Heurestic Closures for the Simulations of Neoclassical Tearing Modes.[1]

T.A. Gianakon

S.E. Kruger C.C. Hegna

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Thesis

- **Neoclassical viscous stress-tensor closures are presented which produce poloidal ion flow damping and a nonlinear threshold for the neoclassical tearing mode consistent with theoretical predictions.**

Outline

- **MHD equations.**
- **Neoclassical viscous stress-tensor forms: CGL and poloidal flow damping.**
- **Ion stress-tensor results.**
- **Nonlinear island evolution equation.**
- **Nonlinear threshold for neoclassical tearing modes.**
- **Future Work.**

The magneto-hydrodynamic form of the two-fluid equations are:

- **The momentum equation**

$$\rho\left(\frac{\partial\vec{V}}{\partial t}+\left(\vec{V}\cdot\vec{\nabla}\right)\vec{V}\right)=\vec{J}\times\vec{B}-\vec{\nabla}p-\vec{\nabla}\cdot\Pi-\nu_{\perp}\rho\nabla^{2}\vec{V}$$

- **The total pressure equation**

$$\frac{\partial p}{\partial t}+\left(\vec{V}\cdot\vec{\nabla}\right)p+\Gamma p\left(\vec{\nabla}\cdot\vec{V}\right)=(\Gamma-1)\left[Q-\vec{\nabla}\cdot\vec{q}-\Pi:\vec{\nabla}\vec{V}+\vec{J}\cdot(\vec{E}+\vec{V}\times\vec{B})\right]$$

- **The generalized Ohm's Law,**

$$\vec{E}\ =\ -\underbrace{\vec{V}\times\vec{B}}_{Ideal\ \ MHD}+\underbrace{\eta\vec{J}}_{Resistive\ \ MHD}+\frac{1}{\epsilon_0\omega_{pe}^2(1+\nu)}\underbrace{\left[\frac{\partial\vec{J}}{\partial t}+\vec{\nabla}\cdot(\vec{V}\vec{J}+\vec{J}\vec{V})\right]}_{Electron\ \ Inertia}$$

$$+\frac{1}{ne(1+\nu)}\left[\underbrace{(1-\nu)\vec{J}\times\vec{B}}_{Hall\ \ Term}-\underbrace{\vec{\nabla}(p_e-\nu p_i)}_{Diamagnetic\ \ Term}-\underbrace{\vec{\nabla}\cdot(\Pi_e-\nu\Pi_i)}_{Closures}\right]$$

- **The pre-Maxwell equations**

$$\nabla\cdot\vec{B}=0\qquad\nabla\times\vec{B}=\mu_0\vec{J}\qquad\frac{\partial\vec{B}}{\partial t}=-\nabla\times\vec{E}$$

The heat flux closure is critical to the simulation of neoclassical tearing modes

- A simple Braginskii form is used to provide the necessary pressure equilibration along perturbed field lines:

  $$\vec{q} = -\chi_{||}\vec{b}\vec{b}\cdot\vec{\nabla}p - (\chi_{\perp} - \chi_{||})\vec{\nabla}p,$$

  where $\vec{b}$ denotes a unit vector in the direction of the total magnetic field.

- Finite parallel and perpendicular diffusion effects introduces a nonlinear threshold for destabilization of the neoclassical tearing modes.

  Cross field diffusion transit time$\tau_{\perp} = (W_d/2)^2/\chi_{\perp}$

  Parallel-diffusion transit time $\tau_{||} = 1/k_{||}^2\chi_{||}$

  The parallel wave number in the large-aspect ratio limit is given by $k_{||} \simeq 0.5mW_d/q^2(dq/dr)$.

- A balance of the two transit times yields

$$W_d = 1.5\sqrt{8}\left(\frac{\chi_{\perp}}{\chi_{||}}\right)^{0.25}\left(\frac{m}{Rq^2}\frac{dq}{dr}\right)^{-0.5}$$

The Chew-Goldberger-Low (CGL) closure form originates from flux-averaged neoclassical theory.

- **In all collisionality regimes, the dominant parallel viscous stress has a Chew-Goldberger-Low (CGL) form that is expressed as**

$$\vec{\vec{\pi}}_\alpha \simeq \vec{\vec{\pi}}_{||\alpha} = \left(\frac{\vec{B}\vec{B}}{B^2} - \frac{\vec{\vec{I}}}{3}\right)(p_{||} - p_\perp)_\alpha,$$

$p_{||}$ is the parallel pressure.

$p_\perp$ is the perpendicular pressure.

The subscript alpha indicates electron's or ions.

- **The pressure anisotropy for this approximation is expressed as**

$$f_\alpha = (p_{||} - p_\perp)_\alpha = -2m_\alpha n_\alpha \mu_\alpha \frac{\langle B^2 \rangle}{\left\langle \left[\frac{\vec{B}\cdot\nabla B^2}{B^2}\right]^2 \right\rangle} \frac{\vec{v}_\alpha \cdot \nabla B^2}{B^2};$$

μ is a poloidal flow damping frequency.

