GEOPHYSICAL RESEARCH LETTERS, VOL. 22, NO. 12, PAGES 1481-1484, JUNE 15, 1995

Nonlinear vortex structures with diverging electric fields and their relation to the black aurora

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Abstract. Large, radially divergent electric fields have been observed by the FREJA satellite and have been associated with black auroral curls. We interpret these structures as two-dimensional solitary nonlinear plasma waves. In accordance with observation, we suggest that these are slowly propagating vortex structures traveling along elongated density depletions perpendicular to the magnetic field. We demonstrate that these structures are localized in space (the order of 1 km) and can have large, outwardly directed radial electric fields (the order of 1 V/m)—characteristics consistent with FREJA observations.

Introduction

The Atmospheric Explorer C first measured divergent electric field structures at low altitudes associated with a fallout in electron precipitation [*Burch et al.*, 1976]. The absence of precipitation in these regions corresponds well with the events termed "black aurorae" observed optically from the ground [*Hallinan and Davis*, 1970]. Recently the FREJA satellite detected similar electric field structures in the nightside/morningside auroral region [*Marklund et al.*, 1994, 1995]. Some of these structures are particularly interesting because they are very intense (approximately 1 V/m), have narrow spatial widths (approximately 1 km), and are radially divergent. Particle measurements also indicate a fallout of precipitating electrons associated with these events and a weak downward directed field-aligned current.

In this paper we interpret the FREJA electric field observations mentioned above in terms of a two-dimensional electrostatic solitary structure moving in the E-W direction. The solitary structure is characterized by a narrow spatial scale and large radially divergent electric field in accordance with the observations[*Chang and Johnson*, 1994a, 1994b].

Model

The particular event discussed by *Marklund et al.* [1994] is a two dimensional vortex like electric field structure. It is natural to consider the structure to be two-dimensional in the plane perpendicular to the magnetic field. Such a solution does not take into account the parallel dynamics and the associated implications for particle precipitation, but it does provide a reasonable framework to describe the large divergent perpendicular electric fields and the small scale perpendicular structure observed by FREJA.

The observations of *Marklund et al.* [1994] lie within a region of diffuse precipitation as is typical of black auroral curls. The wave data associated with the local FREJA observations at the time of the event indicate that the electric field is embedded in a region of density depletion [*Eriksson*, private communication]; therefore, we consider wave structures in the presence of an elongated background density depletion which varies with latitude. The scale of this background density depletion is observed to be much larger than the local density depletion associated with the electric field structure [*Marklund et al.*, 1994].

To model the FREJA observations, we take a simplified model illustrated in Figure 1 with the magnetic field along the z direction and a background density which varies latitudinally in the x direction. In accordance with ground based observations of the black aurora [*Hallinan and Davis*, 1970], we assume that the structures travel slowly along the trough of the density depletion in the E-W direction, y, rotating in an anti-cyclonic direction.

Background density depletions give rise to a rich variety of plasma modes and instabilities—most notable are drift waves and Kelvin-Helmholtz instabilities. It is well known that large amplitude vortex solutions exist in the presence of a nonuniform plasma [*Lakhin et al.*, 1988; *Su et al.*, 1991; *Petviashvili*, 1980; *Horihata and Sato*, 1987]. We explore the possible wave structure in the following section.

Basic Equations

The electric field structures at FREJA altitudes are essentially stationary compared with the ion gyroperiod, but the length scale of the structures is comparable to the ion gyroradius, ρ. However, the length scale, L, of the density depletion is much larger than the ion gyroradius. Hence,

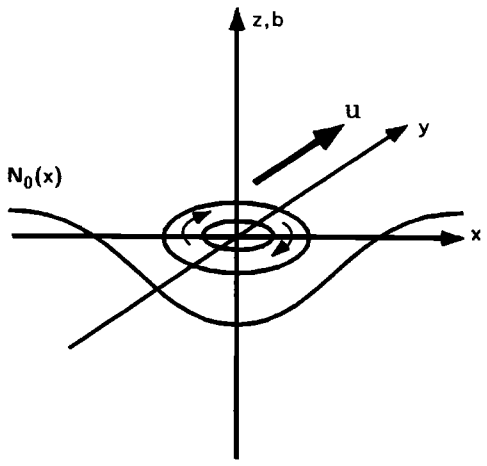

Figure 1. This figure illustrates our model for the electric field structures. The magnetic field is along the z axis, the density varies along the x axis, and the structure travels slowly in the y direction with the velocity u. The electric field points radially outward from the structure so that they are anti-cyclonic.

