Modeling of RF/MHD coupling using NIMROD and (eventually) GENRAY

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The SWIM project — self–consistent RF/MHD interactions

• Major physics issue being addressed by SWIM project (Center for the Simulation of RF Wave Interactions with Magnetohydrodynamics)— How can RF sources optimally be used to suppress or reduce the negative effects of MHD instabilities in fusion plasmas?

• At present, no theoretical framework exists for self–consistently including the effects of arbitrary RF sources in the MHD model

• Relevant issues in developing such a formalism:

—Can a small expansion parameter be found, such that the lowest order distribution function is a local Maxwellian,

$$f_M(\mathbf{x}, \mathbf{v}, t) \equiv n(\mathbf{x}, t) \left(\frac{m}{2\pi T(\mathbf{x}, t)} \right)^{3/2} \exp \left[\frac{-m|\mathbf{v} - \mathbf{V}(\mathbf{x}, t)|^2}{2T(\mathbf{x}, t)} \right] \ ?$$

—If so, how do RF effects enter the fluid equations? What is the proper closure?

—If not, what is the lowest order distribution function? What are the appropriate fluid–like quantities describing the plasma?

• This work will focus on the effects of electron cyclotron current drive — small expansion parameter can be found for this case; existing theoretical approaches can be used

Goal — numerically simulate ECCD stabilization of NTM's

• Experimental efforts to stabilize neoclassical tearing modes (NTM's) via electron cyclotron current drive (ECCD) have been very successful; R. J. La Haye [Phys. Plasmas **13**, 055501 (2006)] gives a detailed overview and many references

• For ECCD, the RF–induced current is relatively small [of the same order as the current driven by the electric field] $\Rightarrow$ small expansion parameter

• General kinetic equation:

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v} \cdot \nabla f_\alpha + \frac{q_\alpha}{m_\alpha}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_\alpha}{\partial \mathbf{v}} = C(f_\alpha) + Q(f_\alpha)$$

• $Q(f_\alpha)$ is a gyrophase–averaged quasilinear diffusion operator

$$Q(f_\alpha) \equiv \frac{\partial}{\partial \mathbf{v}} \cdot \mathcal{D} \cdot \frac{\partial}{\partial \mathbf{v}}$$

where the diffusion tensor $\mathcal{D}$ arises from the RF source.

• $C(f_\alpha)$ is the gyrophase–averaged Fokker–Planck Coulomb collision operator

RF terms appear in the fluid equations

- Taking fluid moments in the conventional manner yields

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \mathbf{v}_\alpha) = 0 \qquad \text{(RF produces no particles)}$$

$$m_\alpha n_\alpha \left(\frac{\partial \mathbf{v}_\alpha}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla)\mathbf{v}_\alpha \right) = n_\alpha q_\alpha (\mathbf{E} + \mathbf{v}_\alpha \times \mathbf{B}) - \nabla p_\alpha - \nabla \cdot \pi_\alpha + \mathbf{R}_\alpha + \mathbf{F}_{\alpha 0}^{rf}$$

$$\mathbf{F}_{\alpha 0}^{rf} \equiv \int m_\alpha \mathbf{v} Q(f_\alpha) d\mathbf{v} \qquad \text{(additional momentum imparted by RF waves)}$$

$$\frac{3}{2} n_\alpha \left(\frac{\partial T_\alpha}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla) T_\alpha \right) + n_\alpha T_\alpha \nabla \cdot \mathbf{v}_\alpha = -\nabla \cdot \mathbf{q}_\alpha - \pi_\alpha : \nabla \mathbf{v}_\alpha + Q_\alpha + S_{\alpha 0}^{rf}$$

$$S_{\alpha 0}^{rf} \equiv \int \frac{1}{2} m_\alpha v^2 Q(f_\alpha) d\mathbf{v} \qquad \text{(additional energy imparted by RF waves)}$$

How RF terms in the fluid equations are calculated

- Need a small expansion parameter to proceed

- For ECCD, electric field imparted by RF is small ($E \ll E_D$, the Dreicer field strength) $\Rightarrow$ RF terms are small

- Assume the lowest order distribution function is a local Maxwellian — $f_\alpha = f_{M\alpha} + \delta f_\alpha$;

$$f_{M\alpha} = n(\mathbf{x}, t) \left(\frac{m}{2\pi T(\mathbf{x}, t)} \right)^{3/2} \exp \left[\frac{-m|\mathbf{v} - \mathbf{V}(\mathbf{x}, t)|^2}{2T(\mathbf{x}, t)} \right]$$

- Source terms in fluid equations come from velocity moments of $Q(f_\alpha)$; become functions of low–order fluid moments in this approximation

$$\mathbf{F}^{rf}_{\alpha 0} \equiv \int m_\alpha \mathbf{v} Q(f_\alpha)\, d\mathbf{v} \approx \int m_\alpha \mathbf{v} Q(f_{M\alpha})\, d\mathbf{v}$$

$$S^{rf}_{\alpha 0} \equiv \int \frac{1}{2} m_\alpha v^2 Q(f_\alpha)\, d\mathbf{v} \approx \int \frac{1}{2} m_\alpha v^2 Q(f_{M\alpha})\, d\mathbf{v}$$

- For a given $Q(f_\alpha)$, lowest–order effect can be calculated

RF effects modify the closure scheme

- Effects of RF must also be included in closure calculations for heat fluxes and stress tensors

- Use a Chapman–Enskog–like approach; assume kinetic distortion δf_α has no density, momentum, or temperature moments

$$\int \delta f_\alpha d\mathbf{v} = \int \delta f_\alpha m_\alpha \mathbf{v} d\mathbf{v} = \int \delta f_\alpha \frac{m_\alpha v^2}{2} d\mathbf{v} = 0$$

- Equation for kinetic distortion:

$$\frac{d\delta f_\alpha}{dt} - C(\delta f_\alpha) - Q(\delta f_\alpha) = -\frac{df_{M\alpha}}{dt} + C(f_{M\alpha}) + Q(f_{M\alpha})$$

reduces to

$$\frac{d\delta f_\alpha}{dt} - C(\delta f_\alpha) = -\frac{df_{M\alpha}}{dt} + Q(f_{M\alpha})$$

- Use fluid equations to evaluate $df_{M\alpha}/dt$

Solve the kinetic distortion equation to get RF–modified closure

- Equation for kinetic distortion:

$$
\begin{aligned}
\frac{d\delta f_\alpha}{dt} - C(\delta f_\alpha) &= Q(f_{M\alpha}) + (\mathbf{v} - \mathbf{V}_\alpha)\cdot[\nabla\cdot\pi_\alpha - \mathbf{R}_\alpha - \mathbf{F}^{rf}_{\alpha 0}]\frac{f_{M\alpha}}{n_\alpha T_\alpha} \\
&+ \left(\frac{m_\alpha(\mathbf{v}-\mathbf{V}_\alpha)^2}{3T_\alpha} - 1\right)\left[\pi_\alpha : \nabla\mathbf{V}_\alpha + \nabla\cdot\mathbf{q}_\alpha - Q_\alpha - S^{rf}_{\alpha 0}\right]\frac{f_{M\alpha}}{n_\alpha T_\alpha} \\
&- \left(\frac{m_\alpha(\mathbf{v}-\mathbf{V}_\alpha)^2}{2T_\alpha} - \frac{5}{2}\right)(\mathbf{v}-\mathbf{V}_\alpha)\cdot\nabla T_\alpha \frac{f_{M\alpha}}{T_\alpha} \\
&+ \frac{m_\alpha f_{M\alpha}}{T_\alpha}\left[(\mathbf{v}-\mathbf{V}_\alpha)(\mathbf{v}-\mathbf{V}_\alpha) - \frac{|\mathbf{v}-\mathbf{V}_\alpha|^2}{3}\mathbf{I}\right] : \nabla\mathbf{V}_\alpha
\end{aligned}
$$

- Time–independent, homogeneous magnetic field; collisional limit — can touch base with Spitzer problem (modified by RF terms)

- More generally, use moment expansion formalism [J.–Y. Ji and E. D. Held, Phys. Plasmas **13**, 102103 (2006)] to make progress

Resistive and neoclassical tearing modes — island widths

- For resistive tearing modes, Rutherford equation predicts algebraic growth of island width in nonlinear regime:

$$\frac{dw}{dt} = 1.22\frac{\eta}{\mu_0}\Delta'$$

- Nonlinear saturation of island width — $\Delta' \rightarrow \Delta'(w)$:

$$\frac{dw}{dt} = 1.22\frac{\eta}{\mu_0}\Delta'(w)$$

- Neoclassical modifications to tearing mode (perturbed bootstrap currents, curvature stabilization, ion polarization currents, resistive interchange, etc.) enter Rutherford equation additively

$$\frac{dw}{dt} = 1.22\frac{\eta}{\mu_0}[\Delta'(w) + \Delta_{NTM}(w)]$$

- Heuristically, with RF included,

$$\frac{dw}{dt} = 1.22\frac{\eta}{\mu_0}[\Delta'(w) + \Delta_{NTM}(w) + \Delta_{rf}(w)]$$