- **The closure as implemented has been partially linearized.**

Poloidal Flow Damping Closure is approximate flux-averaged CGL.

- The suggested form for $\vec{\nabla} \cdot \Pi_\alpha$ is

$$\vec{\nabla} \cdot \Pi_\alpha = \rho_\alpha \mu_\alpha \left\langle B^2 \right\rangle \frac{\vec{V}_\alpha \cdot \vec{e}_\Theta}{\left(\vec{B} \cdot \vec{e}_\Theta\right)^2} \vec{e}_\Theta,$$

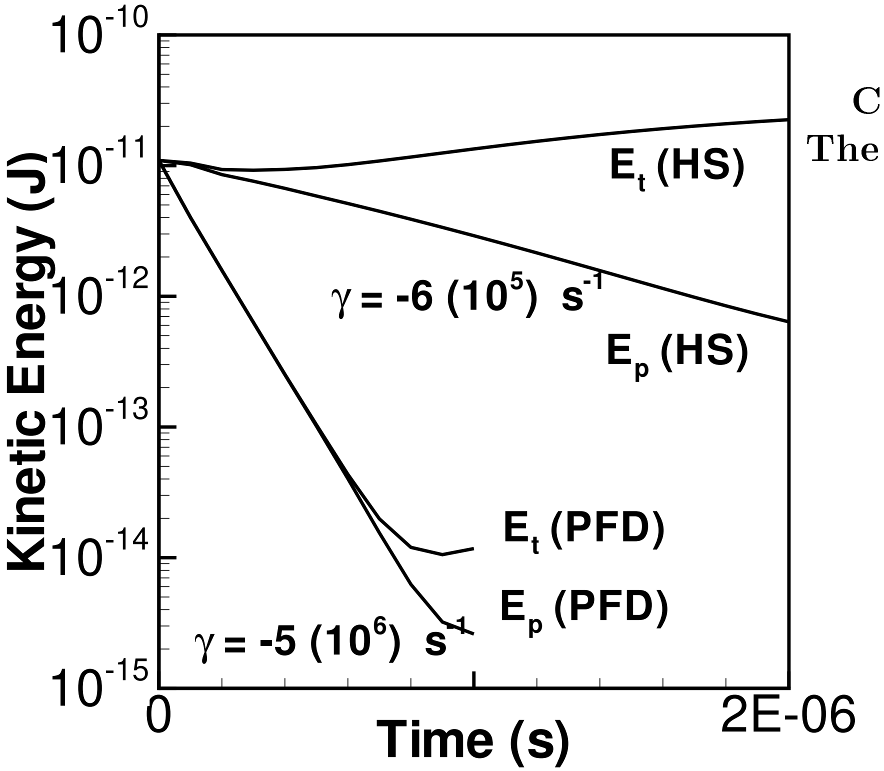
  μ_α is the viscous damping frequency for each species α,
  Depends on the collisionality regime.
  $\vec{e}_\Theta = \mathcal{J} \vec{\nabla}\zeta \times \vec{\nabla}\psi$ and ζ is the axisymmetric toroidal angle,
  ψ is the poloidal flux,
  $\mathcal{J}$ is the Jacobian of the coordinate system.

- The form can be shown to be dissipative.
- Linear layer analysis yields bootstrap current, flow damping, and neoclassical enhancement of the polarization current.
- Additional approximations can be made:

  Diamagnetic approximation expresses electron flow as pressure gradient.
  Hole approximation uses analytic pressure profile about island.

Closures are tested for equilibrium poloidal flow damping.

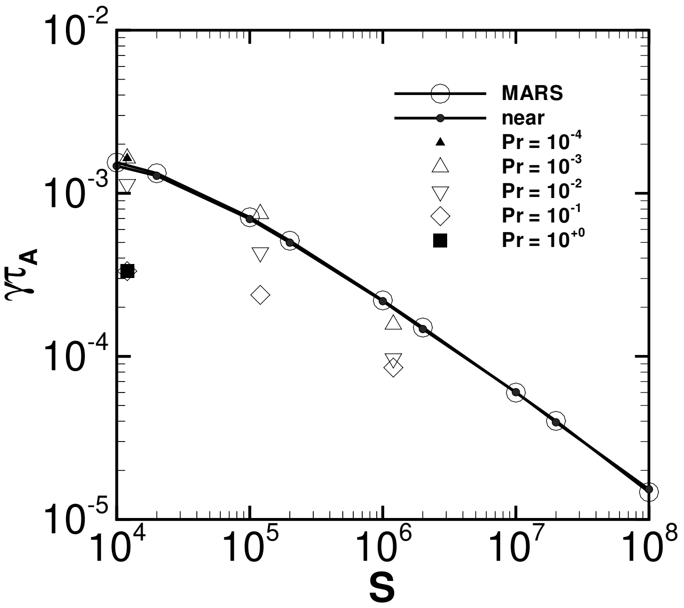
- **Impose a poloidal flow and verify that the poloidal energy is damped.**

