*GRIN: Resistive boundary conditions for *M?3D**

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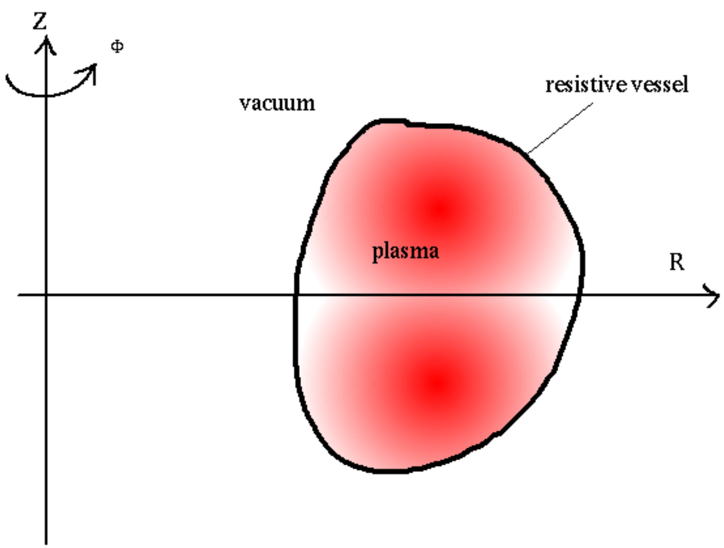
Overview

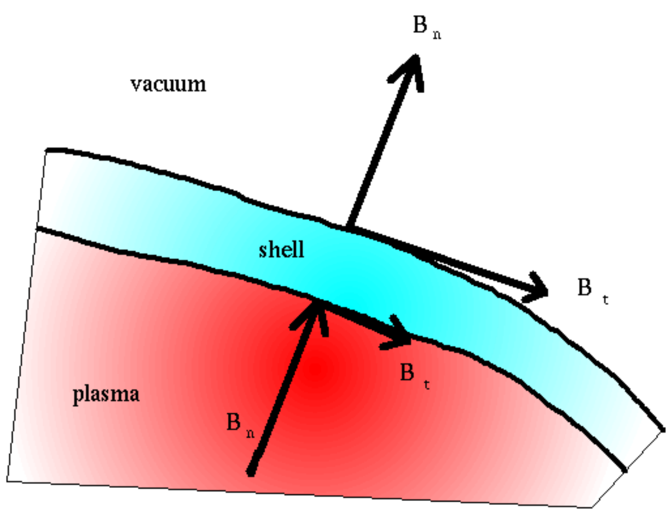
- plasma surrounded by a toroidally symmetric vessel
- vacuum beyond vessel
- vessel's conductivity $\sigma \times$ thickness $\delta \sim$ finite



Strategy

Compute the vacuum 'impedance' $\mathcal{Z} \equiv$ linear matrix relating the normal magnetic perturbation $\mathbf{B}_n$ to the tangential component $\mathbf{B}_t$.

- $\mathbf{B}_n$ continous across shell (thin wall approximation)
- jump $[[\mathbf{B}_t]] \approx \sigma\delta \quad \mathbf{E}\times\mathbf{n}$



*What *M?3D* needs*

Natural boundary conditions involve $\mathbf{E} \times \mathbf{n}$ at the plasma edge.

From $\mathbf{B}_n \rightarrow \mathbf{B}_t^{outside} \rightarrow [[\mathbf{B}_t]] \leftarrow \mathbf{B}_t^{inside}$.

- *M?3D*'s provides $\mathbf{B}_n$ and $\mathbf{B}_t^{inside}$
- GRIN computes $\mathcal{Z}$
- $\mathbf{B}_t^{outside} = \mathcal{Z}\mathbf{B}_n$

The $\mathcal{Z}$ matrix depends on the shell geometry only. It is computed before launching a nonlinear *M?3D* calculation.

