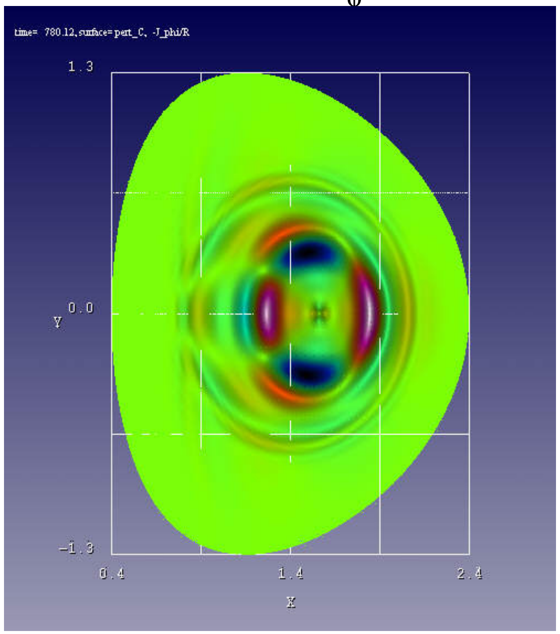
1,1 mode; $\gamma\tau_A \approx (4.52 \pm 0.05) \times 10^{-2}$

n=2 eigenmode

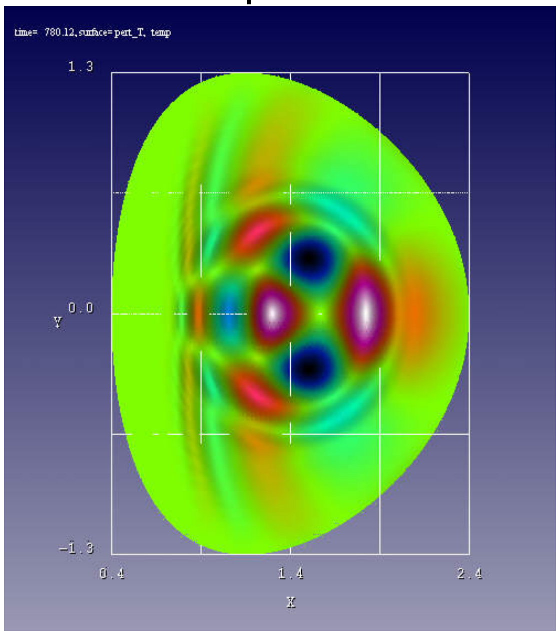
Velocity stream function U



C = -RJ$_\phi$



Temperature



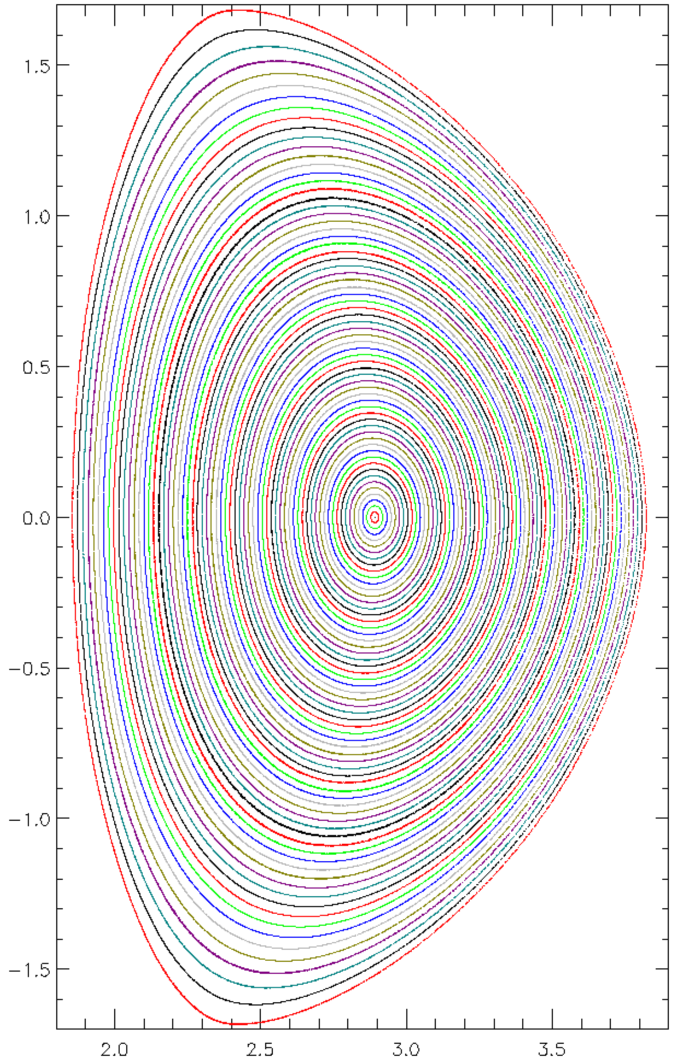
2,2 mode; $\gamma\tau_A \approx (4.015 \pm 0.005) \times 10^{-3}$

