# CDX-U: New Equilibrium & M3D Results;

# DIII-D Error Field Calculations

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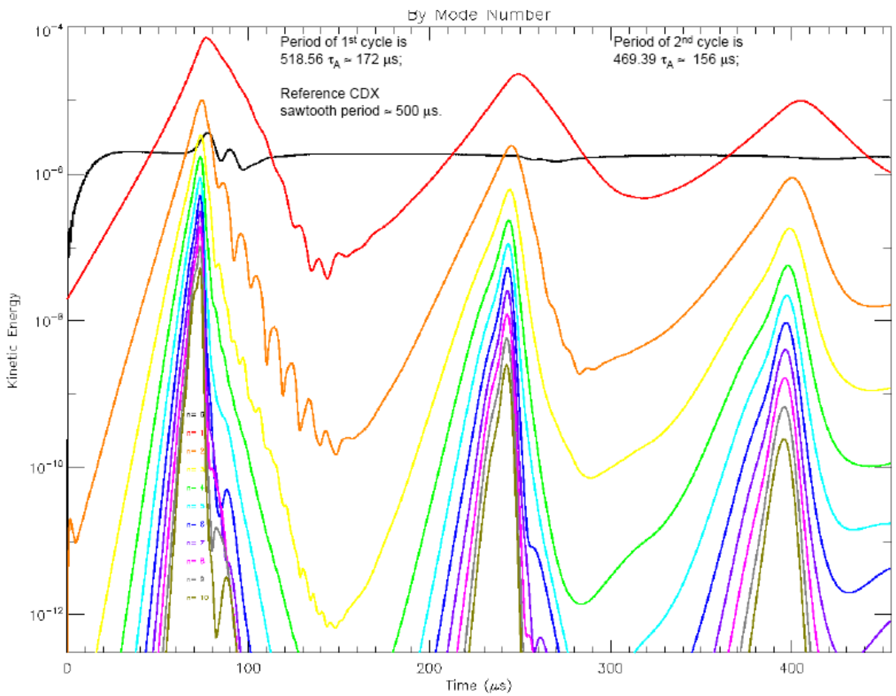
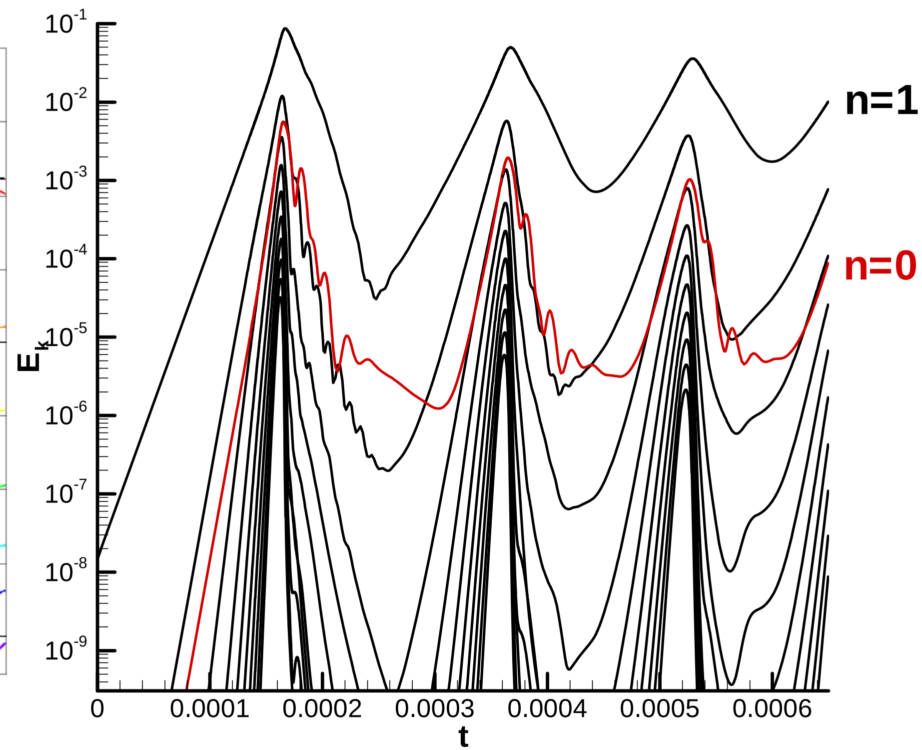
# Previous Nonlinear M3D-NIMROD Comparison




M3D Kinetic energy in first 10 modes




NIMROD Kinetic energy in first 10 modes

Good agreement with each other; period **not** in agreement with experiment.

# Reasons for Proposing a New Benchmark

- M3D and NIMROD results from 1st benchmark agree with each other but not with experiment. Better fidelity to experiment should yield better validation.
  - Replace current source with loop voltage.
  - Replace pressure source with ohmic heating.
  - Use a much more realistic profile for $\kappa_\perp$.
  - Allow resistivity to track evolving temperature profile.
  - Use constant Prandtl number.

- Beginning with an analytically specified equilibrium will make it possible to publish the benchmark as a standard test problem available to other nonlinear MHD codes.

# Specification of Analytic Equilibrium

| **Quantity** | **Value** |
|---|---|
| Major radius $R_0$ | 0.341 m |
| Minor radius $a$ | 0.247 m (aspect ratio = 1.38) |
| Ellipticity $\kappa$ | 1.35 |
| Triangularity $\delta$ | 0.25 |
| Central temperature ($T_e$=$T_i$) | 100 eV |
| Normalized central pressure $\mu_0 p_0$ | $7.5 \times 10^{-4}$ (implies $n_0 = 1.86 \times 10^{19}$ m$^{-3}$) |
| $\alpha$ Parameter in pressure equation* | 0.1 |
| Vacuum value $g_0$ of $R \cdot B_T$ | 0.04252 T·m |
| Effective ion charge $Z_{EFF}$ | 2.0 |
| Loop voltage $V_L$ | 3.1741 V (implies $q_0 \approx 0.82$) |

$$*p(\psi) = p_0\left[\alpha\tilde{\psi} + (1-\alpha)\tilde{\psi}^2\right], \text{ where } \tilde{\psi} \equiv \frac{\psi - \psi_{\text{limiter}}}{\psi_{\text{axis}} - \psi_{\text{limiter}}}.$$

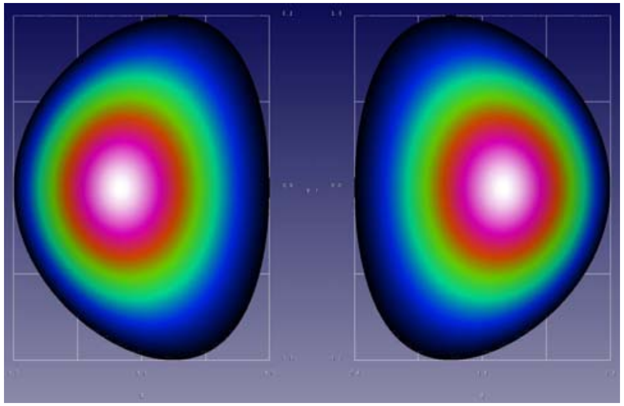
Use equilibrium code to solve Grad-Shafranov equation, with profile of heat conduction coefficient $\chi$ computed self-consistently to keep temperature constant given profile, energy supplied by applied $V_L$.

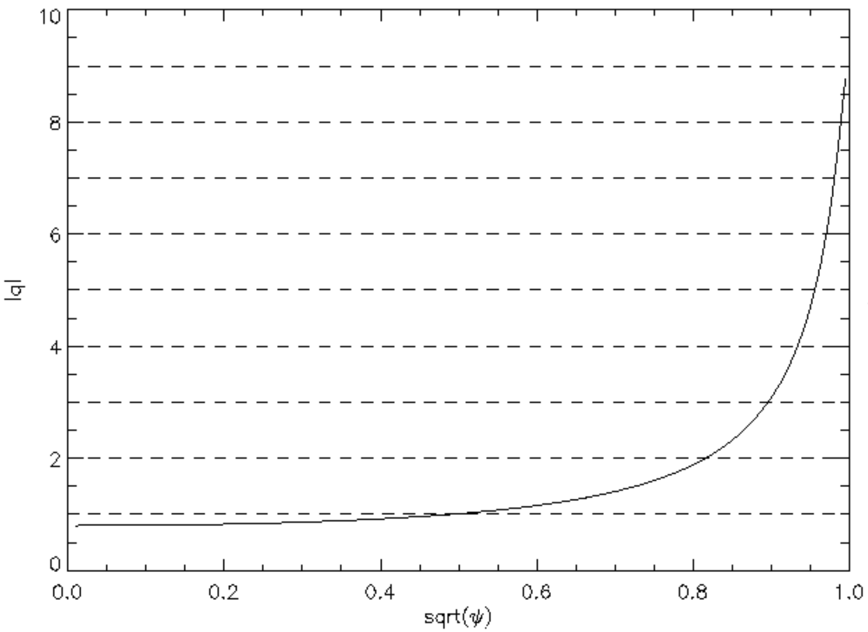
# Form of New Equilibrium

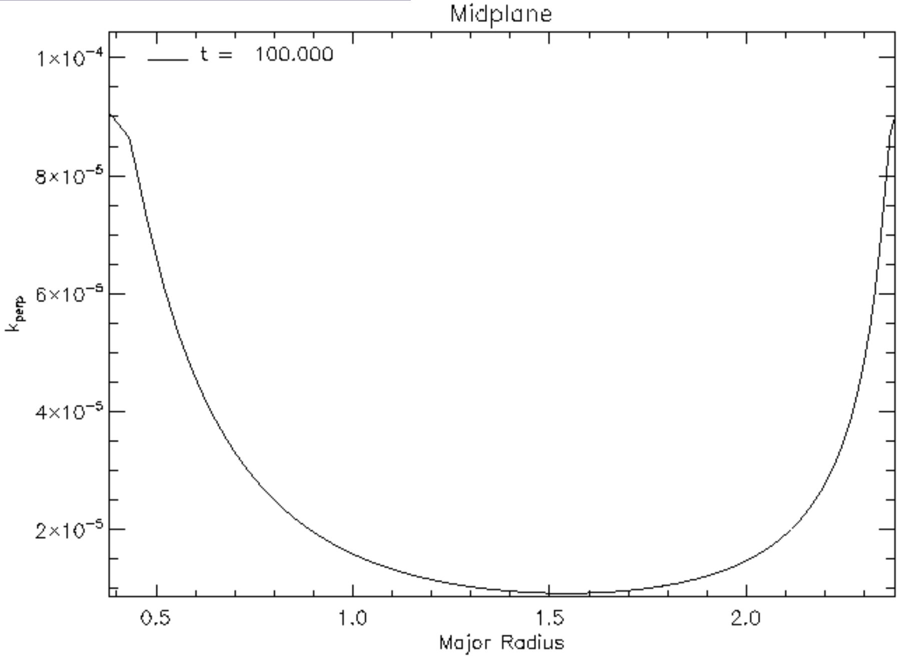
$$R(\theta) = R_0 + a\cos\left[\theta + \delta\sin(\theta)\right]$$
$$z(\theta) = a\kappa\sin(\theta)$$

$$T(\psi) = T_0\tilde{\psi},$$

$$n(\psi) = \frac{p}{2k_B T} = \frac{p_0}{2k_B T_0}\left[\alpha + (1-\alpha)\tilde{\psi}\right]$$






$q_{min} = 0.8023$

Minimum value: $9.21 \times 10^{-6}$

Old case: pkkk $\equiv 9.09 \times 10^{-4}$

# Transport Coefficients

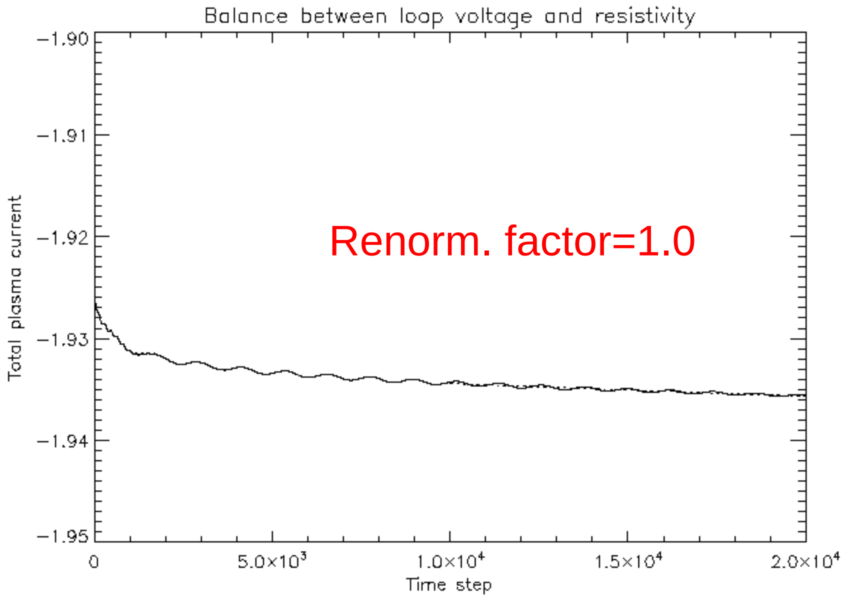
- Evolving Spitzer resistivity $\eta(\boldsymbol{x},t)\propto T^{-3/2}$ with cutoff 100x initial central value; initial central $S$ = $1.94\times10^4$.

