# Resistive linear stability calculations with the M3D-C1 code
# Sherwood Meeting
# 2011 May 1-3

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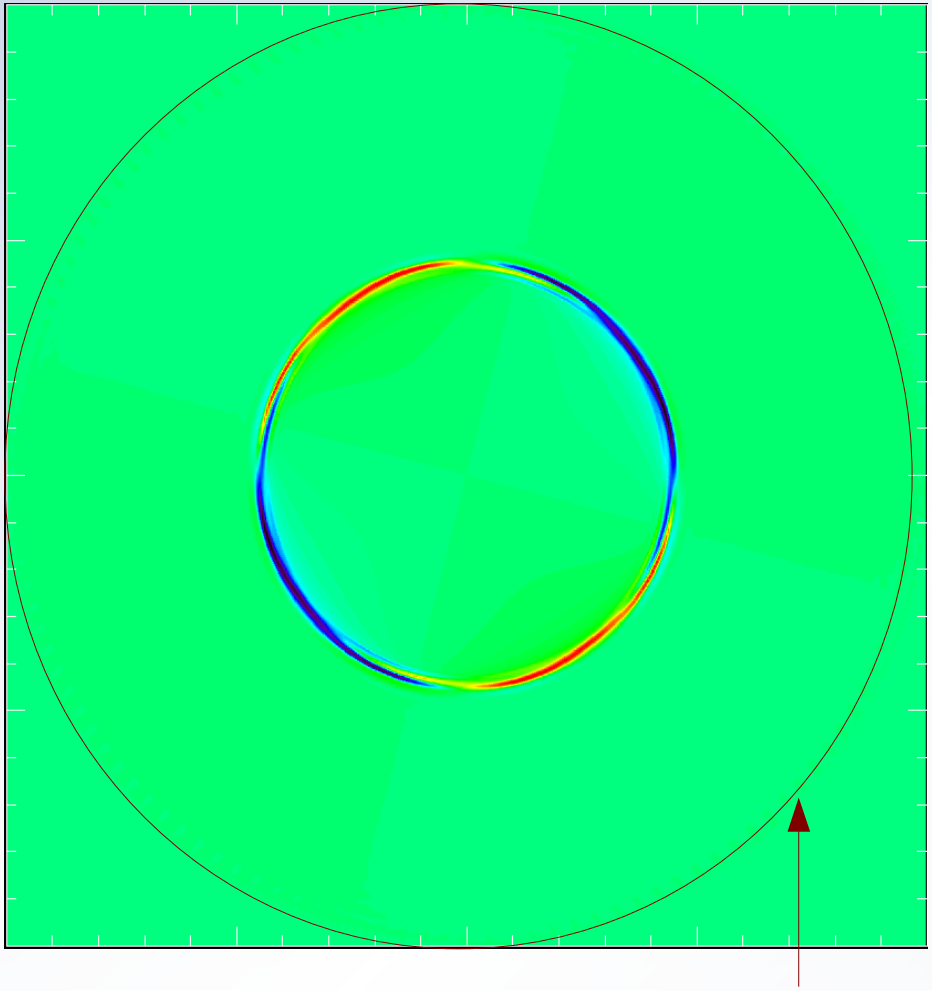
# Resistive linear stability calculations with the M3D-C1 code




- Model : resistive modes in cylinder
- Convergence study of M3DC1
- Asymptotic matching solution
- Comparison
- Conclusion

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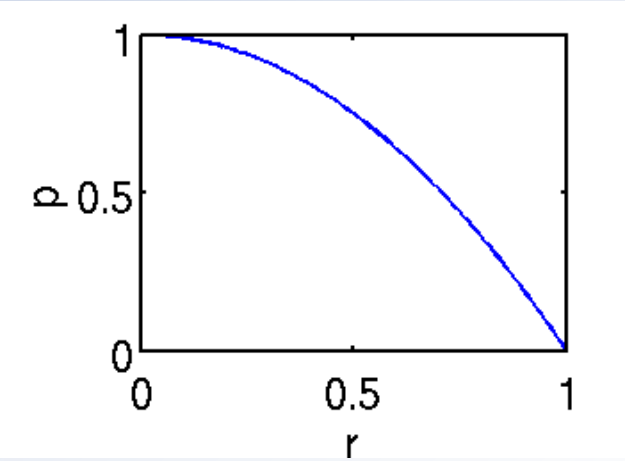
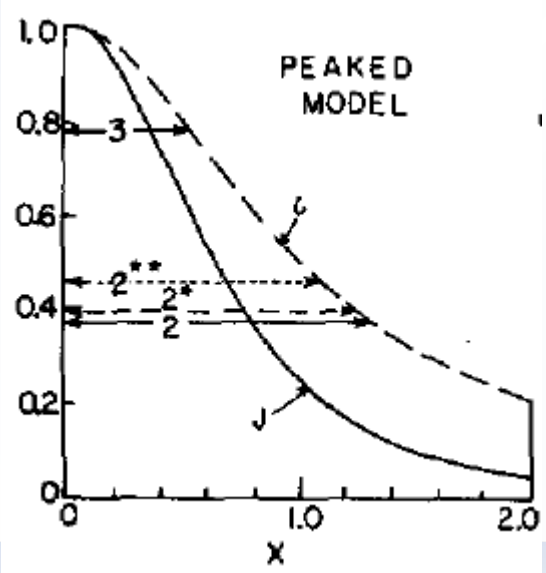
# Model description

