

Introduction to Plasma Physics

**INTRODUCTION
TO
PLASMA PHYSICS**

Robert J Goldston
and
Paul H Rutherford

*Plasma Physics Laboratory
Princeton University*

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Dedicated to
Ruth Berger Goldston
and
Audrey Rutherford

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Preface

Plasmas occur pervasively in nature: indeed, most of the known matter in the Universe is in the ionized state, and many naturally occurring plasmas, such as the surface regions of the Sun, interstellar gas clouds and the Earth's magnetosphere, exhibit distinctively plasma-dynamical phenomena arising from the effects of electric and magnetic forces. The science of plasma physics was developed both to provide an understanding of these naturally occurring plasmas and in furtherance of the quest for controlled nuclear fusion. Plasma science has now been used in a number of other practical applications, such as the etching of advanced semiconductor chips and the development of compact x-ray lasers. Many of the conceptual tools developed in the course of fundamental research on the plasma state, such as the theory of Hamiltonian chaos, have found wide application outside the plasma field.

Research on controlled thermonuclear fusion has long been a world-wide enterprise. Major experimental facilities in Europe, Japan and the United States, as well as smaller facilities elsewhere including Russia, are making remarkable progress toward the realization of fusion conditions in a confined plasma. The use, for the first time, of a deuterium–tritium plasma in the tokamak experimental fusion device at the Princeton Plasma Physics Laboratory has recently produced slightly in excess of ten megawatts of fusion power, albeit for less than a second. In 1992, an agreement was signed by the European Union, Japan, the Russian Federation and the United States of America to undertake jointly the engineering design of an experimental reactor to demonstrate the practical feasibility of fusion power.

This book is based on a one-semester course offered at Princeton University to advanced undergraduates majoring in physics, astrophysics or engineering physics. If the more advanced material, identified by an asterisk after the Chapter heading or Section heading, is included then the book would also be suitable as an introductory text for graduate students entering the field of plasma physics.

We have attempted to cover all of the basic concepts of plasma physics with reasonable rigor but without striving for complete generality—especially where this would result in excessive algebraic complexity. Although single-particle,

fluid and kinetic approaches are introduced independently, we emphasize the interconnections between different descriptions of plasma behavior; particular phenomena which illustrate these interconnections are highlighted. Indeed, a unifying theme of our book is the attempt at a deeper understanding of the underlying physics through the presentation of multiple perspectives on the same physical effects. Although there is some discussion of weakly ionized gases, such as are used in plasma etching or occur naturally in the Earth's ionosphere, our emphasis is on fully ionized plasmas, such as those encountered in many astrophysical settings and employed in research on controlled thermonuclear fusion, the field in which both of us work. The physical issues we address are, however, applicable to a wide range of plasma phenomena. We have included problems for the student, which range in difficulty from fairly straightforward to quite challenging; most of the problems have been used as homework in our course.

Standard international (SI) units are employed throughout the book, except that temperatures appearing in formulae are in units of energy (i.e. joules) to avoid repeated writing of Boltzmann's constant; for practical applications, temperatures are generally stated in electron-volts (eV). Appendices A and C allow the reader to convert from SI units to other units in common use.

The student should be well-prepared in electromagnetic theory, including Maxwell's equations, which are provided in SI units in Appendix B. The student should also have some knowledge of thermodynamics and statistical mechanics, including the Maxwell-Boltzmann distribution. Preparation in mathematics must have included vectors and vector calculus, including the Gauss and Stokes theorems, some familiarity with tensors or at least the underlying linear algebra, and complex analysis including contour integration. Appendix D contains all of the vector formulae that are used, while Appendix E gives expressions for the relevant differential operators in various coordinate systems. Higher transcendental functions, such as Bessel functions, are avoided. Suggestions for further reading are given in Appendix F.

In addition to the regular problems, which are to be found in all chapters, we have provided a disk containing two graphics programs, which allow the student to experiment visually with mathematical models of quite complex plasma phenomena and which form the basis for some homework problems and for optional semester-long student projects. These programs are provided in both Macintosh¹ and IBM PC-compatible format. In the first of these two computer programs, the reader is introduced to the relatively advanced topic of area-preserving maps and Hamiltonian chaos; these topics, which form another of the underlying themes of the book, reappear later in our discussions both of the magnetic islands caused by resistive tearing modes and of the nonlinear

¹ Macintosh is a registered trademark of Apple Computer, Inc.

phase of electron plasma waves.

We are deeply indebted to Janet Hergenhan, who prepared the manuscript in L^AT_EX format, patiently resetting draft after draft as we reworked our arguments and clarified our presentations. We would also like to thank Greg Czechowicz, who has drawn many of the figures, John Wright, who produced the IBM-PC versions of our programs, and Keith Voss, who served for three years as our ‘grader’, working all of the problems used in the course and offering numerous excellent suggestions on the course material.

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Robert J Goldston
Paul H Rutherford
Princeton, 1995

Introduction

After an initial Chapter, which introduces plasmas, both in the laboratory and in nature, and derives the defining characteristics of the plasma state, this book is divided into six 'Units'. In Unit 1, the plasma is considered as an assemblage of charged particles, each moving independently in prescribed electromagnetic fields. After deriving all of the main features of the particle orbits, the topic of 'adiabatic' invariants is introduced, as well as the conditions for 'non-adiabaticity', illustrating the latter by means of the modern dynamical concepts of mappings and the onset of stochasticity. In Unit 2, the fluid model of a plasma is introduced, in which the electromagnetic fields are required to be self-consistent with the currents and charges in the plasma. Particular attention is given to demonstrating the equivalence of the particle and fluid approaches. In Unit 3, after an initial Chapter which describes the most important atomic processes that occur in a plasma, the effects of Coulomb collisions are treated in some detail. In Unit 4, the topic of small-amplitude waves is covered in both the 'cold' and 'warm' plasma approximations. The treatment of waves in the low-frequency branch of the spectrum leads naturally, in Unit 5, to an analysis of three of the most important instabilities in non-spatially-uniform configurations: the Rayleigh–Taylor (flute), resistive tearing, and drift-wave instabilities. In Unit 6, the kinetic treatment of 'hot' plasma phenomena is introduced, from which the Landau treatment of wave–particle interactions and associated instabilities is derived; this is then extended to the non-uniform plasma in the drift-kinetic approximation.