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Interpretation of Experiments on Collisional Drift Modes,* by

B. Coppi,† H. W. Hendel,‡ F. Perkins, and P. A. Politzer

We report the interpretation of experiments on low-temperature alkali plasmas in strong magnetic field[1] in terms of collisional drift modes, in which diffusion over the transverse wavelength, resulting from ion-ion collisions,[2] plays an important role.

Collisional drift modes[3] arise in the presence of a density gradient perpendicular to the magnetic field and result from the combined effects of ion inertia,[4] electron-ion collisions, and mean electron kinetic energy along the magnetic field lines.[5]

The principle experimental results are the consistent observations of a single-mode steady-state oscillations, identified as drift modes, and, at certain critical values of the magnetic field strength, of abrupt changes of both the azimuthal mode number and frequency[6] of the oscillations. These experimental results are explained in terms of a theory which finds abrupt stabilization of a particular mode with decreasing magnetic field as a result of diffusion over the transverse wavelength[2] due to ion-ion collisions.[7] This diffusion, in fact, suppresses the instability when the ion gyroradius reaches a critical size relative to the transverse wavelength of the mode. Although the observed relative drift wave amplitudes are not small ($n_1/n_o \approx e\phi/KT \sim 20\%$), the linearized approximation of the theory succeeds in predicting both the measured instability frequencies and the measured dependence of instability onset for the various modes on plasma density and other parameters.

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Figure 1 displays the experimental results for fixed neutral beam density and plasma temperature—i.e., for nearly constant ion density. For most values of magnetic field only one single mode is detected, but in the mode transition regions two separate modes are observed. The modes are localized, Fig. 2, in the radial direction with the amplitude maximum at approximately 1/3 of the plasma radius. For these localized modes we carry out the theory simulating cylindrical geometry by a one-dimensional "slab" model, with density gradient in the x direction and magnetic field in the z direction. Only electrostatic modes ($\underset{\sim}{E} = - \underset{\sim}{\nabla}\phi$) are considered—an approximation valid when $\beta = 8\pi\ P/B^2 << 1$, as is the case here ($\beta \lesssim 10^{-6}$). The time-independent electric field is ignored because it only produces a Doppler shift. The following equations can be used, and with standard notation

$$nM \frac{\partial \underset{\sim}{u}_{1i\perp}}{\partial t} - \mu \nabla^2 \underset{\sim}{u}_{1i\perp} = - \underset{\sim}{\nabla}_{\perp} p_1 + (\underset{\sim}{J}_1 \times \underset{\sim}{B}/c) \tag{1a}$$

$$- \nabla_{\parallel} (n_1 KT_e - en\phi_1) - \nu_{ei} n m u_{1e\parallel} = 0 \quad , \tag{1b}$$

$$\frac{\partial n_1}{\partial t} + \underset{\sim}{u}_{1i\perp} \cdot \underset{\sim}{\nabla} n + \underset{\sim}{u}_i \cdot \underset{\sim}{\nabla} n_1 = 0 \quad , \tag{1c}$$

$$\frac{\partial n_1}{\partial t} + \underset{\sim}{u}_{1e\perp} \cdot \underset{\sim}{\nabla} n + n \nabla_{\parallel} \cdot u_{1e\parallel} = 0 \quad , \tag{1d}$$

$$\underset{\sim}{u}_{1e\perp} = - \underset{\sim}{\nabla}\phi_1 \times \underset{\sim}{B}\, cB^{-2} \quad , \quad J_{1\parallel} = - en u_{1e\parallel} \quad , \tag{1e}$$

$$\underset{\sim}{\nabla}_{\perp} \cdot \underset{\sim}{u}_{1i\perp} = 0 \, , \; \underset{\sim}{\nabla} \cdot \underset{\sim}{J}_1 = 0 \, , \; \underset{\sim}{u}_i = -KTc(eB^2 n)^{-1} \underset{\sim}{\nabla} n \times \underset{\sim}{B} \quad . \tag{1f}$$

Perturbed quantities are indicated by the subscript 1. Ion motion and electron inertia along the lines of force have been ignored.[8] Ion-ion collisions enter through the coefficient μ, which is given by $\mu = nKT\nu_{ii}/4(\Omega_i^2 h_\pi^2)$ for the experimental condition,[9] with Ω_i the ion gyrofrequency and h_π a dimensionless coefficient tabulated in Ref. 9.

