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Detecting Causality In Space Plasmas With Entropy Based Measures

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Objective to Understand

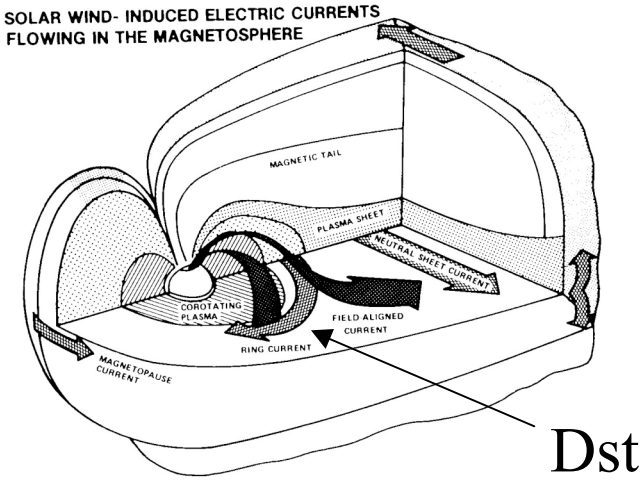
- Underlying dynamics of geospace systems
  - Driven vs Internal Response
  - Causality
  - Dimensionality
- Nonlinear behavior
  - Loading/unloading processes
- Nonstationarity
  - Short term: e.g. substorms
  - Long term: solar cycle changes
- Predictability
  - Horizon
  - Models

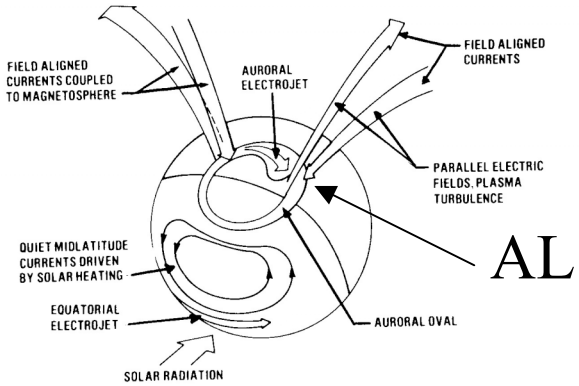
Methodology

- Consider the evolution of variables that define the
  - "State" of the magnetosphere:
    - Geomagentic indices: Kp, Dst, AL, …
    - Physical Measures: Particle flux, b2i, auroral power
  - Physical process
    - Ion fluxes ⇔ion outflows
    - Bulk flows ⇔ reconnection
- Determine the dependence on:
  - External solar wind drivers: n,V,B,P,…
  - History of the magnetospheric state

Defining the State of the Magnetosphere

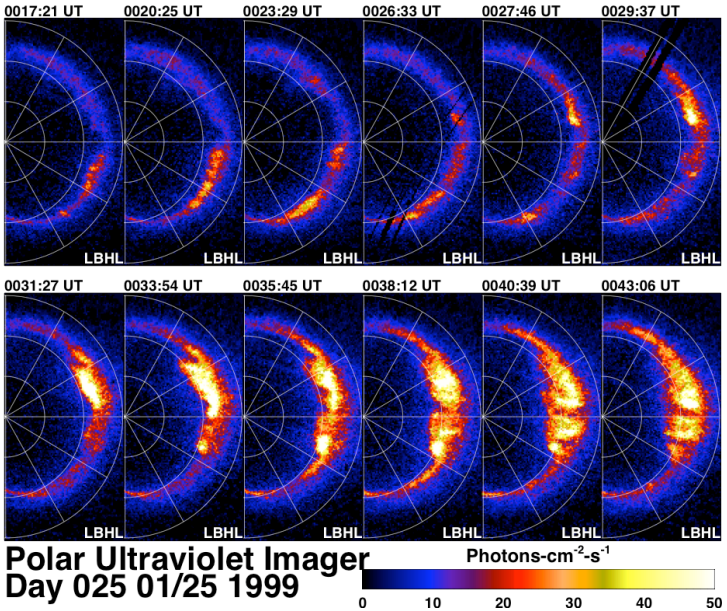
- Magnetic indices
- Historical record (1932 to present)
- Related to physical current distributions

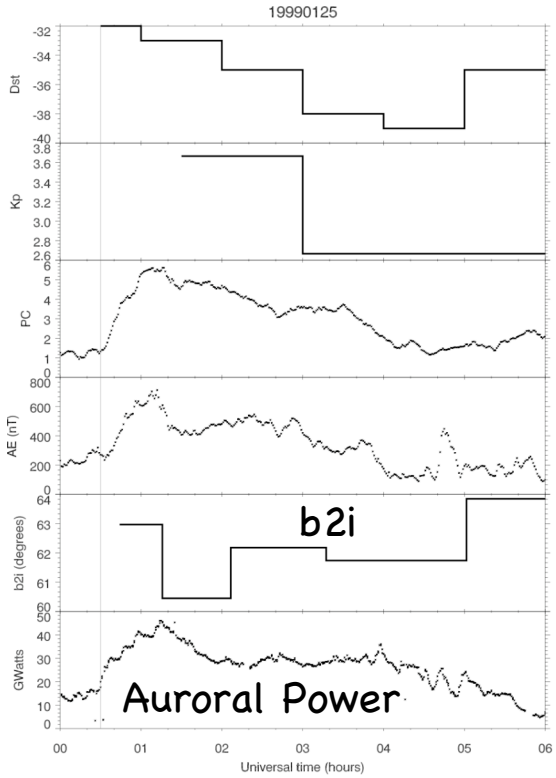




Defining the State of the Magnetosphere

- auroral power
- polar cap potential
- b2i---tail stretching
- energetic particle flux



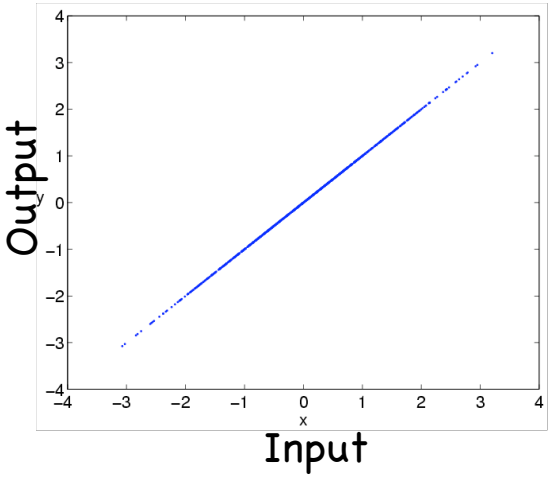


Statistical Measures of Dependency

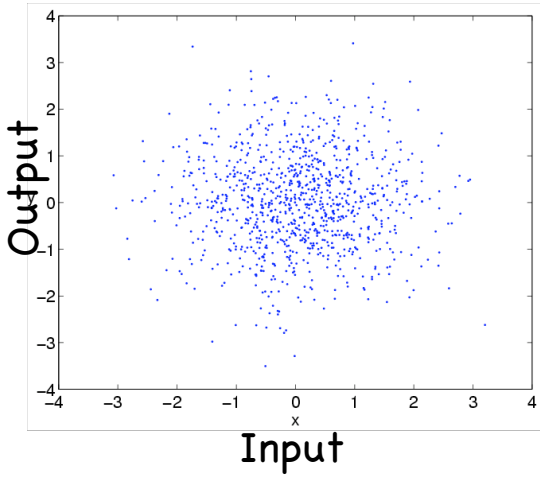
1. Correlation function
2. Mutual Information
3. Cumulants

$$P(x,y) = P(x)P(y) \ ?$$

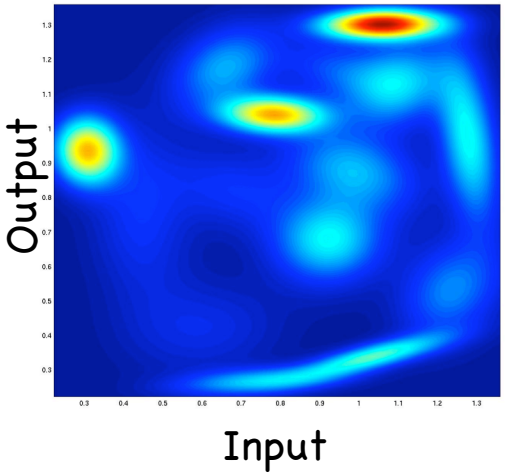
Correlated Data



Uncorrelated Data



High Order Correlations



Entropy and Mutual Information

$$H(x) = -\sum_{\aleph_1} p(x)\log p(x)$$

$$H(y) = -\sum_{\aleph_2} p(y)\log p(y)$$

$$H(x,y) = -\sum_{\aleph_1,\aleph_2} p(x,y)\log p(x,y)$$

$$I(x,y) = H(x) + H(y) - H(x,y)$$

$$x \in \{1,\ldots,N\} \equiv \aleph_1$$

$$y \in \{1,\ldots,M\} \equiv \aleph_2$$

Mutual information is commonly used as an alternative to correlation functions which have limitations for nonlinear systems. Generalization to higher dimensions is called redundancy.