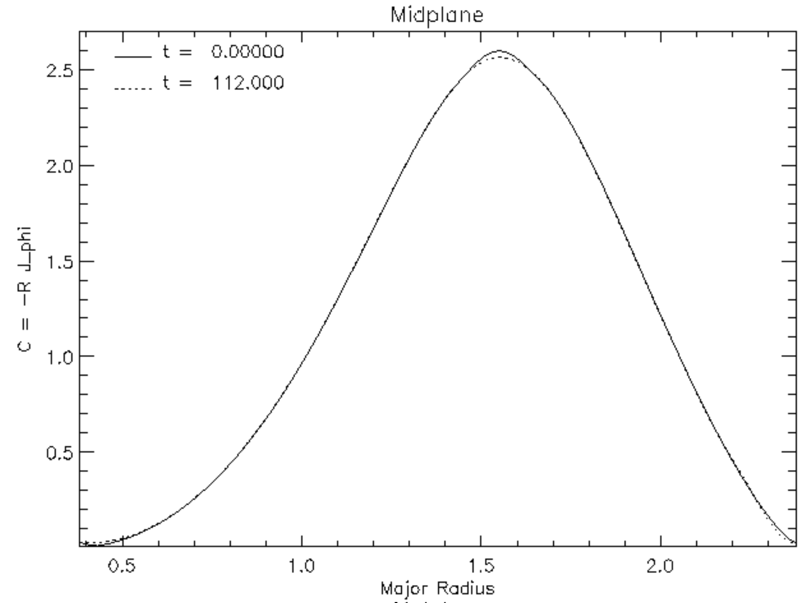
- Constant Prandtl number 10 (evolving axisymmetric viscosity).

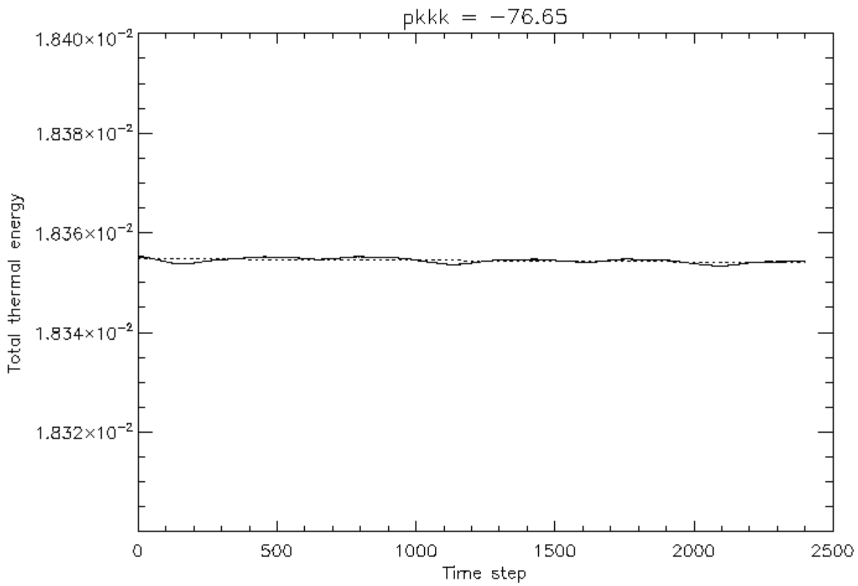
- Perpendicular heat diffusivity $\kappa_\perp$ read from self-consistent steady state computed with equilibrium code; central value renormalized to about 2.03 $m^2/s$ to maintain steady-state.

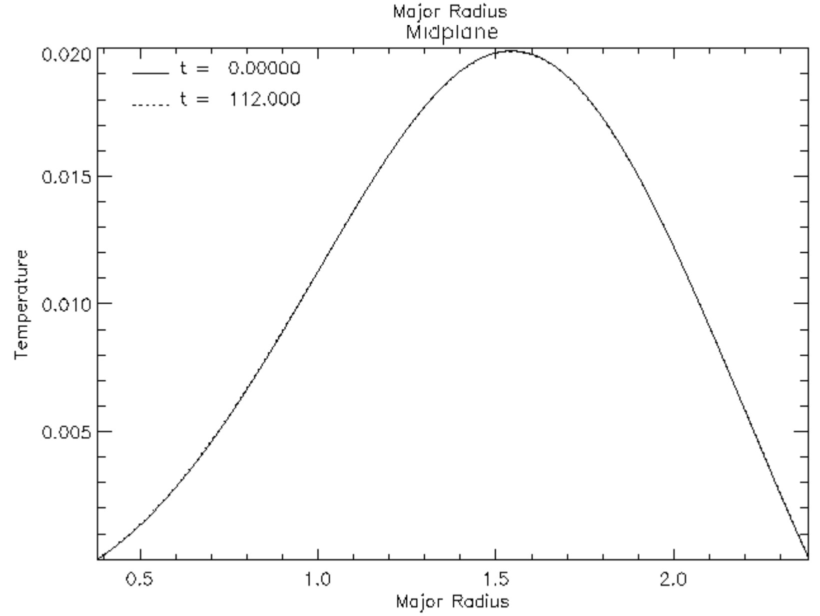
- Parallel heat conduction as in previous case ($v_{Te}$ = 6 $v_A$).