Physical effects captured by Rutherford equation are independent

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- RF modifications to Rutherford equation enter additively, on the same footing as other neoclassical modifications

- Physical effects can be treated independently

- Consequently, can study effects of RF on (ordinary) resistive tearing mode simulations as a prototype problem — no need to start with (more complicated) neoclassical simulations

- Mock up RF effects by heuristically modifying Ohm's law; move progressively to more complicated models for RF interaction

Eventual goal — self–consistent coupling of MHD (NIMROD), Fokker–Planck (CQL3D), and RF codes (GENRAY – short term, AORSA or TORIC – long term)

Insert an ad hoc term (general RF effects) in Ohm's law

- MHD: Ohm's law (from electron momentum equation):

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{J}$$

- Determine effect of current drive (not self-consistent): ad hoc force on electrons

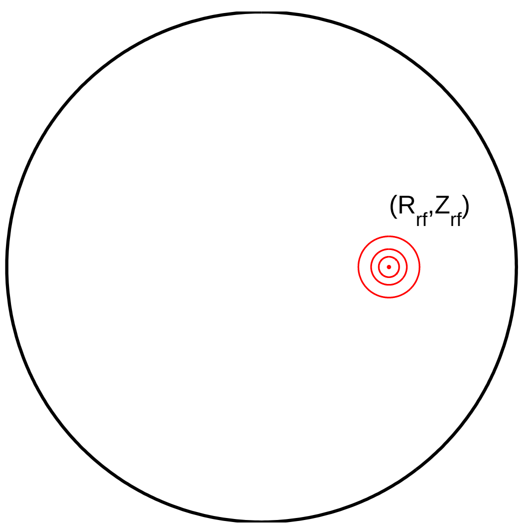
$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{J} - \frac{\mathbf{F}_{\rm rf}}{e}$$

- Should modify plasma equilibrium. Assume forcing term has form

$$\mathbf{F}_{\rm rf}(R, Z, \xi, t) = e\eta\lambda_{\rm rf} \exp\left[\frac{-\left[(R - R_{\rm rf})^2 + (Z - Z_{\rm rf})^2\right]}{w_{\rm rf}^2}\right] \frac{\mathbf{B}}{\mu_0} f(t) g(\xi)$$

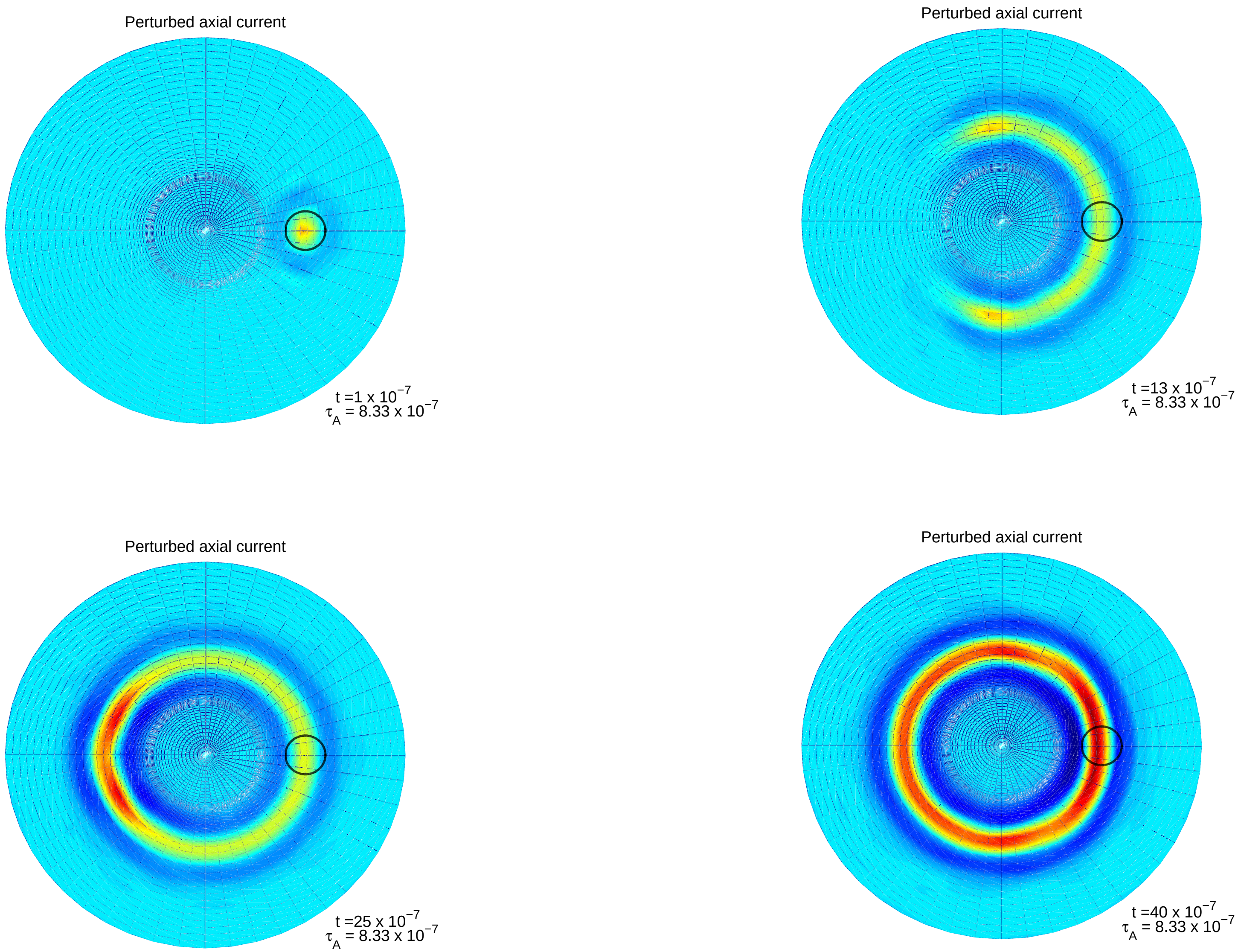
- Can also consider this as $\mathbf{E} \to \mathbf{E} - \eta \mathbf{J}_{\rm rf}$, where $\eta \mathbf{J}_{\rm rf}$ is an emf-per-unit-length inducing magnetic field in the plasma (mocking up effects of current density $\mathbf{J}_{\rm rf}$).

- Variable parameters $\lambda_{\rm rf}, R_{\rm rf}, Z_{\rm rf}, w_{\rm rf}$ (amplitude, location, spatial width), along with time dependence $f(t)$ and toroidal dependance $g(\xi)$.



Rapid current equilibration occurs on the flux surfaces

- Begin with time–independent perturbation; $\lambda_{\rm rf} = 16.0, w_{\rm rf} = 0.1, R_{\rm rf} = 0.5, Z_{\rm rf} = 0$:



Current equilibration occurs on the Alfvén timescale

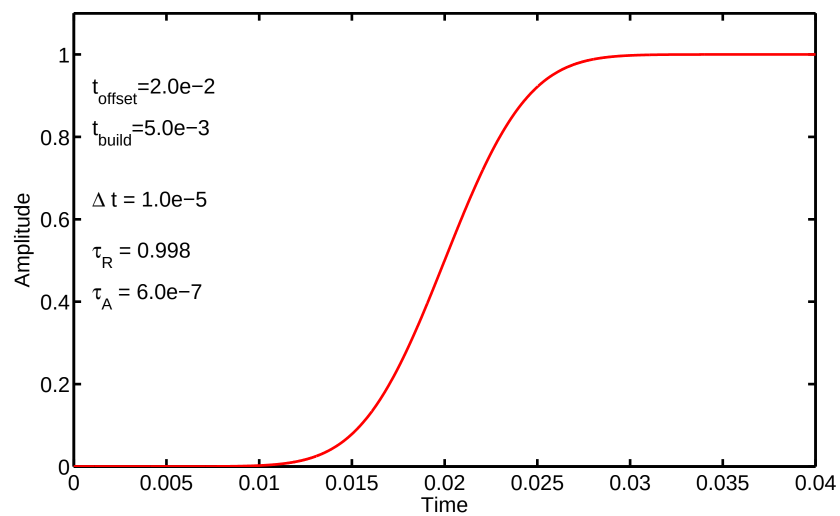
- Current equilibrates over a flux surface on the Alfvén timescale.