Pressure



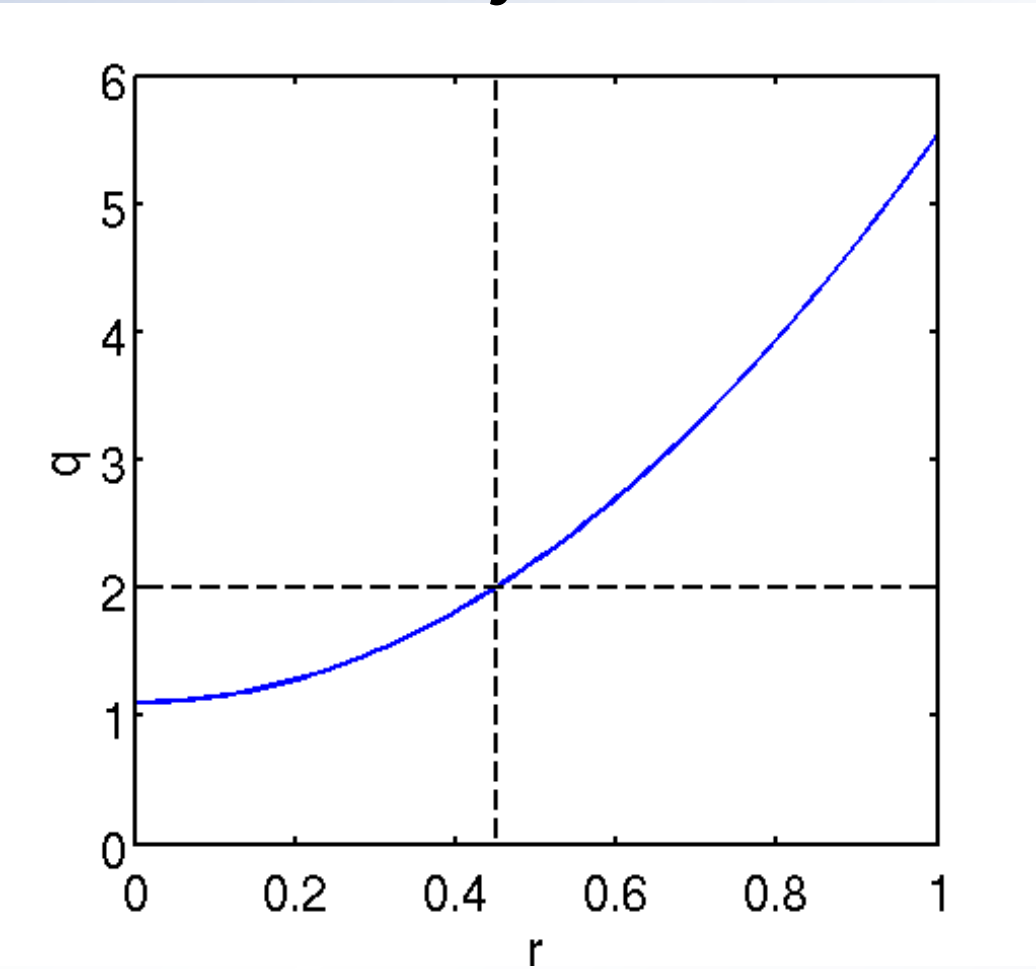

Current




- Resistive MHD in a cylinder
- No flow, axisymmetric equilibrium ==> 2 input profiles : current (or safety factor) and pressure
- Parabolic pressure profile
- Current profiles from FRS73

# Model description (2)

Safety factor




- Monotonic q profile => up to one rational surface for each (m,n)
- Focus on m=2, n=1 rational surface (rs)
- q0 > 1 => no internal mode
- Periodicity : k=1/R, R=10
- Minor radius a =1
- Conductive wall at r=a, no vacuum

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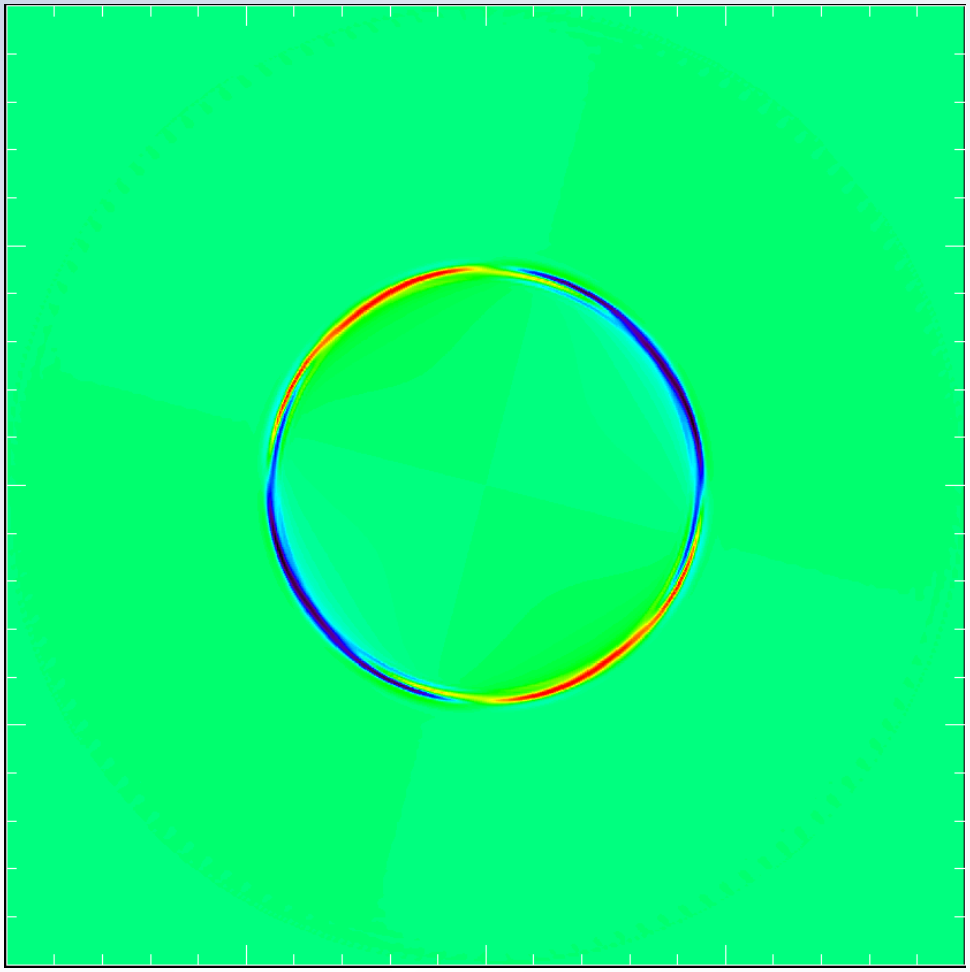
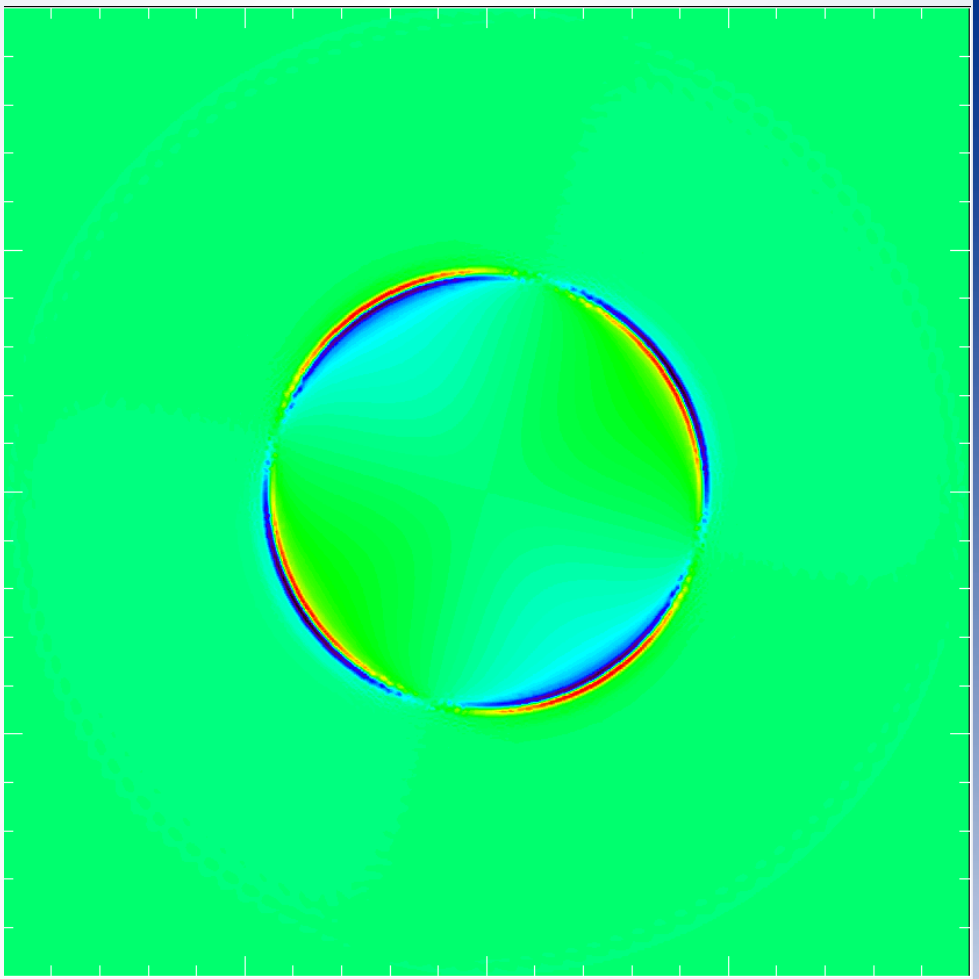
# M3DC1 – eigenfunctions m=2 n=1 resistivity=1.e-6