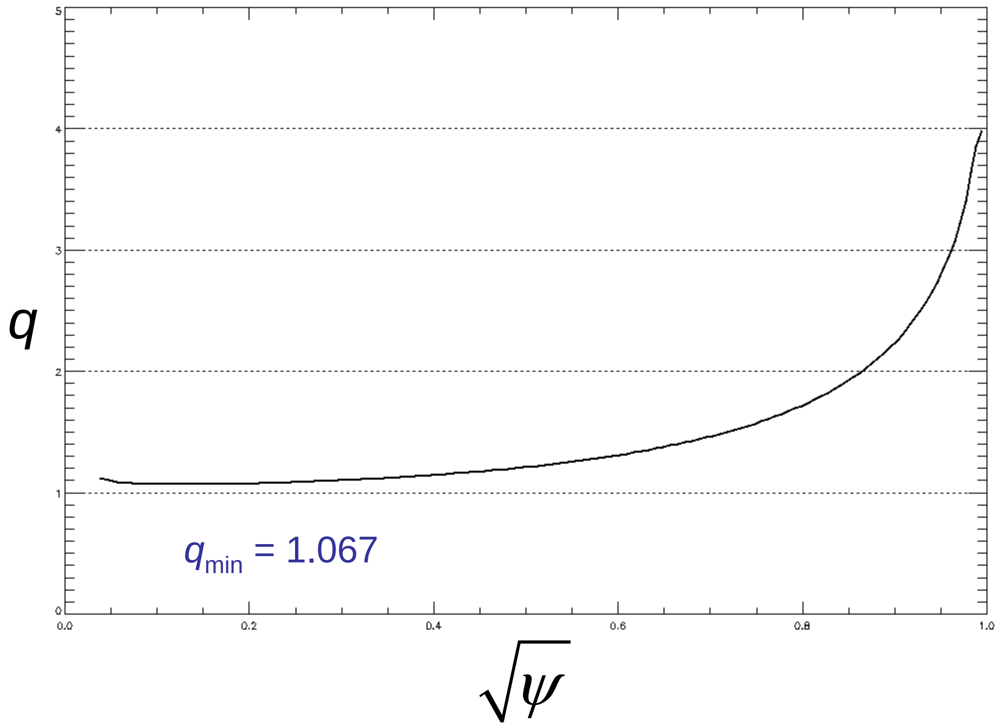
II. DIII-D Error Fields

Initial study

- Begin with a DIII-D equilibrium.
- Add an m=2, n=1 perturbation of specified amplitude to initial poloidal flux on plasma boundary.
- Measure plasma displacements, singular currents with linear code; infer island widths.
- Evolve M3D nonlinearly until saturation of n=1 islands; compare widths to linear result.

DIII-D Equilibrium





Initial Perturbation

- Add helical perturbation to poloidal flux function ψ on boundary of the form

$$\tilde{\psi}_{boundary}(\theta,\varphi) = \tilde{\psi}_0 \cos(\varphi - 2\theta)$$