# Conservation properties



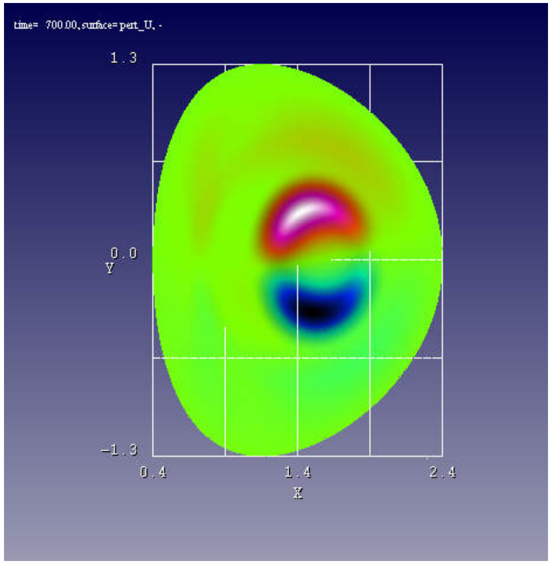



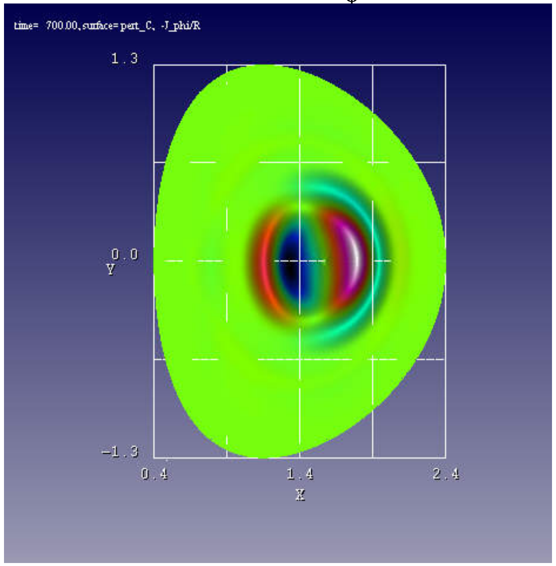



# $n$=1 eigenmode

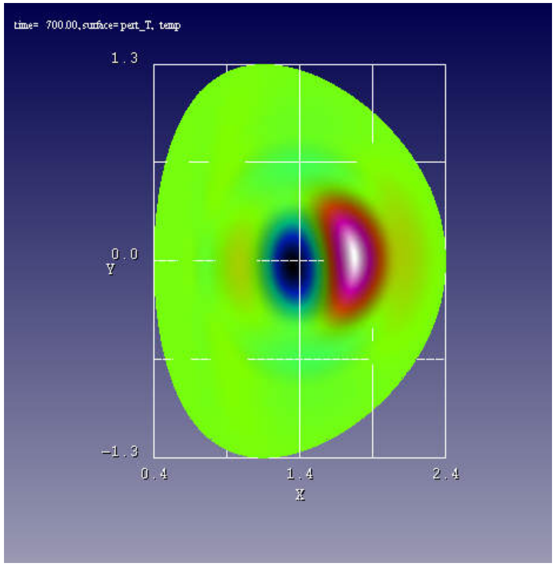
Velocity stream function U




C = -RJ$_\phi$




Temperature

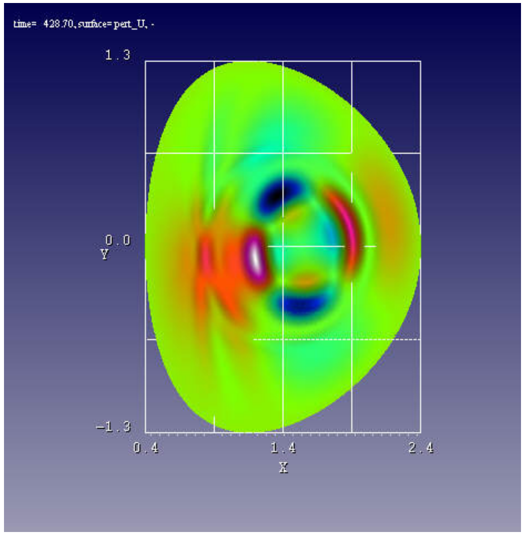


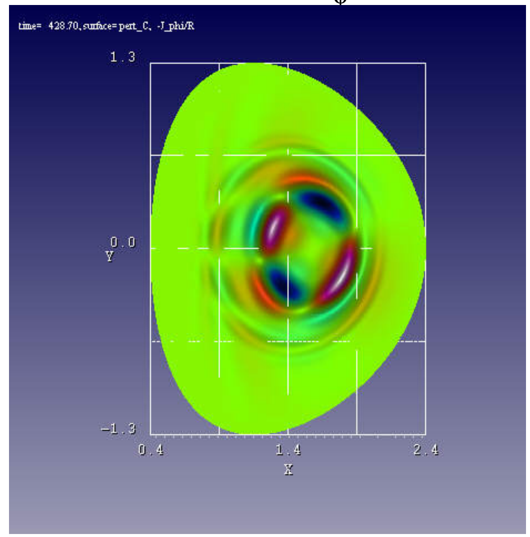

1,1 mode; $\gamma\tau_A \approx (1.415 \pm 0.0005) \times 10^{-2}$

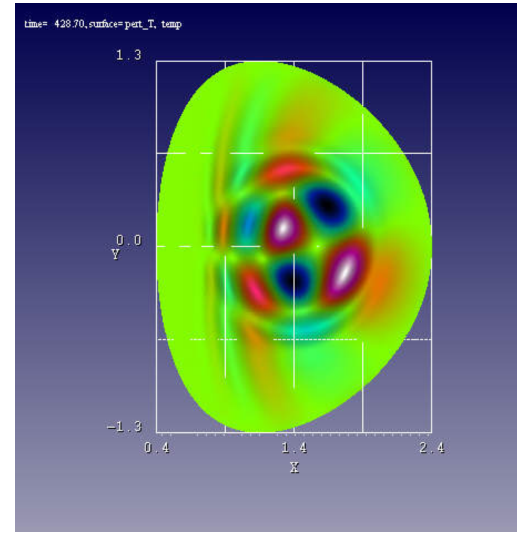
# Higher *n* eigenmodes

Velocity stream function U | $C = -RJ_\phi$ | Temperature

*n*=2

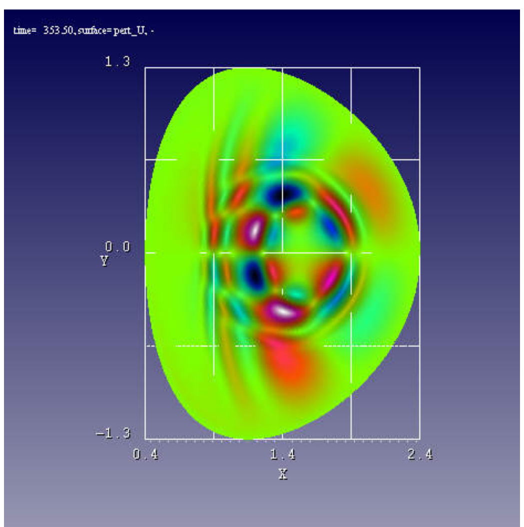