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Paper number 95GL01419
0094-8534/95/95GL-01419$03.00

$$\frac{1}{\Omega_i}\frac{\partial}{\partial t} \sim \frac{\mathbf{v}\cdot\nabla}{\Omega_i} << 1; \frac{\rho}{L} << 1 \tag{1}$$

We normalize the potential, density, velocity and position

$$\Psi = \frac{e\phi}{T_e}, N = \frac{n}{n_0}, \mathbf{V} = \frac{\mathbf{v}}{c_s}, \mathbf{R} = \frac{\mathbf{r}}{\rho_s}, \tau = \Omega_i t \tag{2}$$

where T_e is the electron temperature, n_0 is a background density, $c_s^2 = T_e/m_i$ is the ion acoustic sound velocity, and $\rho_s = c_s/\Omega_i$ is the ion acoustic gyroradius.

Initial analysis of the observed magnetic fluctuations indicates that they are not strongly correlated with the strong electric field structures [*Marklund*, private communication], therefore we limit ourselves to the study of electrostatic structures. The ion motion is described by the normalized continuity and momentum equations

$$\frac{d\log N}{d\tau} = -\nabla\cdot\mathbf{V} \tag{3}$$

$$\frac{d\mathbf{V}}{d\tau} = -\nabla\Psi + \mathbf{V}\times\mathbf{b} \tag{4}$$

where $\mathbf{b}$ is the direction of uniform background magnetic field. To lowest order in the gyrofrequency, the ion velocity consists of the "$\mathbf{E}\times\mathbf{B}$" drift and the polarization drift

$$\mathbf{V} = \mathbf{b}\times\nabla\Psi - \frac{d}{dt}\nabla\Psi \tag{5}$$

We follow the standard theory for analyzing solitary vortex solutions of the nonlinear drift equations [*Lakhin et al.*, 1988; *Su et al.*, 1991, 1992; *Horihata and Sato*, 1987]. From the continuity equation, we form a conserved quantity

$$q \equiv \nabla^2\Psi - \log N \tag{6}$$

such that ignoring compression due to the polarization drift

$$\frac{dq}{d\tau} = 0, \frac{d}{d\tau} \approx \frac{\partial}{\partial\tau} + \mathbf{b}\cdot(\nabla\Psi\times\nabla) \tag{7}$$

Conservation of q may be interpreted as the conservation law of enstrophy in a compressible fluid [*Hasegawa and Mima*, 1978]. To proceed, we specify that the structure drifts along the density trough in the y direction so that $\Psi(x,y,t) = \Psi(x,\eta)$ where $\eta = y - ut$ and u is the normalized velocity of the structure (Mach number). Then (7) becomes

$$\frac{1}{u}\frac{\partial\Psi}{\partial\eta}\frac{\partial q}{\partial x} = (\frac{1}{u}\frac{\partial\Psi}{\partial x} - 1)\frac{\partial q}{\partial\eta} \tag{8}$$

so that

$$q(x,\eta) = g(x - \Psi/u) \tag{9}$$

for an arbitrary function g.

In the simplest model we assume that quasineutrality is valid. More elaborate models for the electron and ion distributions coupled with the Poisson equation would be necessary if we were to include parallel dynamics. However, our approximation is sufficient to explain the large-scale perpendicular fields. We specify the electron distribution function from kinetic theory so that

$$N = N_e(\Psi(x,\eta); x, \eta) \tag{10}$$

Equations (6), (9), and (10) are closed for $\Psi(x,\eta)$ once we specify the form of N and g. The electron distribution function is constructed from the constants of the motion. A Boltzmann distribution for electrons gives a density $N = N_0(x)\exp(\Psi/T(x))$. Another interesting case involves trapped electrons. Large electric field structures which develop slowly compared to an electron transit time trap the low energy electron core. Because trapped electrons can maintain the potential at large values with only modest density enhancements [*Schamel*, 1979; *Bohm et al.*, 1990] it is interesting and important to consider their effect. A model distribution which allows continuous variation from a Boltzmann distribution to "adiabatic trapping" [*Bohm et al.*, 1990] to an electron hole [*Schamel*, 1979] is obtained from the free and trapped populations

$$\begin{aligned} f_{fr}(x,v) &= N(x)\exp(-(v^2 - 2\Psi)/2) \quad |v| > \sqrt{2\Psi} \\ f_{tr}(x,v) &= N(x)\exp(\beta(v^2 - 2\Psi)/2) \quad |v| < \sqrt{2\Psi} \end{aligned} \tag{11}$$