where φ is the toroidal angle, θ is the geometric poloidal angle defined by

$$\tan(\theta) = \frac{z}{R - R_0}$$

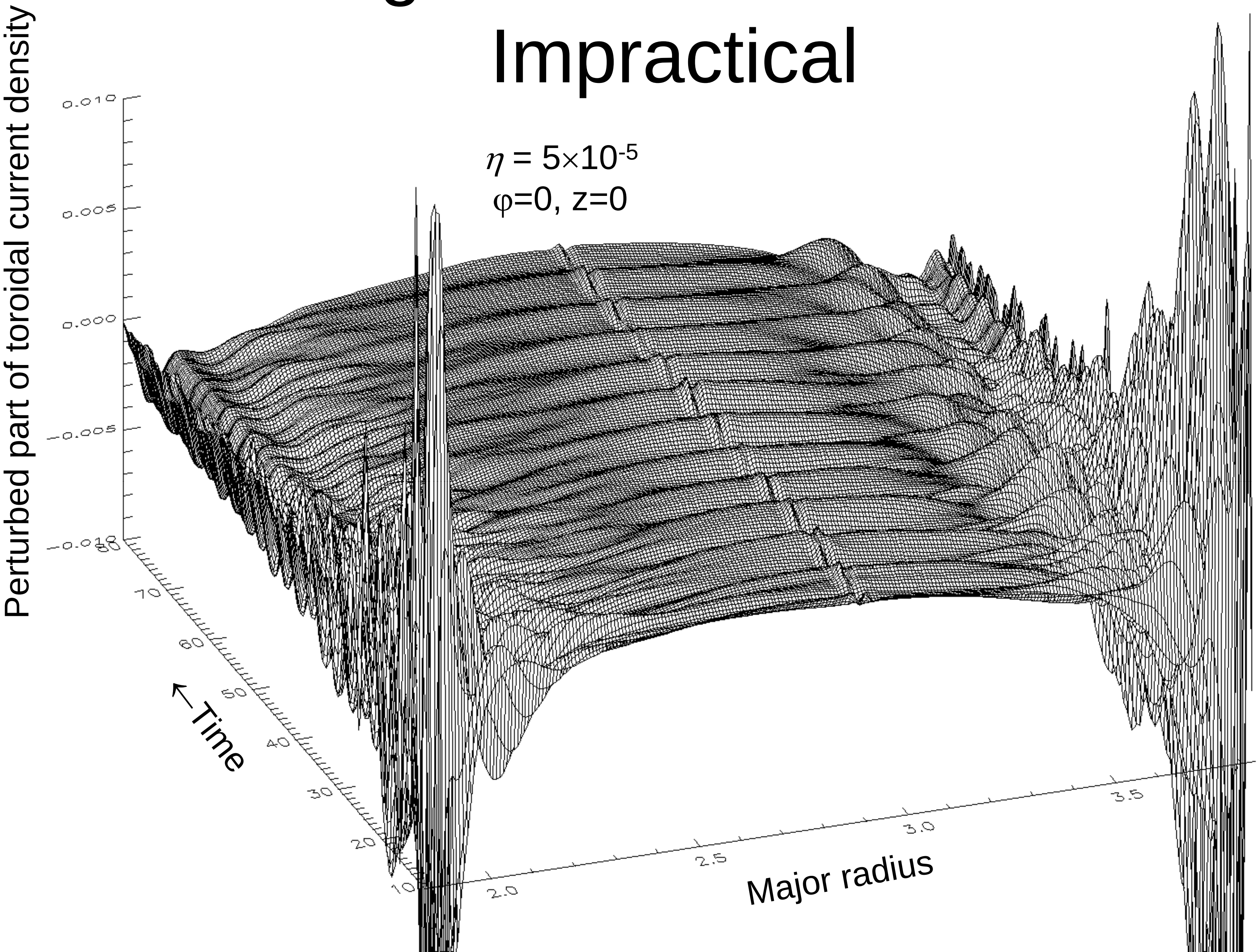
(normalized major radius R_0=2.89), and the equilibrium flux is ψ = 0 on the boundary and ψ = -0.506 on the magnetic axis.

- To generate 2,1 islands large enough to resolve numerically, choose

$$\tilde{\psi}_0 = 7.5\times10^{-3} \qquad \left(\frac{\tilde{\psi}_0}{|\psi_0|} = 1.48\times10^{-2} \right)$$

- Do not perturb initial boundary current density.

Tracking Ideal Current Sheets is Impractical



Initial State

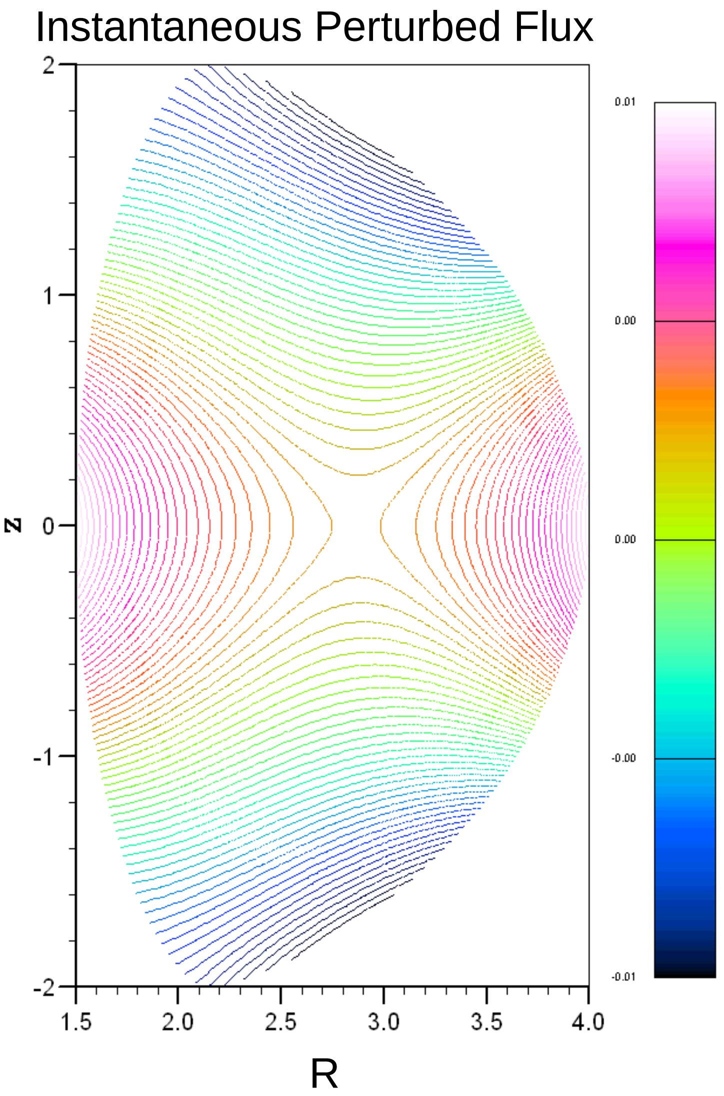
- Begin by solving the Poisson equation

$$\frac{\partial^2 \psi}{\partial R^2} - \frac{1}{R}\frac{\partial \psi}{\partial R} + \frac{\partial^2 \psi}{\partial z^2} = -RJ_\phi$$

for ψ subject to the perturbed boundary condition, where J_ϕ is the unperturbed equilibrium toroidal current density.