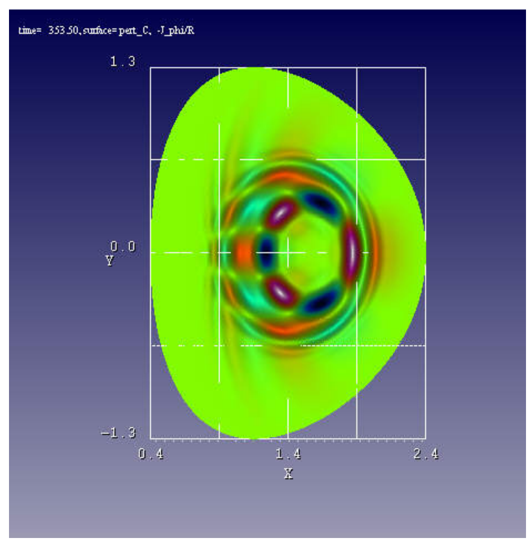



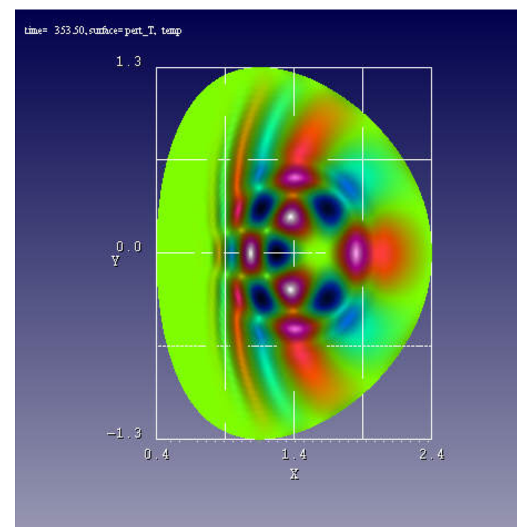




2,2 mode; $\gamma\tau_A \approx (3.90 \pm 0.05) \times 10^{-4}$

*n*=3



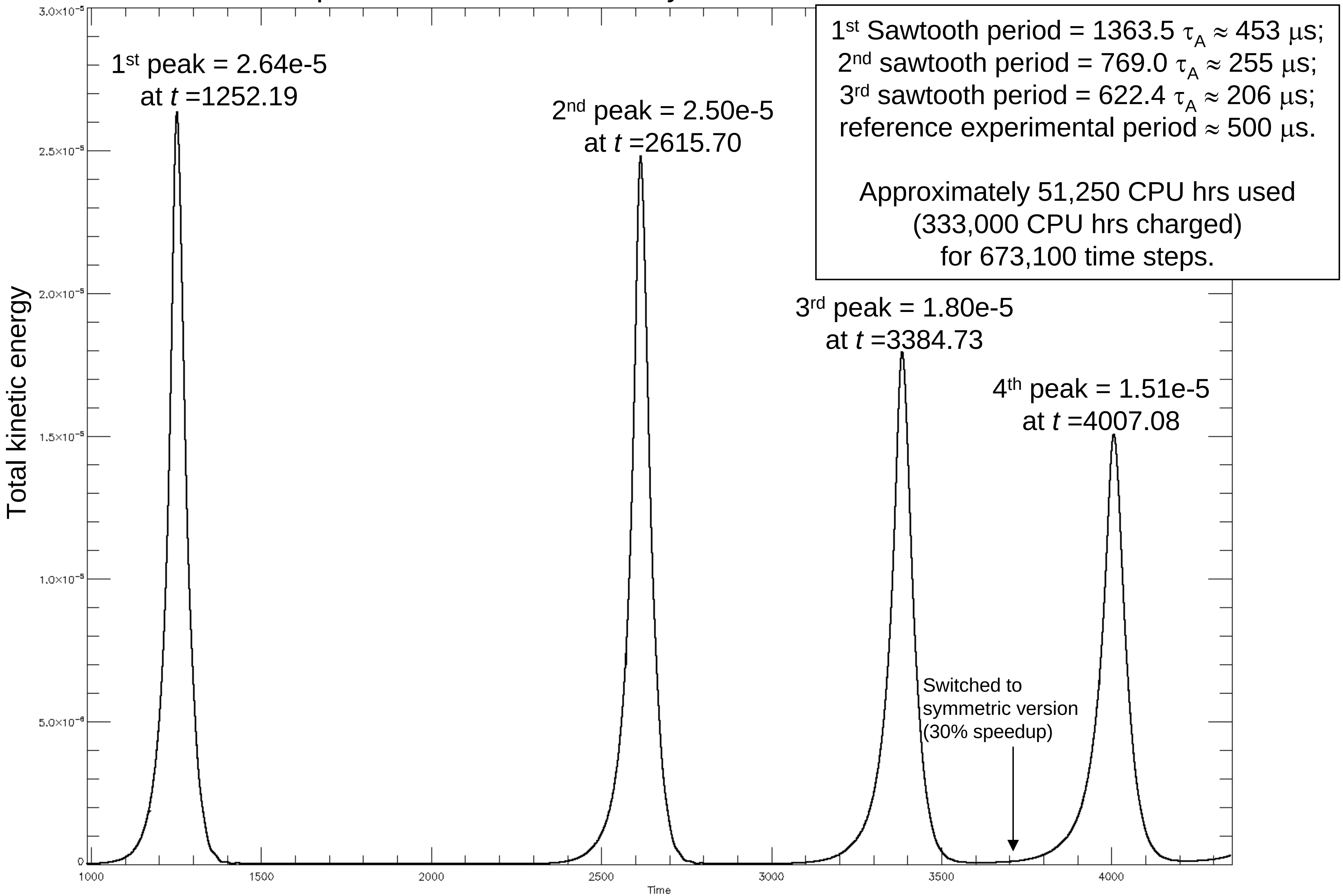



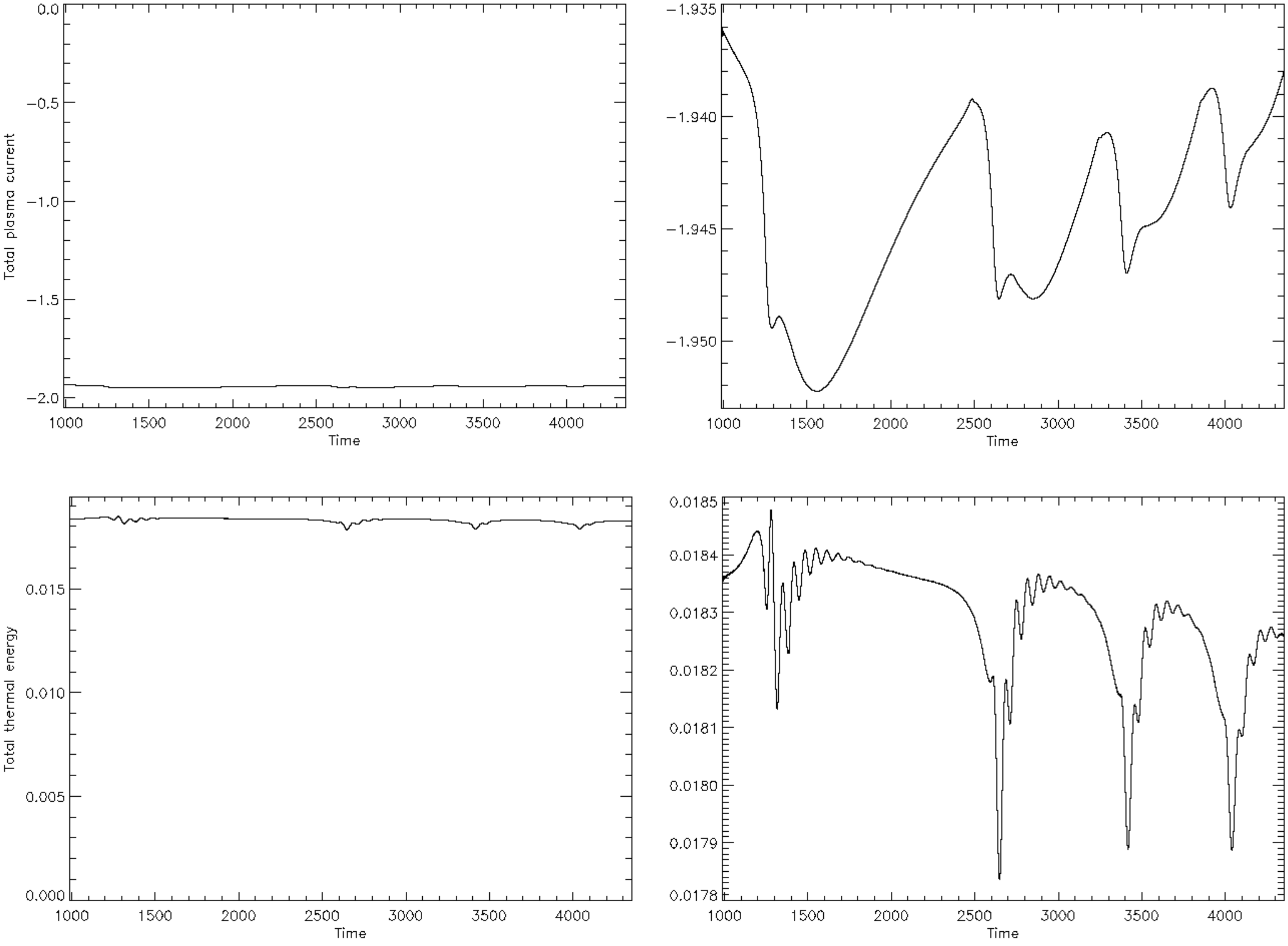

3,3 mode; stable

# Nonlinear results

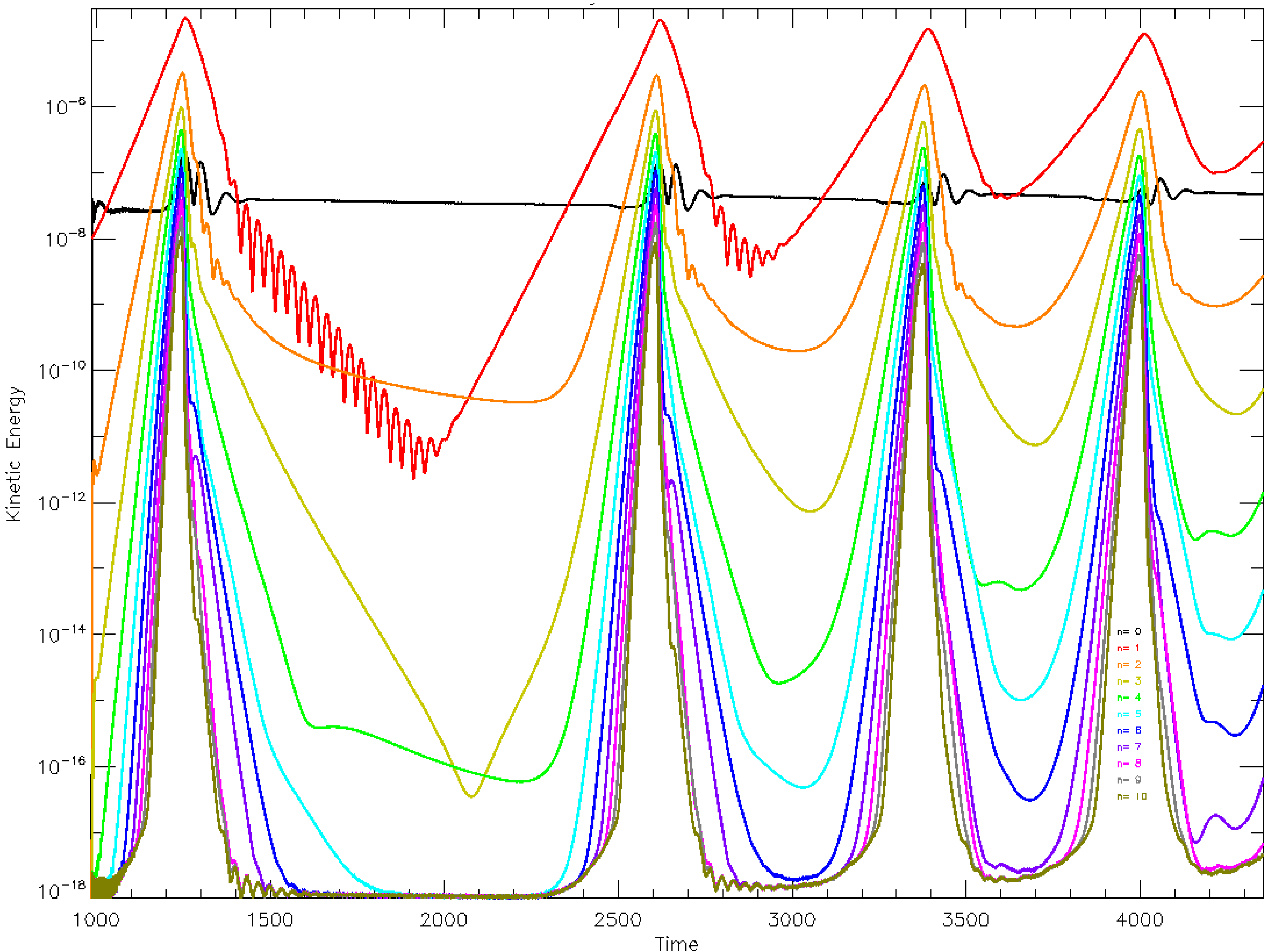
24 planes, 81 radial zones, sym 5 on 144 Franklin cores



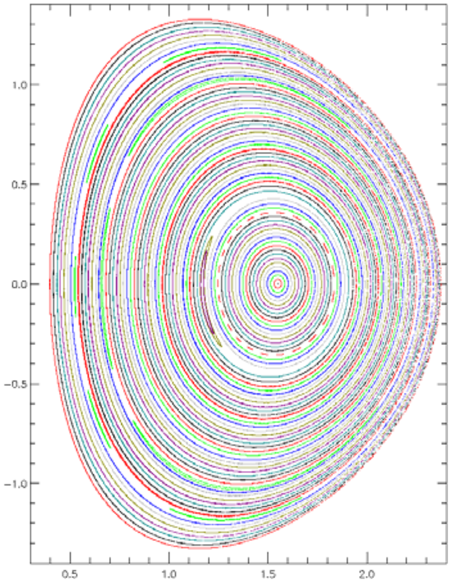
# Nonlinear Conservation



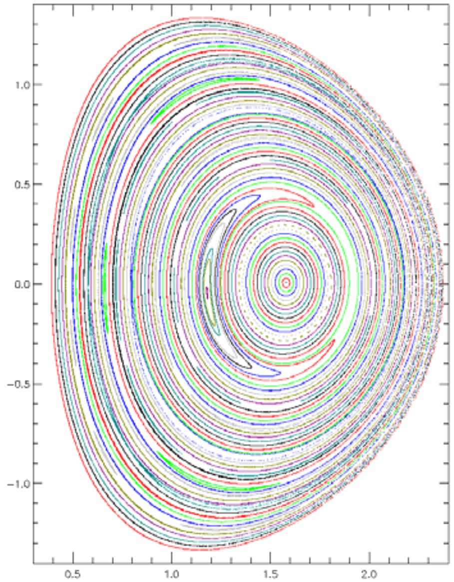
# Nonlinear Mode History (KE)