Radial velocity



Toroidal current



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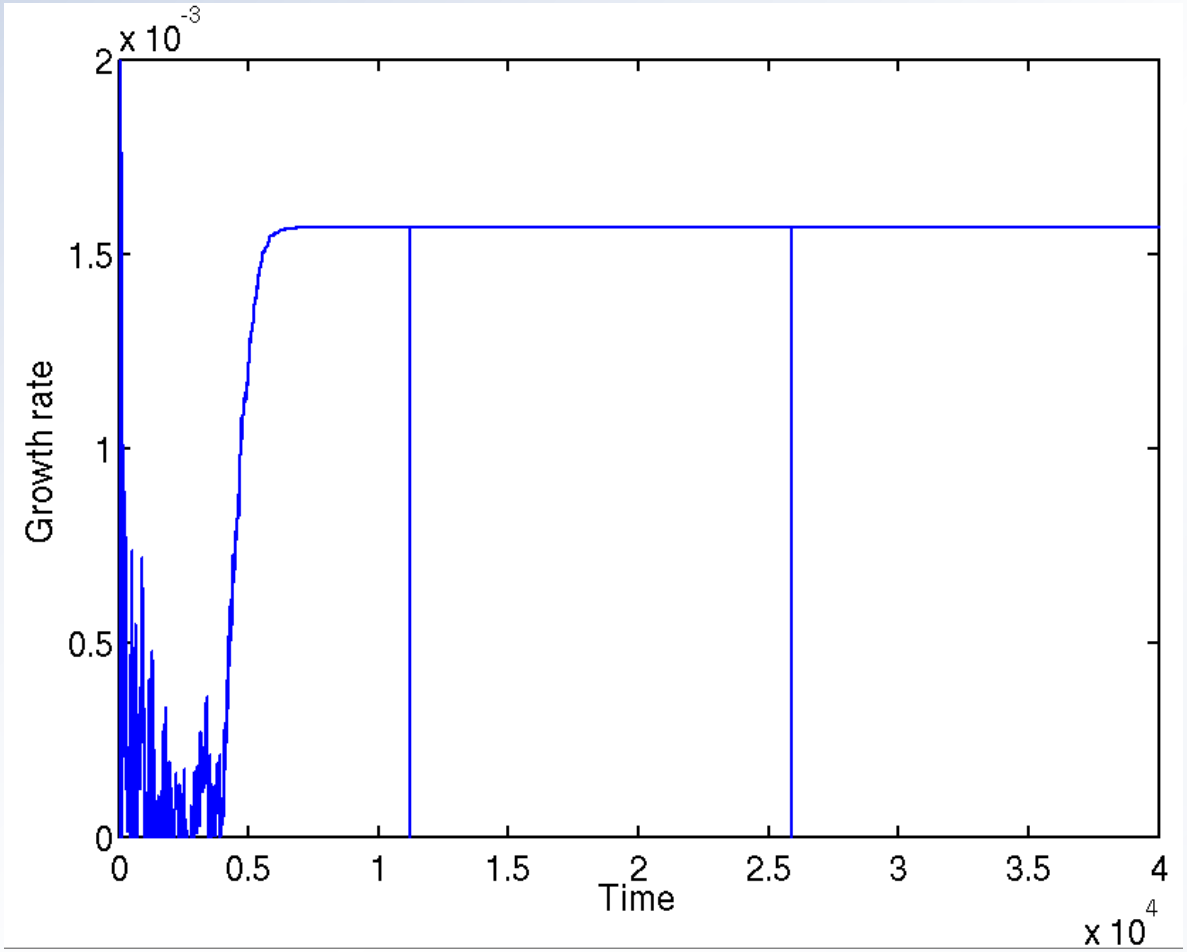
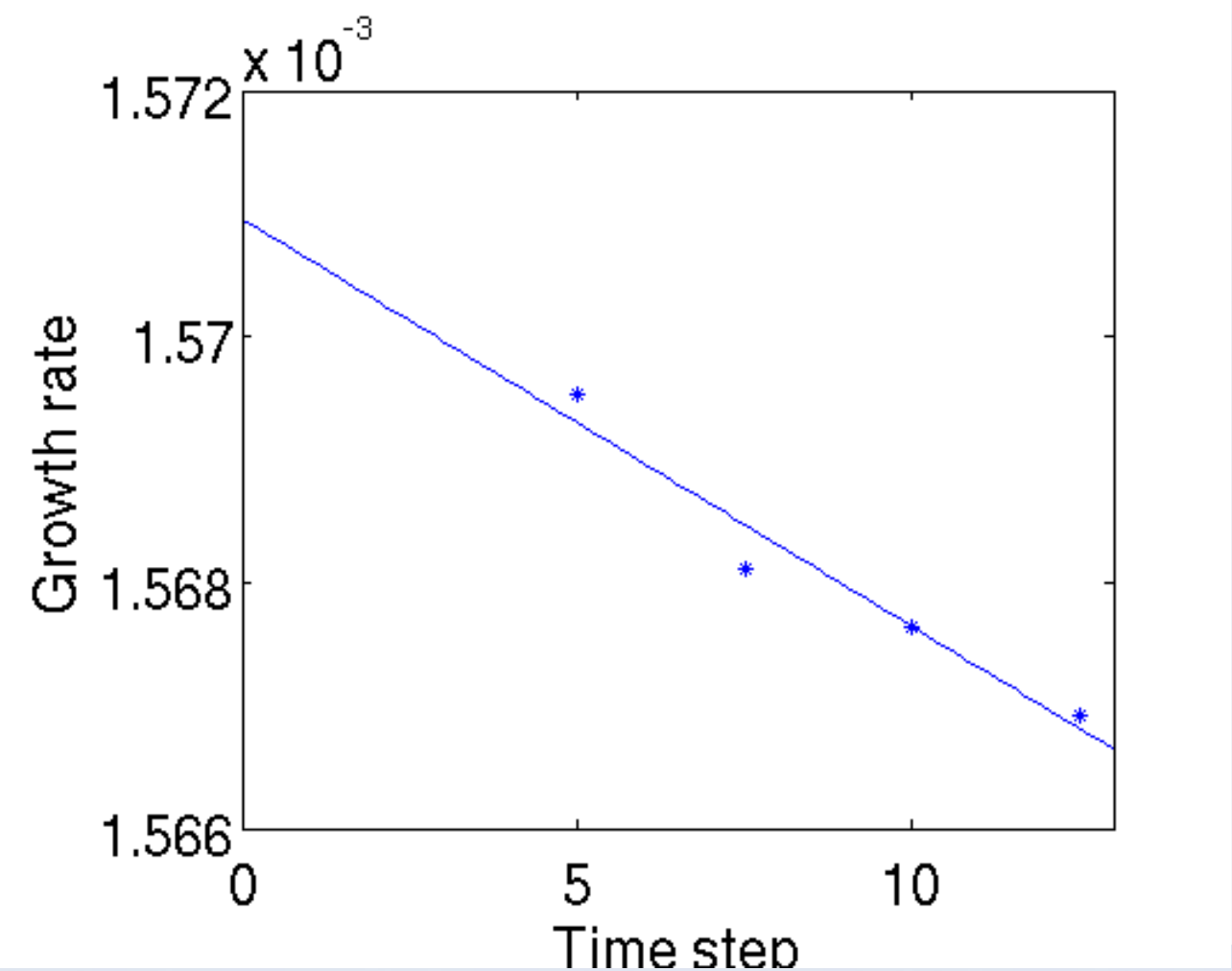
# M3DC1 - Growth rate