- Not necessary to average ad hoc forcing term over flux surface — force balance does this for us

- Specify the time dependence $f(t)$:

$$f(t) = \left[\frac{1}{2} + \frac{1}{2}\mathrm{erf}\left(\frac{t - t_{\mathrm{offset}}}{t_{\mathrm{build}}}\right)\right]$$

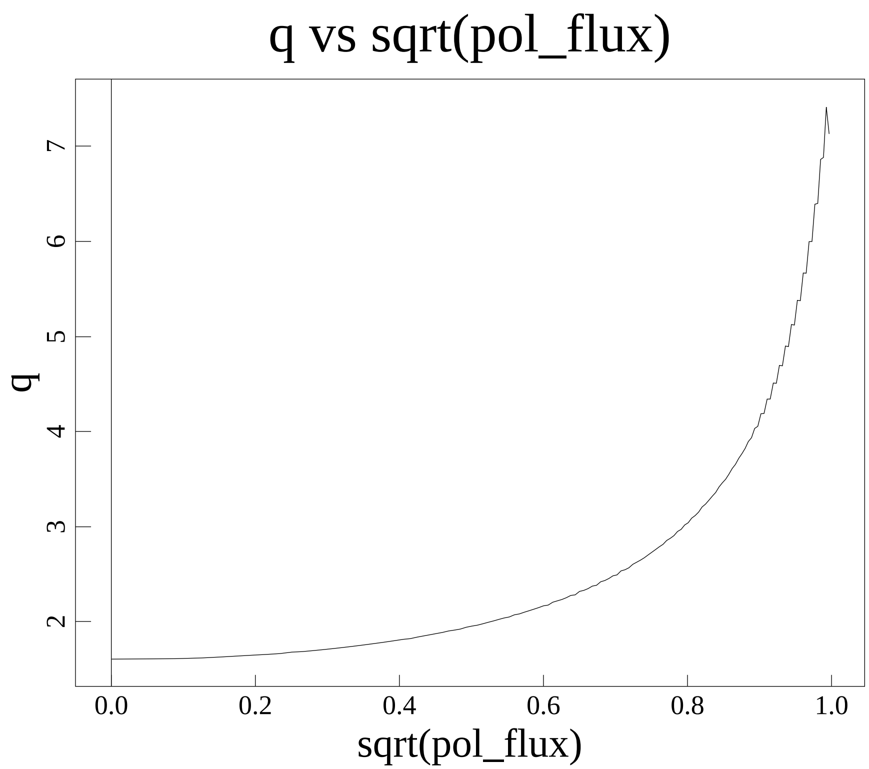
- Build up current on timescale t_{build} to suppress transient Alfvén waves (t_{build} short compared to resistive diffusion timescale τ_R, but long compared to Alfvén time τ_A)

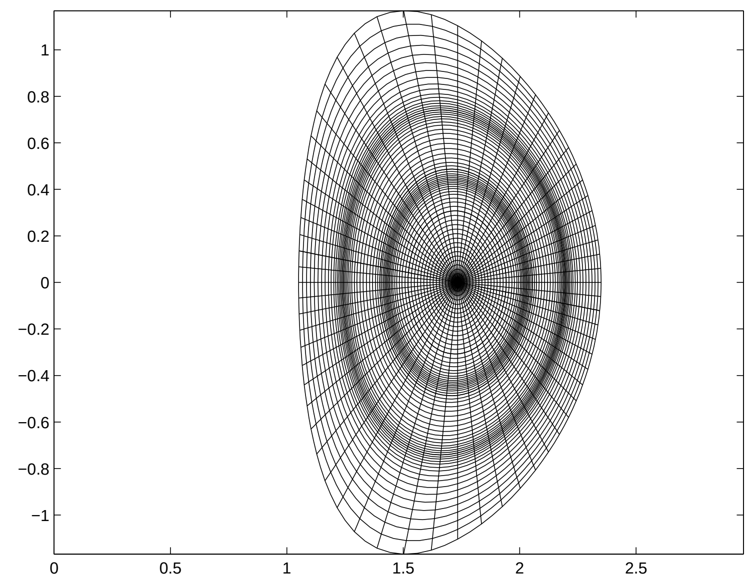


- Initial perturbation has amplitude comparable to random current fluctuations; $\lambda_{\mathrm{rf}} = 8.0 \times 10^{-4}, w_{\mathrm{rf}} = 0.1, R_{\mathrm{rf}} = 0.5, Z_{\mathrm{rf}} = 0$

Use DIII-D-like geometry

- Grid packing used to resolve rational surfaces ($q = 2, q = 3$) more accurately



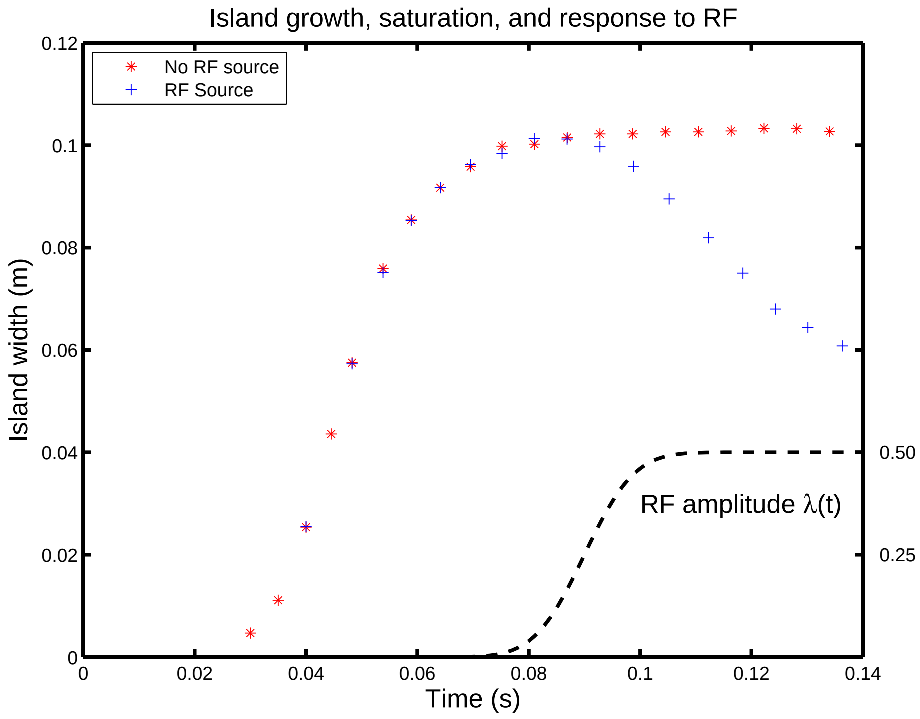


- Benchmark: DIII-D-like simulations of Sovinec *et al.* [J. Comp. Phys. **195**, 355 (2004)].

- Use axially symmetric $\mathbf{F}_{\rm rf}$ (a "ring" of current in the tokamak); examine effects on axially symmetric component of equilibrium ($n = 0$)

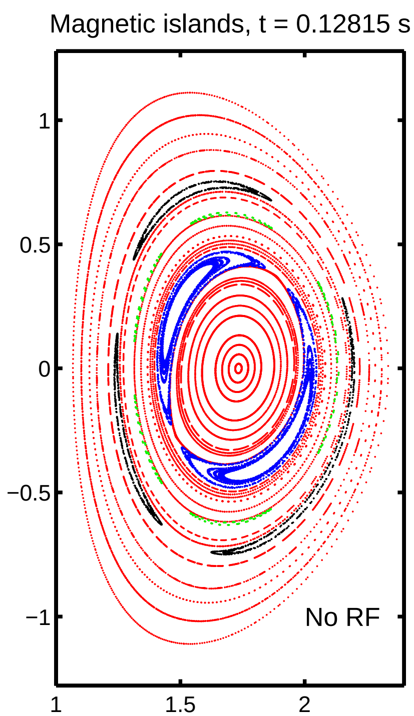
The RF term modifies the width of the magnetic islands

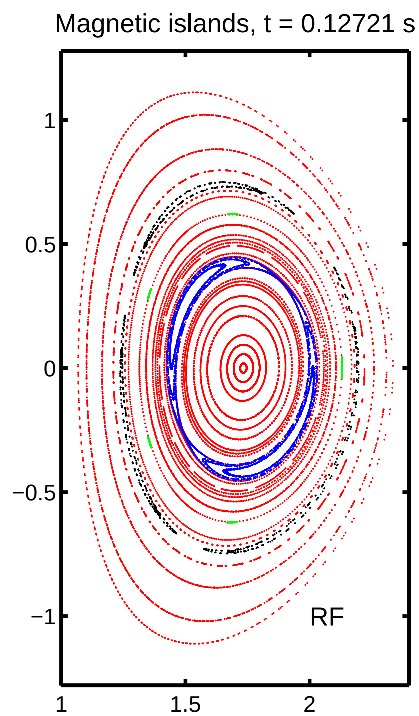
- Now, study resistive tearing modes; let the islands grow up and saturate, and then turn on the RF term