Multivariate Cumulants

$$C_{i,\ldots,j} = \langle x_i \cdots x_j \rangle$$ ← Correlation Tensors

Cumulants

↓

$$K_i = C_i = \langle x_i \rangle$$

$$K_{i,j} = C_{i,j} - C_i C_j = \langle x_i x_j \rangle - \langle x_i \rangle \langle x_j \rangle$$

$$K_{i,j,k} = C_{i,j,k} - C_{i,j} C_k - C_{j,k} C_i - C_{i,k} C_j + 2\, C_i C_j C_k$$

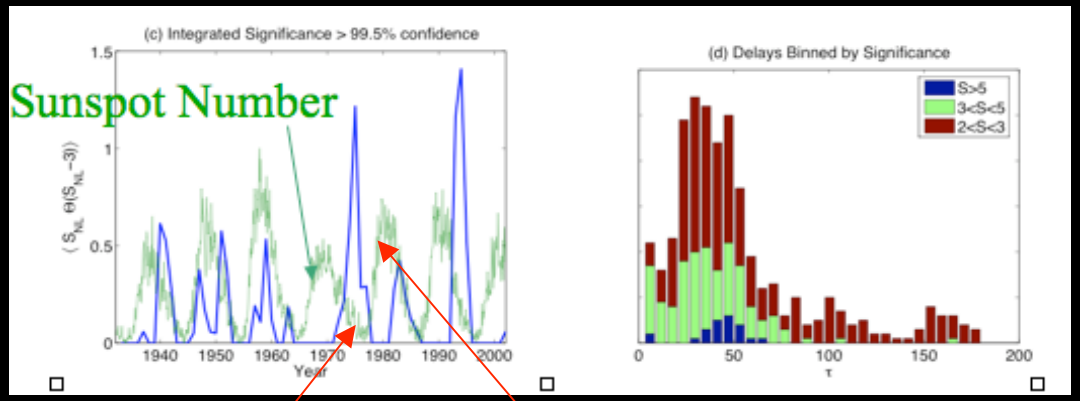
$$\begin{aligned} K_{i,j,k,l} = {} & C_{i,j,k,l} - C_{i,j,k} C_l - C_{i,j,l} C_k - C_{i,l,k} C_j - C_{l,j,k} C_i \\ & - C_{i,j} C_{k,l} - C_{i,l} C_{k,j} - C_{i,k} C_{j,l} \\ & + 2\left(C_{i,j} C_k C_l + C_{i,k} C_j C_l + C_{i,l} C_j C_k + C_{j,k} C_i C_l + C_{j,l} C_i C_k + C_{k,l} C_i C_j \right) \\ & - 6\, C_i C_j C_k C_l \end{aligned}$$

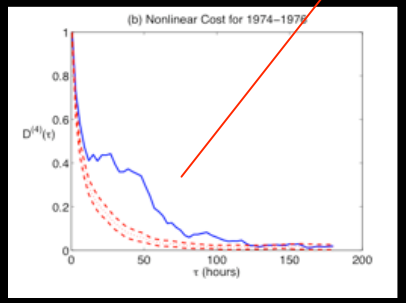
Discriminating Statistics Detect Nonlinearity

- D_{MI} measures deviance from linear correlations
- Truncating D_C at second order vs higher order provides information about nonlinear dependency
- Practical Point: D_C gives better statistics for limited and noisy data

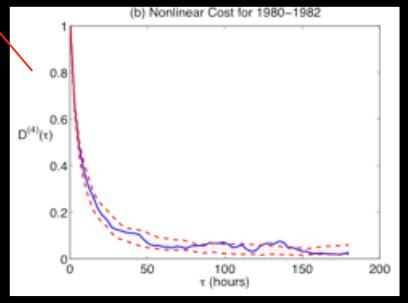
Transport of Information

- Solar Cycle Nonlinearity (Mutual Information and Cumulants)
- Compare with Surrogates that have the same Linear Properties