The free and trapped densities are respectively

$$\begin{aligned} N_{fr} &= N_0(x)\exp(\Psi)\mathrm{erfc}(\sqrt{\Psi}) \\ N_{tr} &= N_0(x)\frac{i}{\sqrt{\beta}}\mathrm{erf}(i\sqrt{\beta\Psi}) \quad \beta > 0 \\ N_{tr} &= N_0(x)\frac{1}{\sqrt{|\beta|}}\mathrm{erf}(\sqrt{|\beta|\Psi}) \quad \beta < 0 \end{aligned} \tag{12}$$

The limiting forms of the densities are Boltzmann for $\beta = -1$, adiabatic trapping for $\beta = 0$, and an electron hole as $\beta \to \infty$.

An appropriate form for the function g is obtained from the requirement that $\Psi \to 0$ as $|x| \to \infty$. One function which satisfies this form is $\log N_0(x)$ [*Horihata and Sato*, 1987]. Hence, we obtain the differential equation,

$$\nabla^2\Psi - \log N_0(x) - \log N_v(\Psi) = \log N_0(x - \Psi/u) \tag{13}$$

where N_v represents the contribution to the density in the absence of density gradients ($N = N_0(x)N_v(\Psi)$). To model a density depletion we take the approximation, $N_0(x) = \exp(x^2/L^2)$ which appropriately models a density depletion at x = 0 and is reasonable as long as the structure is well confined spatially to $|x| << L$ as is the case for the solution.

Boltzmann Electrons. For the Boltzmann distribution we find

$$\nabla^2\Psi - \kappa(x)\Psi + \frac{\Psi^2}{u^2L^2} = 0 \tag{14}$$

where $\kappa(x) = 1/T - 2x/uL$. Because κ is a slowly varying function of x it is appropriate to consider a WKB approximation to the solution. For low amplitudes, equation (14) implies that Debye shielding dominates and no wave structures exist. For larger amplitudes the density gradients focus

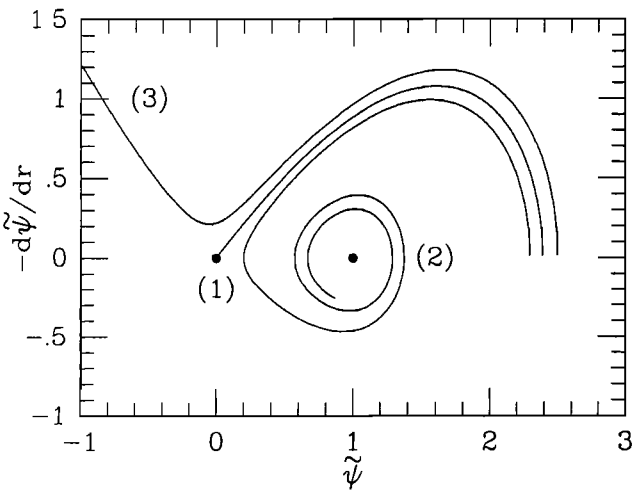

Figure 2. Phase space plot for the radially symmetric solution (16). Solitary solutions have $\tilde{\psi}_{max}$=2.392... and approach the fixed point $\tilde{\psi} = 0$ (1). Solutions with smaller $\tilde{\psi}_{max}$ are oscillatory about the fixed point at $\tilde{\psi} = 1$ (2), while solutions with larger $\tilde{\psi}_{max}$ are divergent (3).

the structure in the center of the region of depleted density and overcome the Debye shielding.