The island widths are visibly reduced due to the RF

- In the absence of RF, large $(2,1)$ islands form; $(3,1)$ and $(5,2)$ also visible
- In the presence of RF, the $(2,1)$ island is reduced in size

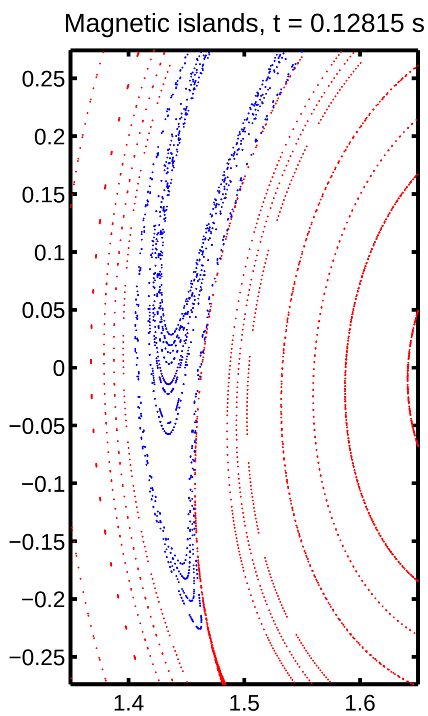


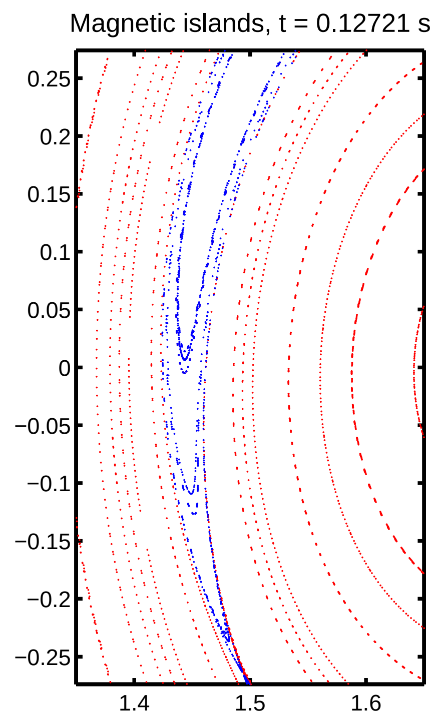


Closeup of reduced island width near X–point

No RF

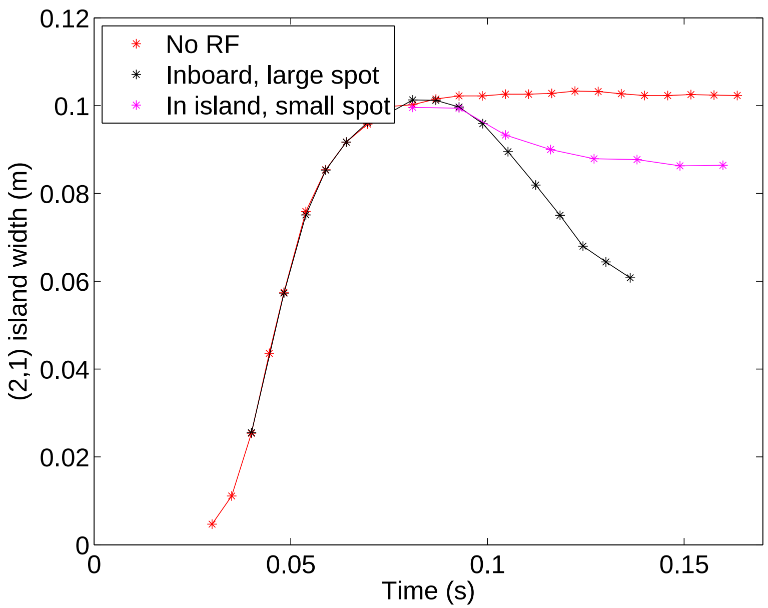
RF

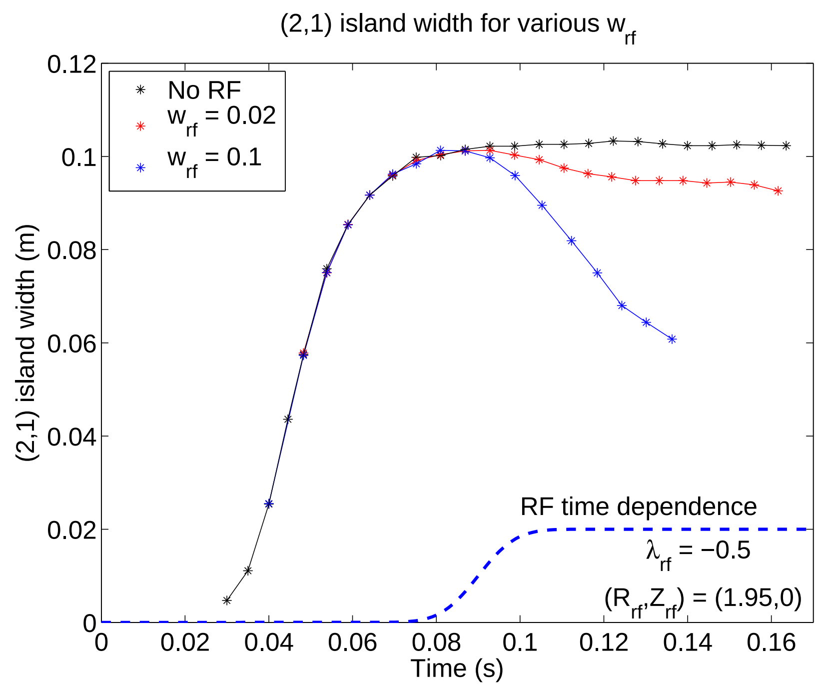




RF localization and location modifies the island widths

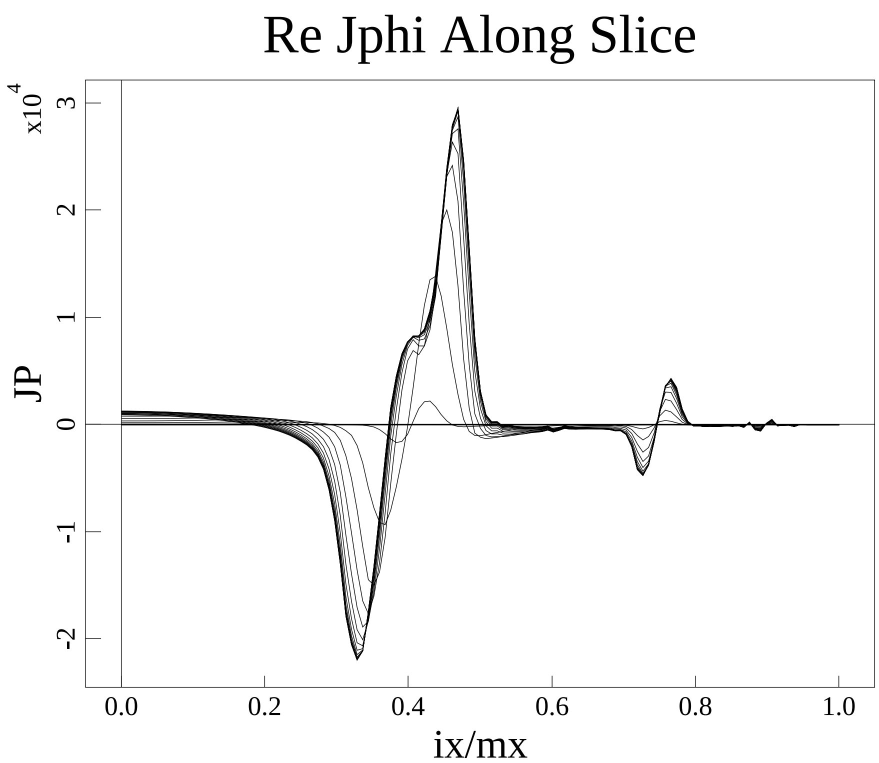
- Island width is very sensitive to current profile and location