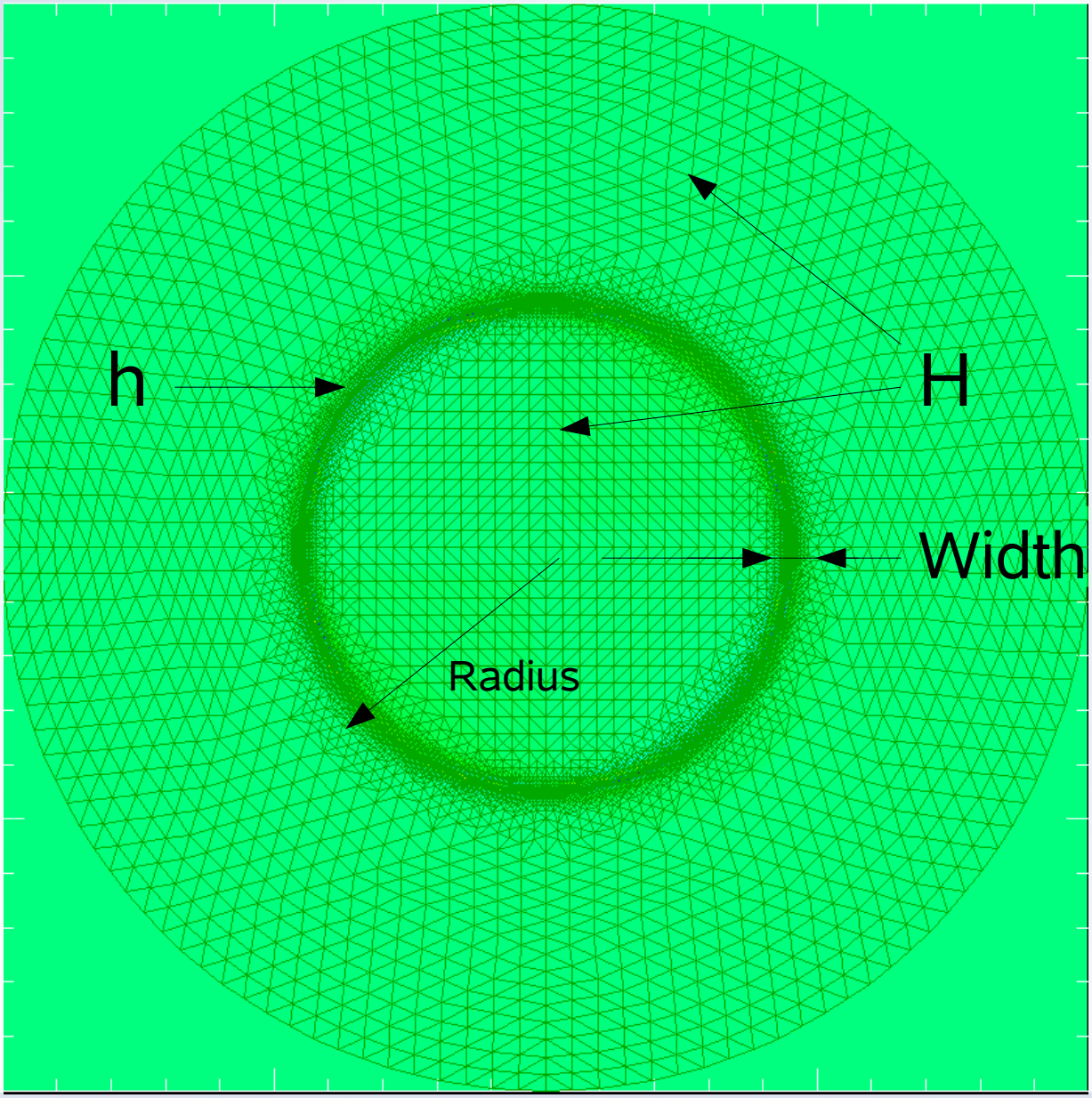

- Growth rate calculated from the time derivative of the kinetic energy

# M3DC1 - time step convergence

- ~10e-3 relative change in growth rate when time step divided by 2



# M3DC1 – mesh parameters

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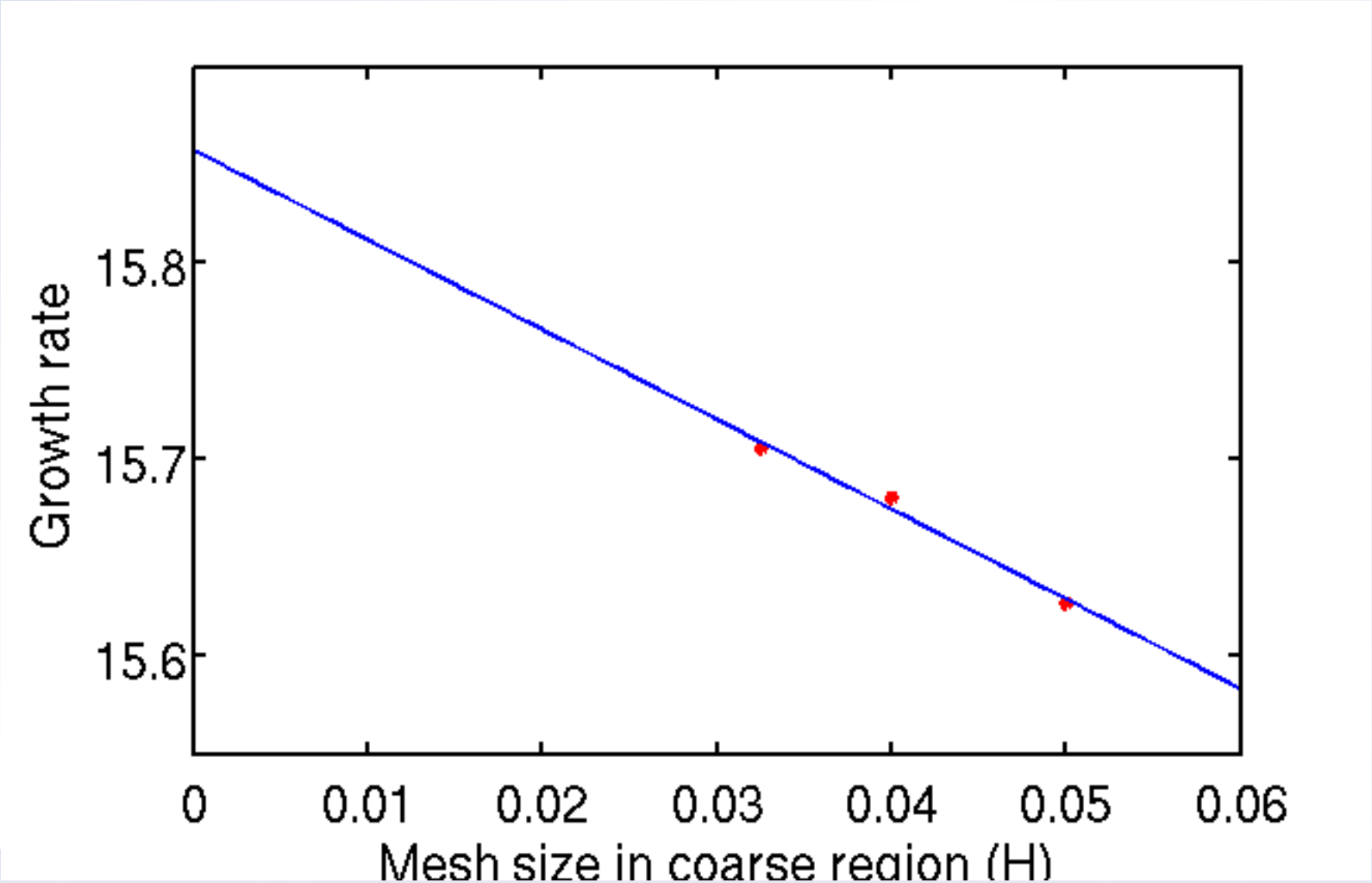
# M3DC1 – spatial convergence

- Width and ratio h/H fixed
- Scaling with resistivity

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# Asymptotic Matching - Linear stability

- Calculation of the equilibrium from the pressure and current profiles

$$\frac{dp}{dr} + \frac{d}{dr}\frac{1}{2}B_z^2 + \frac{B_\theta}{r}\frac{d}{dr}rB_\theta = 0$$

- Perturbation in magnetic field, velocity (displacement) and pressure

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# Linearized perturbation equation

$$\cancel{\rho_0 \gamma^2 \vec{\xi}} = (\vec{\nabla} \times \vec{B_0}) \times \delta\vec{B} + (\vec{\nabla} \times \delta\vec{B}) \times \vec{B_0} - \vec{\nabla}\delta p$$

$$\delta\vec{B} = \vec{\nabla} \times \left(\vec{\xi} \times \vec{B_0} - \cancel{(\eta/\gamma)\vec{\nabla} \times \delta\vec{B}}\right)$$

