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Can Ion Cyclotron Waves Propagate to the Ground?

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Abstract. This paper demonstrates how ion cyclotron waves generated in the equatorial magnetosphere can propagate to the ground and resolves a discrepancy between theory and observation that has lasted over a decade. We present a full wave analysis for ion cyclotron waves propagating from the equatorial magnetosphere to the ground. Our analysis demonstrates that strong mode coupling occurs near the He^+ and O^+ resonance locations which allows the equatorial waves to penetrate to ionospheric altitudes with substantial amplitude and shifted polarization consistent with observational studies.

Introduction

Ion cyclotron waves are important for transferring energy from protons to heavy ions, as remote indicators of proton dynamics, and in regulating proton anisotropy [*Chang et al.*, 1986; *Thorne and Horne*, 1994; *Anderson et al.*, 1996a]. However, for over a decade it has been a matter of contention how these waves could be generated in the equatorial magnetosphere and be observed on the ground so reliably.

The wave observations of *Perraut et al.* [1984] which compared ion cyclotron wave activity in the equatorial magnetosphere with wave activity detected by ground based magnetometers started an interesting conflict between theory and observation. The significant correlation between waves in these two regions contradicted the trailblazing theoretical work of *Rauch and Roux* [1982]. They considered a hydrogen plasma with a sizable helium component, solved the ray-tracing equations based on a cold plasma model, and found that predominately left hand polarized waves below the helium gyrofrequency are guided along the magnetic field to the ground. Waves generated between the helium and hydrogen gyrofrequencies were found to propagate along field lines into the region of stronger magnetic field where polarization reverses and the wave reflects at the Buchsbaum resonance and does not reach the ground. This work was extended to warm plasmas in a series of papers [*Horne and Thorne*, 1990; *Rönnmark and André*, 1991] with similar results. However, these result are inconsistent with the observations at least 50% of the time [*Perraut et al.*, 1984].

Indeed, substantial evidence has accumulated confirming the original assessment of *Perraut et al.* [1984]. Large statistical surveys of ion cyclotron waves in the magnetosphere (based on AMPTE/CCE magnetic field data and DE2 data at low altitudes) and ionosphere provide convincing evidence that ionospheric Pc 1 wave activity in the noon sector recorded below 65° is most likely ion cyclotron waves generated in the equatorial magnetosphere because there is a good correlation between wave frequency and L-shell value [*Anderson et al.*, 1992a, 1992b; *Erlandson and Anderson*, 1996]. The case studies of *Hansen et al.* [1995] which correlated the Viking data [*Erlandson et al.*, 1990] in the northern hemisphere (60° – 70° latitudes) at altitudes up to a few Earth radii with conjugate ground based data at Mawson, Antarctica (70°S latitude) and *Anderson et al.* [1996b] which demonstrated strong temporal correlations between Pc 1 bursts at the south pole and equatorial magnetosphere by AMPTE/CCE provide clear evidence that Pc 1 waves propagate all the way from the equator to the ground.

Until recently most theoretical analyses of ion cyclotron wave propagation made use of the ray-tracing technique [*Rauch and Roux*, 1982; *Rönnmark and André*, 1991; *Horne and Thorne*, 1990; *Thorne and Horne*, 1993, 1994] which has provided an important first step in understanding the nonlocal features of electromagnetic ion cyclotron waves. Ray-tracing analysis has demonstrated important features about how waves grow and how the polarization changes as the waves propagate. These calculations have helped to identify some important effects that need to be included in studying these waves such as electron Landau damping as well as ion cyclotron damping near the ion-ion hybrid frequency and second harmonic of the oxygen gyrofrequency.

Part of the problem associated with whether waves propagate from the equatorial magnetosphere to the ground is that ray-tracing breaks down near the crossover-cutoff-resonance frequencies in a multi-ion species plasma. To resolve these difficulties we have performed a full wave analysis, and we present results which suggest that waves mode convert, tunnel through stop bands, and reach the ground.