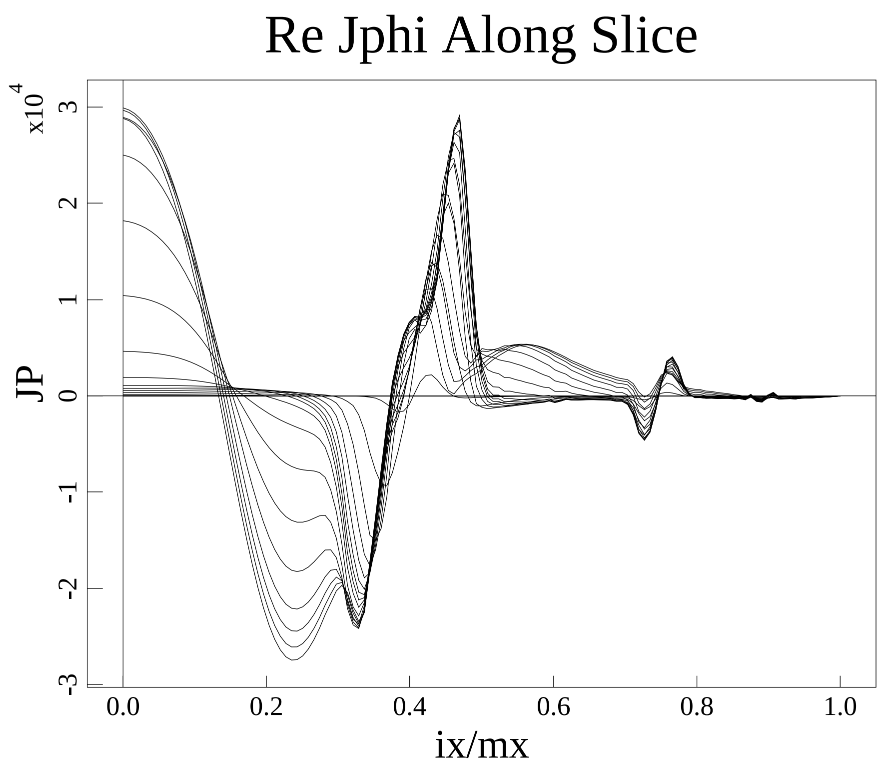




The RF source term affects the perturbed current

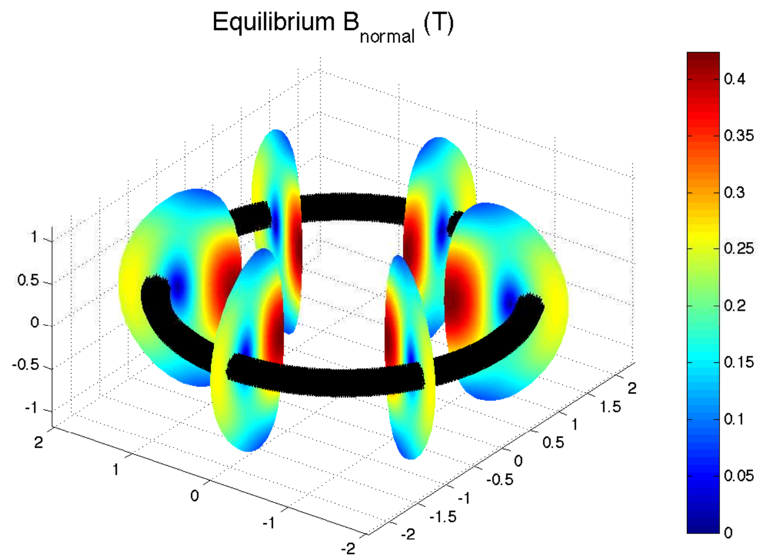
- Look at time evolution of perturbed toroidal current in RF and non–RF case
- Without RF, a steady state is reached as the tearing mode saturates
- When RF is turned on, the on–axis current peaks in response

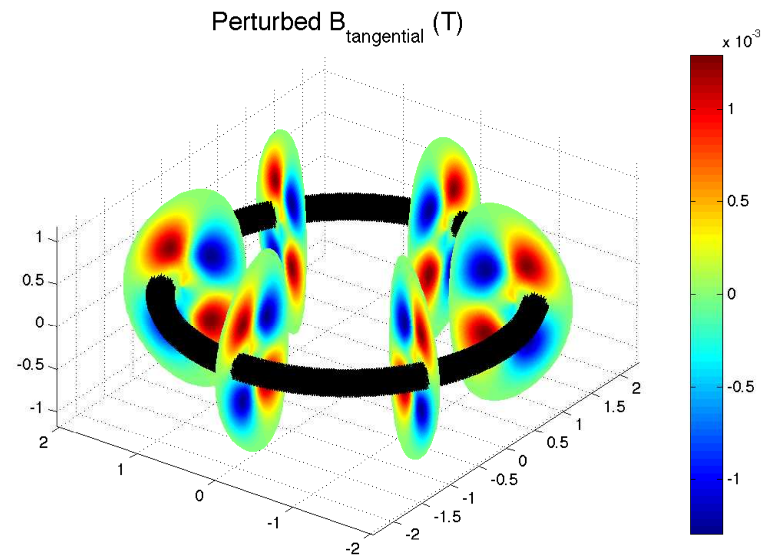




The RF source term modifies the plasma Δ'

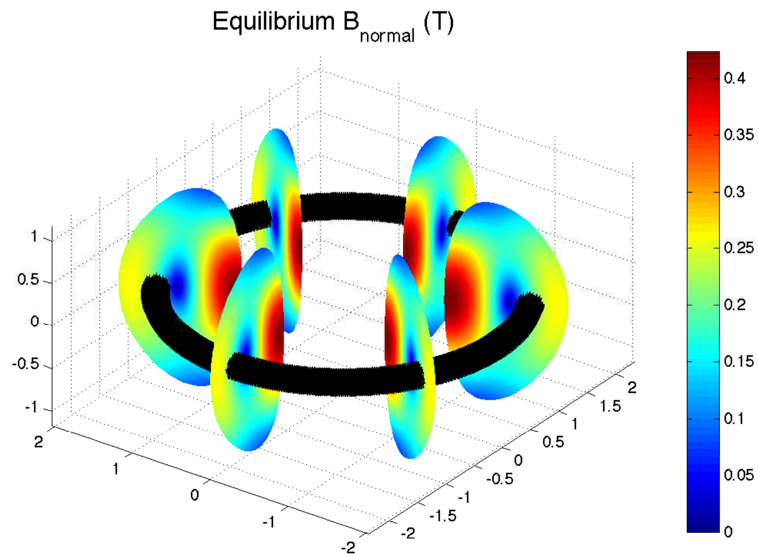
- Numerical experiment — begin with a toroidally symmetric equilibrium strongly unstable to $(2, 1)$ tearing mode
- Initially, allow only $n = 0$ modes in the plasma (suppressing tearing mode growth), ramp up the toroidally symmetric RF term, and allow plasma to come to new RF–modified equilibrium

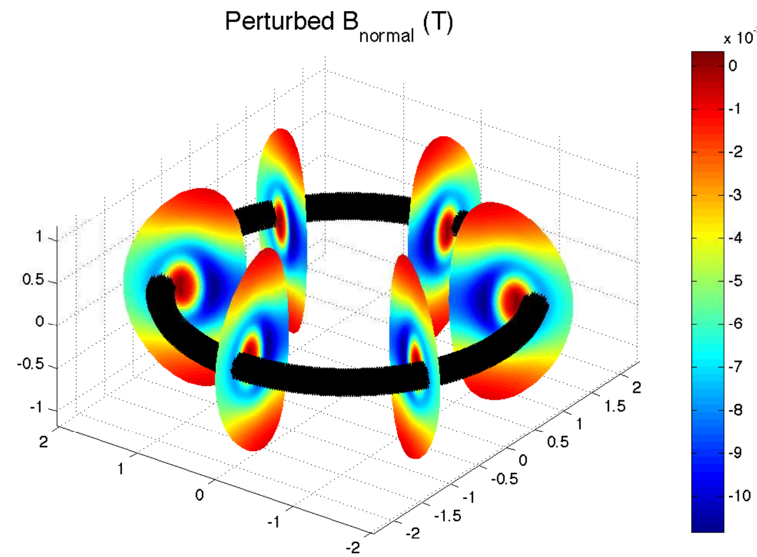




The RF source term modifies the plasma Δ'

- Leaving RF term on, then allow higher–order Fourier modes in the plasma. For the case previously considered, the plasma is now stable to the $(2, 1)$ tearing mode
- More detailed numerical analysis upcoming — PEST3





Incorporating toroidally asymmetric RF terms

- For GENRAY interfacing (and other physical models of ECCD), RF term needs to be toroidally localized; suppose that for some δ,

$$
\begin{aligned}
\lambda_{\mathrm{rf}} &\sim 1 \qquad\qquad \pi - \delta \leq \xi \leq \pi + \delta \\
&\sim 0 \qquad\qquad \text{otherwise}
\end{aligned}
$$

- Needs to be compatible with NIMROD's Fourier representation in the toroidal direction;

$$
\begin{aligned}
A_m &= \sum_{n=-N}^{N-1} \hat{A}_n e^{imn\pi/N} \qquad ; \qquad m \in [-N, N-1] \\
\hat{A}_n &= \sum_{m=-N}^{N-1} \frac{A_m}{2N} e^{-imn\pi/N} \qquad ; \qquad n \in [-N, N-1]
\end{aligned}
$$

approximating

$$
A(\xi) = \sum_{n=-\infty}^{\infty} \hat{A}_n e^{in\xi} \qquad\qquad \hat{A}_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} A(\xi) e^{-in\xi} d\xi
$$

- Fourier coefficients $\hat{A}_n \sim (-1)^n \sin(n\delta)/n\pi$ for above case

Toroidal representation — Issues

- How many coefficients are required for a given δ? Are important coefficients being cut off by NIMROD's dealiasing scheme?

