Calculations of Linear Two-Fluid Plasma Response to Nonaxisymmetric Fields

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Understanding Time-Independent 3D Plasma Response Requires Non-Ideal Physics

- **When is response ideal? When is it vacuum?**
- **Does the response vary smoothly between these two limits?**
- **If so, what is the relevant dimensionless parameter?** $\tau_R \Omega$?
- **To what extent are other effects important?**
  - Viscosity, two-fluid, FLR, etc..
- **How can this inform our interpretation of experimental results?**

Progress in 3D Response Since Last Meeting

- **Implementation of time-independent method**
  - Response to Boozer's concerns
- **Better understanding of results, especially in two-fluid case**
- **Application to several DIII-D experiments**
  - Lanctot, Mordijck, Buttery

Time-Dependent Method

- **$B(t) = B_0 + B_1(t)$**
  - B_0 is the axisymmetric equilibrium field
  - $B_1(0)$ is the "vacuum field" from non-axisymmetric coils (I-coils).
- **Conducting-wall boundary condition**
  - **n•B** is held constant in time on simulation domain boundary (approximately vacuum vessel).
- **Simulation is time-advanced until the steady-state is reached.**
- **Final B_1 is applied field + plasma response.**
- **Caveats:**
  - Equilibrium must be stable (otherwise response is ∞)
  - More computationally intensive than time-independent calculation

Time-Independent Method

- **Time-dependent time-step:**

$$\frac{\mathbf{x}^{n+1}-\mathbf{x}^{n}}{\delta t} = A\cdot\left[\theta\mathbf{x}^{n+1} + (1-\theta)\mathbf{x}^{n}\right]$$

- **Time-independent solution ($x^{n+1}=x^n$) is**

$$A\cdot\mathbf{x}^{n+1} = 0$$

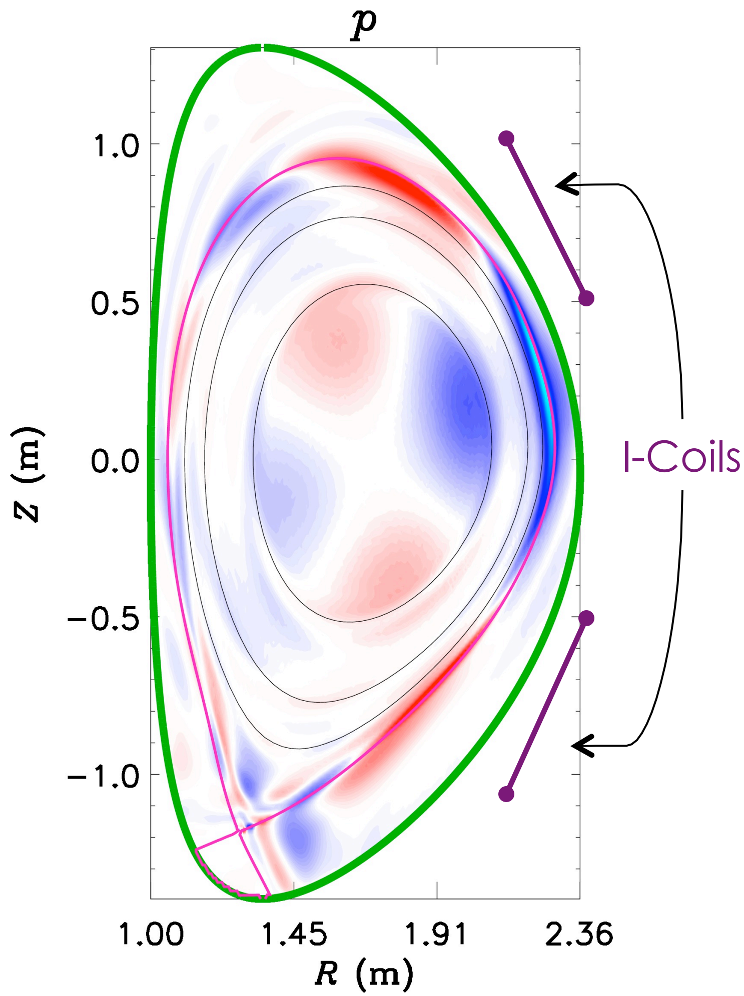
(BCs make some elements of RHS nonzero)