$$\delta p = -\vec{\xi} \cdot \vec{\nabla} p_0 - 5/3 p_0 \vec{\nabla} \cdot \vec{\xi}$$

2 regions :

- Inner resistive layer ar the rational surface where variation of equilibrium quantities can be neglicted
- Outer region where resistivity and inertia can be neglicted (growth rate must scale like fractional power of resistivity)

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# Outer equation

$$\frac{d}{dr}\left(f\frac{d}{dr}\xi_r\right) - g\xi_r = 0$$

- We assume the following angular dependance for all perturbations : $e^{i(m\theta - nkz)}$
- Following Newcomb(1960), we eliminate the other perturbations to get an ODE on the radial displacement only
- Need for special treatment at the singular point where f vanishes, i.e. at the rational surface (where q = m/n)

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## Outer equation

$$\frac{d}{dr}\left(f\frac{d}{dr}\xi_r\right) - g\xi_r = 0$$

$$x = r - rs$$

$$f(r) = \sum_m f_m x^m$$

$$g(r) = \sum_m g_m x^m$$

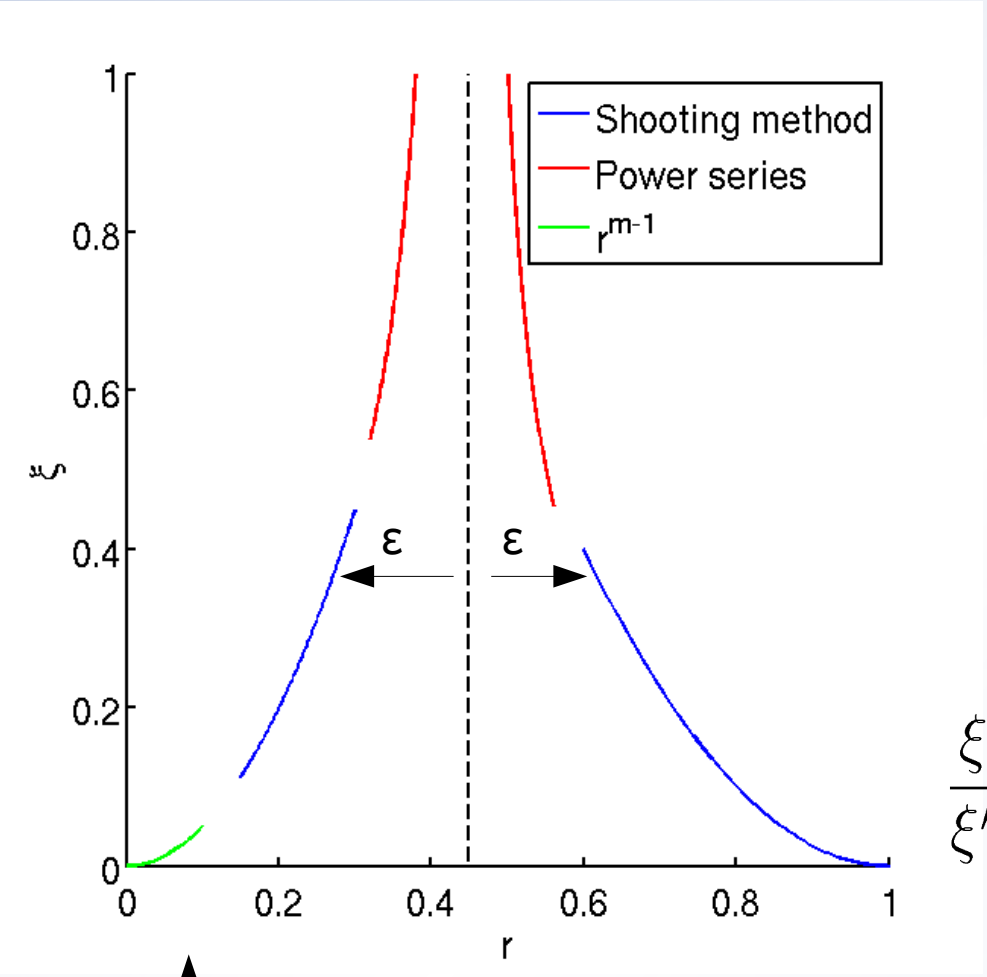
$$\xi(r) = |x|^\sigma \sum_m \xi_m x^m$$