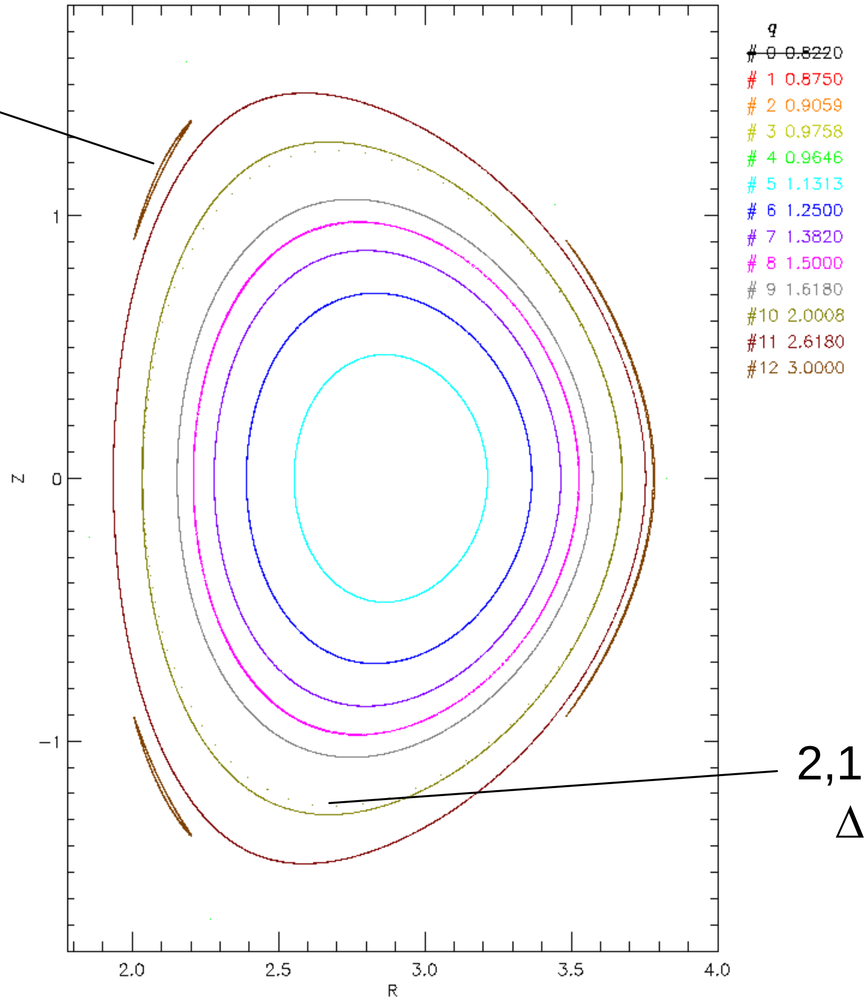
- Because the initial current remains unperturbed, the resulting state represents the superposition of the equilibrium field (including external and plasma currents) and the error field, without the plasma reponse.

- Time-evolving from this state with various choices of resistivity η will show the effect of the plasma response on the islands.