To first order, the vortex structures are azimuthally symmetric. If we were to consider other modes, the azimuthal wave number could be included in the parameter κ. With the scaling

$$\Psi(r) = \kappa u^2 L^2 \tilde{\psi}(\kappa^{1/2} r) \tag{15}$$

we obtain the dimensionless equation

$$\nabla^2 \tilde{\psi} - \tilde{\psi} + \tilde{\psi}^2 = 0 \tag{16}$$

The one dimensional analogue of this equation admits periodic solutions and a soliton solution which may be solved exactly. The radially symmetric solution has two fixed points at $\tilde{\psi} = 1, 0$ which correspond to oscillatory solutions and a soliton. A phase space plot ($d\tilde{\psi}/dr$ vs. $\tilde{\psi}$) of the solutions in Figure 2 demonstrates these properties. For a maximum potential less than 2.392 the solutions oscillate about $\tilde{\psi} = 1$. For a larger maximum potential the solutions diverge, and for $\tilde{\psi}_{max}$=2.392... the solution represents a soliton. The spatial structure of the electric field is shown for these solutions in Figure 3.

In the preceding analysis we took a depleted background density profile and expanded about the minimum. Near the minimum the expansion is even so that the linear term corresponding to the drift velocity, $v_{dy} = -TdN/dx$, vanishes. The character of our solutions is therefore determined by the curvature of the density. For a density depletion, the electric field is outwardly directed in the radial direction and the rotation anti-cyclonic. Furthermore, our choice for the background density profile corresponds to a density minimum and does not limit the amplitude of the structure as long as its spatial extent is well contained within the background density scale length, L, which is the case for the FREJA observations.

Our analysis differs from a situation with a linear background density profile [*Su et al.*, 1991, 1992] where the character of the solution is determined by the difference between v_{dy} and u. For the linear profile, the rotation of the vortex is anti-cyclonic (cyclonic) depending on whether the velocity of the structure is greater (less) than the drift velocity of the plasma. Those solutions also have a larger spatial size with a width of several $\sqrt{\rho_s L_n}$ where L_n is the scale of the density gradient.

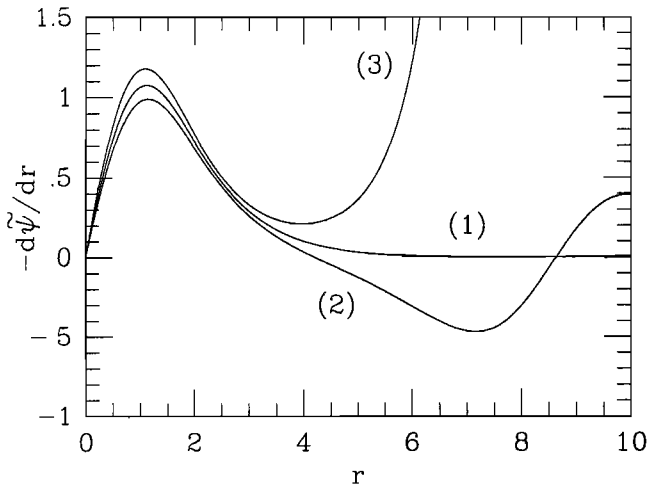

Figure 3. Electric field as a function of radius. For the solitary structure, (1), the field is radially divergent.

Trapped Electrons. For trapped electrons the equations become explicitly dependent upon the gradient scale length which can be important at relatively large values of Ψ as the density actually decreases with increasing Ψ for $\beta > 0$. The rescaled analogy of (16) is approximately

$$\nabla^2 \tilde{\psi} - \frac{1}{\Lambda} \log(N_v(\Lambda \tilde{\psi})) + \tilde{\psi}^2 = 0 \tag{17}$$

where $\Psi = \Lambda \tilde{\psi}$ and $\Lambda \equiv 1/u^2 L^2$. The phase space and real space analogues of Figures 2-3 are essentially the same except that the fixed points are at 0 and a value less than 1. The central amplitudes are reduced and the width of the structure is slightly increased. For large values of Λ the trapped electron potential cannot balance the nonlinear term due to the density gradients therefore only small potential solutions are allowed. However, for $\Lambda \sim 10$, a typical value based upon the observations, and for the most realistic values of β ($0 < \beta < 1$) the amplitude of the potential remains the order of unity. It is interesting to note that whereas for parallel double layers and also 1D electrostatic shocks the trapped population increases the potential substantially [*Schamel*, 1979], in this case there is a modest decrease in the size of the potential. We point out that for $\beta \sim 1$ the central density is depleted by as much as 20 percent of the ambient background density. This is entirely the result of the trapped electron population which decreases for large potentials unlike the Boltzmann case. We emphasize that there is an observed decrease in not only the background density, but also a further decrease in the density at the location of the electric field structure. The observations are consistent with the existence of a strongly trapped electron population. It would therefore be important to consider measurements of the electron distribution function at the location of these events to determine whether the density depletions result from strong electron trapping.