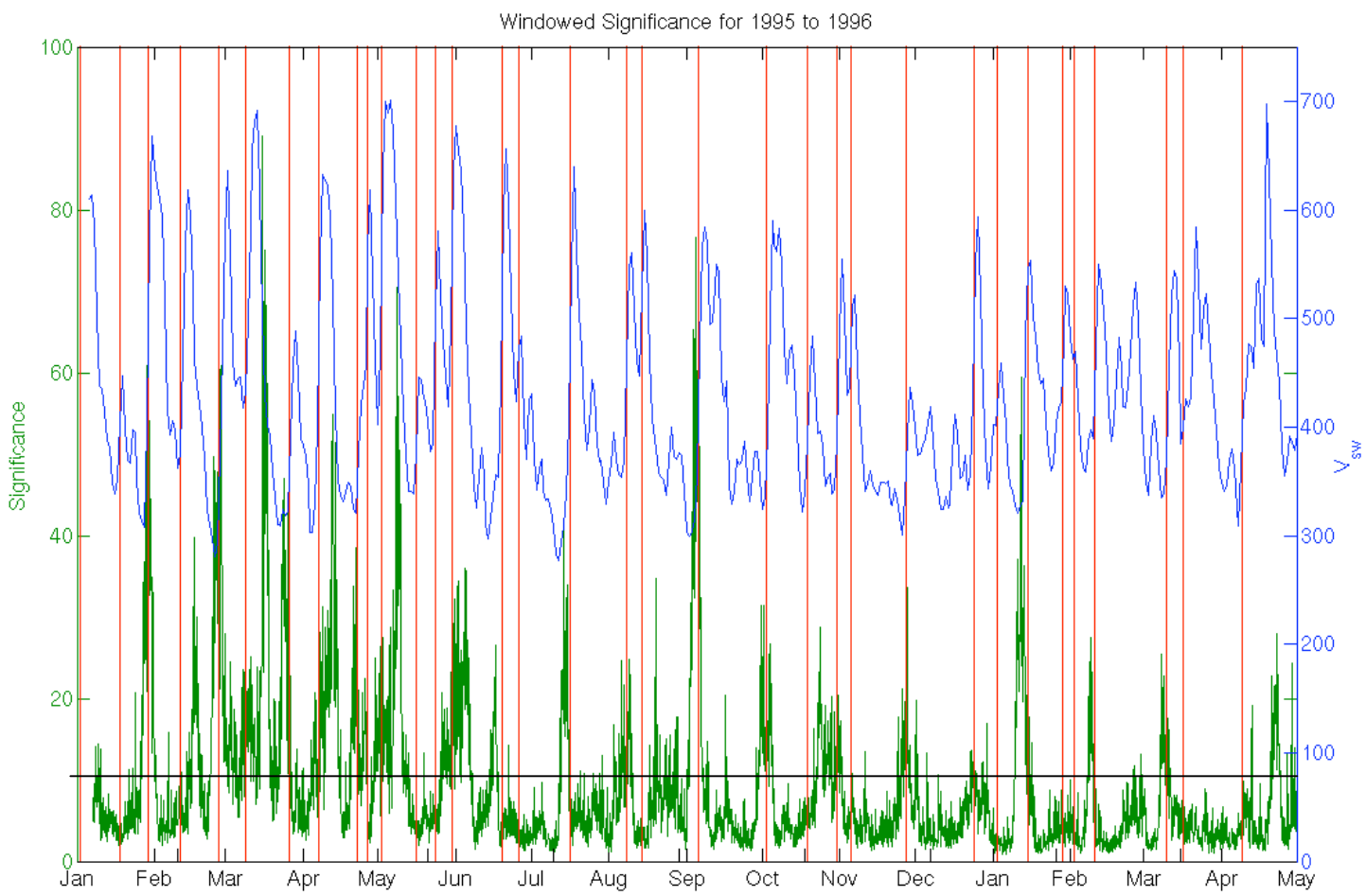


Solar Minimum

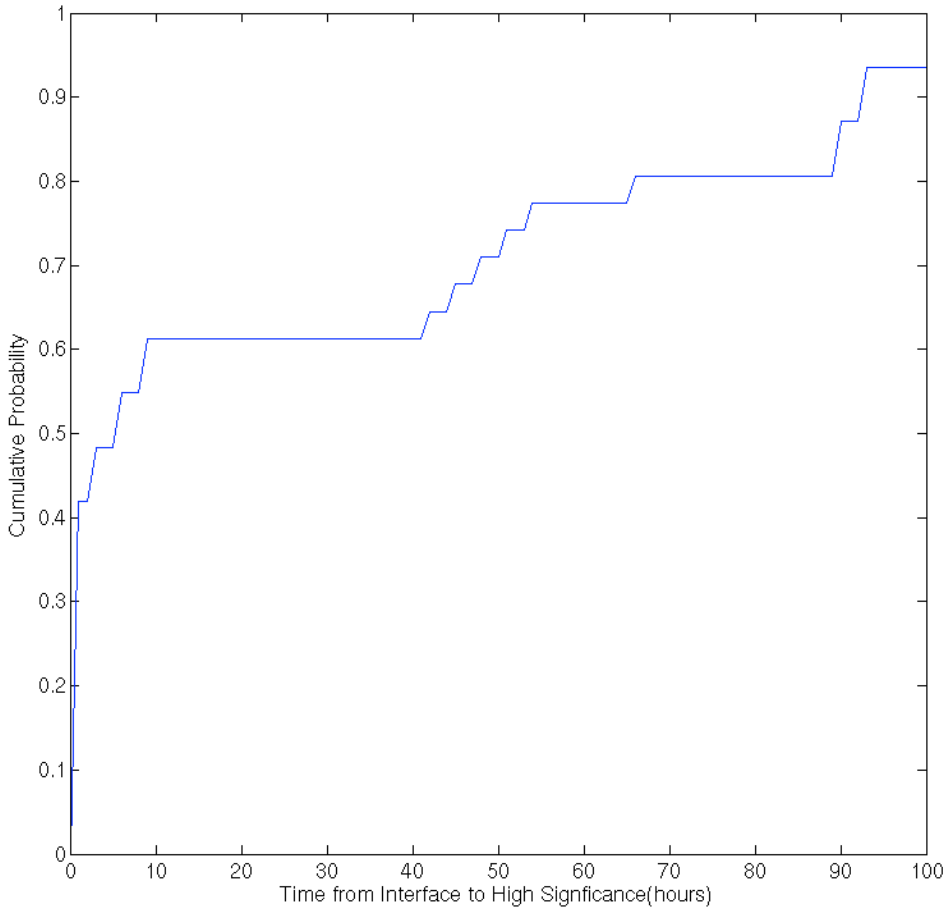


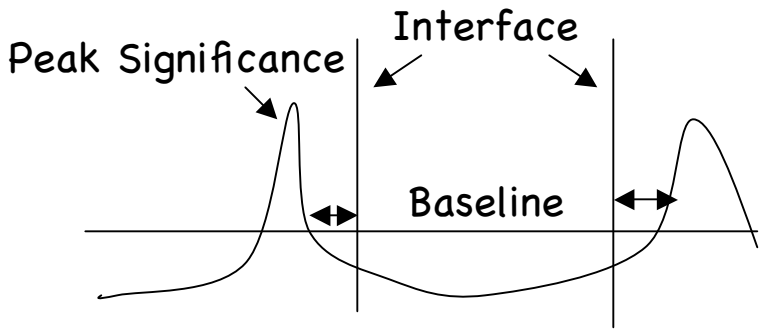
Solar Maximum

High Speed Streams in the Declining Phase of Solar Cycle



How Well Are Peaks Correlated with High Significance?





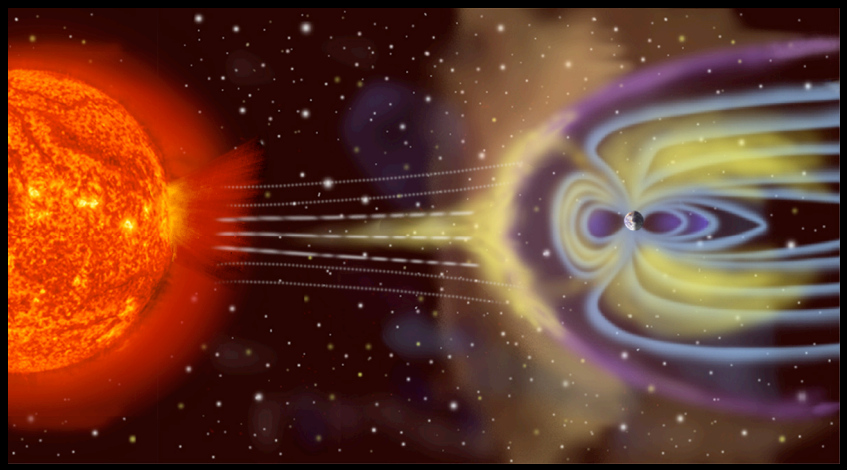
40% of interfaces occur simultaneous with high Significance

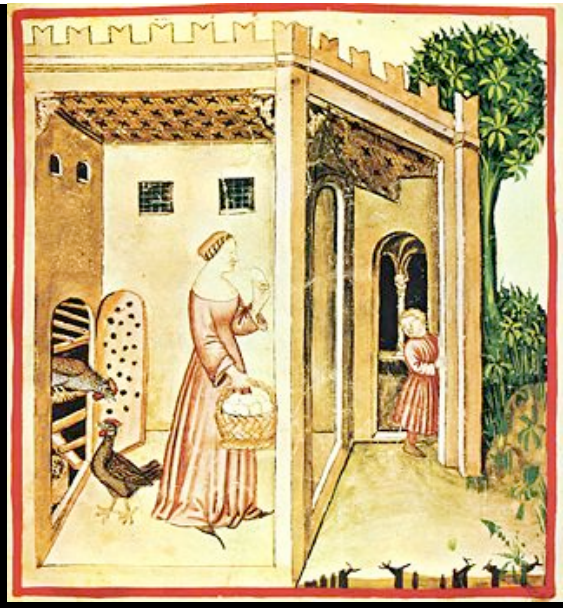
60% of interfaces occur within 1 day of high Significance

80% of interfaces occur within 2 days of high Significance

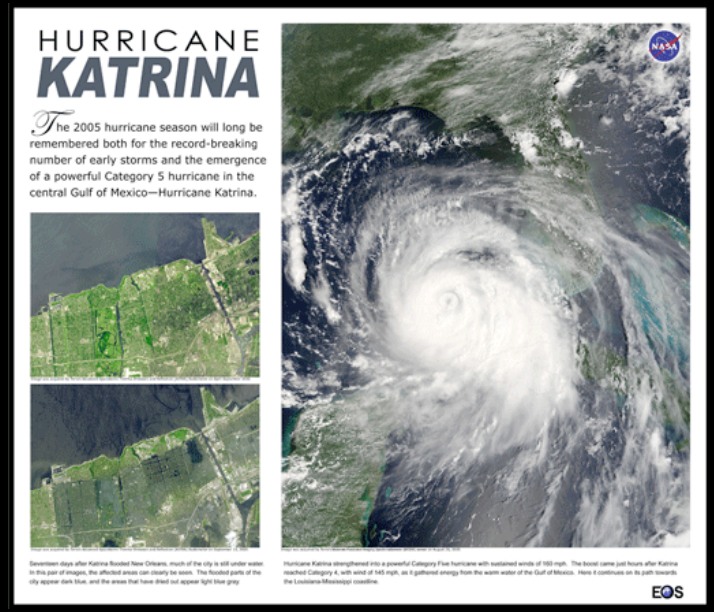
Results of the Analysis

- Identified nonlinearity in magnetospheric dynamics during the declining phase of the solar cycle
- Identified a timescale (information horizon)
- The nonlinear response is an internal magnetospheric response to solar wind velocity enhancements
- Suggests an important nonlinear coupling sensitive to solar wind velocity





Causality



HURRICANE
KATRINA

The 2005 hurricane season will long be remembered both for the record-breaking number of early storms and the emergence of a powerful Category 5 hurricane in the central Gulf of Mexico—Hurricane Katrina.

Seventeen days after Katrina flooded New Orleans, much of the city is still under water. In this pair of images, the affected areas can clearly be seen. The flooded parts of the city appear dark blue, and the areas that have dried out appear light blue gray.

Hurricane Katrina strengthened into a powerful Category Five hurricane with sustained winds of 160 mph. The boost came just hours after Katrina reached Category 4, with wind of 145 mph, as it gathered energy from the warm water of the Gulf of Mexico. Here it continues on its path towards the Louisiana-Mississippi coastline.