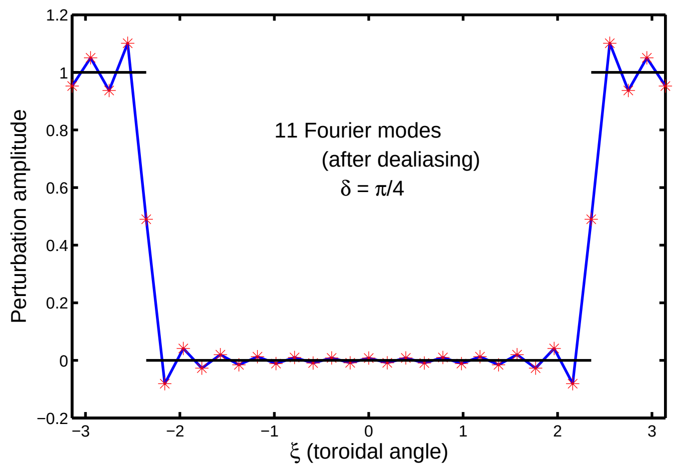
- Does force balance over flux surfaces occur, as in the toroidally symmetric case?

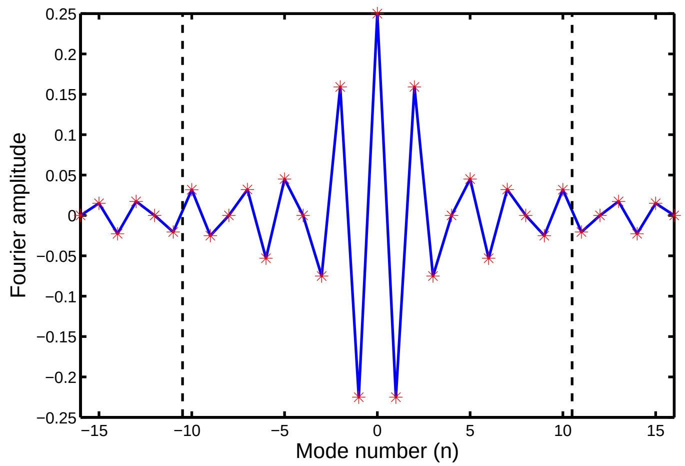
- How are the magnetic islands affected?

- How should the RF amplitude be modulated in time to optimally influence island width reduction?

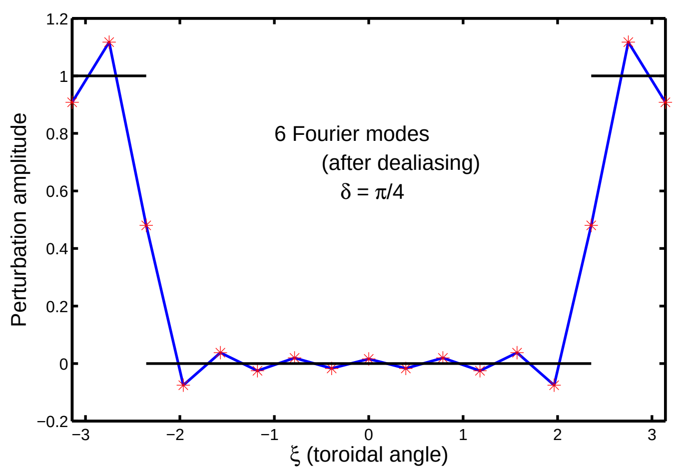
Few modes are required for moderate toroidal localization

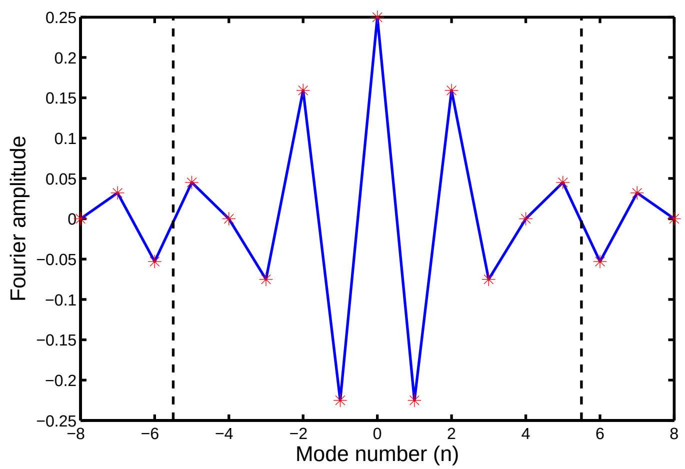
- For $\delta = \pi/4$, keeping eleven modes gives reasonable resolution (six $\rightarrow$ tolerable)





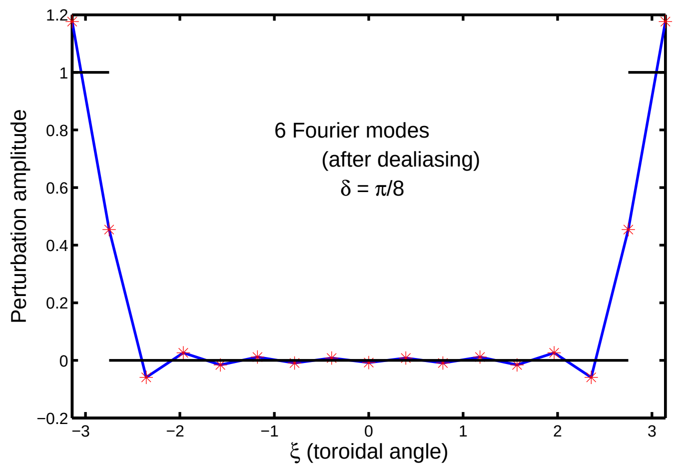
- "Important" spatial scales cut off if not enough modes are used

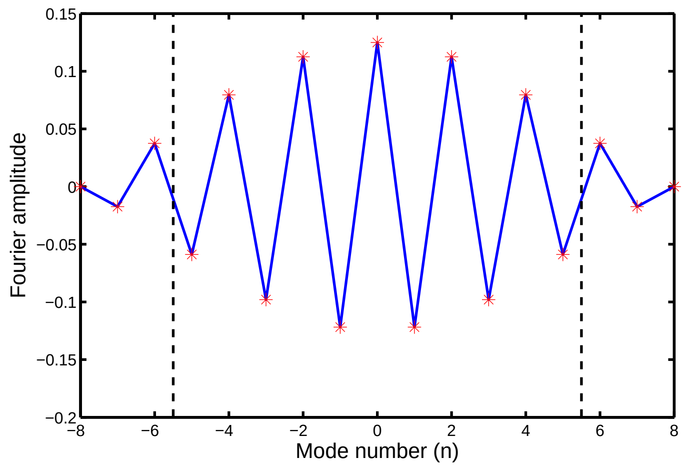




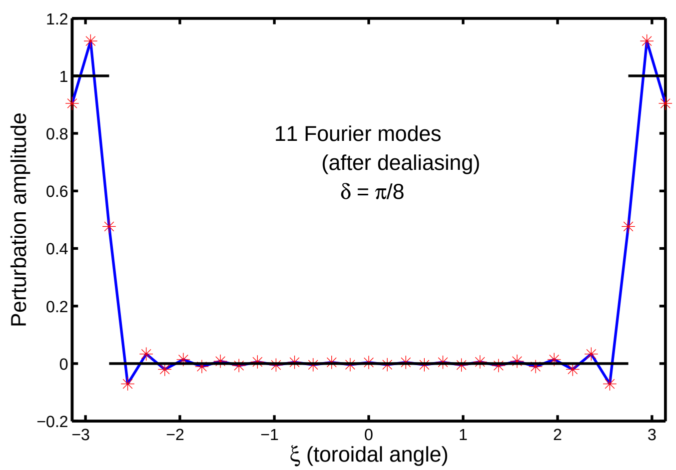
Many modes are required for highly toroidally localized sources