We assume a WKB-type solution $\phi = \phi_1 \exp\,(i \int k_x\, dx + ik_y y + ik_{\parallel} z + i\omega t)$ and consider modes localized in the x direction. In the experiments

$k_{\parallel} \approx \pi/L$, where L is the plasma column length. This localization will restrict us to modes with $m > 1$, since $m = 1$ modes are equivalent to an off-axis shift of the whole plasma column and are therefore not localized. By expressing $\underset{\sim}{J}_{1\perp}$ and $\underset{\sim}{J}_{1\parallel}$ in terms of ϕ and using the same method outlined in Ref. 10, one derives for $T_e = T_i$ the dispersion relation

$$i(\omega + k_y v_d)\left(\frac{k_{\parallel}^2 KT}{m_e \nu_{ei}} - \frac{\nu_{ii} b^2}{4}\right) = b\left(\omega - k_y v_d - \frac{i\nu_{ii} b}{2}\right)\omega \quad , \qquad (2)$$

where $v_d = -(1/n)(dn/dx)(KTc/eB)$ is the electron diamagnetic velocity, $b = \frac{1}{2}(k_x^2 + k_y^2)a_i^2$ assumed to be smaller than unity, and $a_L = (2KT/M)^{1/2}$ $1/\Omega_i$ is the ion Larmor radius. In deriving Eq. (2) we have also neglected terms of order $k_{\parallel}^2 KT(m_e \nu_{ei} k_y v_d)^{-1}$ in comparison with unity. This dispersion relation can be solved numerically, and reveals two important points. First, there exists a critical value of b given by $b_c^2 = 4k_{\parallel}^2 KT(m_e \nu_{ei} \nu_{ii})^{-1}$ such that the linearized growth rate is positive only for $b < b_c$. In view (a) of Fig. 1 the theoretical growth rates γ are shown for the measured m-numbers, and the rapid rise of $\gamma = -\,\mathrm{Im}(\omega)$ with increasing magnetic field, once the condition $b < b_c$ is satisfied, is evident. Second, the growth rate γ is maximized for the dimensionless parameter $\Sigma_o = k_{\parallel}^2 KT(m_e \nu_{ei} k_y v_d b)^{-1}$ slightly above unity and $b < b_c$, m_e being electron mass. At this point the magnitude of the linearized growth rate is $\gamma \approx 0.2\, k_y v_d$ and is comparable to the instability frequency $\mathrm{Re}(\omega) \approx 0.5\, k_y v_d$. View (b) of Fig. 1 shows that the observed frequencies are proportional to $k_y v_d$. More detailed measurements[11] have given a proportionality factor $\simeq 1/2$. These considerations indicate that the criterion $b = b_c$, which can be written as

$$\frac{(k_x^2 + k_y^2)^{1/2}}{B} = \frac{k_{\perp}}{B} = \left(\frac{4e^4 k_{\parallel}^2}{M^2 c^4 KT m_e \nu_{ei} \nu_{ii}}\right)^{1/4} \propto (T/n)^{1/2} \frac{1}{M^{3/8}} \quad , \qquad (3)$$

describes the onset of instability (M = ion mass). In Fig. 3 we plot the experimental values of $B/k_\perp$ at instability onset versus $n^{1/2}$ and find agreement of experiment and this theoretical prediction, including the numerical coefficient. In addition, other measurements have shown dependence on plasma temperature and ion mass, as predicted by the transition criterion, Eq. (3). The amplitude of the oscillation has been measured as a function of radial position. The extent of the oscillation in the radial direction was found to increase for decreasing values of m, as expected from the analytic solution, where $\partial/\partial x \sim k_y$ (see Fig. 4). The position of the amplitude maximum is found not to coincide with the position of maximum density gradient, as we can infer from the theoretical dependence of the growth rate on n and dn/dx as a function of the radius (Fig. 2). Although the linearized theory cannot predict the large (nonlinear) experimental amplitudes, we note that the measured amplitudes, as a function of magnetic field, follow closely the theoretical growth rate, indicating how to pursue a nonlinear analysis.[12]