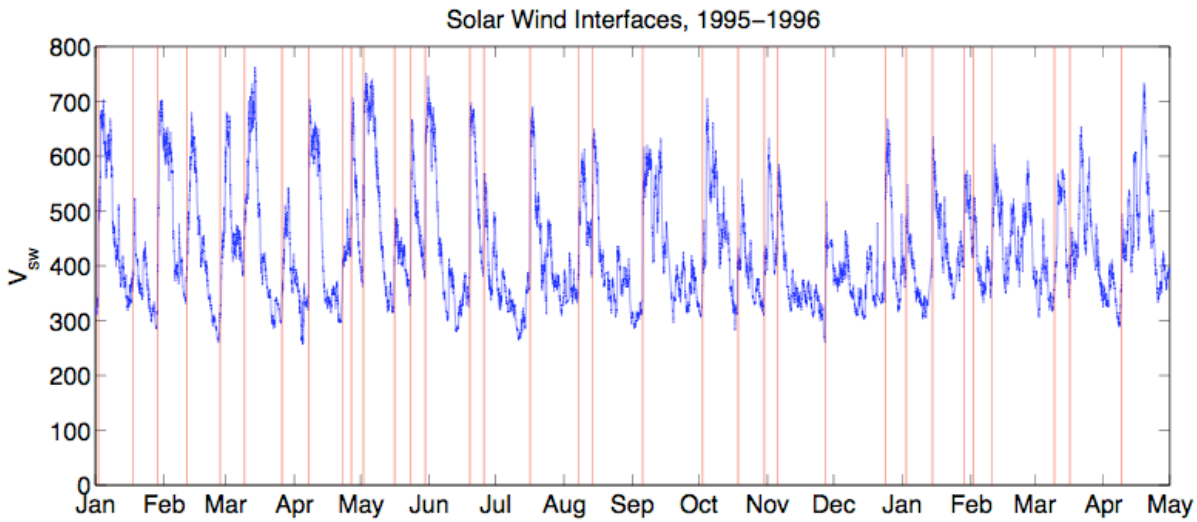
EOS

Tools to Study Causality

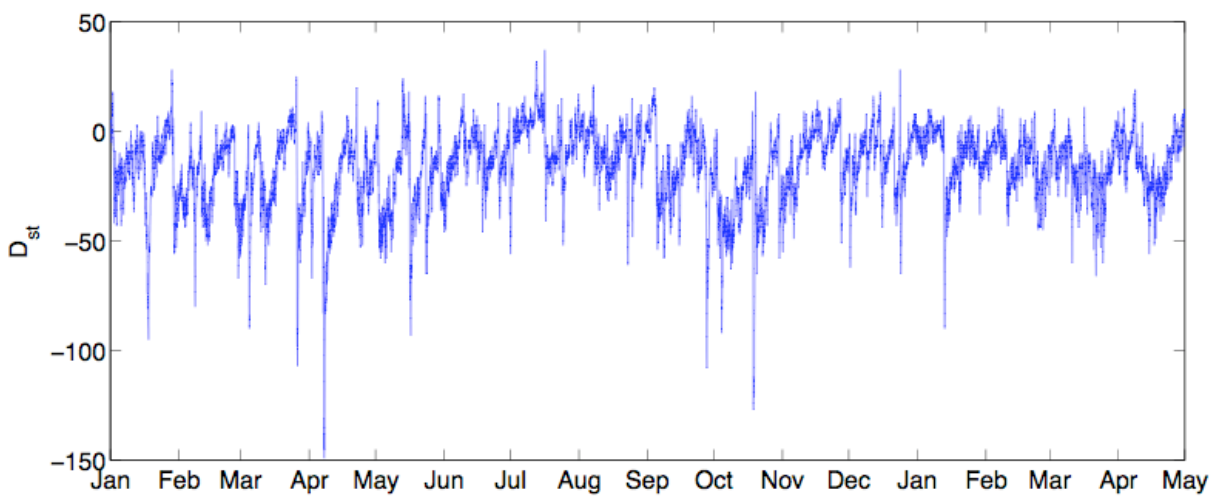
- Cross Correlations
  - $C(\tau) = \Sigma_t X_{t+\tau} Y_t$
- Mutal Information
  - $MI(\tau) = \Sigma_t p(X_{t+\tau}, Y_t) \log(p(X_{t+\tau}, Y_t)/p(X_{t+\tau})p(Y_t))$
- Transfer Entropy [Schreiber, 2000]
  - $T(\tau) = \Sigma_t p(X_{t+\tau}, X_t, Y_t) \log(p(X_{t+\tau} | X_t, Y_t)/p(X_{t+\tau} | X_t))$

Data Set (Solar Min HSS)

Driver

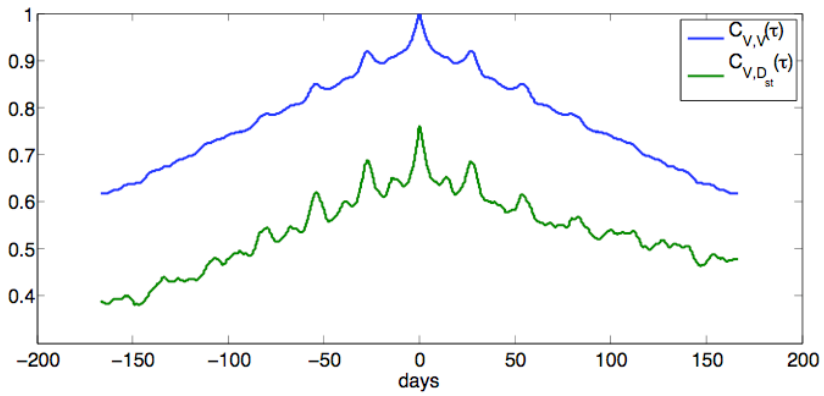


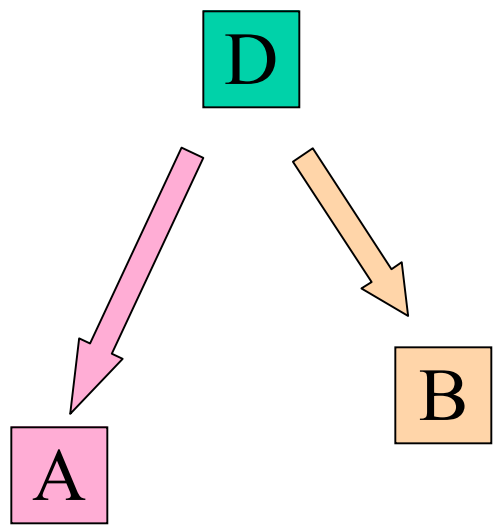
Response



Issues

- Cross Correlation
  - Linear
  - Highly correlated in both directions
  - Common driver but no coupling
- Mutual Information
  - Highly correlated in both directions
  - Common driver but no coupling

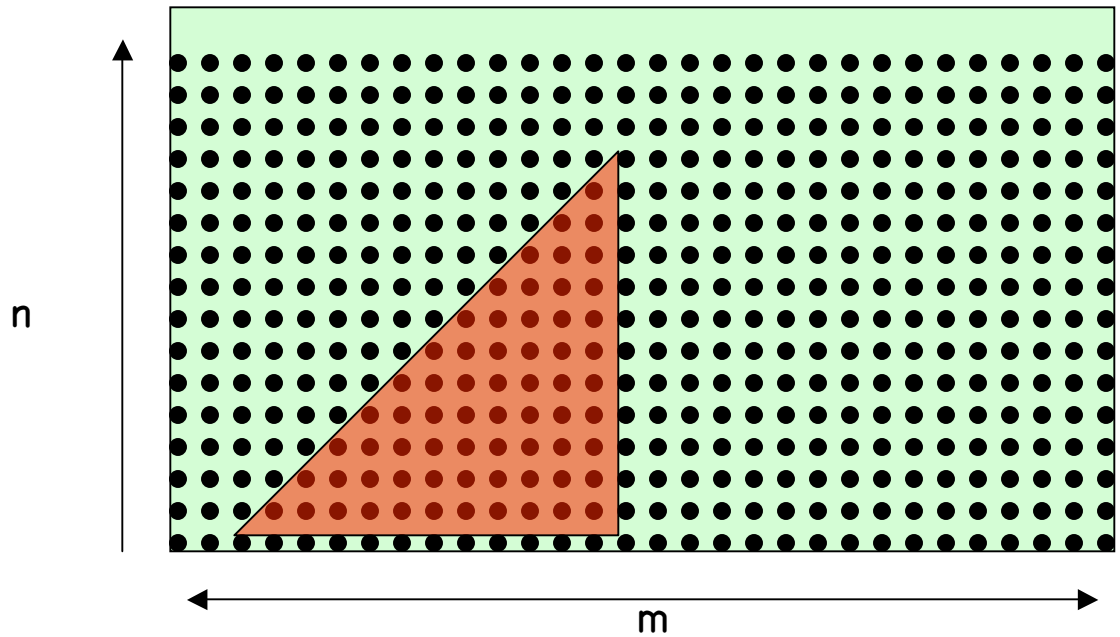




Transfer Entropy

- Definition
  - $T_{X\rightarrow Y}(\tau) = \Sigma_t\, p(X_{t+\tau},X_t,Y_t)\log(p(X_{t+\tau}|\, X_t,Y_t)/p(X_{t+\tau}|X_t))$
  - $T_{Y\rightarrow X}(\tau) = \Sigma_t\, p(Y_{t+\tau},Y_t,X_t)\log(p(Y_{t+\tau}|\, Y_t,X_t)/p(Y_{t+\tau}|Y_t))$
- Strongly Directional
- Eliminates Internal Correlations
- Distinguish Common Driver