Initial State Has Magnetic Islands

Poincaré section, φ=0 plane

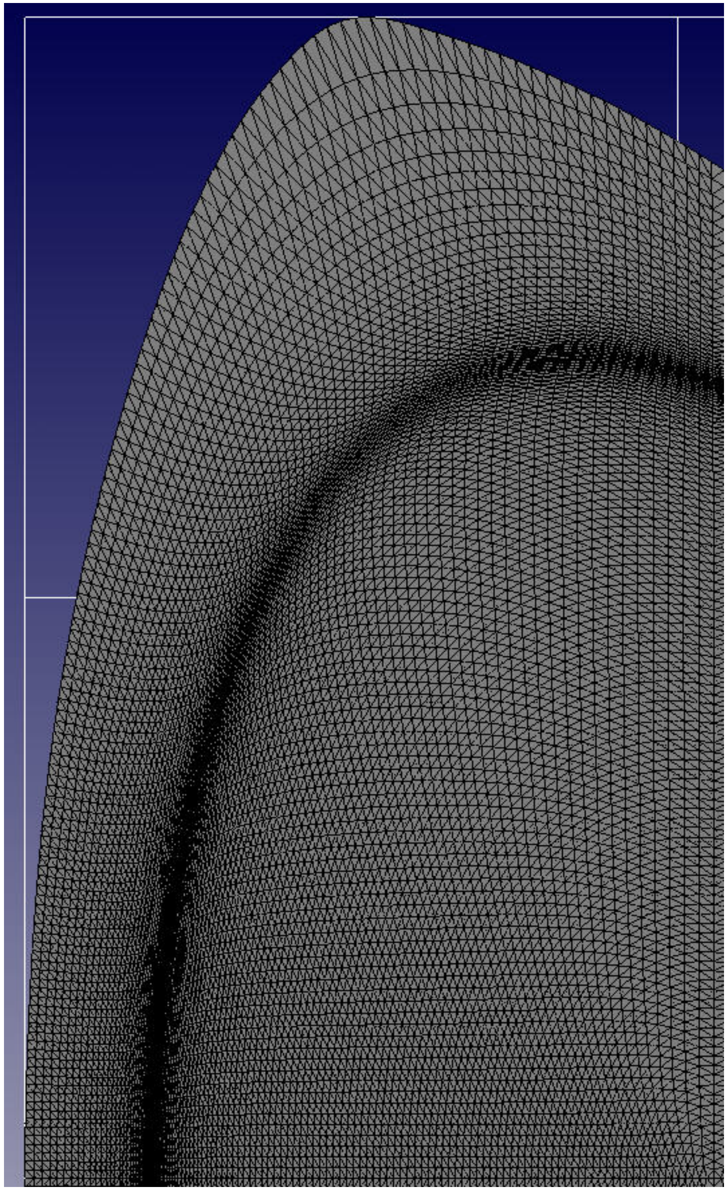


3,1 island width
$\Delta\psi = 1.33\%$

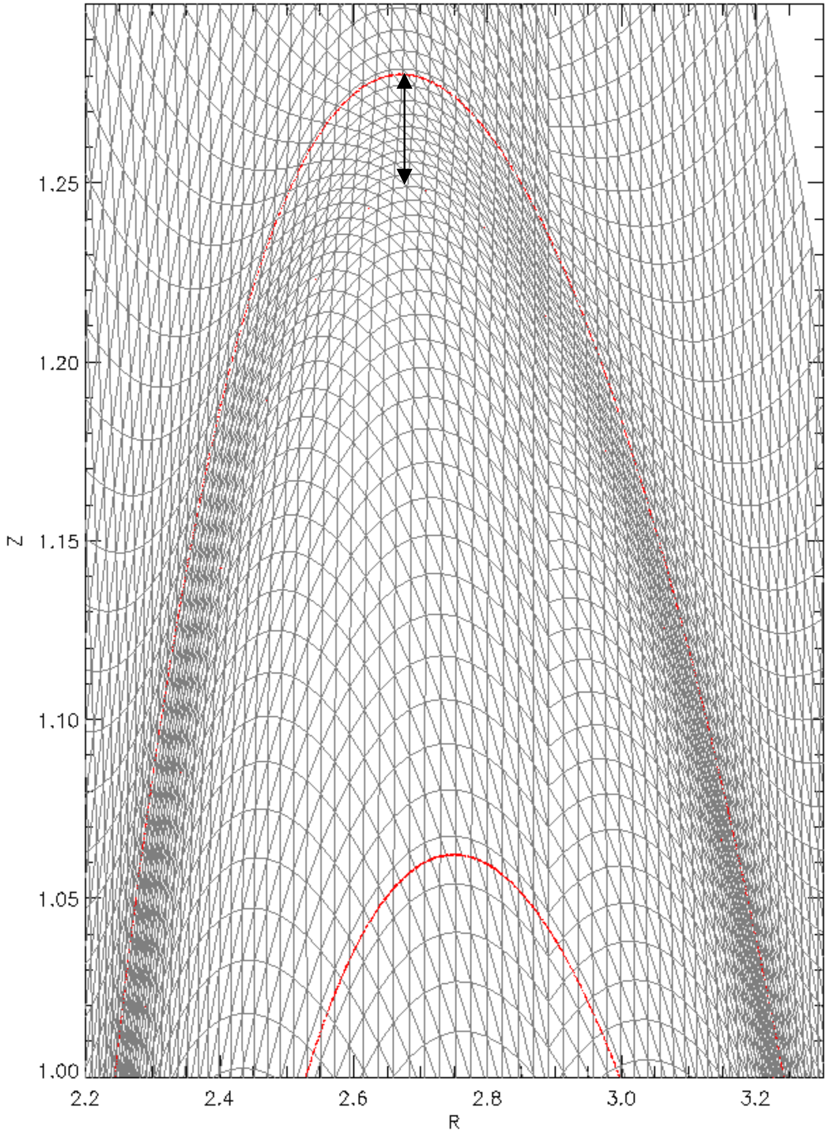
2,1 island width
$\Delta\psi = 1.09\%$