- **This is equivalent to our time-dependent calculation, if we remove d/dt terms and set θ =1**

- **Caveats:**
  – Doesn't work with semi-implicit methods
  – Only makes sense for linear calculations
  – System may be very poorly conditioned
  – Says nothing about dynamics

Response Calculated For DIII-D Discharges

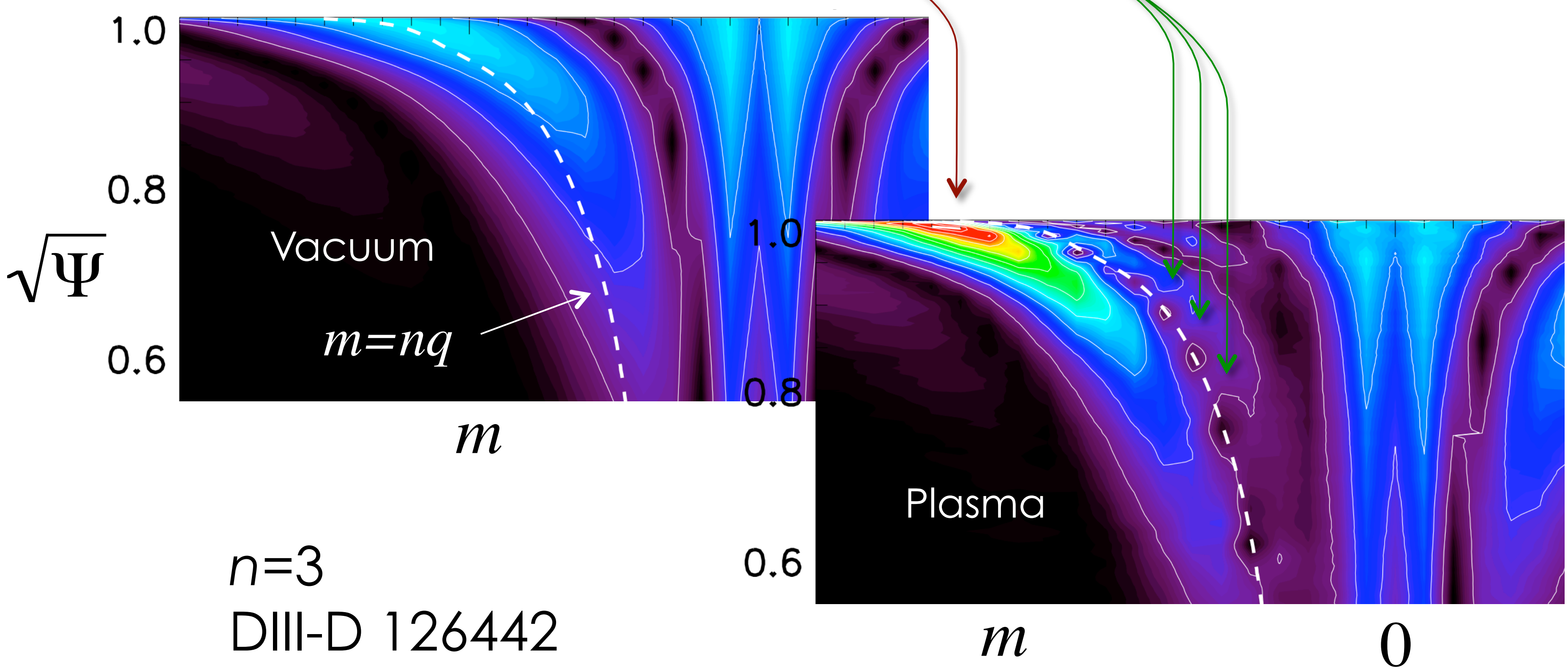
- Use EFIT reconstructions of DIII-D shots 135758--73
- Equilibrium is recalculated self-consistently using $\rho\,\Omega^2 = \alpha\, p_0$
- Numerical considerations require I-Coils to be outside wall (unlike experiment)
- Wall excludes plasma response, invalidating magnetics diagnostics