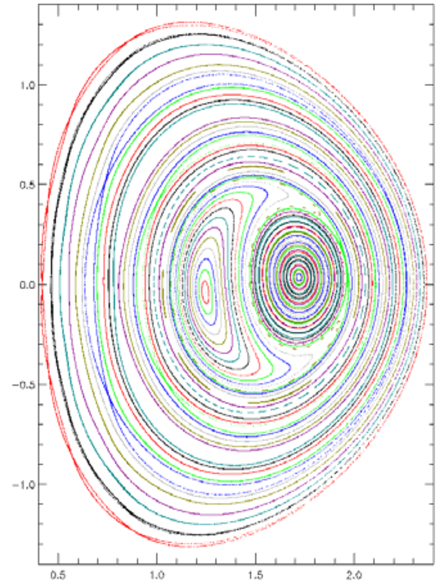
# Poincaré Plots



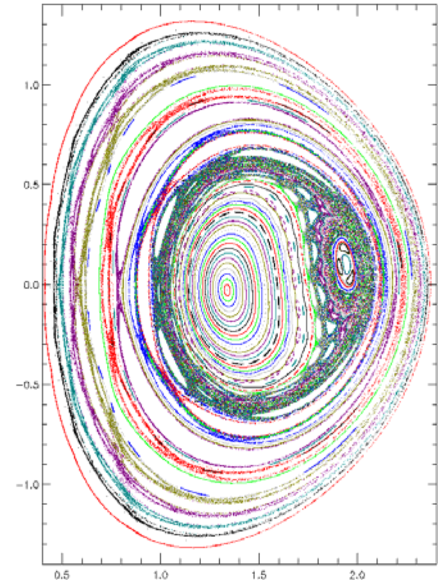
$t$ = 987.80; $q_{min}$ = 0.8022



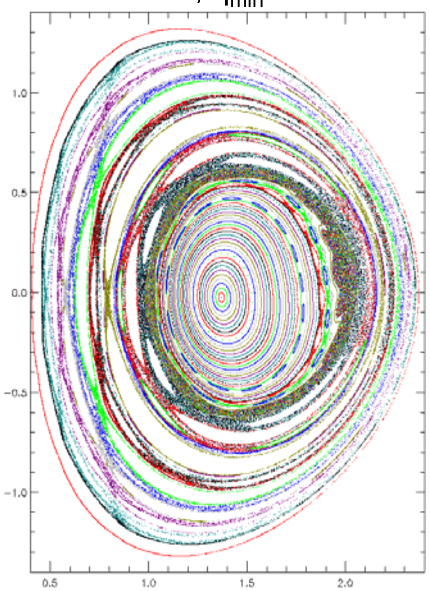
$t$ = 1109.55; $q_{min}$ = 0.7965



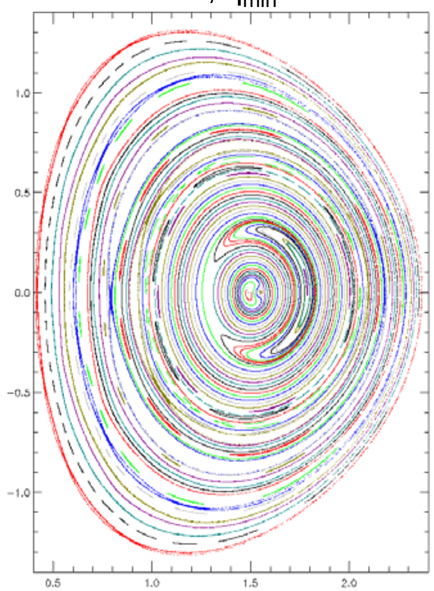
$t$ = 1207.05; $q_{min}$ = 0.7955



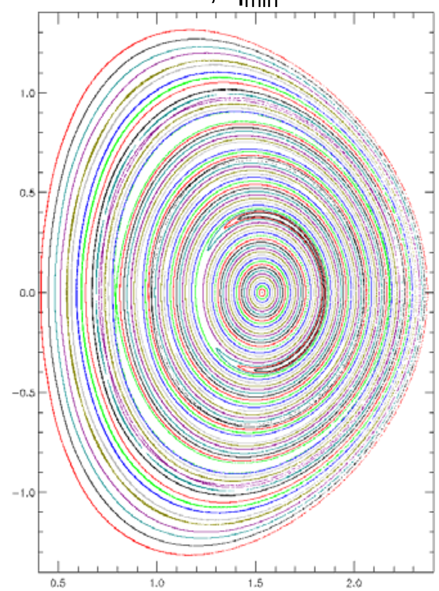
$t$ = 1244.55



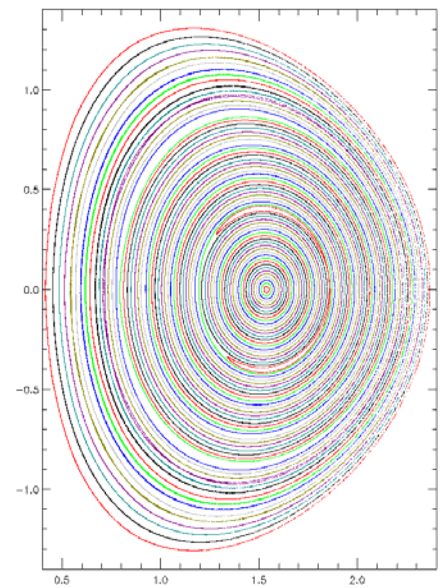
$t$ = 1252.05; $q_{min}$ = 1.0329



$t$ = 1357.05; $q_{min}$ = 0.9953

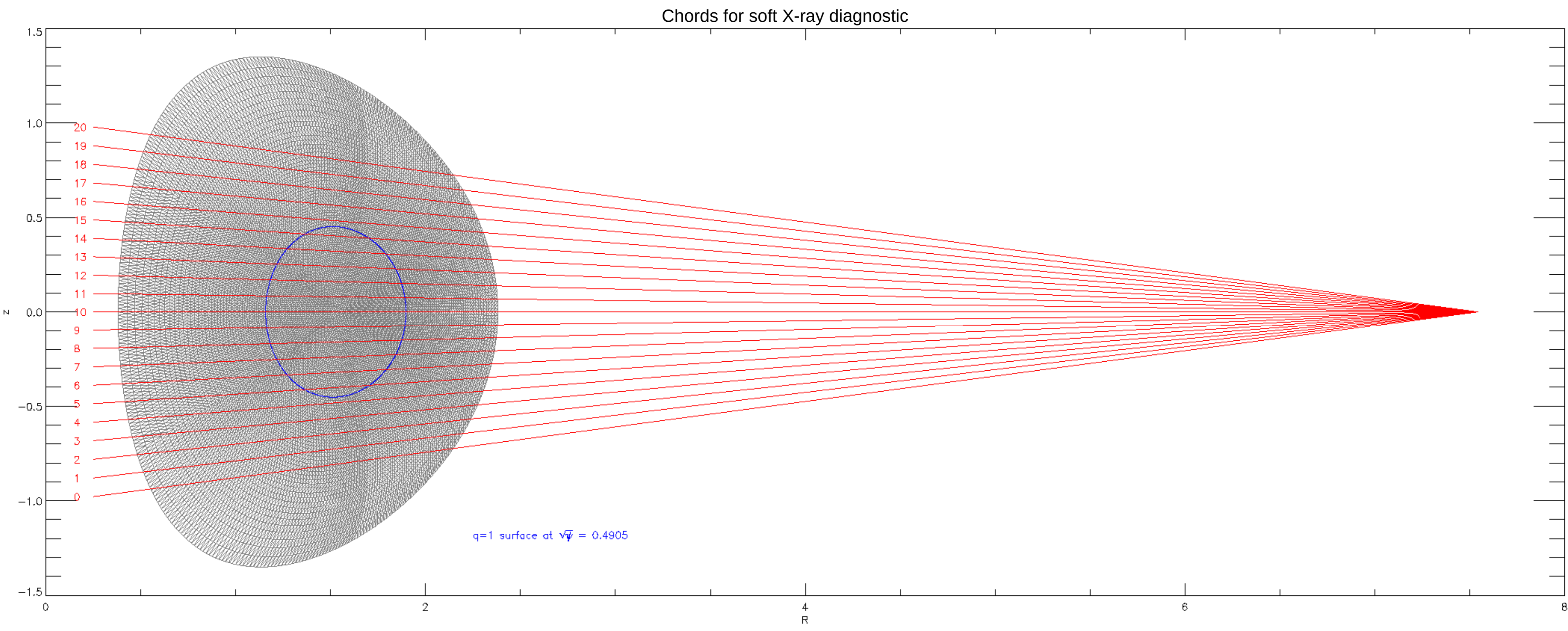


$t$ = 1452.05; $q_{min}$ = 0.9518



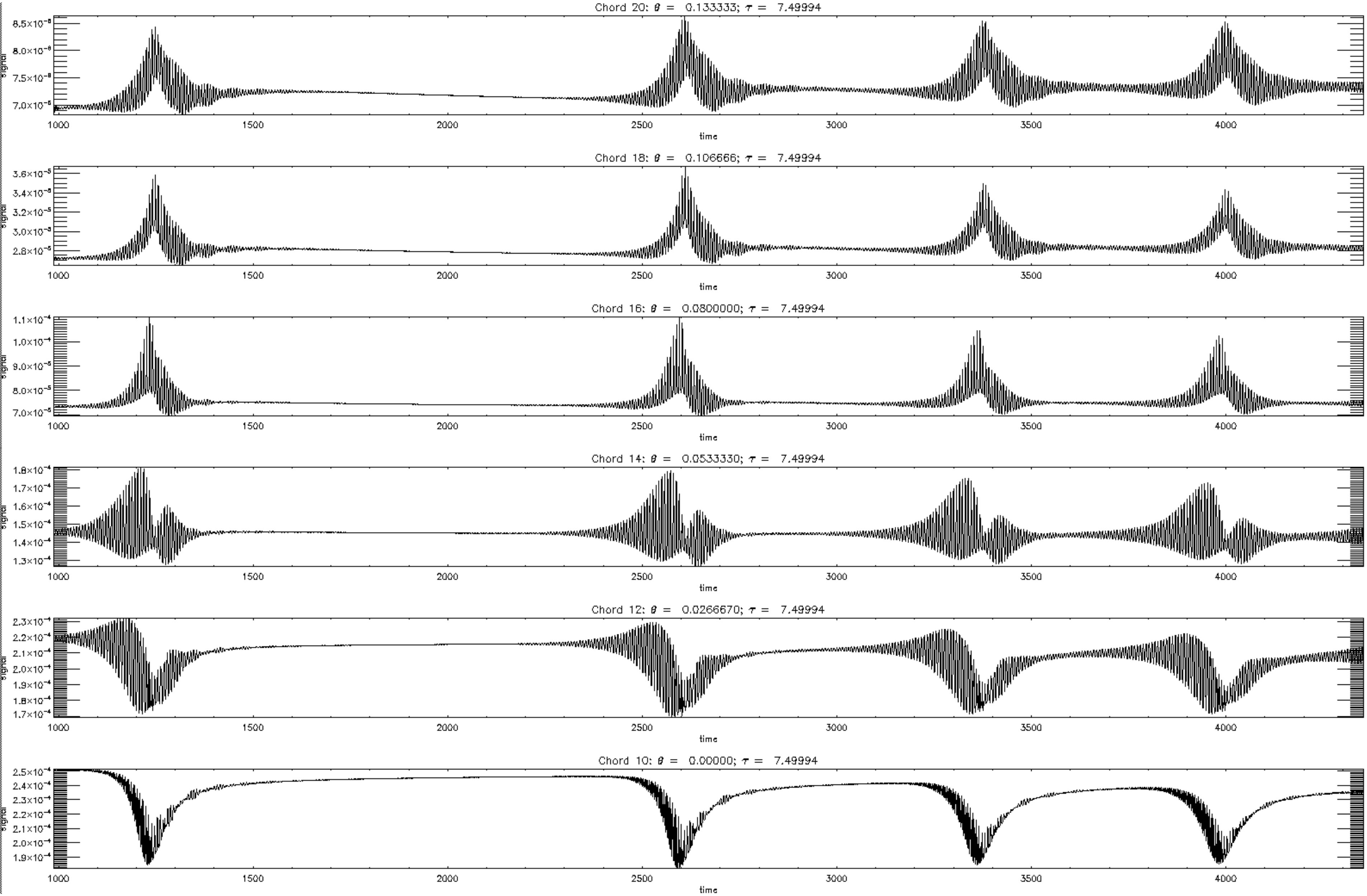
$t$ = 1531.43; $q_{min}$ = 0.9211

# Simulated Temperature Diagnostic

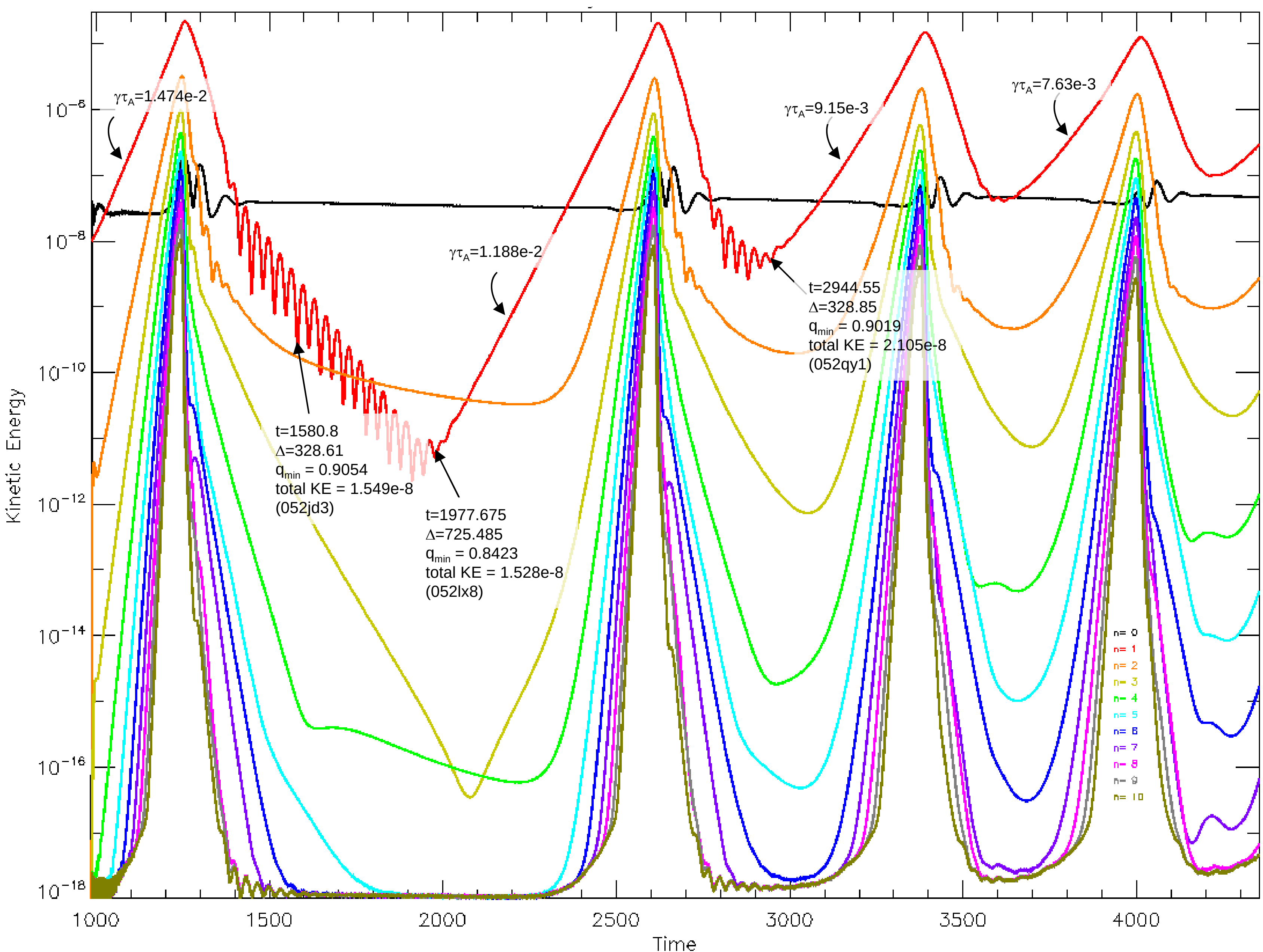



Simulated signal is simply $\int p^2 d\ell$ along a chord through the plasma.
Rotation frequency: two planes every 0.625 $\tau_A \rightarrow$ period = 7.5 $\tau_A$.

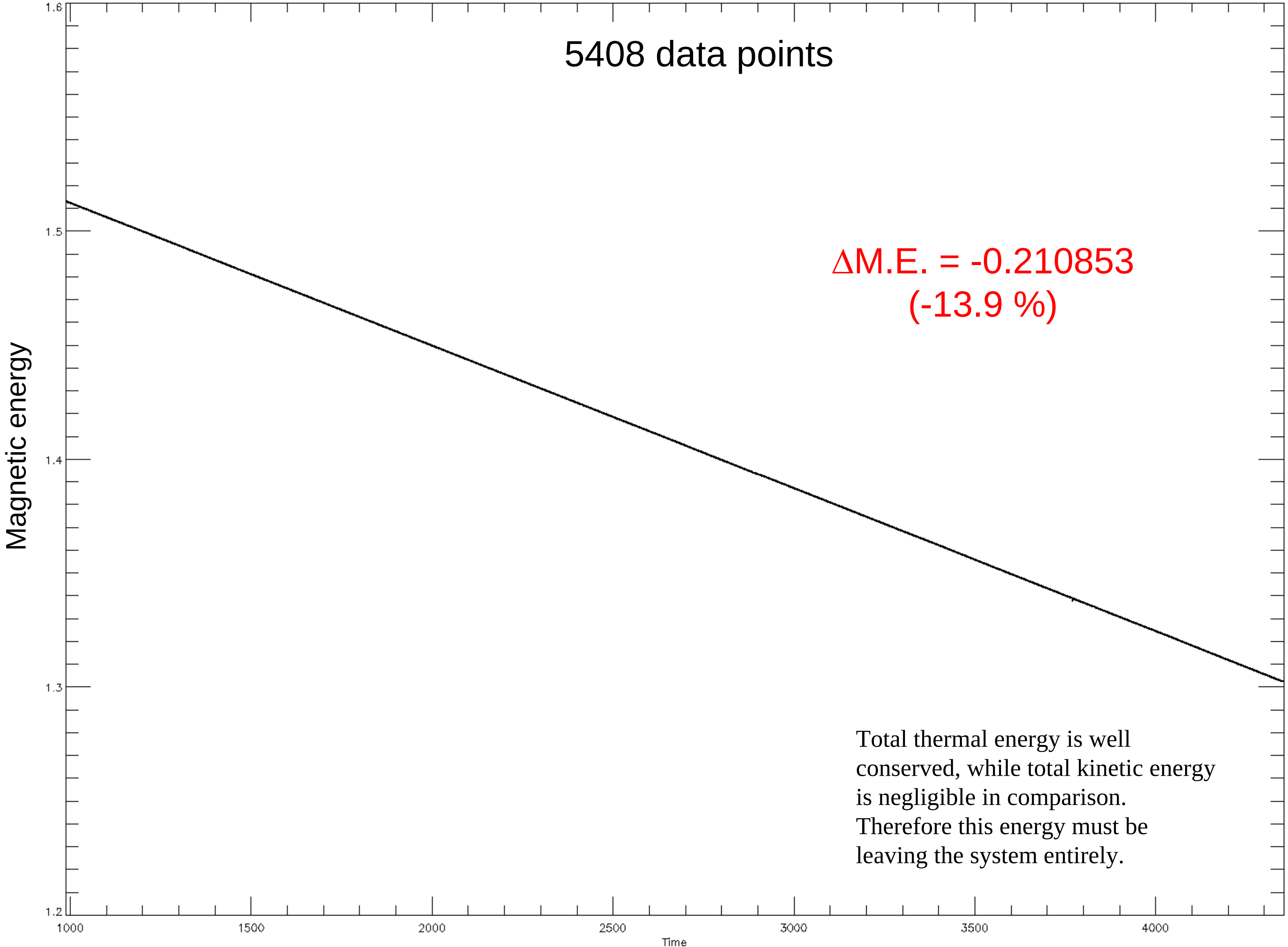
# Soft X-ray Signals (Integrated p²)