Limitations of Ray Tracing

To assess the limitations of ray tracing as well as the importance of mode conversion, tunneling and absorption, we examine the full wave equations. Maxwell's equations combined with the fluid equations describe wave propagation in a nonuniform plasma medium [*Stix*, 1962]. Using a one-dimensional approximation with variation along the magnetic field direction, these equations reduce to a set of coupled ordinary differential equations for the electric field. For low frequencies, the electric field parallel to the magnetic field becomes negligible. Wave propagation of the circularly polarized electric field components perpendicular to the magnetic field, $E_\pm = E_x \pm iE_y$, is determined by

$$\epsilon^2\Psi'' \equiv \epsilon^2\frac{d^2\Psi}{d\zeta^2} = -\mathbf{M}\Psi \tag{1}$$

$$\mathbf{M} = \begin{pmatrix} r-\kappa_\perp^2/2 & \kappa_\perp^2/2 \\ \kappa_\perp^2/2 & l-\kappa_\perp^2/2 \end{pmatrix}; \Psi = \begin{pmatrix} E_- \\ E_+ \end{pmatrix} \tag{2}$$

Typical characteristics of the medium are contained in the parameters: $\epsilon = 1/k_{A0}L$., $\kappa_\perp = k_\perp/k_{A0}$, $\kappa_\parallel = k_\perp/k_{A0}$, $k_{A0} = \omega/v_{A0}$, where v_{A0} is the Alfvén velocity evaluated at some reference point which we will take to be the equatorial magnetosphere. The dimensionless coordinate $\zeta = z/L$

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where z is a field aligned distance and L is a typical background gradient scale length which will be taken as $1R_E$. For compactness we have introduced $'$ notation for $d/d\zeta$. The rescaled Stix functions for three ion species are

$$\begin{matrix} r_{(+)} \\ l_{(-)} \end{matrix} = \pm \frac{v_{A0}^2}{v_A^2} \frac{\Omega_{c1}\Omega_{c2}\Omega_{c3}}{\omega_{co1}\omega_{co2}} \frac{(\omega \pm \omega_{co1})(\omega \pm \omega_{co2})}{(\omega \pm \Omega_{c1})(\omega \pm \Omega_{c2})(\omega \pm \Omega_{c3})} \tag{3}$$

where $+$ is used for r and $-$ is used for l. The two cutoff frequencies ω_{co1} and ω_{co2} depend primarily on the concentration of ion species and are in the range $\Omega_{c1} < \omega_{co1} < \Omega_{c2} < \omega_{co2} < \Omega_{c3}$ where Ω_c's are the ion cyclotron frequencies of three ion species with mass satisfying $m_3 < m_2 < m_1$. We have taken the Fourier transform in the direction perpendicular to the magnetic field.

The parameter $\epsilon = \lambda/2\pi L$ is a good measure of the validity of the WKB approach which is valid when the wavelength, λ, is much smaller than the background gradient scale L. (For typical observed Pc 1 wave frequencies of 0.2 to 1 Hz, $\epsilon < 0.05$ in the equatorial magnetosphere and the WKB approach is valid.) The solutions take a form $E_+ = \exp(S(\zeta)/\epsilon)$, where $S(\zeta) = S_0(\zeta) + S_1(\zeta)\epsilon + S_2(\zeta)\epsilon^2 + \ldots$. Solving Eq. 1 to lowest order in ϵ, we find

$$-S_0'^2 = s - \kappa_\perp^2/2 \pm \sqrt{d^2 + \kappa_\perp^4/4}, \tag{4}$$

where $s = (r+l)/2$ and $d = (r-l)/2$. Note that $\epsilon E_+' = [S_0' + \mathcal{O}(\epsilon)]E_+$. In homogeneous plasma $E_+ \sim \exp(\pm i\kappa_\parallel \zeta/\epsilon)$ so $\epsilon E_+' = \pm i\kappa_\parallel E_+$. Hence, $\pm iS_0'$ can be identified as the local parallel wavevector, and the truncation $S(\zeta) = S_0(\zeta)$ is the geometric optics solution employed by ray tracing.