* Lanctot *et al.*, *Phys. Plasmas* **17**:030701 (2010)

Plasma Screens Resonant Fields, Enhances Non-Resonant Fields

- **Generally, two types of responses are seen**
  - Screening
  - Kink excitation



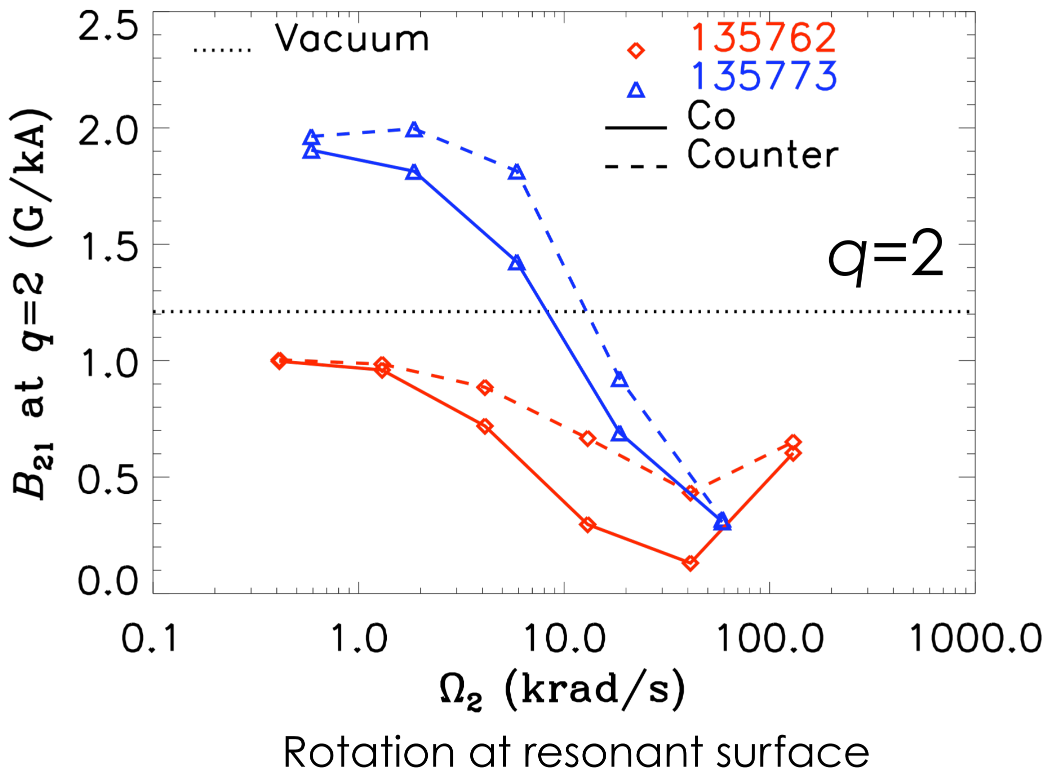