To explain the observation of one single mode at a given magnetic field, invoking of nonlinear effects (such as mode locking)[13] is not strictly required because, as indicated by the dispersion relation, Eq. (2), only one mode has appreciable growth rate outside the mode transition region.[14]

Several important considerations of a general nature arise from this work. First, the agreement of the theoretically prodicted frequencies with observations, coupled with the fact that the mode amplitudes maximize when $\Sigma_o \approx 1$, implies that the large growth rates predicted by the linearized theory $[\gamma \approx 0.2 k_y v_d \sim \mathrm{Re}(\omega)]$ can occur at proper values of density and magnetic field even though the transverse wavelength is larger than the ion gyroradius. This feature is not shown by the collisionless drift waves and can make the collisional ones suitable to explain anomalous Bohm-type diffusion.[15] Secondly, the role of ion-ion collisions, which has been predicted to stabilize collisional interchange modes in plasma confinement configurations having magnetic shear,[2] has been demonstrated experimentally in this simple geometry in agreement with theory. Finally, we notice that the maximum amplitude reached by the instability is such that the perturbed $\underset{\sim}{E} \times \underset{\sim}{B}$ drift velocity is of the order of the diamagnetic drift velocity ($\sim 2 \times 10^3$ cm/sec). This point is also in agreement with a theoretical estimate of the amplitude saturation level reached by the instability.[15]

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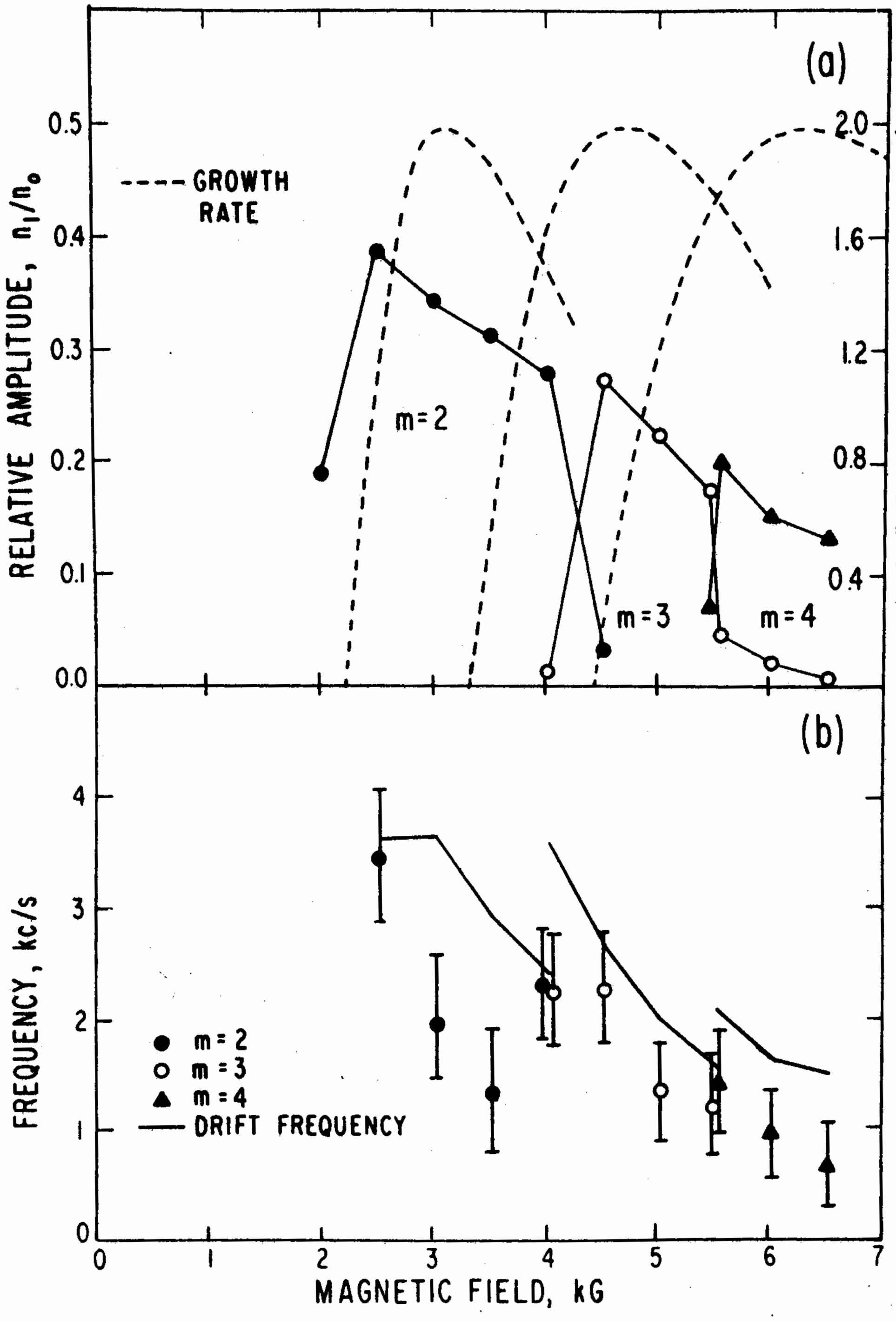