Suydam criterion

$$\sigma_{l,s} = -1/2 \pm \sqrt{1/4 + g_0/f_2}$$

$$\xi_{l,s} = |x|^{\sigma_{l,s}} \sum_m \xi_m^{l,s} x^m$$

# Outer equation




$$\xi \propto r^{m-1}$$

$$\xi(r) = C_l\left(\xi_l(r) + \frac{C_s}{C_l}\xi_s(r)\right)$$

$$\xi'(r) = C_l\left(\xi_l'(r) + \frac{C_s}{C_l}\xi_s'(r)\right)$$

$$\frac{\xi(r)}{\xi'(r)} = \frac{\xi_l(r) + \frac{C_s}{C_l}\xi_s(r)}{\xi_l'(r) + \frac{C_s}{C_l}\xi_s'(r)}$$

$$\frac{\xi(rs+\epsilon)}{\xi'(rs+\epsilon)} = \frac{\xi_l(rs+\epsilon) + \Delta^{right}\xi_s(rs+\epsilon)}{\xi_l'(rs+\epsilon) + \Delta^{right}\xi_s'(rs+\epsilon)}$$

$$\Delta' = \Delta^{right} + \Delta^{left}$$

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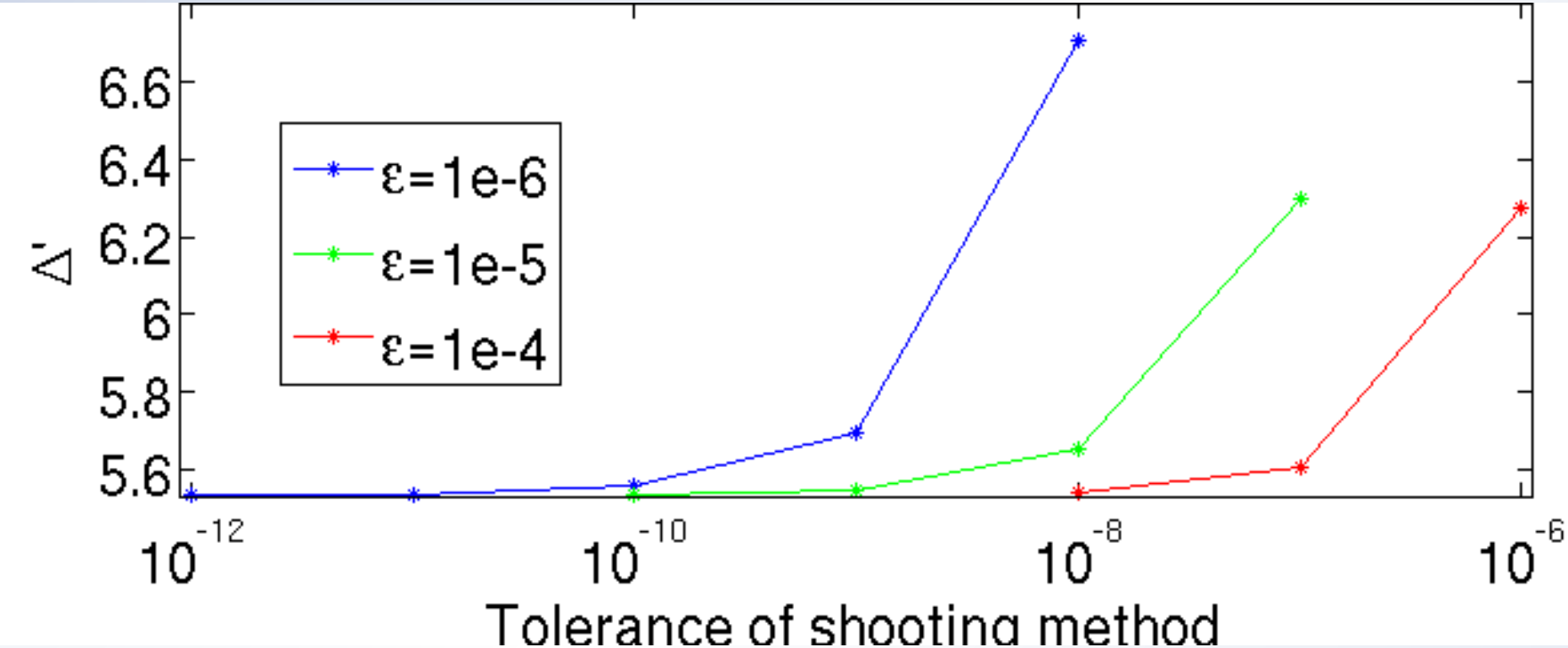
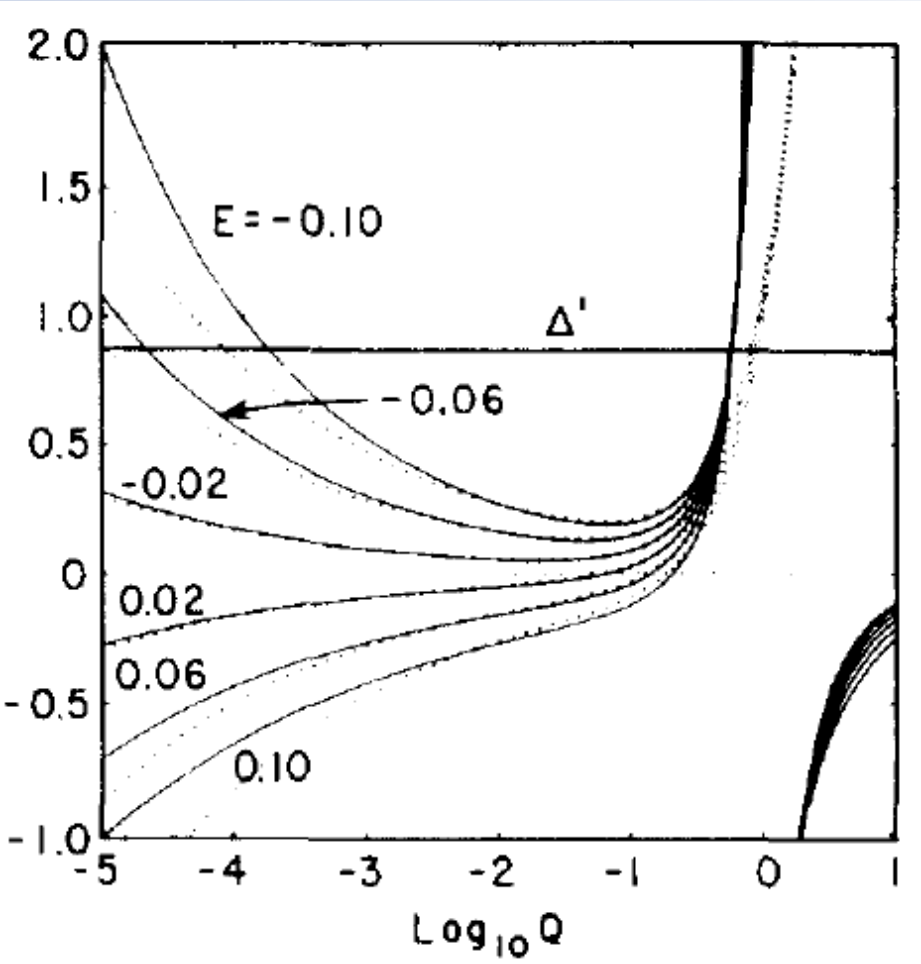
# Outer region : $\Delta'$