# Nonlinear Mode History (KE)

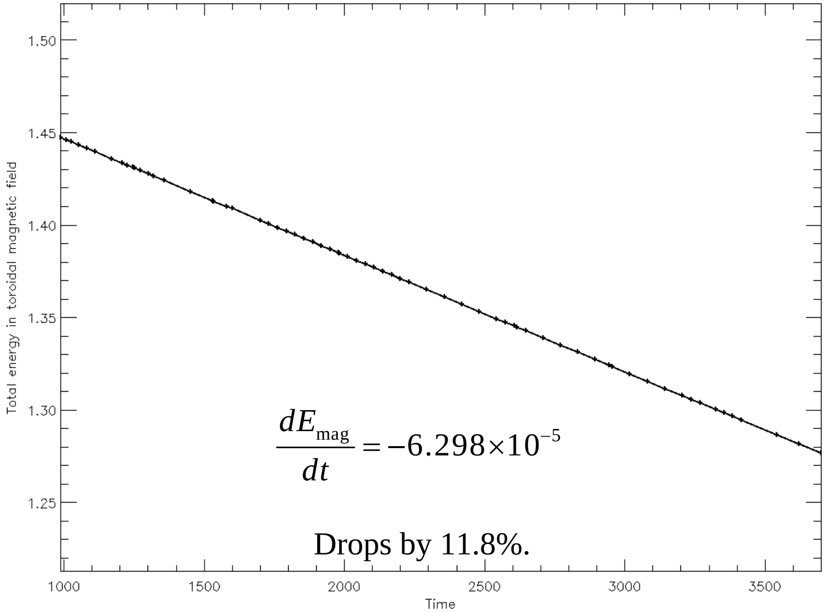


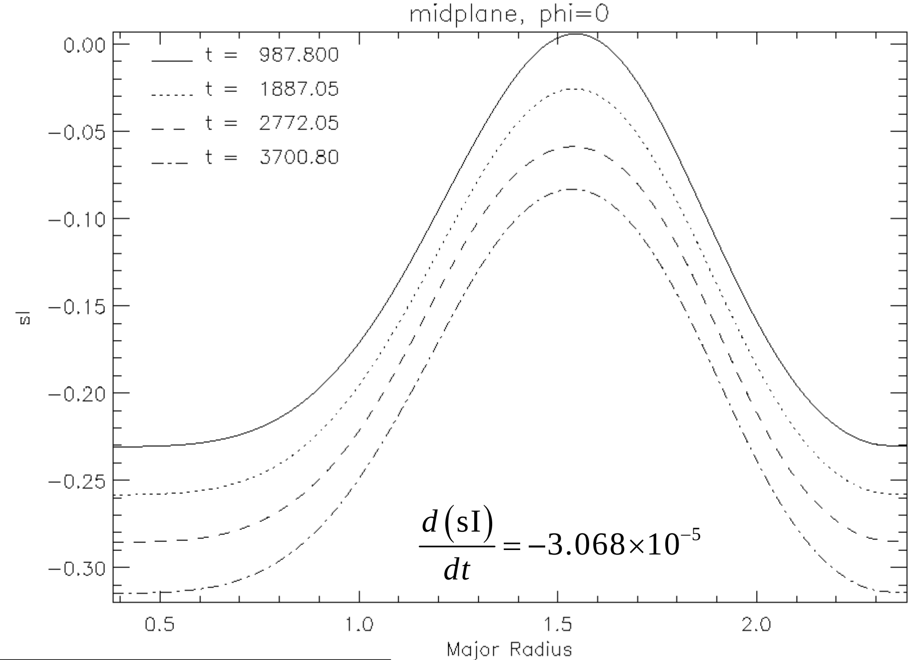
# Total Magnetic Energy History

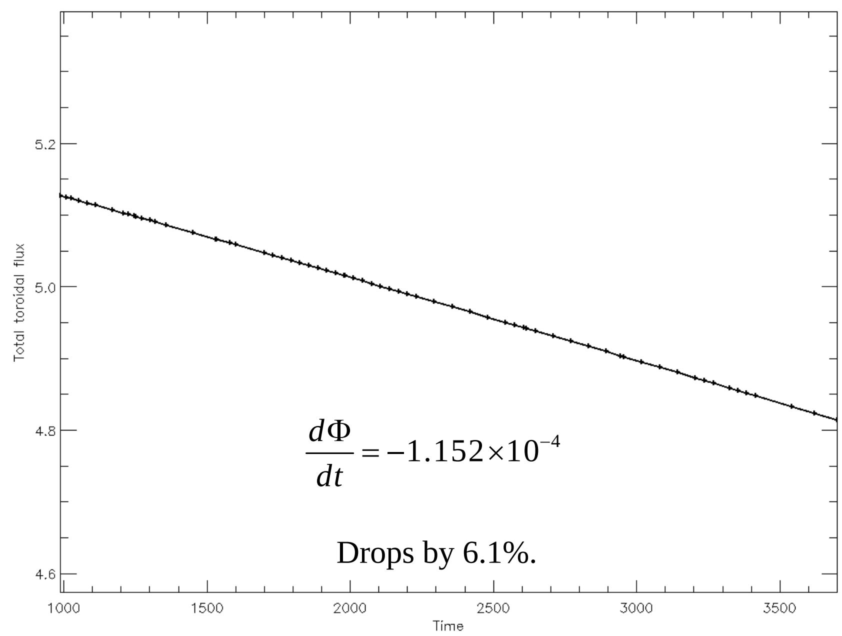



Total thermal energy is well conserved, while total kinetic energy is negligible in comparison. Therefore this energy must be leaving the system entirely.