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Fig. 1. (a) Comparison of observed mode amplitudes vs theoretical growth rates for m = 2, 3, 4. The absolute magnetic field scale for the theoretical calculations has been adjusted to give the best fit to the data. (b) Comparison of the observed frequencies (after the subtraction of the doppler shift due to the radial electric field) with the drift frequency $k_y v_d/2\pi$ computed from experimentally determined values of k_y, n, and dn/dx. The experimental uncertainty in the drift frequency is ± 0.5 kc/s.

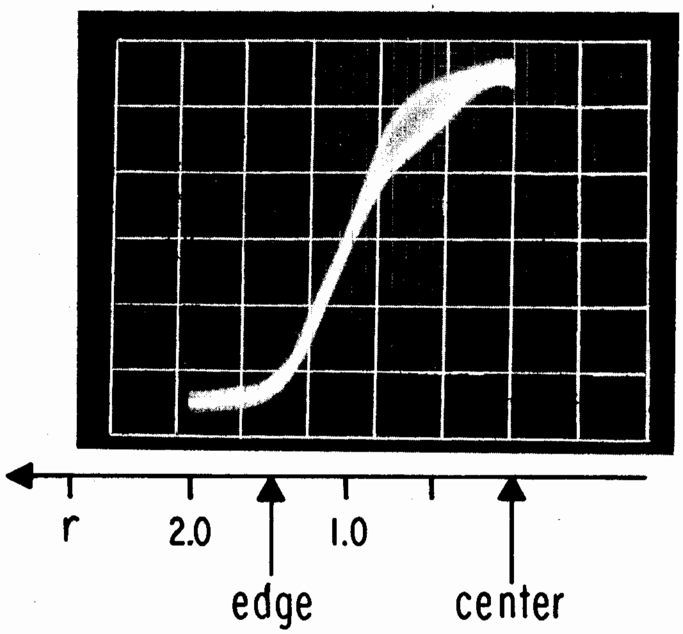

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Fig. 2. The observed oscillation amplitude (in white) and density (center line of white area) vs radius. The positions of the center and edge of the plate are shown.