n=3
DIII-D 126442

Rotation (Usually) Improves Screening; Response Depends on Beta

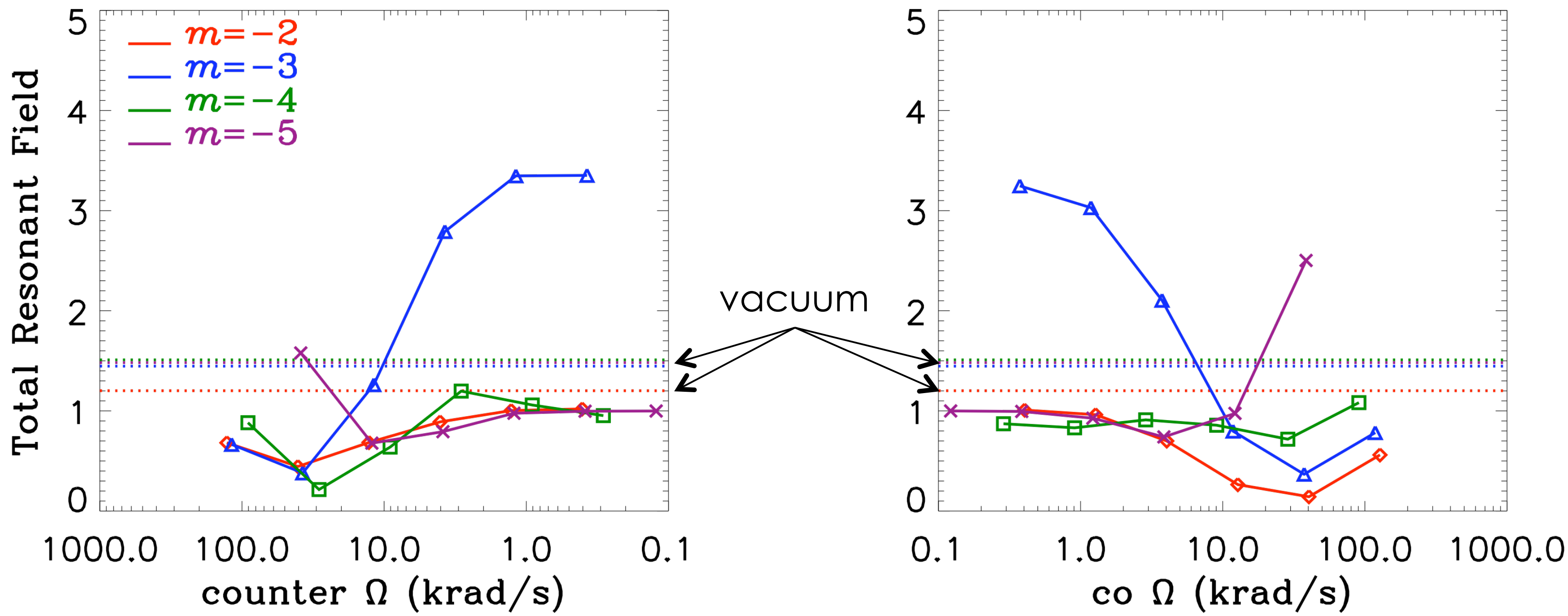
- **Co-current rotation is found to screen better than counter-current (even in resistive 1F model)**