It is beneficial to inspect the geometrical optics solutions for a dipole field. In Figure 1 we plot the square of the dimensionless local parallel wavevector, $\kappa_\parallel^2 = -S_0'^2$, along the magnetic field line ($z = 1\mathrm{R_E}$ corresponds to the ionosphere and $z = 5\mathrm{R_E}$ to the equatorial magnetosphere) for a plasma consisting of 94% H^+, 5% He^+, and 1% O^+. Note that an additional factor of V_A^2/V_{A0}^2 multiplies the wavevector so that the wave refracts strongly at lower altitudes becoming oblique. The wavevectors are plotted for different values of propagation angle, $\theta = \tan^{-1}(\kappa_\perp/\kappa_\parallel)$, evaluated in the equatorial region at $z = 5\mathrm{R_E}$. For parallel propagation ($\theta = 0$) the two branches are the functions r (dashed) and l (solid). The function r is well behaved, but the function l has resonances at Ω_{He^+} and Ω_{O^+} and cutoffs at ω_{co1} and ω_{co2}. Stop gaps, where $\kappa_\parallel^2 < 0$, appear between the cutoff ($\kappa_\parallel = 0$) and the resonance ($\kappa_\parallel \to \infty$), and ray tracing predicts reflection at the cutoffs. At the local crossover locations, $r = l$, and $d \to 0$. For oblique propagation the modes couple near the crossover point as is evidenced in their change of predominant polarization. For highly oblique propagation ($\kappa_\perp \to \infty$) the mode $\kappa_\parallel^2 \to -\kappa_\perp^2$ is evanescent. The propagating mode $\kappa_\parallel^2 \to s$ has resonances at the cyclotron frequencies and cutoffs at the Buchsbaum frequencies defined by $r + l = 0$. The stop gap for perpendicular propagation is smaller than that for parallel propagation.

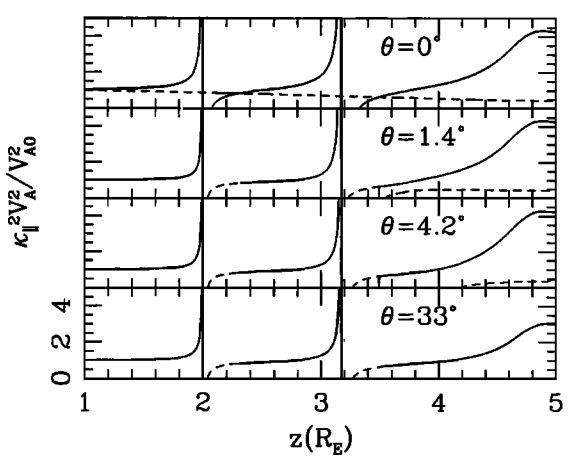

Figure 1. Normalized square of the local parallel wavevector as a function of altitude (z in $\mathrm{R_E}$). Predominant left hand polarized (LHCP) mode is indicated as solid lines and predominant right-hand polarization as dashed lines. The wave is launched at $z = 5\mathrm{R_E}$ as a LHCP ion cyclotron mode. As the wave propagates toward the Earth ($z = 1$) it propagates through He^+ ($z = 3.18\mathrm{R_E}$) and O^+ ($z = 2\mathrm{R_E}$) resonance locations.

In the problem at hand, an ion cyclotron wave generated in the equatorial region propagates from $z = 5\mathrm{R_E}$ to smaller z. Geometrical optics suggests that the wave reflects before reaching the ionosphere, *i.e.* the solid line at $z = 5\mathrm{R_E}$ always encounters a cutoff ($\kappa_\parallel = 0$) before reaching ionospheric altitudes. According to ray tracing, the wave cannot access such low altitudes and will reflect before reaching the helium resonance. Ray tracing results are not in agreement with observations because they neglect the coupling processes at the crossover frequencies as well as tunneling and absorption near the cutoff and Buchsbaum frequencies.