Example: Spatio-Temporal Lattice

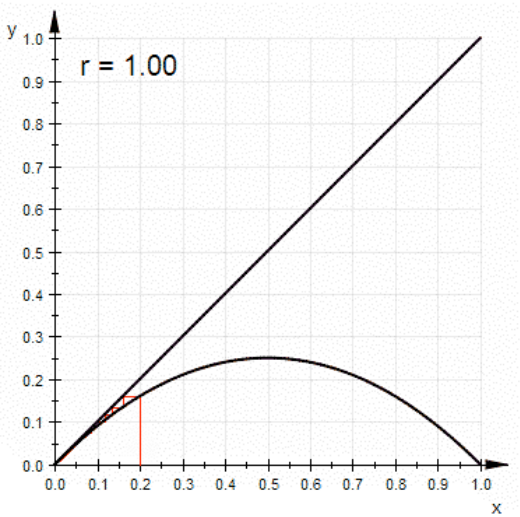


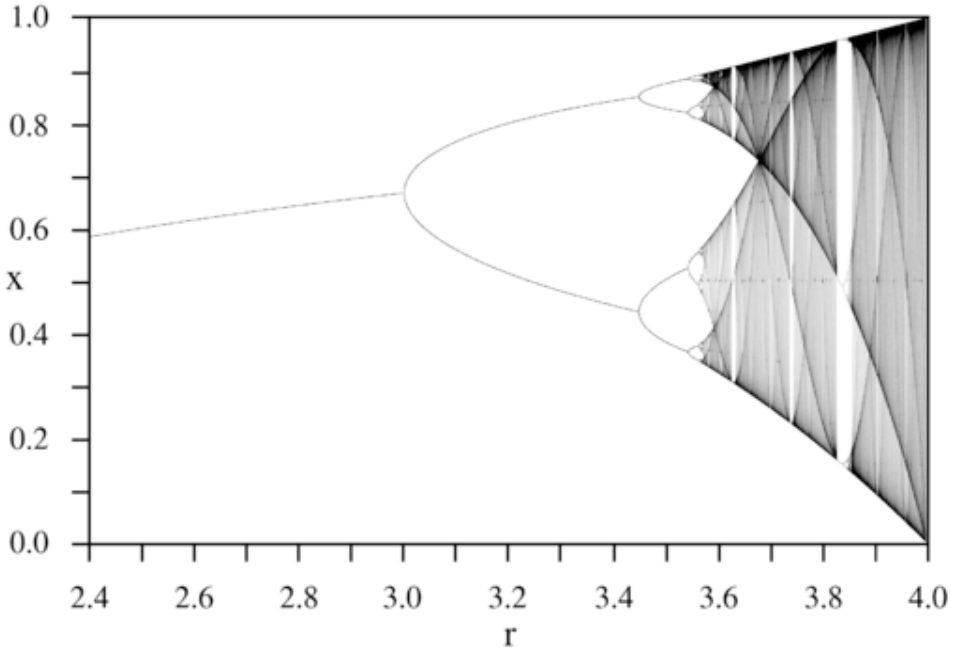
Periodic BC

Map: $x_{n+1,m} = f(\varepsilon x_{n,m-1} + x_{n,m})\}$

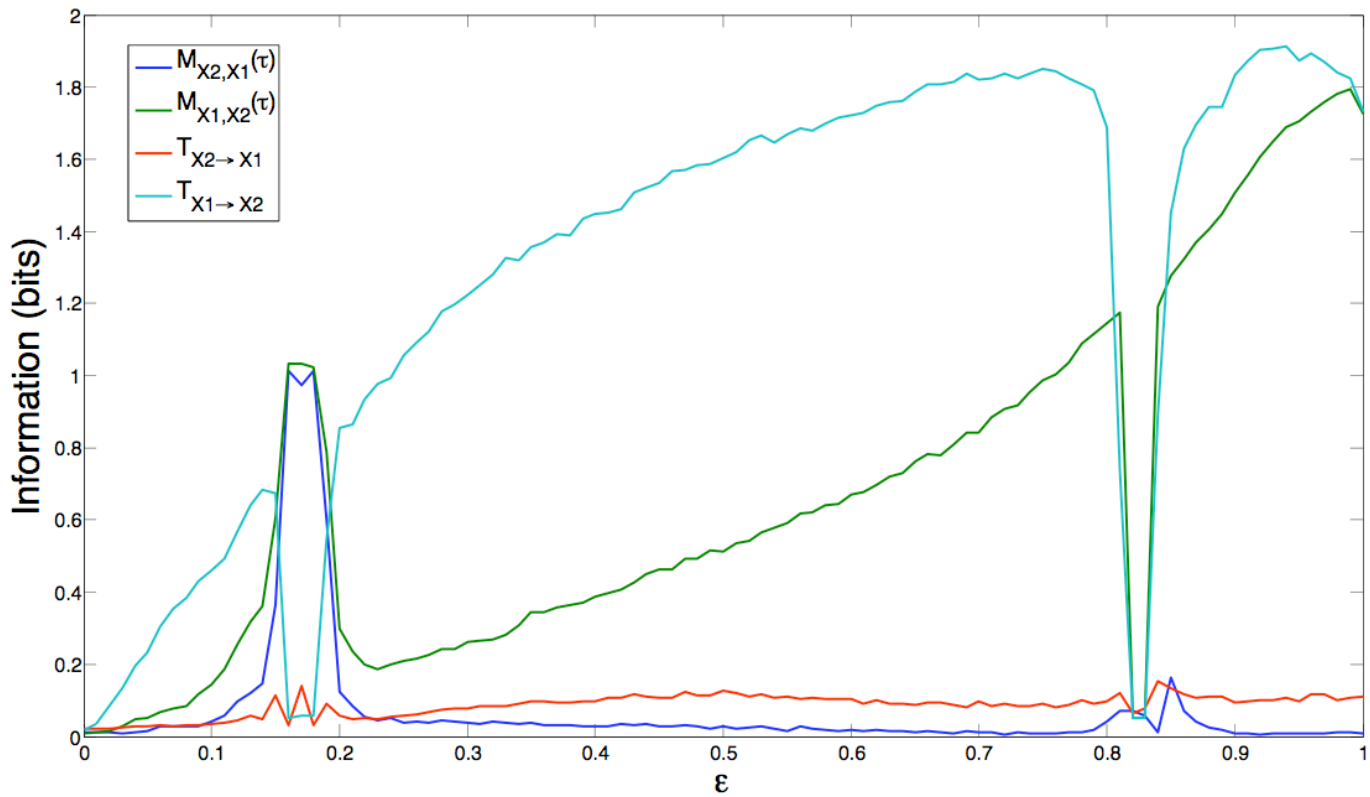
Ulam/Logistic Map

- $f(x) = r\,x(1-x)$
- $f(x) = 2-x^2$



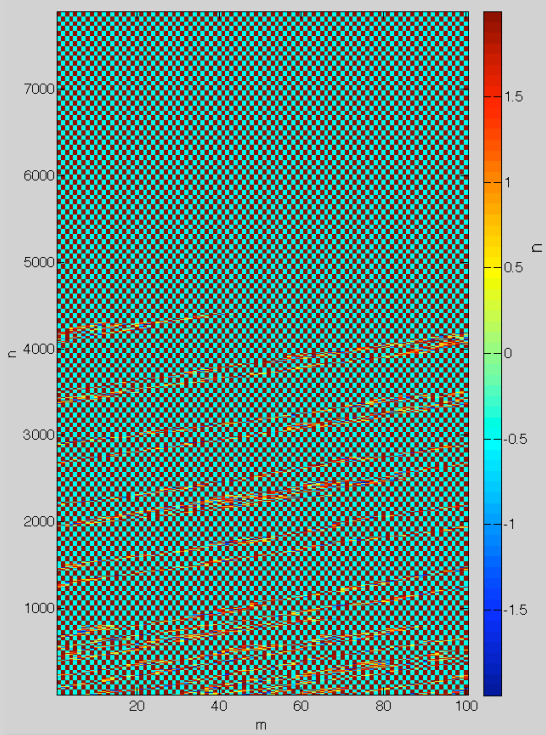


Transfer Entropy



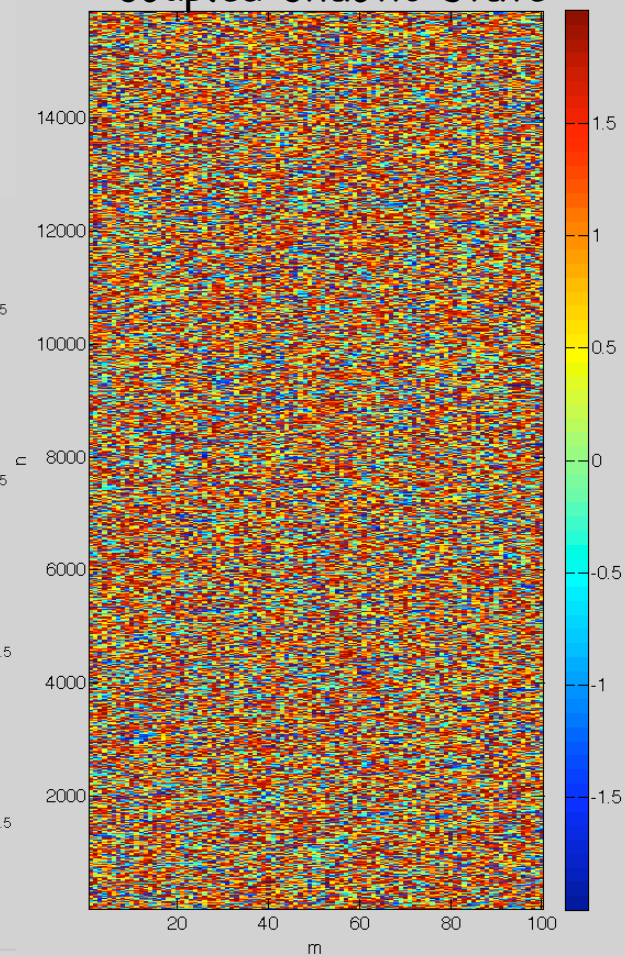
Mappings