- **In low-β case, plasma screens even without rotation**

- **In high-β case without rotation plasma amplifies resonant fields**

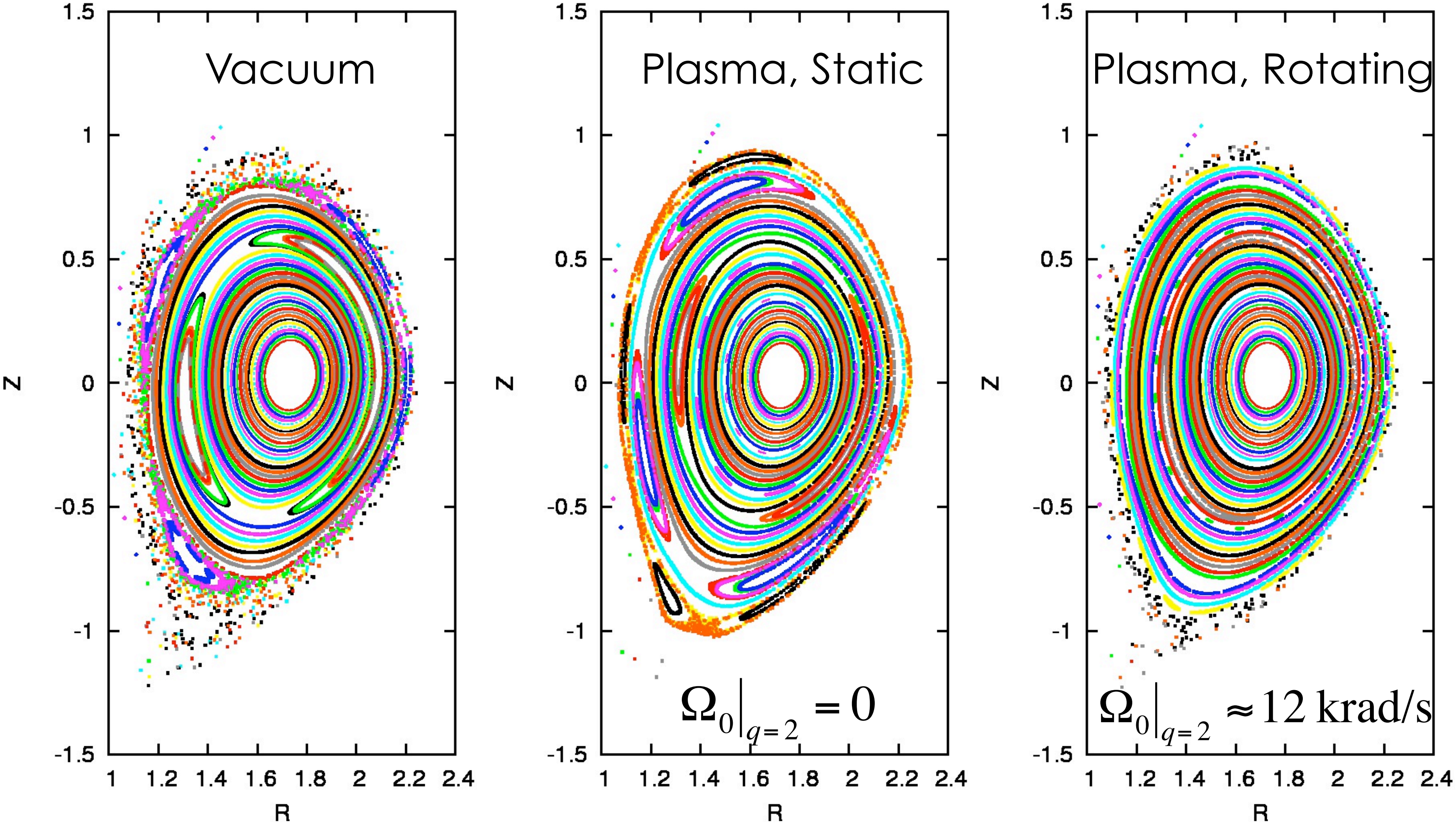


Shot 135762 Responds Most Strongly at *q*=3 Surface



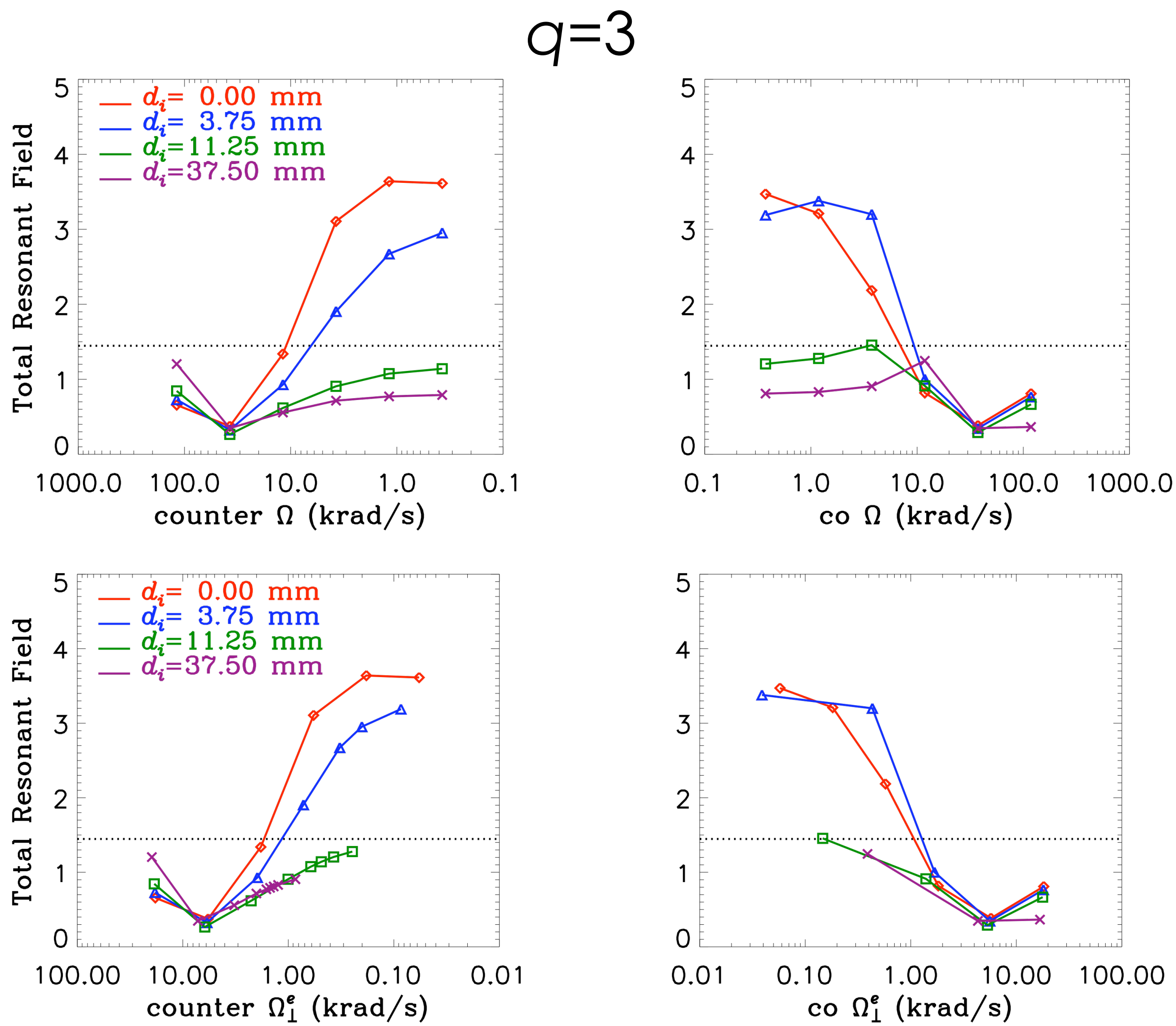
- **Rotation generally suppresses resonant response**
- **Strong rotation enhances resonant response near edge (especially q=5)**

Rotation Improves Core Screening; But Stochasticizes Edge



Core Penetration Depends on Perpendicular Electron Velocity

- **2F terms introduced by raising d_i**
- **Core penetration correlates better with perp. electron rotation (bottom) than with ion rotation (top).**



What is "Perpendicular" Electron Velocity?

- **For incompressible flow,** $\mathbf{v} = R^2\omega(\psi)\nabla\varphi + \lambda(\psi)\mathbf{B}$
  - Note that $\omega(\psi) \neq \Omega = \mathbf{v}\cdot\hat{\varphi}/R$ unless $\lambda(\psi) = 0$
- **The perpendicular angular velocity is defined as**

$$\Omega_\perp = \frac{\mathbf{v}}{R}\cdot\frac{\mathbf{B}\times\nabla\psi}{|\mathbf{B}\times\nabla\psi|}$$