The fundamental problem with ray-tracing may be understood solving Eq. 1 to next order in ϵ. Ignoring slow z variations in r we find

$$S_1' = -\left(\frac{S_0''}{2S_0'}\right)\left(\frac{6S_0^2 + r + l - \kappa_\perp^2}{\pm\sqrt{d + \kappa_\perp^4/4}}\right) \equiv -\left(\frac{S_0''}{2S_0'}\right)\mathcal{F}(\zeta) \tag{5}$$

The validity of ray-tracing requires the ordering $S_0 \gg S_1\epsilon$ which breaks down in two cases: (1) near the crossover frequency where $d \to 0$ and there is a double root for parallel propagation ($k_\perp = 0$), and (2) when one of the two modes is cutoff ($S_0 \to 0$) which occurs at either the cutoff or Buchsbaum resonance locations. Note that $S_0' \sim \kappa_\parallel \sim 1/\lambda$ and $S_0''/2S_0' \sim 1/L$ so that the criterion $\epsilon S_1 \ll S_0$ becomes $\mathcal{F}\lambda \ll \lambda'/\lambda \sim L$. Ray tracing approaches typically gauge validity by checking $\lambda \ll \lambda'/\lambda \sim L_B$ which is satisfied near the crossover frequency where $d \to 0$. Typically $\mathcal{F} \sim 1$; however, $\mathcal{F} \gg 1$ near $d \to 0$, and the WKB approximation is violated. Although for non-parallel ($\kappa_\perp \neq 0$) propagation $d^2 + \kappa_\perp^4/4$ does not vanish, it is small and must be considered. Ray-tracing ignores the contribution of S_1 and therefore does not contain the effects of mode conversion, and the technique will give incorrect results even though the widely applied criterion $\lambda \ll L_B$ remains valid.

Johnson et al. [1995] analyzed these equations near the cutoff-crossover-resonance for a hydrogen-oxygen plasma and demonstrated that ion cyclotron waves undergo significant tunneling near the cutoff frequency for parallel propagation and even greater tunneling for perpendicular propagation. The tunneling and absorption are significant for minority ion species concentrations of 0.5–5%—concentrations which are observed [*Anderson et al.*, 1992b]. The mode conversion processes responsible for the tunneling, reflection, and absorption cannot be described by ray-tracing.

Results of Full Wave Analysis

To understand how magnetospheric wave activity with frequency between the helium and hydrogen cyclotron frequencies can be detected at the ground, we take a one-

dimensional cold plasma model with density and magnetic field variation only along magnetic field lines with a three ion species plasma with concentrations used in Figure 1 which are similar to the concentrations deduced by *Anderson et al.* [1996a]. On the high altitude side we impose down-going ion cyclotron waves and at the ground we impose boundary conditions for a perfect conductor, $\mathbf{E}_\perp \to 0$.

We have examined solutions of the full wave equation for a wave incident on the LHCP branch. As expected, the solutions may be well understood in terms of Figure 1. For parallel propagation ($\theta = 0$), the wave propagates to the cutoff and is completely reflected in the LHCP branch. For oblique propagation the wave couples strongly near both crossover frequencies with the other propagating mode. A significant fraction of wave energy can propagate through the evanescent layer in the RHCP mode allowing it to penetrate below the ion cyclotron resonance location where it couples with the LHCP wave which is linearly polarized due to large Alfvén velocity. The linearly polarized wave reflects off the ionosphere and propagates back into the magnetosphere to the He^+ and O^+ resonance locations. As this linearly polarized wave approaches the resonances it becomes predominantly left hand polarized and most of the reflected wave power is absorbed.