Resolving the Islands

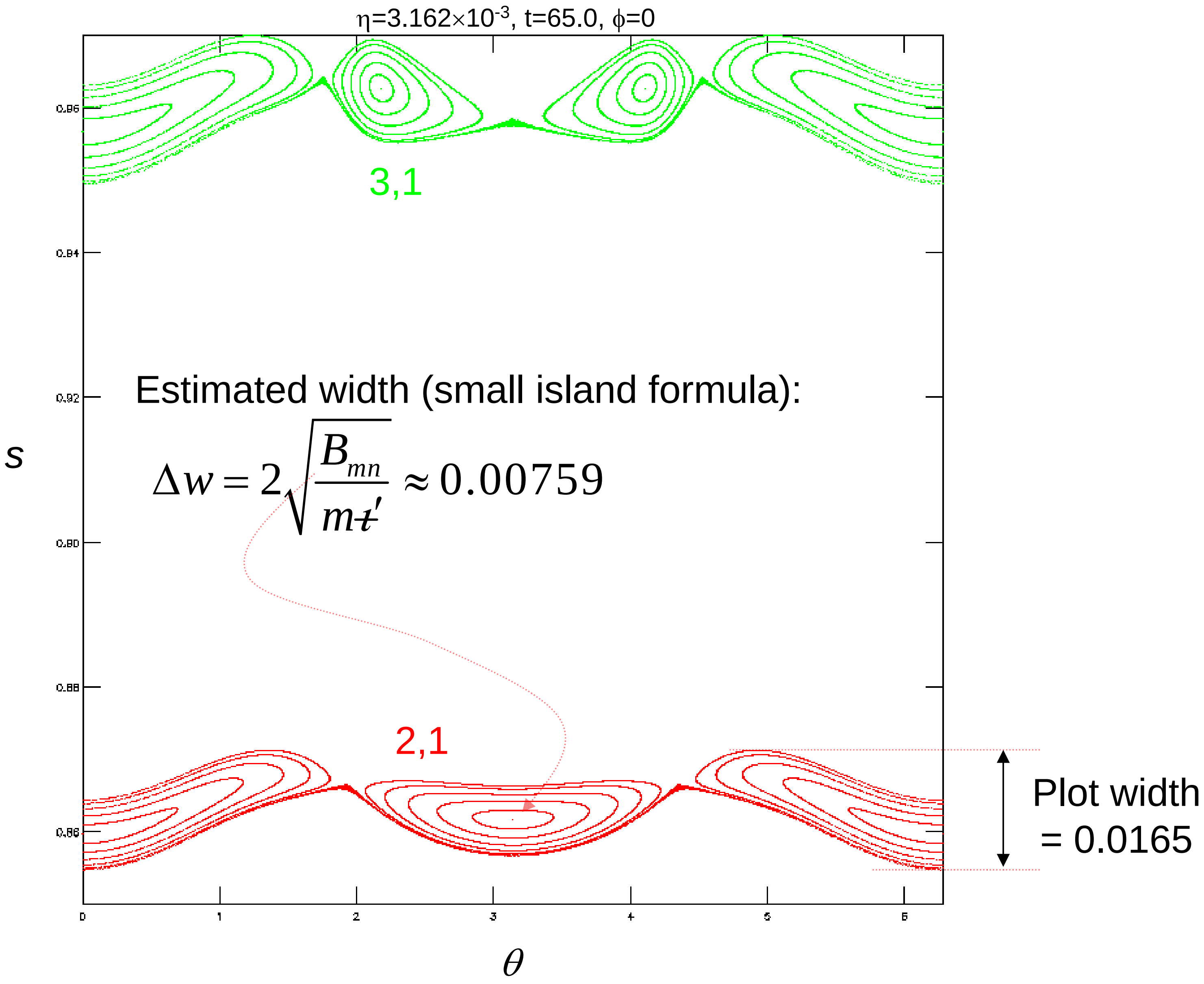
Poloidal mesh has 128 radial, 512 θ zones; packed x9 around q=2 surface.



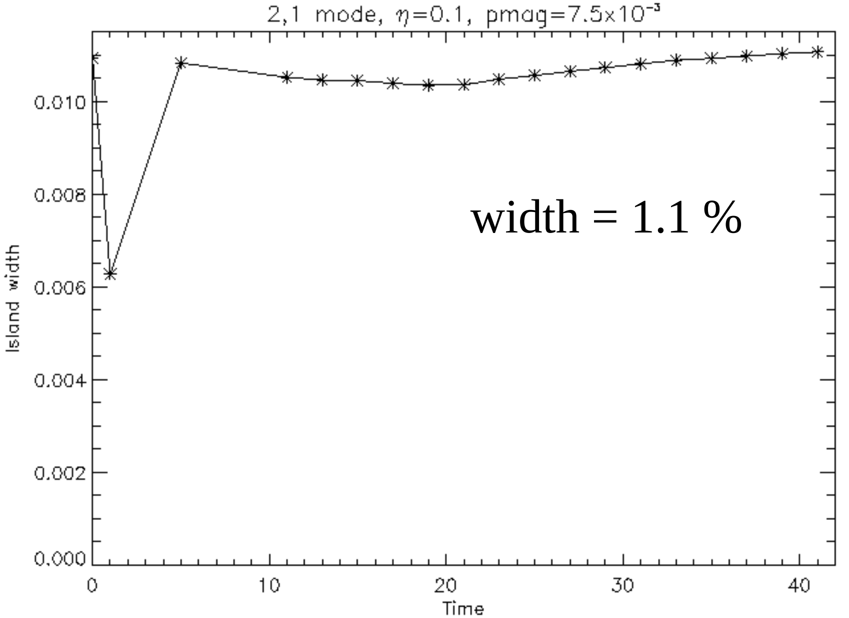
2,1 island spans nine zones → resolved.