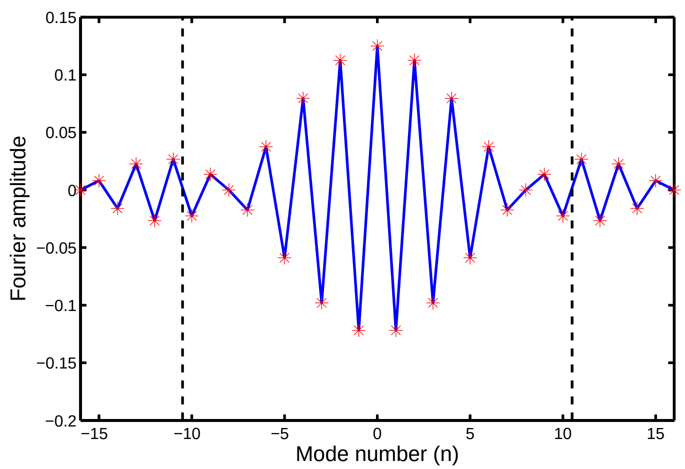
- For $\delta = \pi/8$, many modes are required for adequate resolution





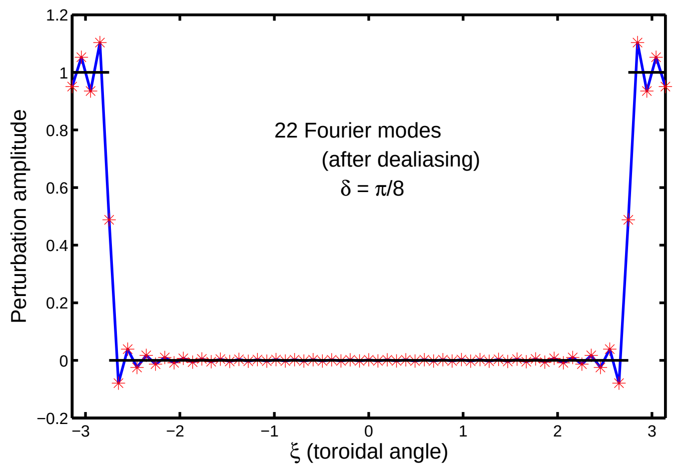
- The previous resolution of 11 modes is arguably inadequate here

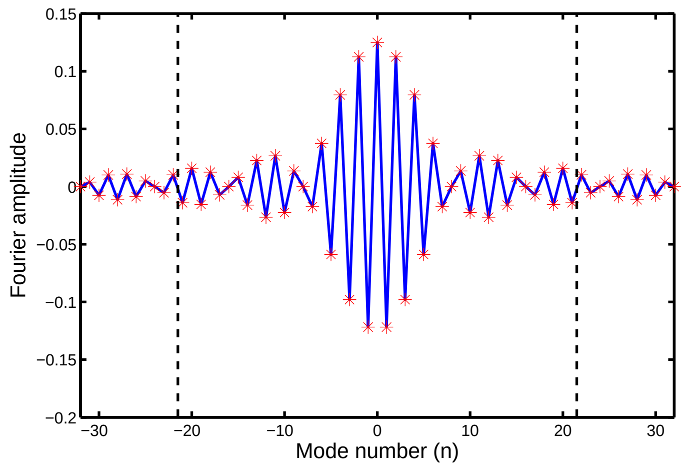




Many modes are required for highly toroidally localized sources

- For $\delta = \pi/8$, 22 modes gives acceptable spatial resolution

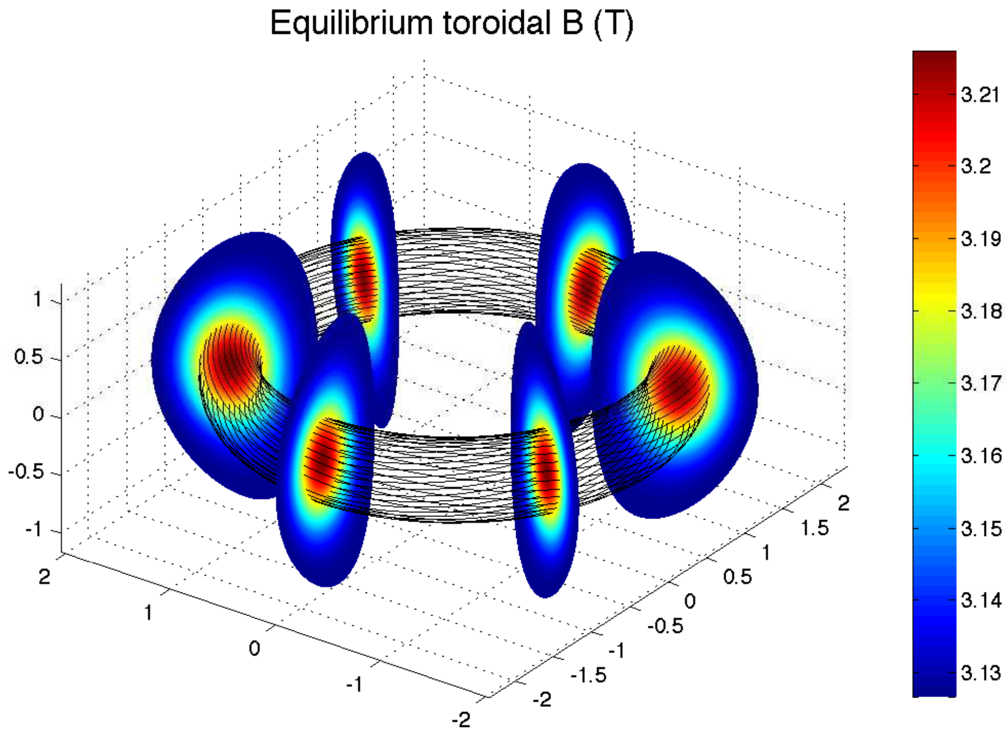




- "Tighter" toroidal resolution of source increases complexity of simulations

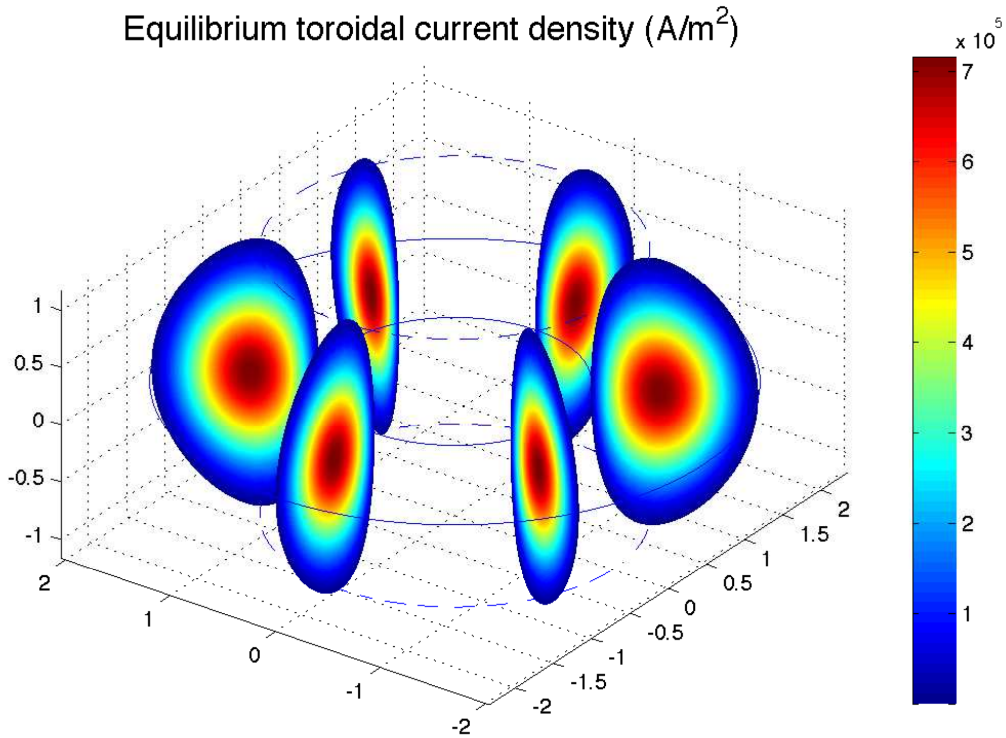
New NIMROD graphics development using Matlab

- High–quality visualizations of NIMROD simulations can be generated by interfacing Matlab with existing post–processing routines (nimplot, nimfl)