This qualitative behavior can be seen in the full wave solution presented in Figure 2 where we plot the wave properties for $\theta = 4.3°$. We plot the wave energy in the LHCP electric field in the top panel, the wave energy in the RHCP component in the second panel, the fraction of LHCP in the third panel, and the parallel Poynting flux in the bottom panel. Notice that the incident wave is predominantly LHCP. The wave shifts polarization before reaching Ω_{He^+} and the wave energy is almost entirely in the RHCP mode. The wave propagates through the resonance in the RHCP branch. Below the He^+ resonance the RHCP again couples with the LHCP wave and the wave is a linearly polarized Alfvén wave. Although little absorption occurs at the O^+ resonance, higher O^+ density and/or higher wave frequency would give more efficient absorption. The Poynting flux is negative indicating that wave energy is flowing from the equator toward the ionosphere. Power is absorbed at the two resonances corresponding to jumps in the Poynting flux. Note that the polarization near the ground is significantly different than in the equatorial region. This difference is consistent with the studies of *Perraut et al.* [1984] and *Hansen et al.* [1995] which also found that polarizations of the ground based waves did not generally match the polarizations of the equatorially observed waves and tended to be more linear.

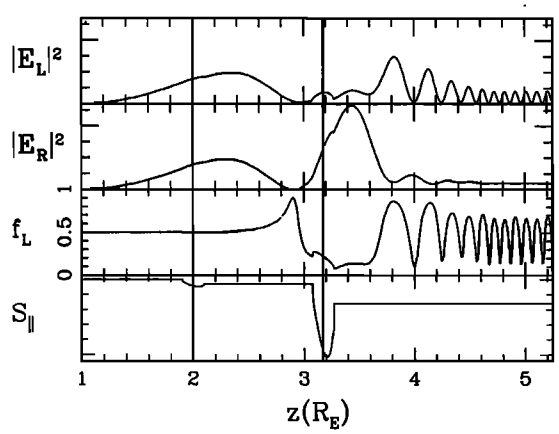

Figure 2. Wave solution as a function of z (altitude) for $\theta = 4.3°$. The top panel shows energy in the LHCP electric field, the second panel shows energy in the RHCP electric field, the third panel shows the fraction of left-hand polarization, and the bottom panel shows the Poynting flux which is 0 at $z = 1R_E$ and negative elsewhere indicating that wave energy is flowing earthward. Notice the strong coupling between the propagating modes near the helium crossover frequency location.

A good gauge of the importance of mode conversion and energy absorption is the energy budget of the propagating modes. In Figure 1 we find that there are two propagating modes—the solid LHCP mode and the dashed RHCP mode. Each mode can sustain both upward and downward propagating waves. An ion cyclotron wave is launched downward in the solid LHCP mode at $z = 5R_E$. It propagates earthward, undergoes strong coupling with the RHCP mode, and eventually is reflected or absorbed (in this model, no energy is dissipated at the ionosphere). At $z = 5R_E$ the reflected wave may be decomposed into upgoing LHCP (solid) and RHCP (dashed) waves. The fractional wave power reflected in these two branches is R_L and R_R respectively (defined as the ratio of the Poynting flux of the reflected wave to that of the incident wave). A fraction, A, of wave power is absorbed. Strong coupling near the crossover frequencies is indicated by a significant component R_R and is a good indicator that ray tracing is not appropriate. Absorption is strong near the resonances when A is large and is generally indicative that wave power has reached low altitudes and has been reflected as a heavily damped ion-cyclotron wave.