B *in vacuum*

- poloidal and toroidal slit $\Rightarrow m = 0$ and $n = 0$ **B** components are continuous!
- no external $n \neq 0$ currents
- $\boldsymbol{\nabla} \cdot \mathbf{B} = 0 \Rightarrow \mathbf{B} = \boldsymbol{\nabla} \lambda$
- $\boldsymbol{\nabla}^2 \lambda = 0$
- $\lambda = \sum_{n=1}^{N} \lambda_n \exp in\phi$

Green's functions

Following Chance [*Phys. Plasmas* **4**, 2161 (1997)], write Green's identity

$$\oint d\ell R \left\{ G_n \frac{\partial \lambda}{\partial n} - \frac{\partial G_n}{\partial n} \lambda \right\} + 2\alpha\lambda = 0.$$

where

$$G_n(R, Z; R'Z') = \left(\frac{1}{\pi R' R} \right)^{\frac{1}{2}} \Gamma(\tfrac{1}{2} + n)\, p^{\frac{n}{2} + \frac{1}{4}} \frac{F\left(n + \frac{1}{2}, \frac{1}{2}, n + 1 | p\right)}{\Gamma(n+1)}$$

is the toroidally averaged Green's function for the Laplace equation...

Green's functions cont'd

$$p \equiv \frac{s-1}{s+1}, \qquad s \equiv \frac{\lambda}{\sqrt{\lambda^2-1}}$$

$$\lambda \equiv 1 + \frac{(R'-R)^2 + (Z'-Z)^2}{2R'R}$$

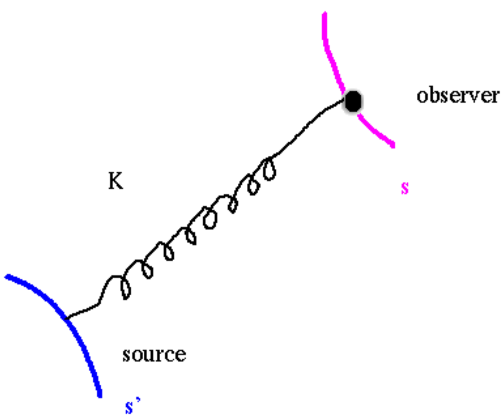
and α is either 0 if (R', Z') is outside the contour, 1 if it is inside, or $\frac{1}{2}$ if (R', Z') lies exactly on the contour.

GRIN in a nutshell

A package to compute

- $\int d\ell' K(\ell, \ell')\alpha'(\ell')$
- $\int_s d\ell\alpha(\ell) \int_{s'} d\ell' K(\ell, \ell')\alpha'(\ell')$

for arbitrary open or closed segments s and s', and K a user-supplied kernel with a singularity as $\ell \to \ell'$.



segments and contours

Good...

...and bad segments

A contour $\equiv$ collection of segments.

Example of GRIN usage

Define the geometry (segments)

```
/* Define segments */

Vec tb(11), ta(5);
segment sa, sb;
ta.space(0., 1.);
tb.space(0., TwoPi);
Vec xa(5); xa = 2.;
Vec ya(5); ya.space(-0.1, 0.1);
sa.load(ta, xa, xb, not_periodic);
sb.load(tb, xb, yb, periodic);
```

Example of GRIN usage cont'd

Compute $\int d\ell' K(\ell, \ell')\alpha'(\ell')$

```
/* set observer point */

double tobs = 0.5;

/* call procedure */

/* greenLaplaceCartesian = Green's fct
   alpha is the basis fct (ext proc) */

double res = greenIntegral(
      greenLaplaceCartesian,
      sa, tobs, sb, alpha);
```

Example of GRIN usage cont'd

Compute $\int_s d\ell \alpha(\ell) \int_{s'} d\ell' K(\ell,\ell')\alpha'(\ell')$

```
/* beta, alpha are the basis fcts */

double res = twoGreenIntegral(
     greenLaplaceCartesian,
     sb, beta, sa, alpha);
```

Summary

- GRIN code is presently being revived after a 2-year hibernation
- GRIN has similar capability to VACUUM by Chance, but is designed for maximum flexibility (can be used to solve problems outside plasma physics) $\Rightarrow$ possibility to add passive shells.

Thank you for your attention.