# Matching with inner region




Glasser, Jardin, Tesauro 1984

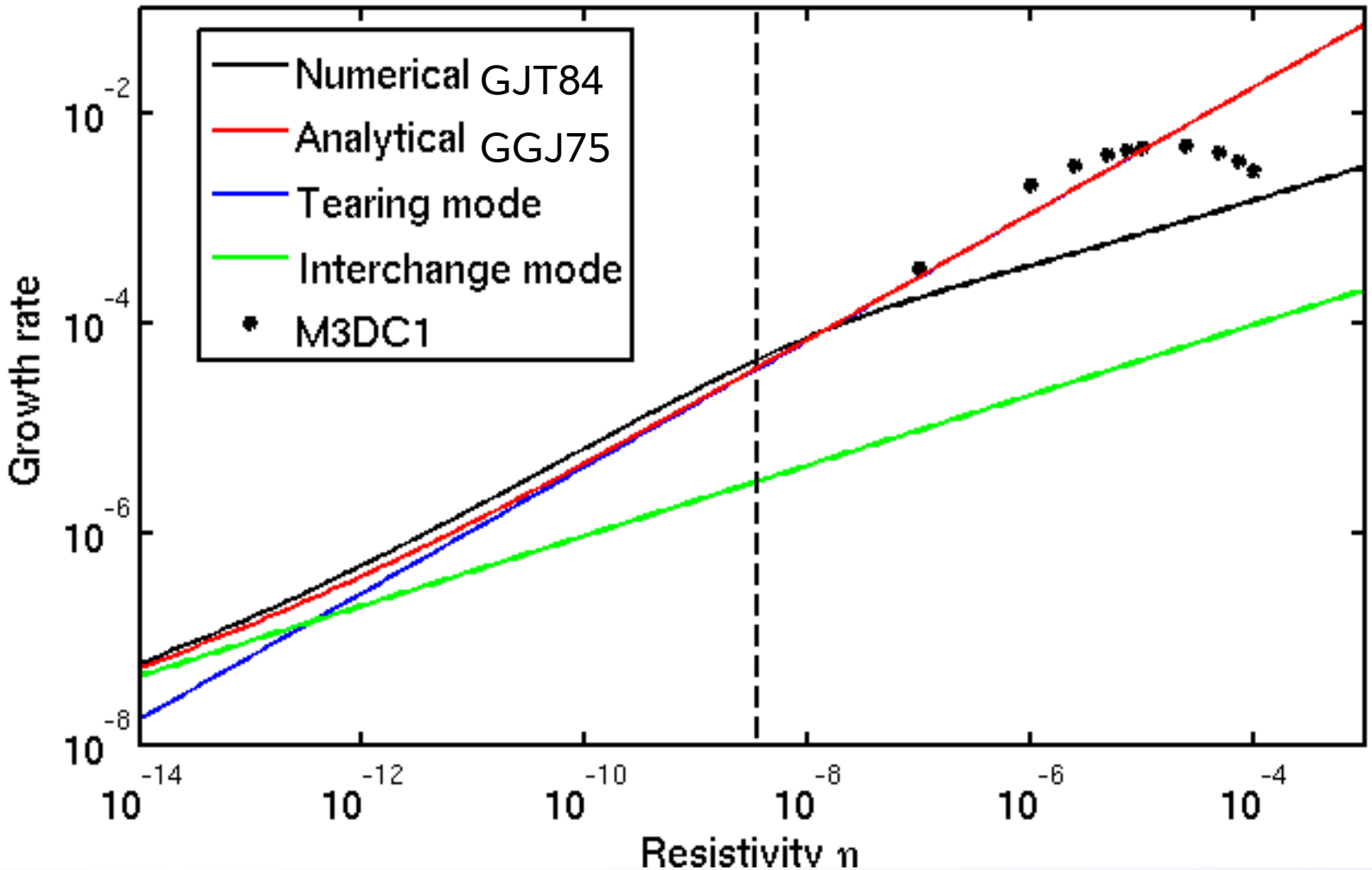
- 2 solutions for inner region
  - Numerical from GJT84
  - Analytical from GGJ75
- Dispersion relation