In Figure 3 we present the coefficients as functions of θ evaluated at $z = 5R_E$. Because wave growth in the equatorial magnetosphere is maximum for $\kappa_\perp/\kappa_\parallel < 0.1$ [*Oscarsson and André*, 1986] those solutions with $\theta < 6°$ are relevant, although curvature effects may lead to further wave refraction. For parallel propagation ($\theta = 0°$), no coupling occurs and the wave is well described as a simple Budden tunneling problem. The wave solution for this case consists of a wave that is mostly reflected at the cutoff above the He^+ resonance. About one quarter of the energy tunnels through the resonant layer, reflects off the ionosphere and is absorbed by ion cyclotron damping on the way back to the magnetosphere. As θ increases R_R also increases because strong coupling between the two propagating modes occurs. From the wave solution it is possible to determine that the incident wave couples to the RHCP mode and reflects near the

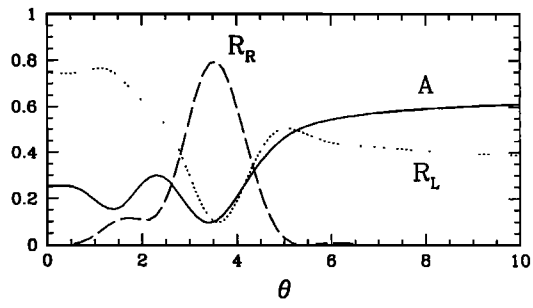

Figure 3. Reflection and absorption coefficients as a function of θ (degrees). Strong coupling between the LHCP and RHCP waves is evidenced by a large reflected wave component R_R. Note the oscillatory behavior of the coefficients and the large absorption that occurs. The coefficients approach an asymptotic value for $\theta \to 90°$.

He^+ resonance location. Most of the reflected wave energy remains on the RHCP branch and R_R becomes sizeable. However, the coefficient R_R peaks and then decreases. This decrease results because coupling between the two modes is no longer efficient above the He^+ resonance when $\theta > 4°$ and is evidenced by movement of the RHCP mode cutoff to high altitude in Figure 1. The coefficients approach asymptotic values corresponding to tunneling at the Buchsbaum frequencies for $\theta > 5°$ [*Johnson et al.*, 1995]. In this case, only one mode propagates, but tunneling is more effective than for parallel propagation because the wave tunnels through the evanescent layer in the RHCP branch which is not singular at the cyclotron resonance. From the wave solution, it is possible to determine that below the resonance the wave couples strongly with the LHCP branch (which at those altitudes is linearly polarized because the Alfvén velocity is large), and it is the reflected LHCP wave that is primarily absorbed. Note that the evanescent gap in Figure 1 is greatly reduced from the parallel propagation case, although the wave changes polarization near the Buchsbaum resonance and tunnels through in the right hand mode. Notice that significant absorption can occur well above the previously thought maximum limit of 25% [*Le Quéau and Roux*, 1992] due to reflection of waves from the ionosphere. For larger as well as smaller concentrations of heavy ions the absorption is reduced. For example, a 10% abundance of He^+ allows for 25% absorption at large θ.

Oscillation of the coefficients as a function of θ occurs because of the reflecting boundary condition at the ionosphere. The condition, $\mathbf{E}_\perp \to 0$, at the ionosphere imposes a node in the wave amplitude and a sequence of minima in the wave amplitude at higher altitudes. When one of these minima of the LHCP electric field component corresponds with the resonance locations, a minimum in absorption occurs. However, one should consider such interference effects with caution because the observed wave packets maintain coherence for less than 20 wave cycles which is a fraction of the propagation transit time of 100 s.

Finally, we point out that a similar analysis may also be applied to propagation of auroral EMIC waves which are excited locally by electron beams with $f \sim$ 10–100Hz [*Lund and LaBelle*, 1997]. Wave signatures can be predicted and compared with ground based magnetometer data.

Summary

In this paper, we have presented a full wave analysis for ion cyclotron waves propagating from the equatorial magnetosphere to the ground. This one dimensional calculation is sufficient to demonstrate that strong coupling between the LHCP and RHCP modes occurs near the He^+ and O^+ resonance locations which allows the equatorial waves to penetrate to ionospheric altitudes. These results explain how waves generated in the equatorial magnetosphere can propagate to the ground and predict the polarization shift which is commonly observed. A natural step for future research would be to include kinetic effects using the the kinetic-fluid model [*Cheng and Johnson*, 1999].

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