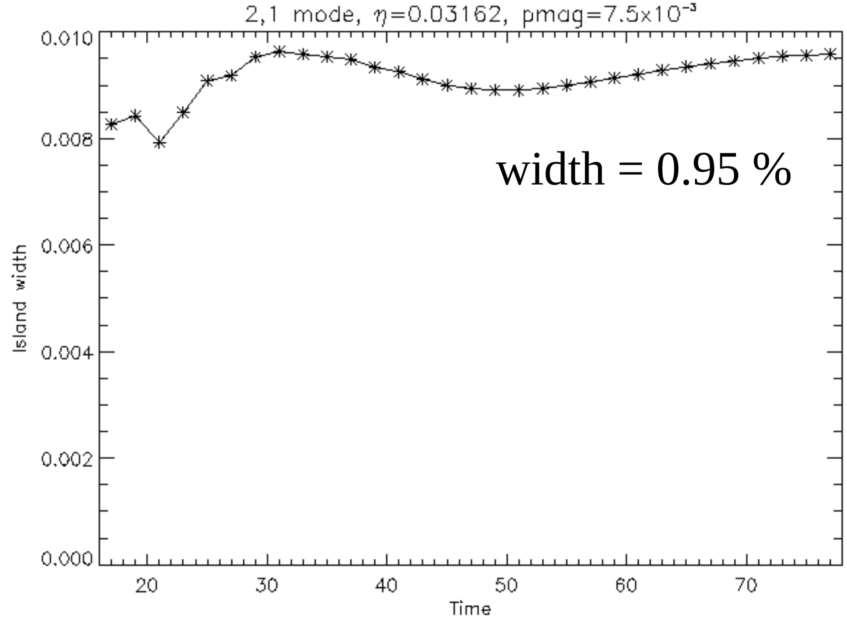


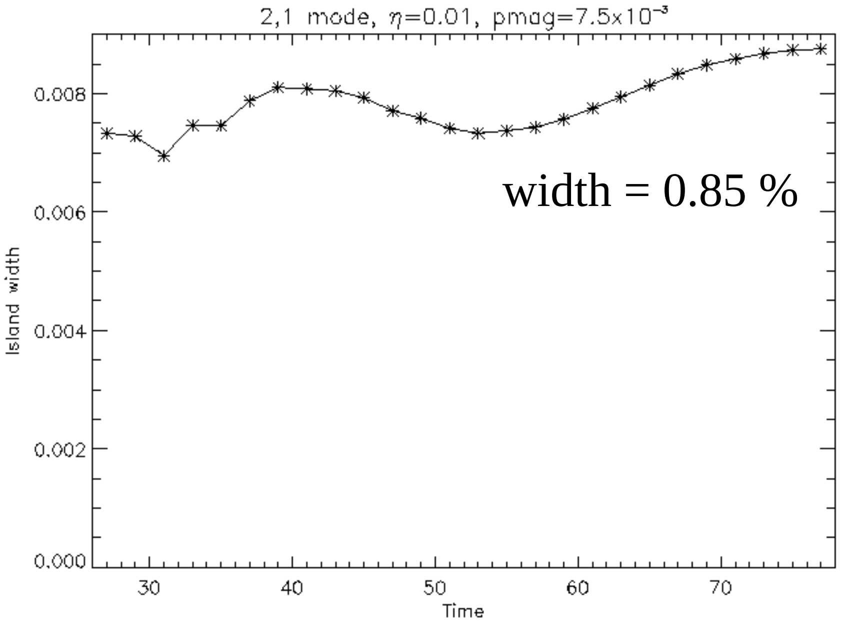
Measuring Island Widths

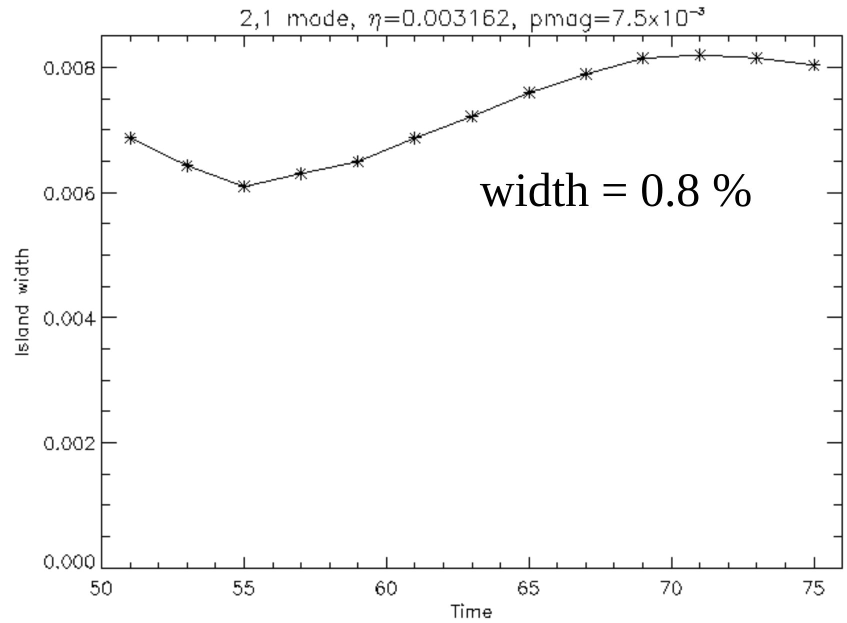


Approaching Steady States

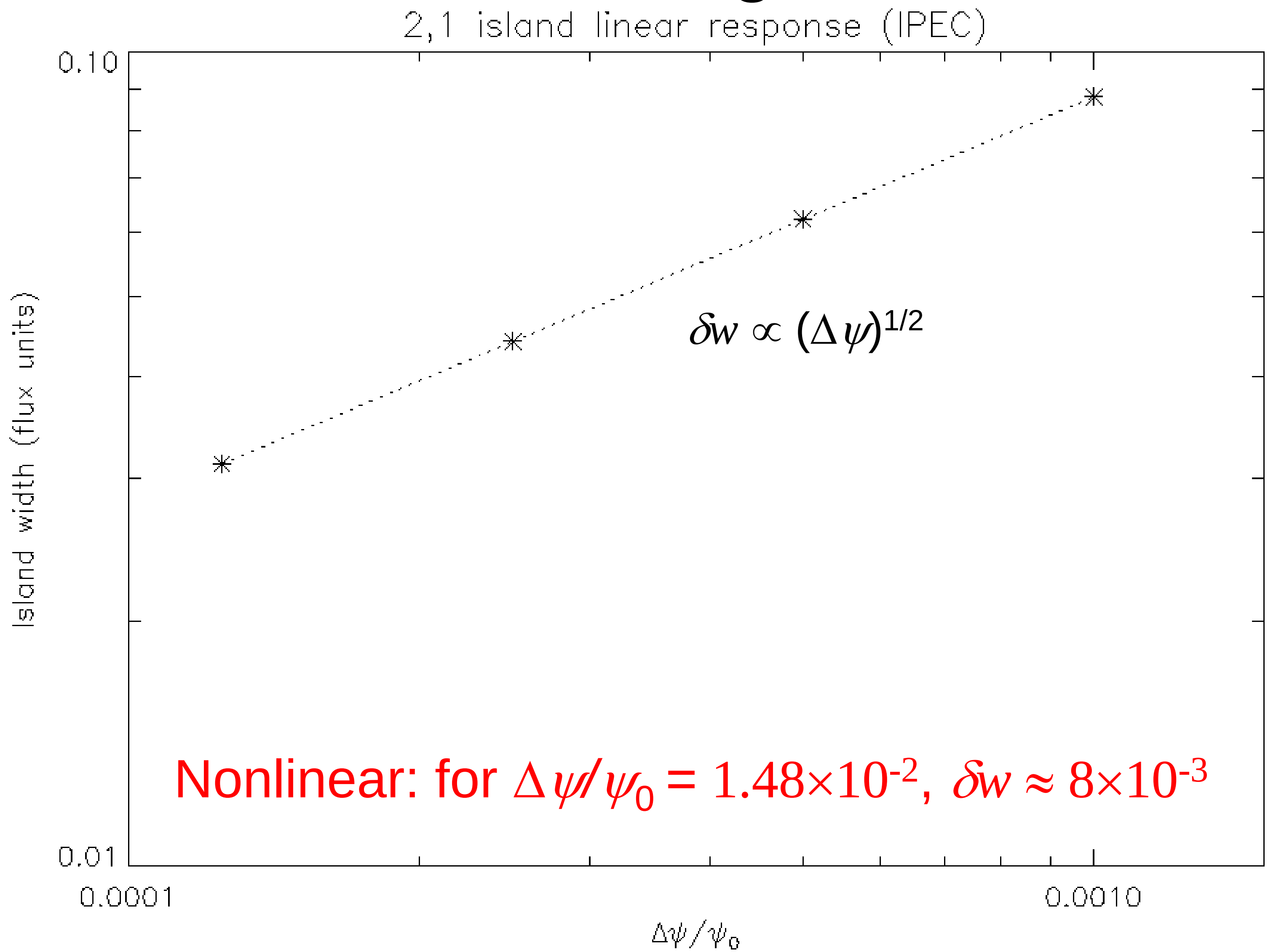








Nonlinear Results Disagree with Linear Scaling



Conclusions

- Nonlinear island width decreases as η decreases.
- Disagreement may be due to differences in boundary conditions, lack of nonlinear convergence, or inadequacy of linear model (nonlinear island saturation).
- More work is needed to resolve disagreement.
- Additional future work to include further scaling studies, and investigate effects of plasma rotation.