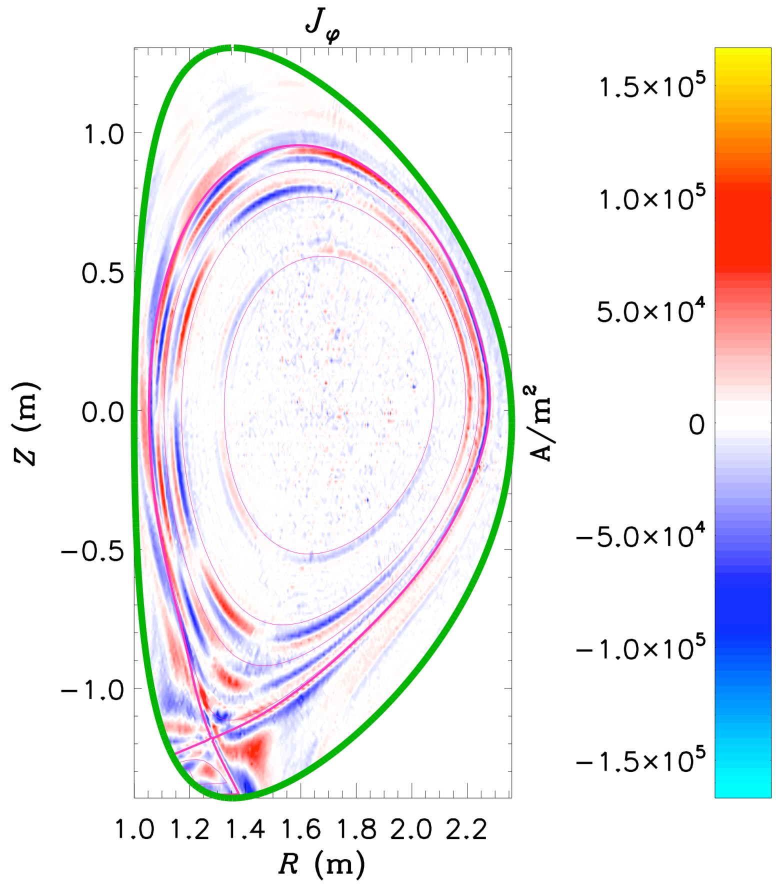
$$= \frac{|\nabla\psi\times\nabla\varphi|}{|B|}\omega(\psi)$$

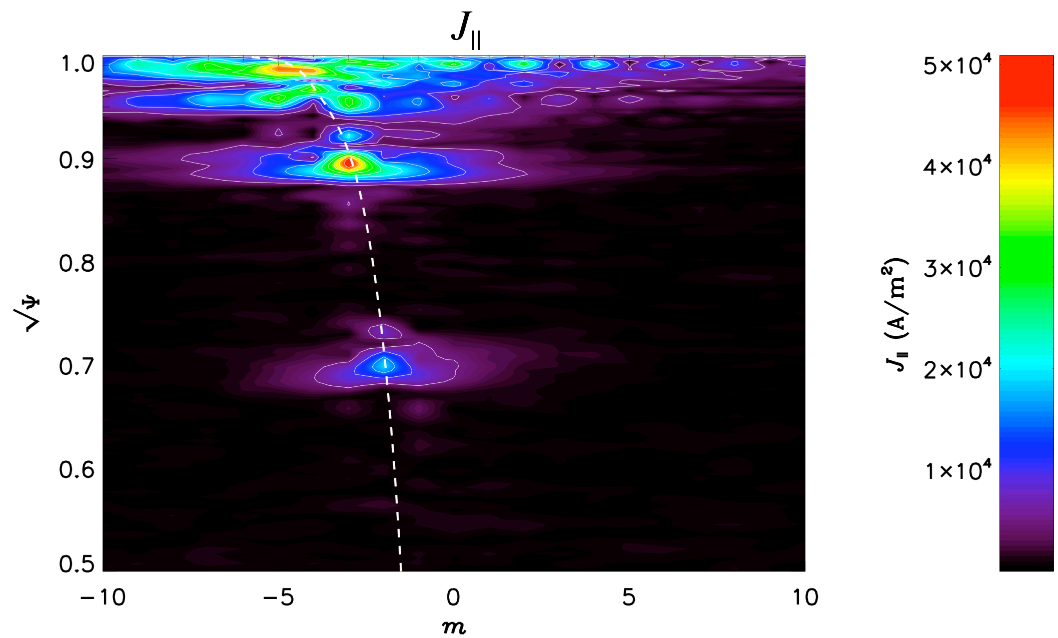
- **From radial force balance:** $\omega(\psi) = \underbrace{\phi'(\psi)}_{\omega_{E\times B}} + \underbrace{\frac{p'(\psi)}{nq}}_{\omega_*}$

$$\omega^e = \omega^i + \omega_*^e - \omega_*^i$$

- $\Omega_\perp$ **vanishes wherever** ω **vanishes, but also at nulls**
  - If $\Omega_\perp$ is the relevant quantity, toroidal rotation is less effective at screening near x-point

Parallel Currents Persist in Steady State





- These are from time-independent calculations, without rotation!