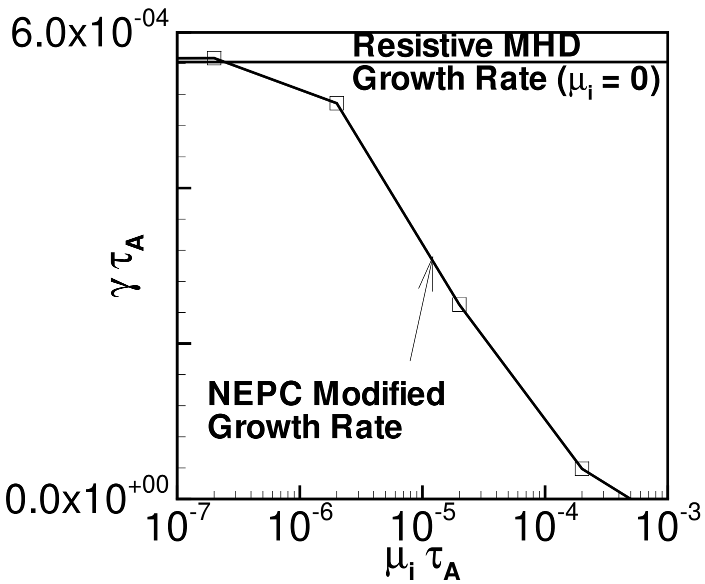


CGL form generates toroidal flow.
The toroidal flow flux-averages to zero.

The ion poloidal flow damping stabilizes a regular tearing mode via neoclassical enhancement of polarization current.

- Equlibrium is the 2/1 tearing unstable M3D/NIMROD PSACI benchmark.
- Damping observed when growth rate on order of damping rate.





Nonlinear Rutherford island evolution equation predicts a stability boundary.

$$\frac{k_0}{\eta^*}\frac{dW}{dt} = \Delta^* + \frac{W}{W^2 + W_d^2}\left(D_{nc} + \frac{D_R}{\alpha_s - H}\right) + ...$$

where W is the full-width of the island.

D_{nc} is the measure of neoclassical tearing mode stability.

$D_R = E + F + H^2$ is the resistive interchange parameter.

α_S and α_R are the small and large Mercier index.

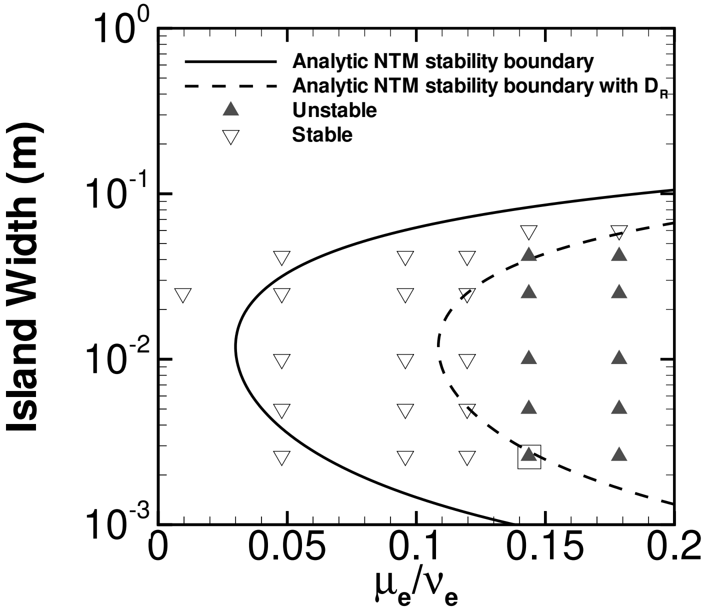
η^* is the resistive diffusion coefficient in flux space.

$\Delta^* = \Delta' |W/2|^{-2\alpha_l} \sqrt{-4D_i}$.

- May be additional effects such as FLR, NEPC.
- Δ' is typically stabilizing.
- D_{nc} is typically destabilizing.
- D_R is typically stabilizing and the anisotropic thermal diffusion may take a different form.

Neoclassical Tearing Mode Stability Boundary agrees with analytics.

- Here, μ_e/ν_e parameterizes the bootstrap current, $D_{nc} \propto \mu_e/\nu_e/(1+\mu_e/\nu_e)$.
- Stability boundary requires inclusion of D_R.
- Discrepancy exists at samll μ_e/ν_e.



Conclusions

- **Two forms of the ion viscous-stress tensor term were presented that reproduce poloidal ion flow damping.**

  The CGL form tends to generate toroidal momentum, but preserves as a flux-surface average.

  The poloidal flow damping form is the preferred form.

  The poloidal flow damping form also can slow down the linear growth of tearing instabilities.

- **The electron stress-tensor approximations succesfully reproduce an NTM.**

  The closure reproduces the nonlinear analytic island evolution equation.

  The diamagnetic approximation has the least restictive time-step.

Future Work

- Two-fluid effects will introduce rotation.

- Flow modifications should lead to more effects from the neoclassical enhancement of the polarization current.

- Unknown effects when pressure is separated into density and temparture.