Period 2 Lattice



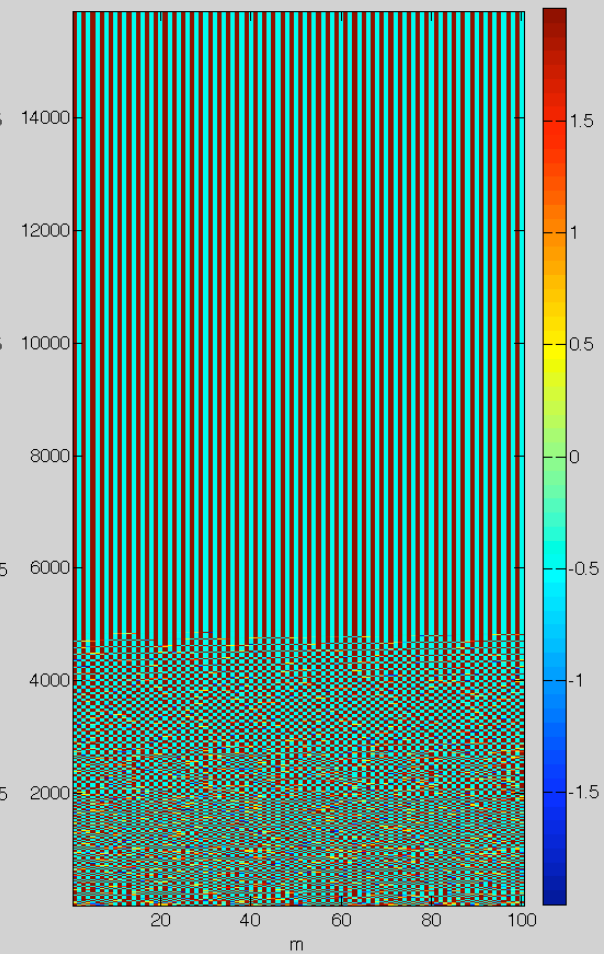
$\varepsilon = 0.18$

Coupled Chaotic State



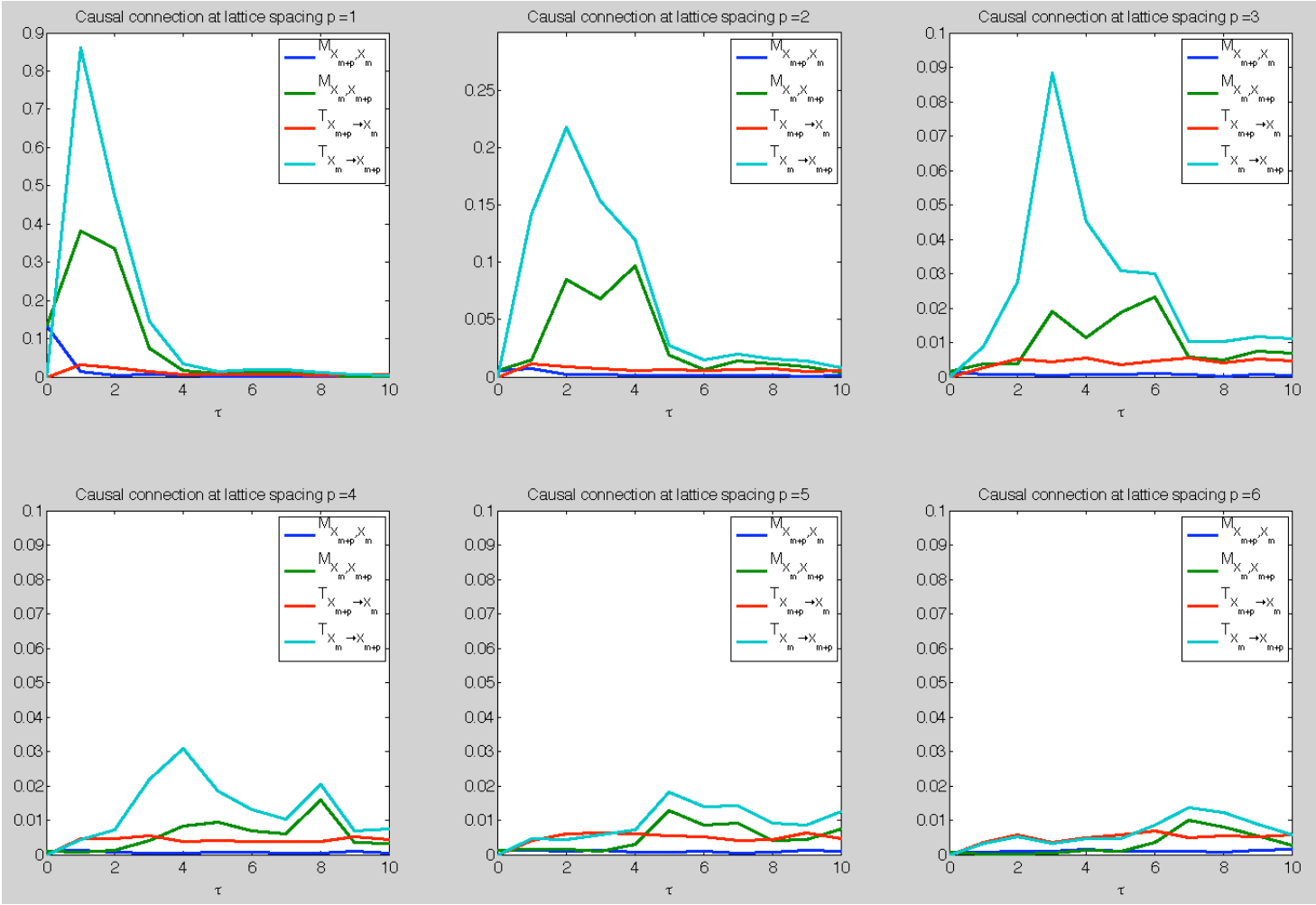
$\varepsilon = 0.5$

Fixed Point State

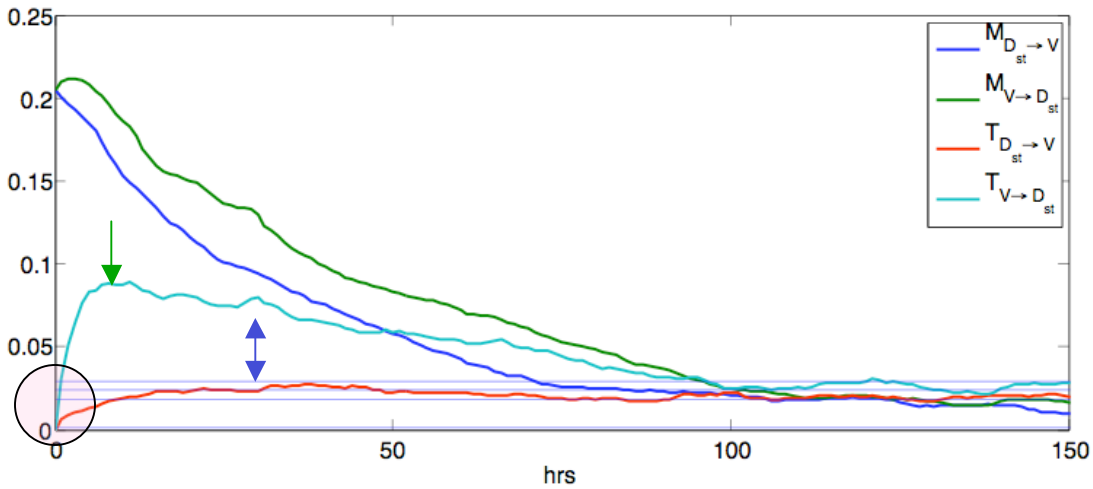


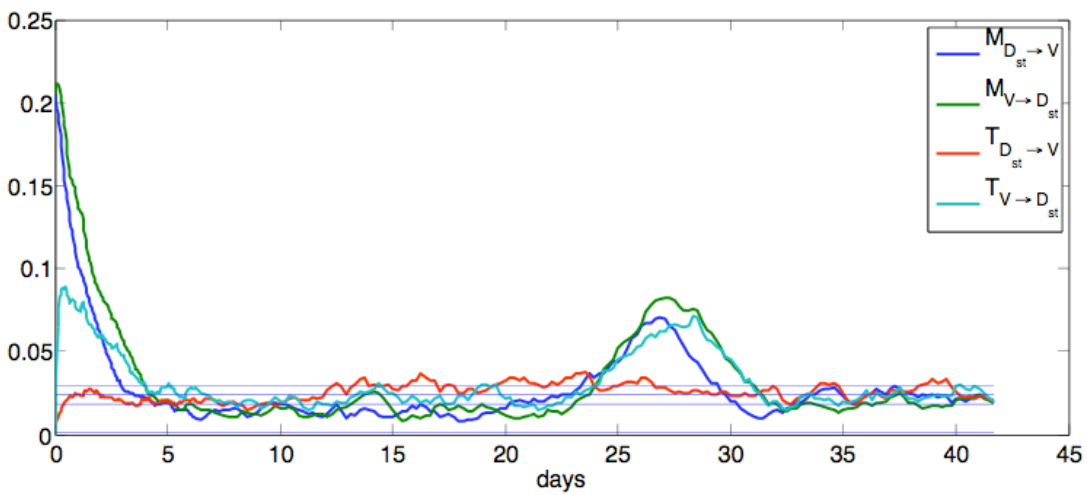
$\varepsilon = 0.82$

Spatio-Temporal Correlations



Example: V_{sw} drives D_{st}





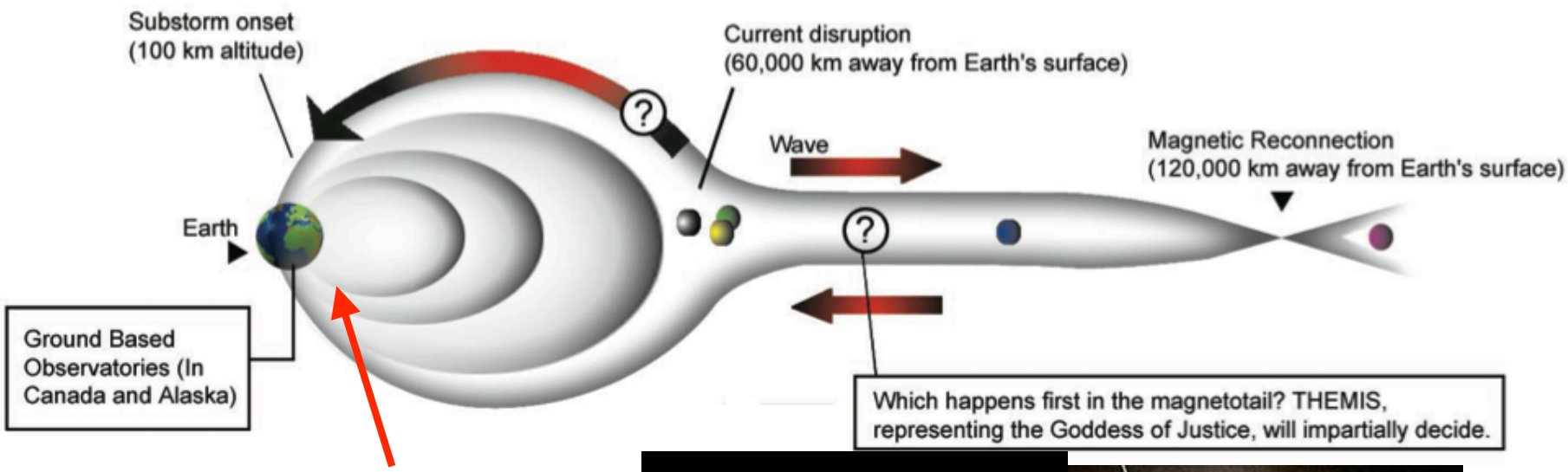