Comparison with Data

To compare our results with data we take parameters characteristic of the auroral region at 1700 km. The scale size of the solutions is determined primarily from the ion acoustic gyroradius. For electron temperatures ranging from 1–50 eV the gyroradius ranges from 30–100 m at these altitudes. Our solutions typically have a width of approximately 5 ρ_s so that the width of these structures is approximately 100–500 m which is consistent with the observations of the FREJA satellite. (The remarkable point about the observations was the thinness of the structures.)

The electric field for our structures can be estimated to be $E \approx 1.5 u^2 L^2 T_e / e\rho_s$. The velocity of black auroral structures tends to be much smaller than the ion acoustic speed. Based upon the observations of *Hallinan and Davis* [1970] the normalized velocity $u \sim 0.1$—much slower than the FREJA satellite so that the structures appear stationary. The background density is depleted by an order of magnitude in the region where these events occur—the background depletion being much larger in spatial extent than the depletion associated with the electric field structures [*Eriksson*, private communication]. In accordance with FREJA plasma wave observations we take the background scale length, $L/\rho_s \sim 50$, and for typical parameters we find electric fields $25T_e/\rho_s \sim$ 0.1–1V/m which is comparable with the FREJA observations.

Future Considerations

Although the nonlinear 2-D structures that we have found exhibit the appropriate spatial size and large divergent elec-

tric fields which characterize the structures observed by *Marklund et al.* [1994], there are a number of features that require further investigation and refinement. It is obvious that parallel dynamics are important for correlating the absence of precipitation with the large perpendicular electric field structures and for providing a consistent explanation of the upward directed field-aligned currents. Assuming that the electric fields map into the ionosphere they will drive Pederson currents out of the electric field structure. Hence, there will be a downward directed field-aligned current as is observed. This interpretation is simply the analogue of the interpretation of the well known inverted V event [*Burch et al.*, 1976]. The obvious improvement for our model is to incorporate proper coupling between these structures and the ionosphere using a 3-D model. Moreover the dropout of electron precipitation is most likely the result of a parallel electric field which is determined by the parallel dynamics. Coupling between the magnetosphere and ionosphere remains an area of active research.

Another important consideration is the time dependence and stability of the structures. In previous studies [*Su et al.*, 1991] it has been shown that such structures can be maintained for about 100 cyclotron periods which is the timescale of the satellite measurements. Individual curls tend to exist for the order of seconds which appears to be consistent with the disruption time of the vortex structures.

The generation mechanism for these structures is an important area for future consideration. The structures can presumably develop as the nonlinear stage of an instability (perhaps Kelvin-Helmholtz, drift-wave or ion acoustic) or perhaps be driven by a process at the solar wind magnetosphere interface. These structures are almost entirely observed on the nightside and are seasonally dependent, so it is important for the generation mechanism to satisfy these characteristics.

Finally, although preliminary observations indicate little correlation between magnetic fluctuations and the large electric field structures it appears that significant field-aligned currents are associated with these structures, and thus, it may be important to also consider electromagnetic effects.

Summary

In this paper we have proposed an explanation of the large, divergent, vortex-like electric fields observed by the FREJA satellite [*Marklund et al.*, 1994]. We analyze a model with an elongated density depletion perpendicular to the Earth's magnetic field. We have found solutions of the relevant equations which correspond to large vortex structures with outwardly directed radial electric fields traveling in the E-W direction along the density trough. We find that the spatial scale of these structures is small (the order of 1 km) and the electric fields can be enormous (the order of 1 V/m). These characteristics correspond well with the FREJA observations.

Acknowledgments. We are indebted to Göran Marklund and Anders Eriksson for useful discussions and for showing us data before it was published. We also thank C.Z. Cheng for useful discussions. This work was partially supported by NASA Grant Nos. NAG5-2255 and NAGW 1532, by the Air Force Office of Scientific Research Grant No. F49620-93-1-0287 and the Phillips Laboratory Contract No. F19628-91-K-0043.

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(received January 12, 1995; revised March 16, 1995; accepted April 6, 1995.)