$$\Delta'(Q) = \Delta'$$

where $\Delta'(Q)$ is given by the solution of resistive MHD in the inner region

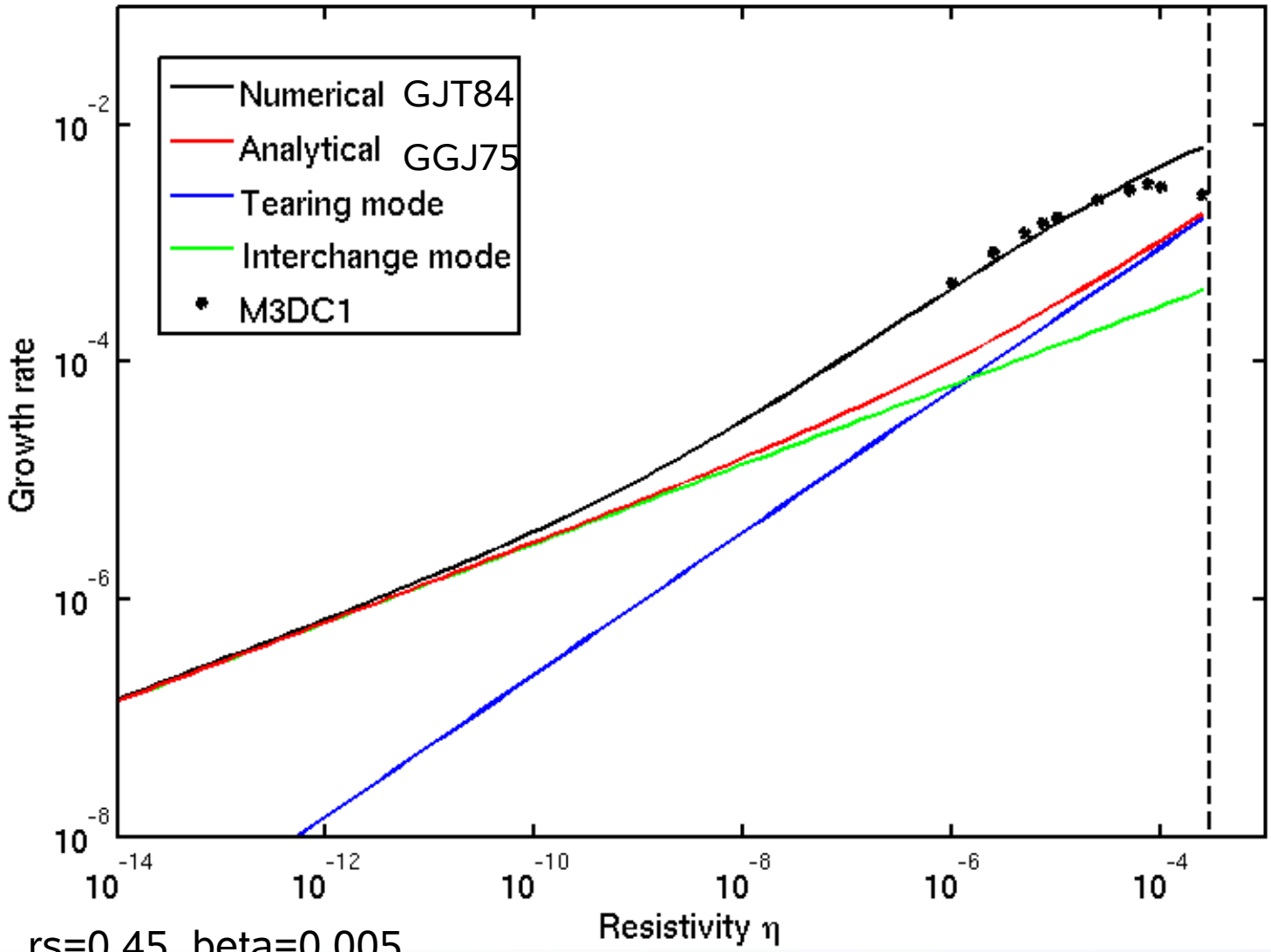
- Pole not predicted by analytical solution

# Comparison




rs=0.2, beta=0.001

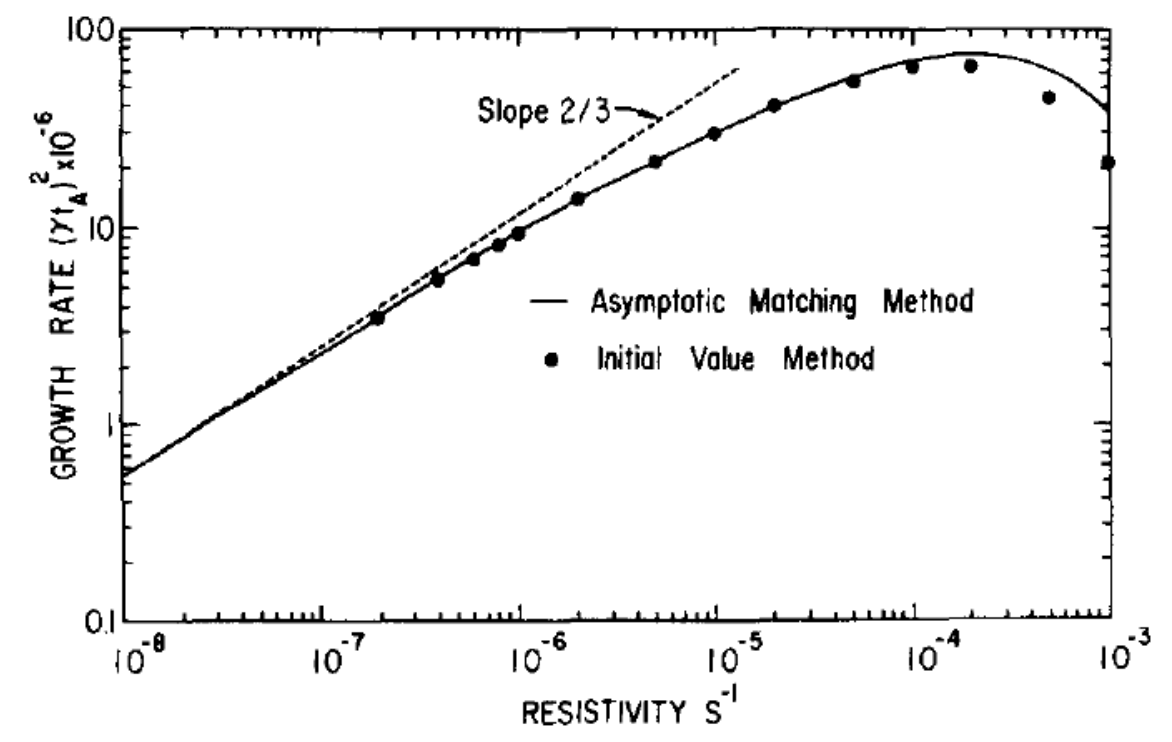
# Comparison (2)




rs=0.45, beta=0.005

## Future work

- Same type of benchmark on toroidal cases – PEST3 code gives $\Delta'$, GJT84 code still gives inner region solution
- Study of tearing modes with M3DC1 non linear capabilities
- Benchmark with interchange mode configuration, with negative $\Delta'$ (asymptotic matching solution should be ok for higher resistivity)




$$\frac{\eta}{\gamma} \xrightarrow{\eta \to 0} 0$$

$$\text{Interchange}: \ \gamma \propto \eta^{\frac{1}{3}}$$

$$\text{Tearing}: \ \gamma \propto \eta^{\frac{3}{5}}$$

DeLucia, Jardin, Glasser

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