- Persistence (2hr)
- Driver (11 hr)
- Unidirectionality

Causality in Space Physics

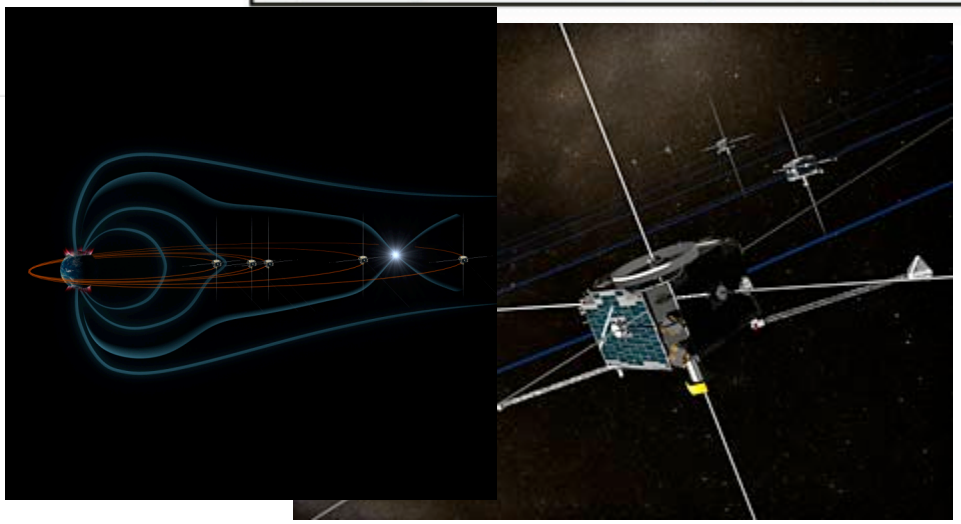
- Solar Wind→Magnetosphere Coupling
- Internal/External Driving of Substorms
- MI Coupling---relation of waves, particle acceleration, heating
- Plasma sheet pressure, Birkeland Currents, Ion Outflows

Substorm Onset

Which comes first: current disruption or reconnection ?