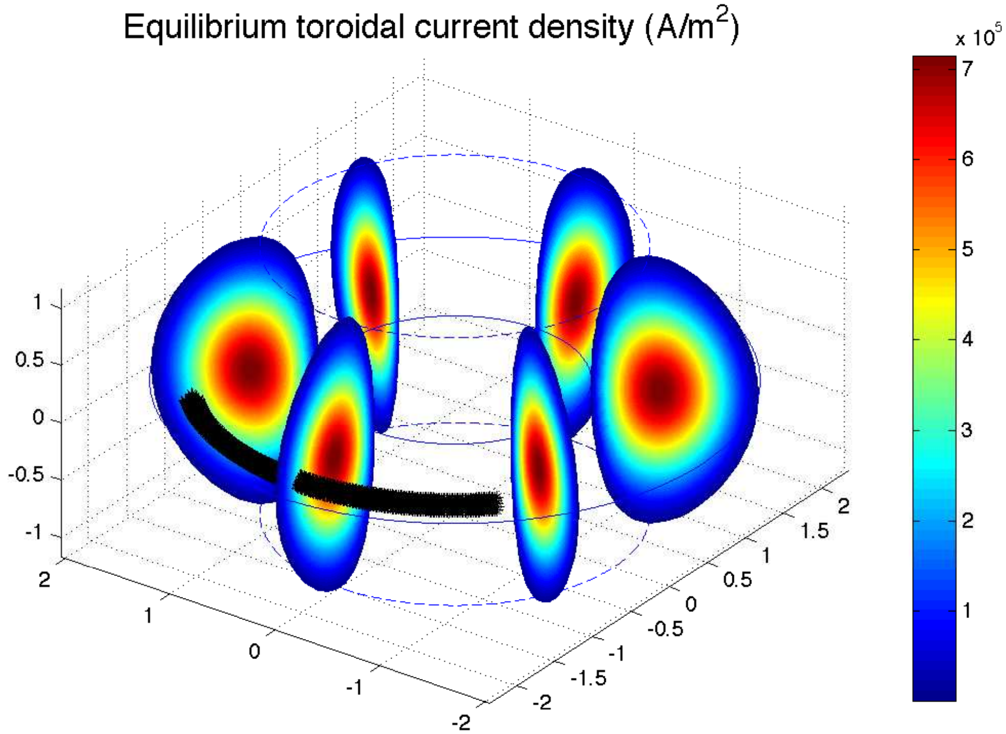
Toroidally asymmetric simulations — first steps

- Begin with a DIII-D–like equilibrium, initially axisymmetric



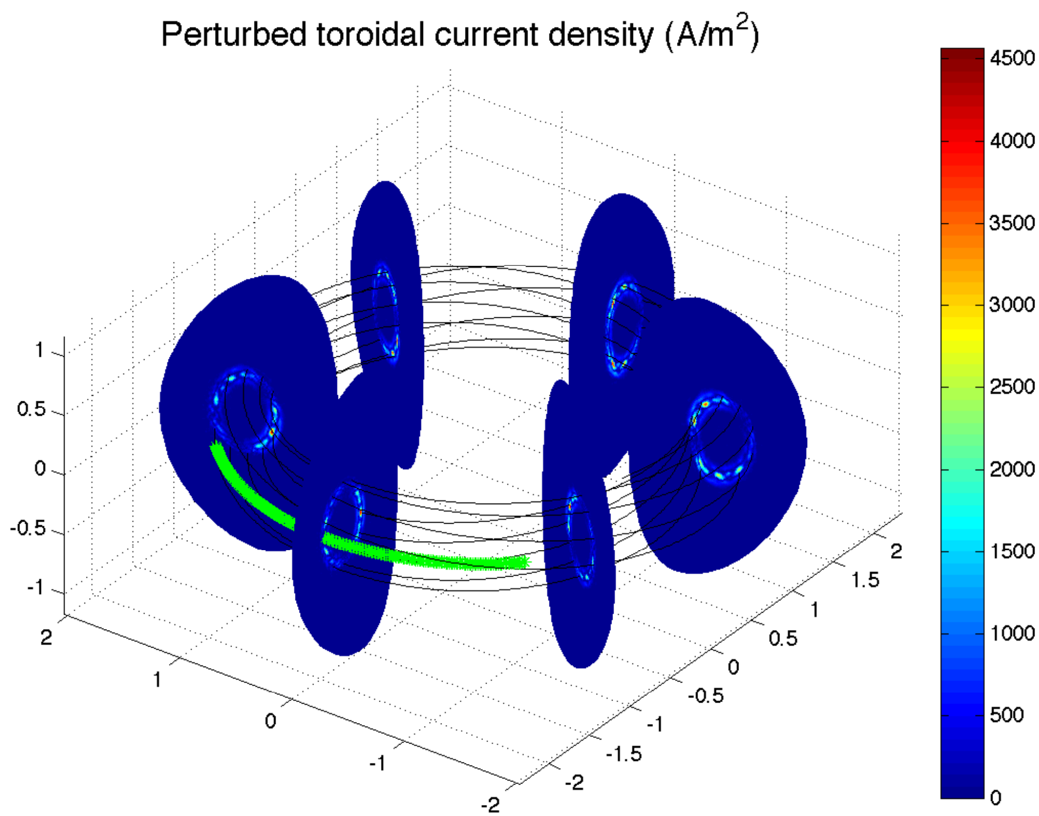
Toroidally asymmetric simulations — ad hoc RF term

- Introduce perturbation with $\delta = \pi/4$ (over one–quarter of the torus — broad enough that reasonably small number of modes can be used, but still introducing toroidal asymmetry)



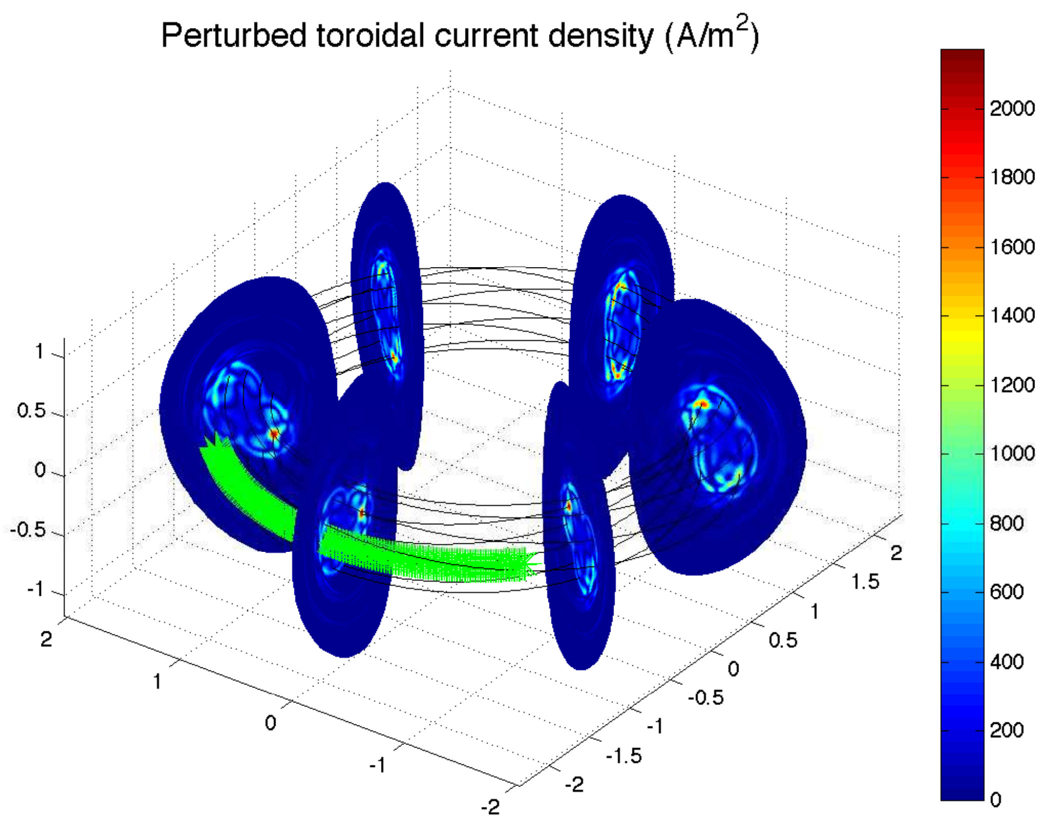
Equilibration of toroidally asymmetric sources

- Equilibration of RF–induced current over flux surfaces proceeds reasonably well for small poloidal cross-sections
- Still occurs on Alfvén timescale



Equilibration of toroidally asymmetric sources

- For large poloidal cross–sections, the equilibration process is more complicated
- Grid resolution (in poloidal plane) may be an issue



Island widths — toroidally asymmetric case

- Currently a work in progress – some numerical convergence issues need to be addressed as RF term is ramped up
- Simulations running at NERSC

Future plans

• Dependence of island width reduction on source amplitude, spatial localization (poloidal cross–section, toroidal extent)

• Further exploration of physics issues associated with island width reduction — how is initial ($n = 0$) component of equilibrium modified by RF? PEST3 studies, etc.

• Further study of flux surface equilibration of RF–induced currents in toroidally nonsymmetric cases — resolve convergence issues for large poloidal cross-sections

• Numerical issues — GENRAY and NIMROD use different representations of the magnetic field (S. Kruger addressing this); convergence issues for saturated islands in toroidally asymmetric case

• Theoretical issues — form of quasilinear operator Q, feasibility of simulating ICRF physics in this theoretical framework