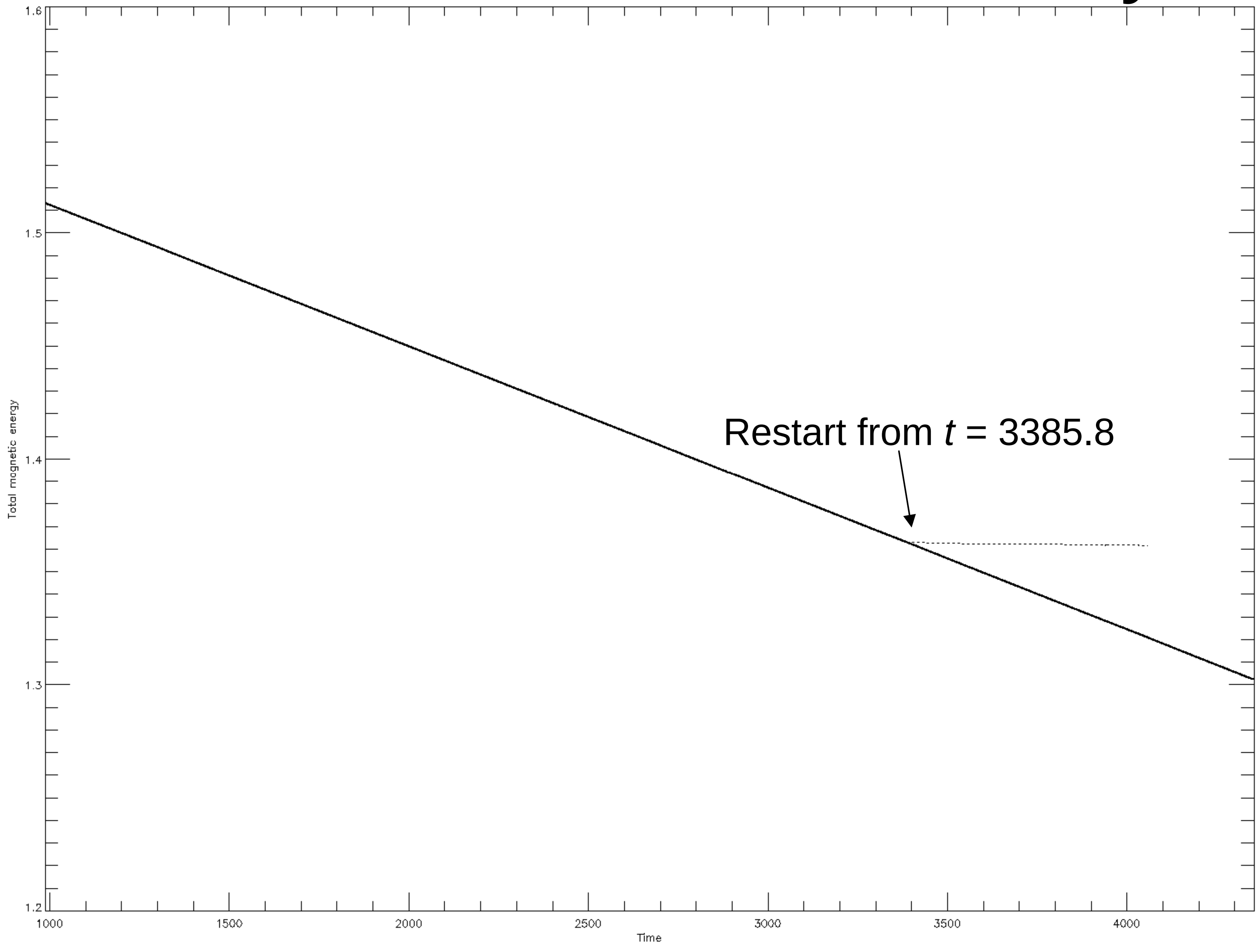
# Steady Drop in Toroidal Field



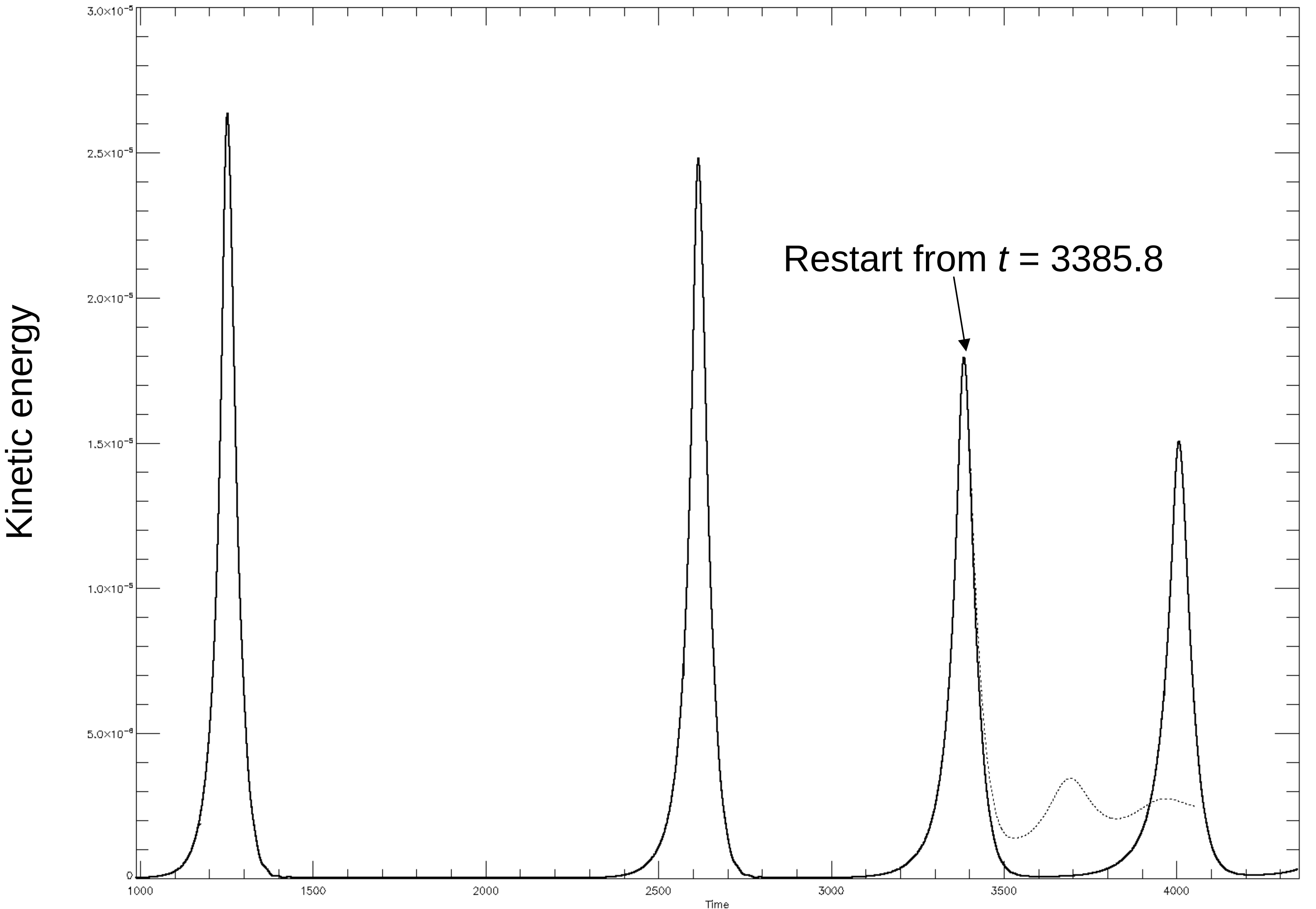



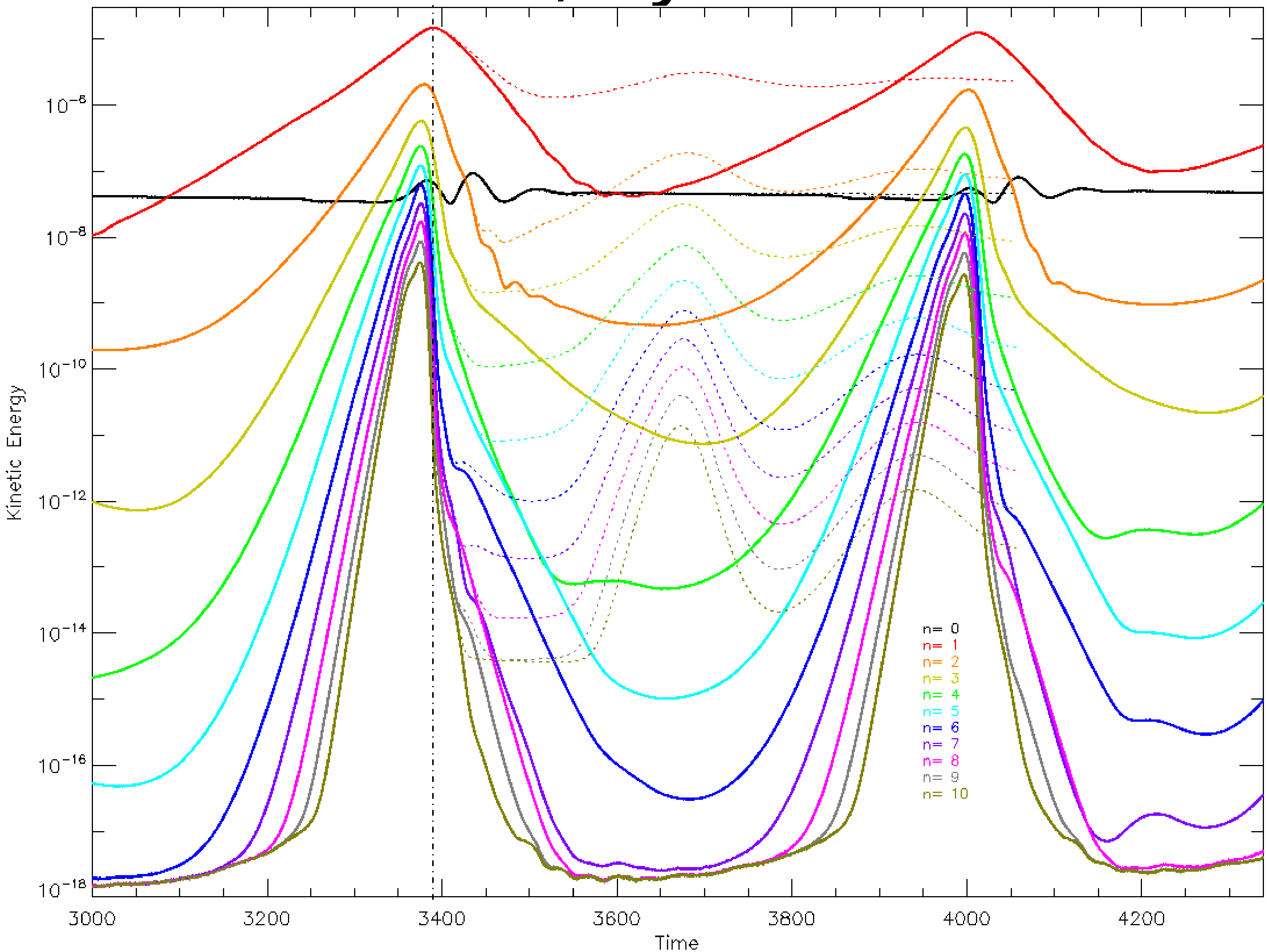
# Total magnetic Energy with Constant `si` on boundary



# Total kinetic energy with constant `si` on boundary



# Constant `si`, by mode number

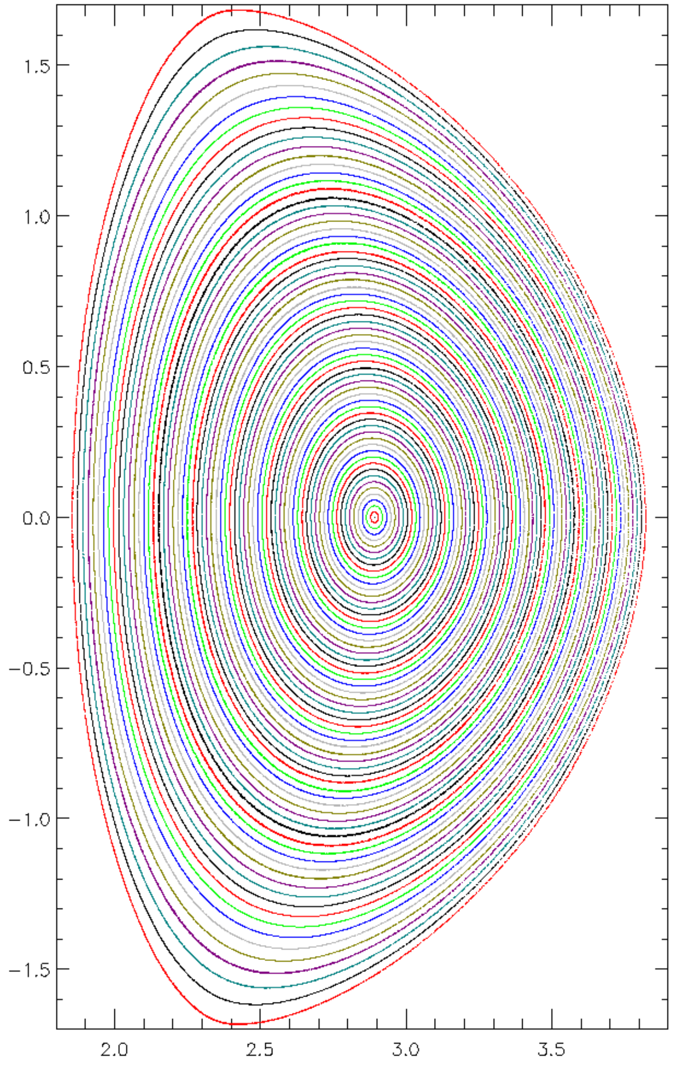


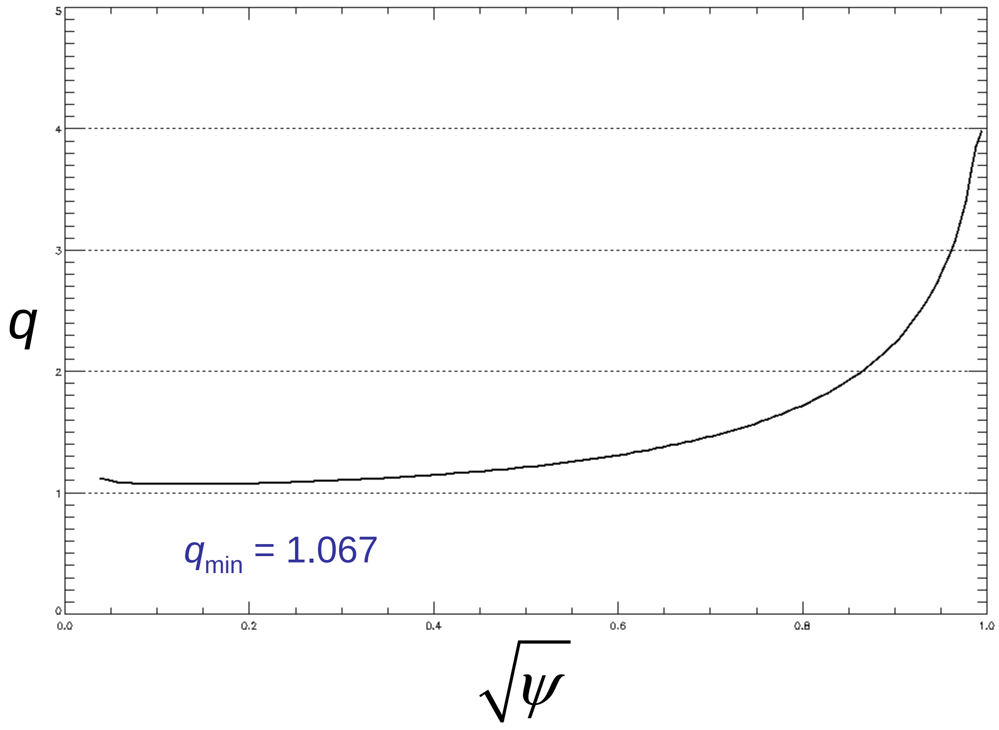
# DIII-D Error Field

# Initial study

- Begin with a DIII-D equilibrium.
- Add an $m$=2, $n$=1 perturbation of specified amplitude to initial poloidal flux on plasma boundary.
- Measure plasma displacements, singular currents with linear code; infer island widths.
- Evolve M3D nonlinearly until saturation of $n$=1 islands; compare widths to linear result.

# DIII-D Equilibrium



# Initial Perturbation

- Add helical perturbation to poloidal flux function $\psi$ on boundary of the form

$$\tilde{\psi}_{boundary}(\theta,\varphi)=\tilde{\psi}_0\cos(\varphi-2\theta)$$

where $\varphi$ is the toroidal angle, $\theta$ is the geometric poloidal angle defined by

$$\tan(\theta)=\frac{z}{R-R_0}$$

(normalized major radius $R_0$=2.89), and the equilibrium flux is $\psi$ = 0 on the boundary and $\psi$ = -0.506 on the magnetic axis.

- To generate measurable 2,1 islands while avoiding stochasticity, choose

$$\tilde{\psi}_0 \leq 2.7\times 10^{-3} \qquad \left(\frac{\tilde{\psi}_0}{|\psi_0|}\leq 5.33\times 10^{-3}\right)$$

- Do not perturb initial boundary current density.

# Initial State

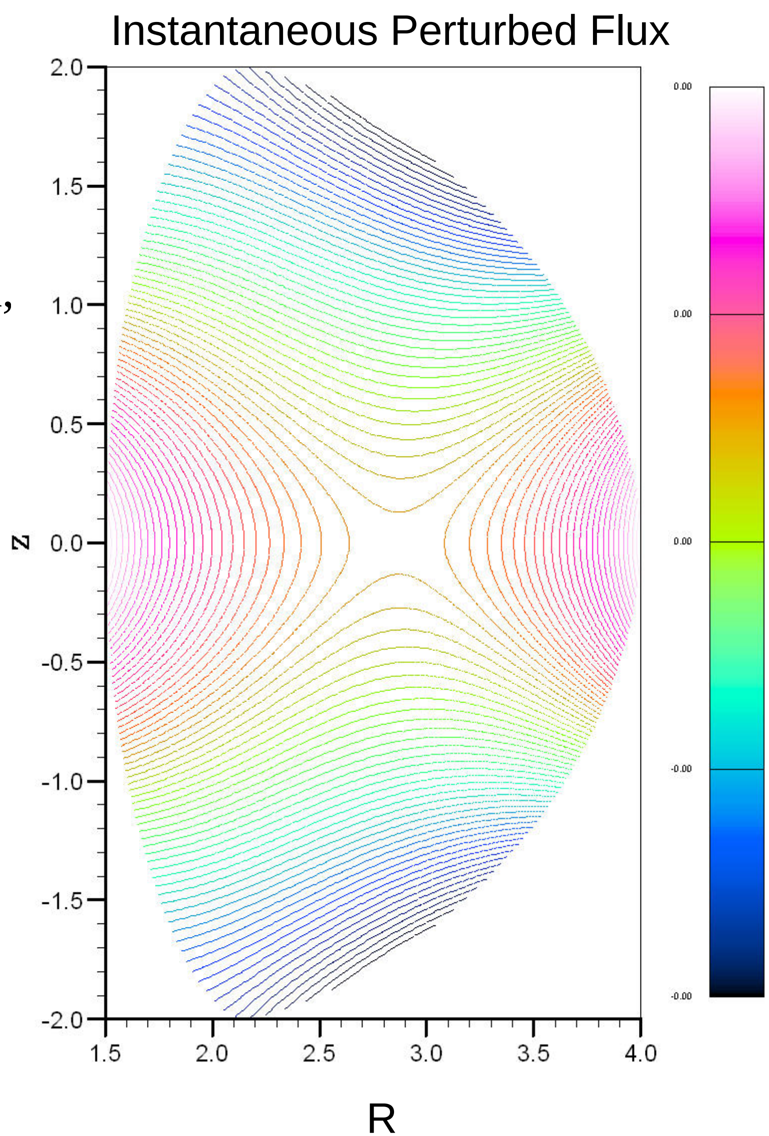
- Begin by solving the Poisson equation

$$\frac{\partial^2 \psi}{\partial R^2} - \frac{1}{R}\frac{\partial \psi}{\partial R} + \frac{\partial^2 \psi}{\partial z^2} = -RJ_\phi$$

for $\psi$ subject to the perturbed boundary condition, where $J_\phi$ is the unperturbed equilibrium toroidal current density.