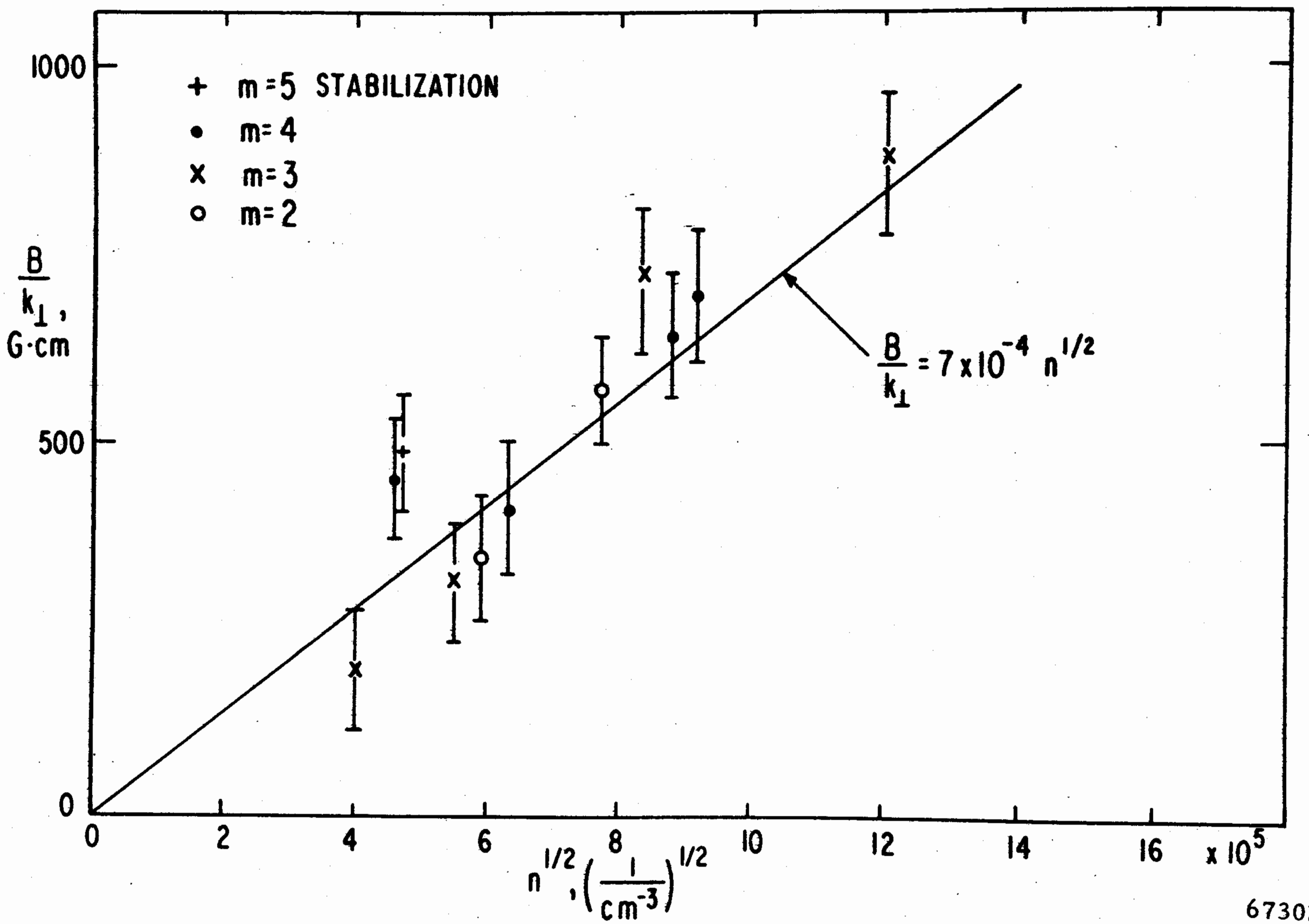

Fig. 3. A comparison between the observed onset of instability and the criterion of Eq.(3). Experiment gives the result $B/k_\perp = 7 \times 10^{-4}\, n^{1/2}$, while theory predicts $B/k_\perp = 8.7 \times 10^{-4}\, n^{1/2}$.