- Currents do not affect energy balance at first order (currents are periodic in ϕ)

Summary and Conclusions

- **M3D-C1 can calculate linear two-fluid response with realistic values of resistivity, rotation, and perpendicular transport**
  - Time-independent and time-dependent methods agree
- **Time-independent parallel currents do exist in resonant layers**
  - *This is true even without rotation*
- **Screening is more complicated than ratio of rotational to resistive time-scales**
  - Depends crucially on proximity to marginal stability
  - Viscosity and thermal diffusion inhibit resonant response (both screening and resonant field amplification)
- **Perpendicular electron velocity appears to be the relevant rotation quantity for screening**

Future Work

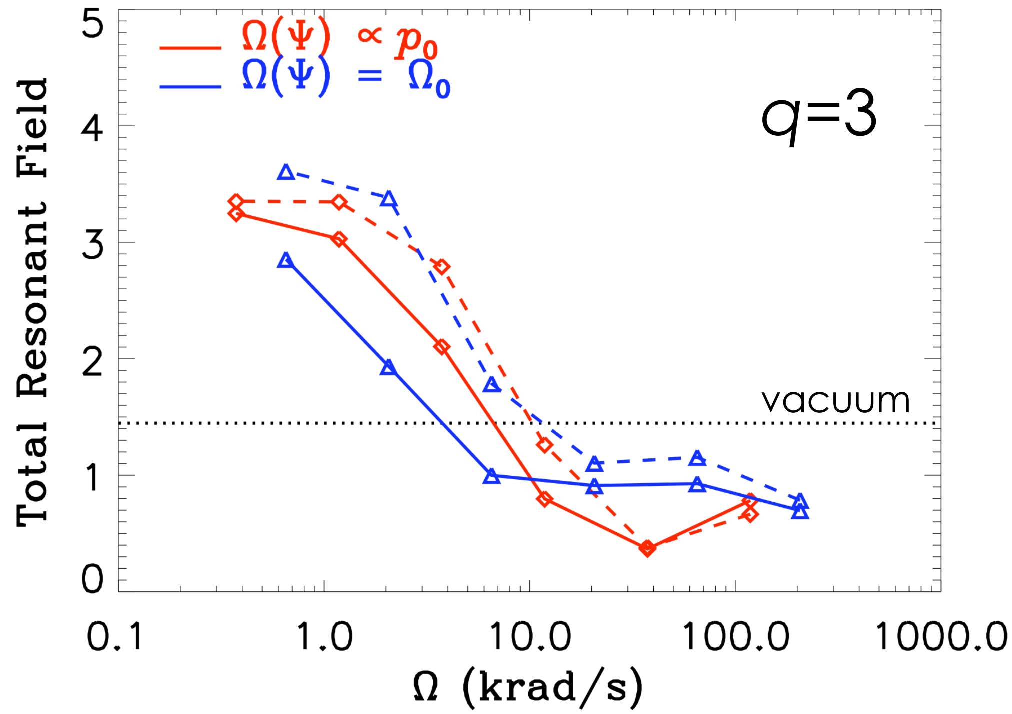
- **Torque calculation**
  - General NTV models can be used to calculate torque generated by linear perturbation *a posteriori*
  - Torque can be calculated self-consistently using Braginskii-like NTV models
- **Nonlinear calculations**
  - M3D-C1 is now capable of 3D nonlinear calculations (see poster 1P24)
  - Unclear how to extend time-independent method to nonlinear model
- **How do we use this to optimize coils? Optimize for what?**

Extra Slides

Rotation Magnitude, Not Shear, Is Responsible For Screening

- **Red**: rotation profile proportional to pressure
- **Blue**: uniform rotation throughout plasma (*i.e.* no shear)

- Rotation screens, even when there is no shear.



Rotation at resonant surface

Rotation-Driven Edge Response Depends on Ion Rotation (Not Electron Rotation)

- Edge mode correlates better with ion rotation (top) than with perp. electron rotation (bottom)
- Not "penetration," but proximity to marginal stability.