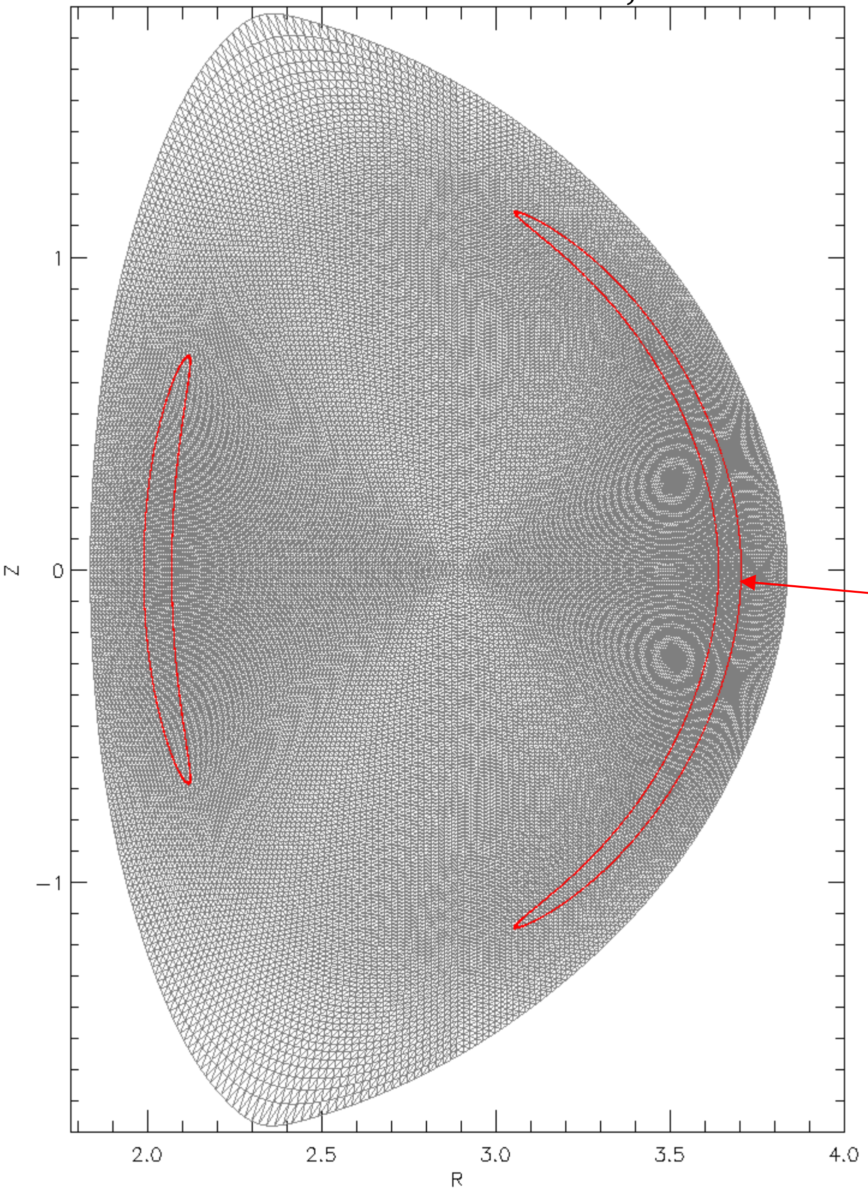
- Because the initial current remains unperturbed, the resulting state represents the superposition of the equilibrium field (including external and plasma currents) and the error field, without the plasma reponse.

- Time-evolving from this state with various choices of resistivity $\eta$ will show the effect of the plasma response on the islands.



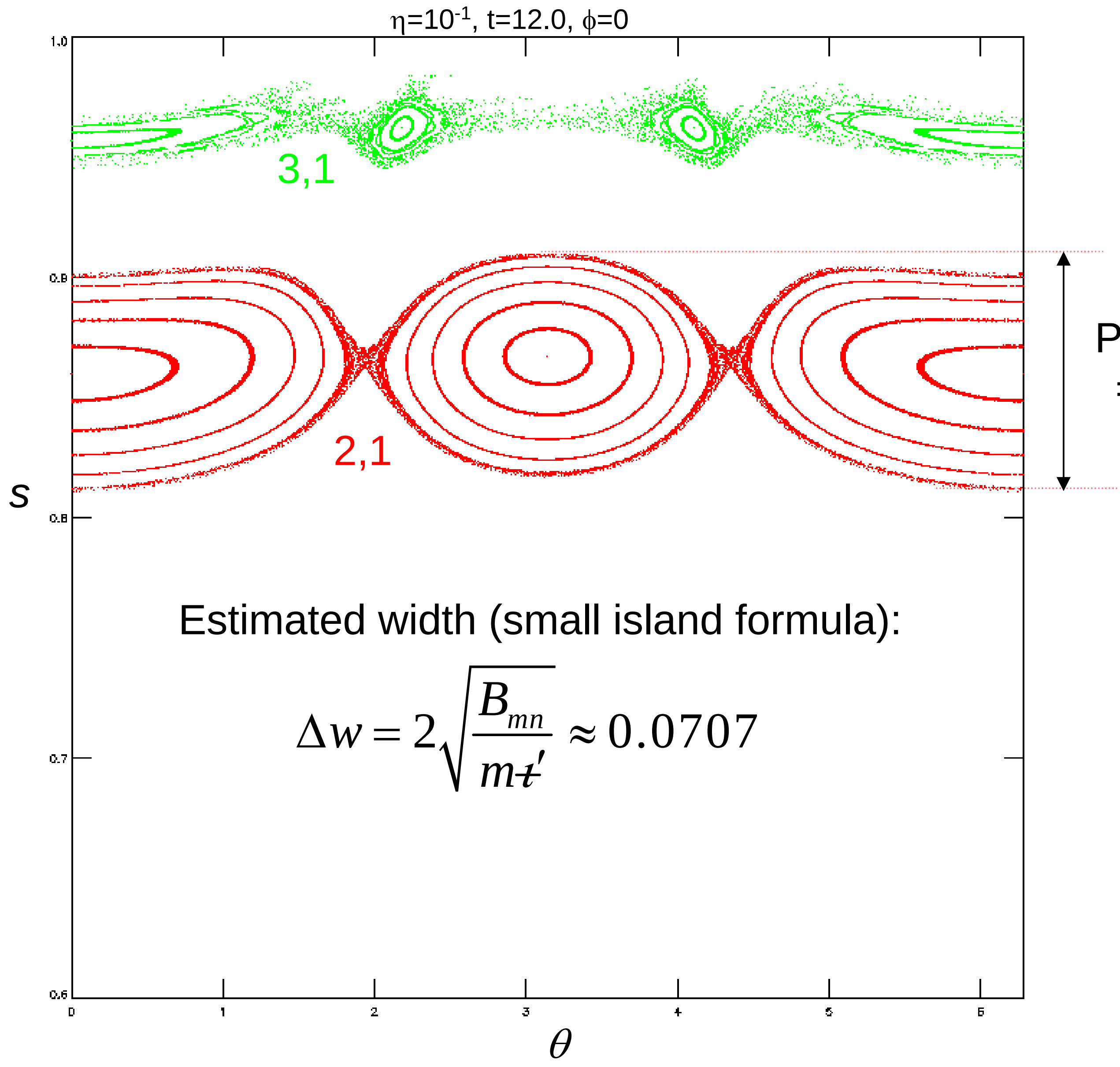
# Resolving the Islands

Poloidal mesh has 128 radial, 512 $\theta$ zones; packed ×2 around $q$=2 surface.




2,1 island spans ~20 zones → resolved.

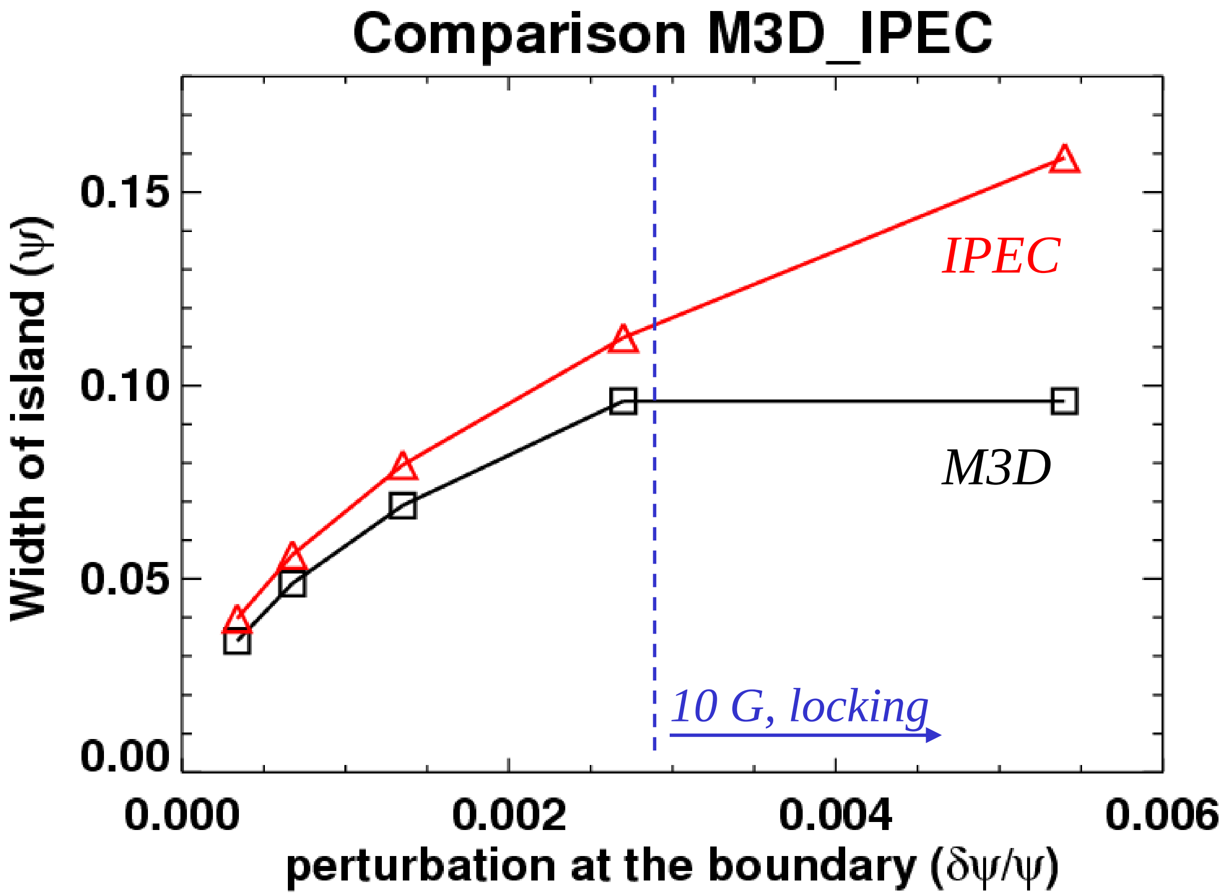
# Measuring Island Widths




Estimated width (small island formula):

$$\Delta w = 2\sqrt{\frac{B_{mn}}{m\iota'}} \approx 0.0707$$

# Widths now agree well with IPEC



# Conclusions

- Island widths agree for sufficiently small perturbations; larger ones show nonlinear effects.

- Additional future work to include further scaling studies, and investigate effects of plasma rotation.