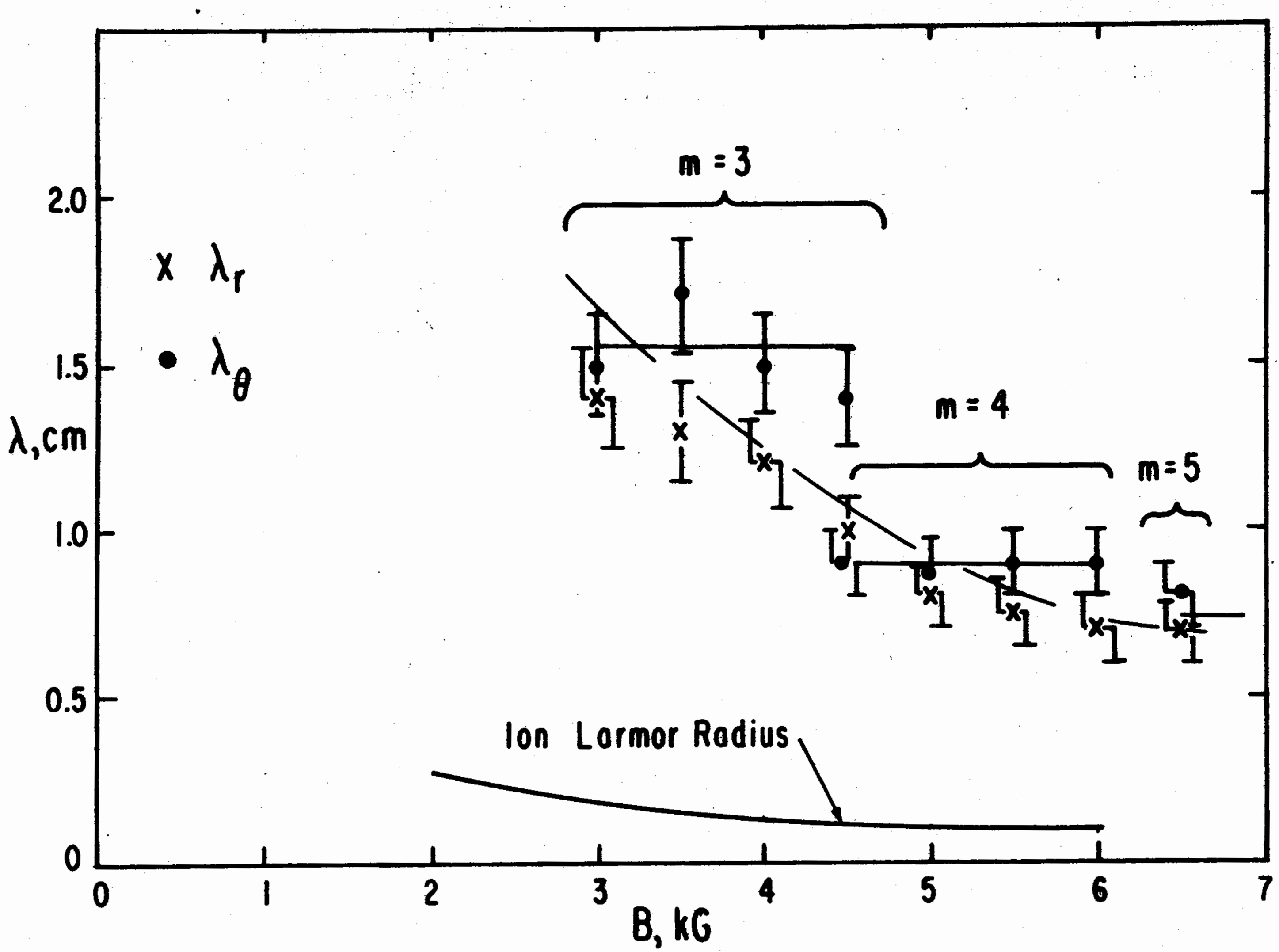

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Fig. 4. Measurements of the azimuthal wavelength λ_θ (solid lines) and radial extent λ_r (dashed line) of the oscillations.