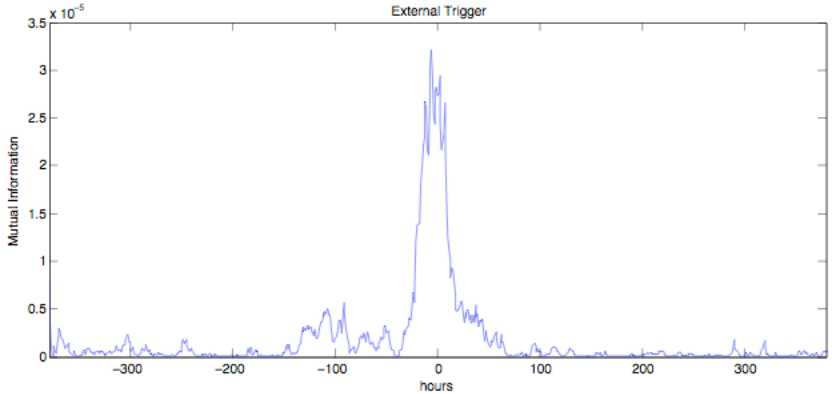
Aurora

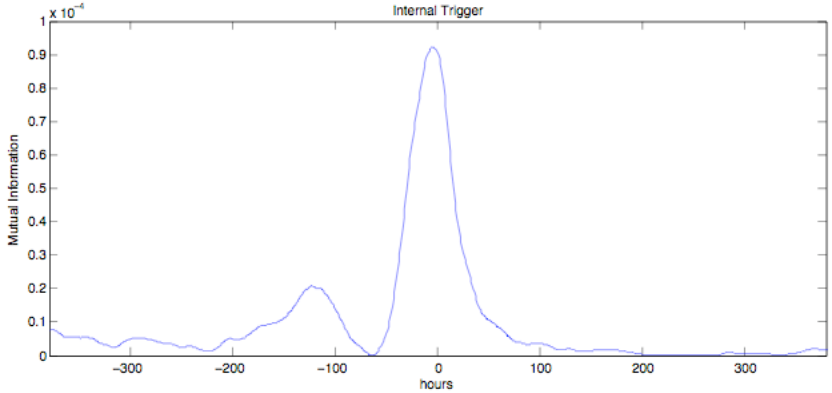


THEMIS

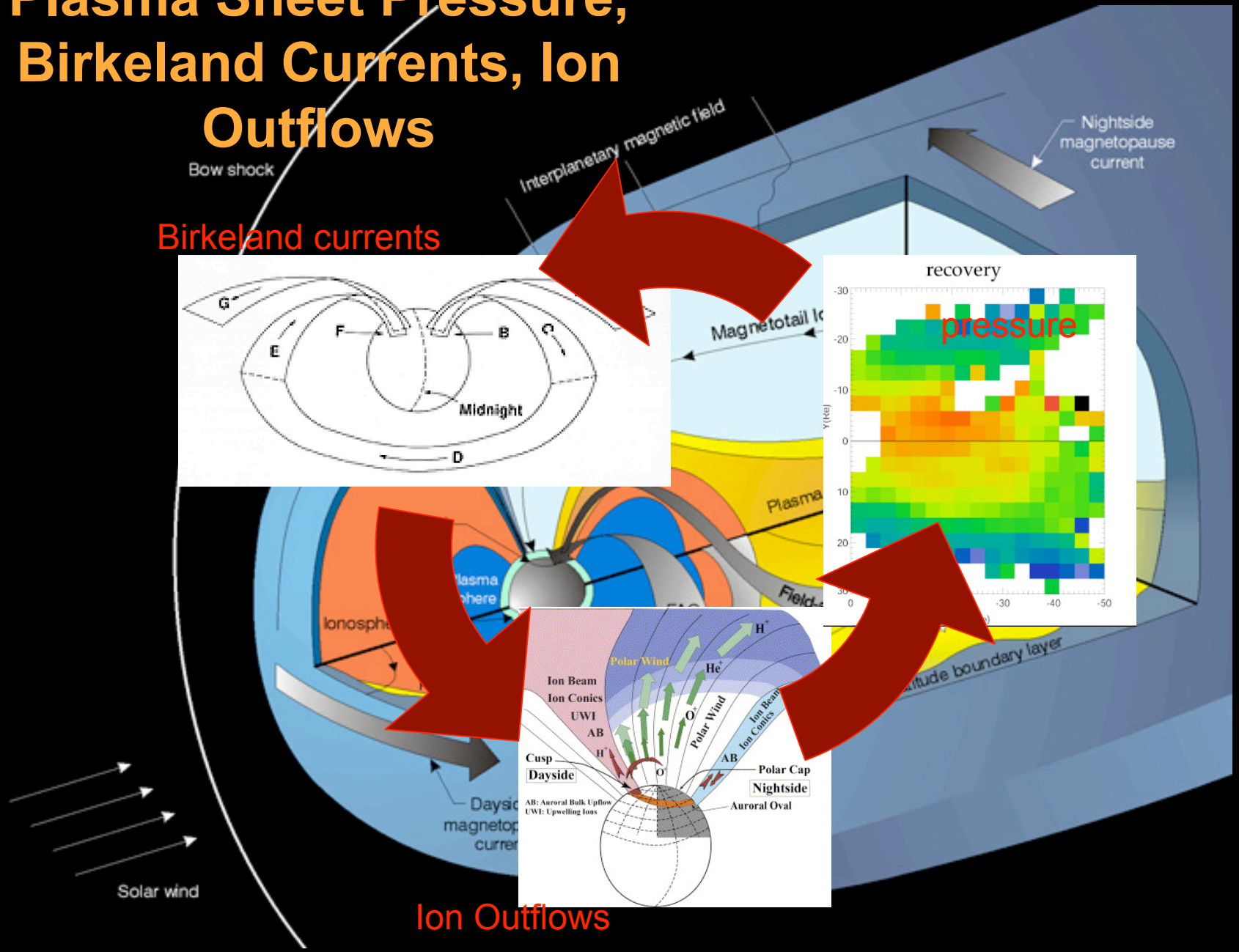
Internal/External Triggering

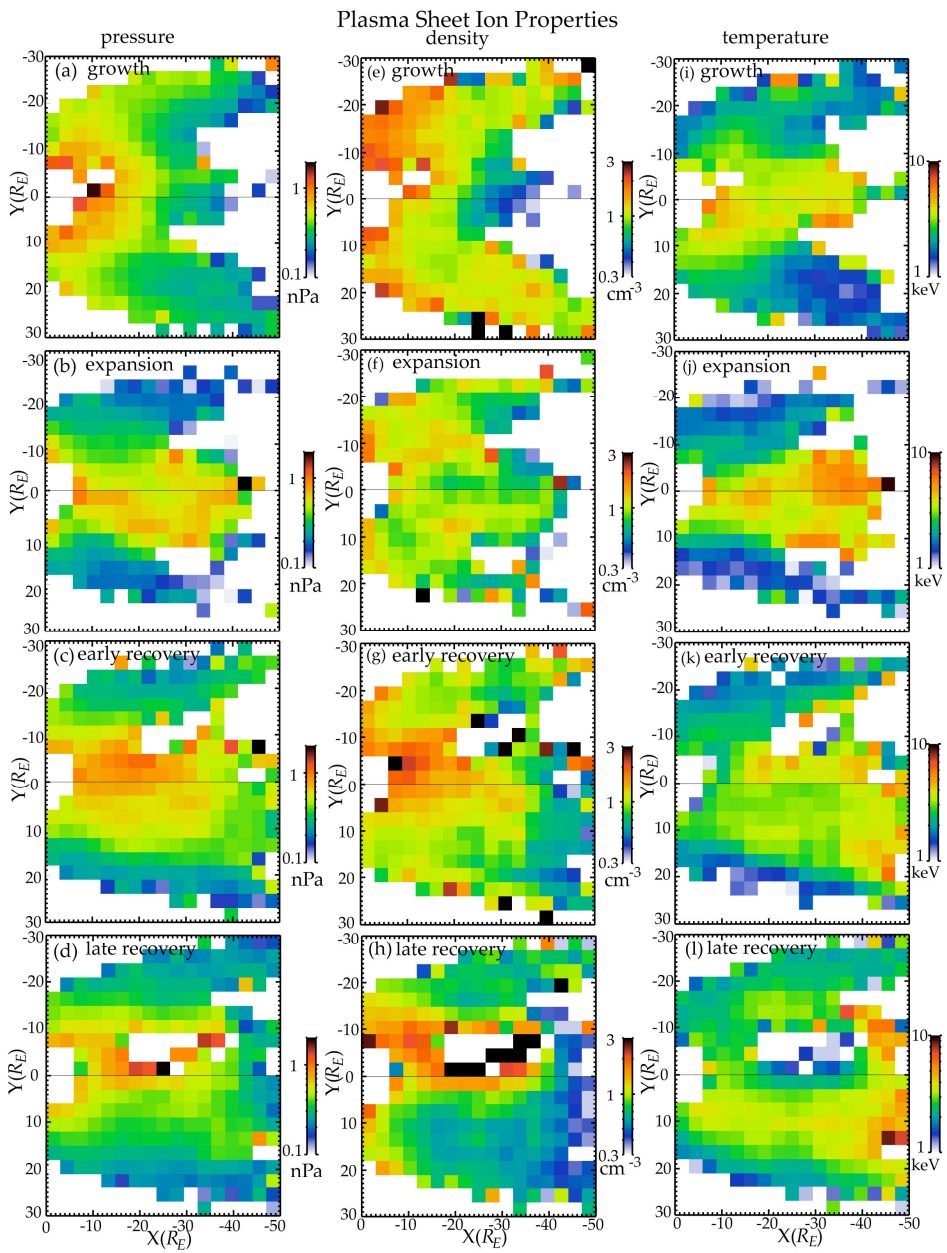
- Northward Turnings
  - > 22 min southward
  - Rapid
  - Sustained
  - Hsu & McPherron, 2003
  - Morley & Freeman, 2007
- Internal
  - IMF southward IMF

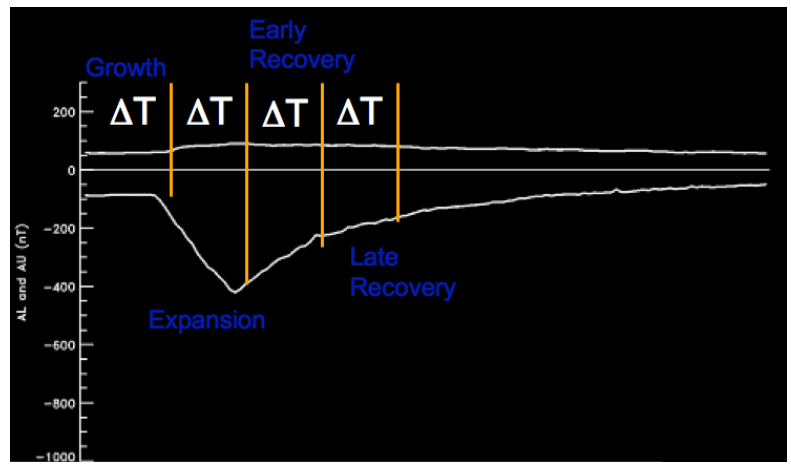




Plasma Sheet Pressure, Birkeland Currents, Ion Outflows

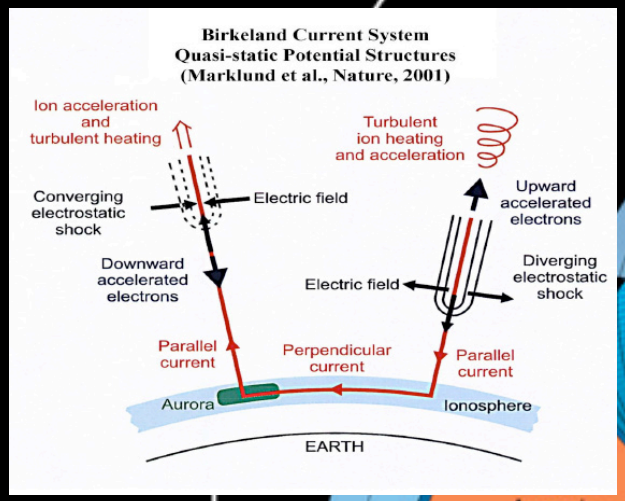


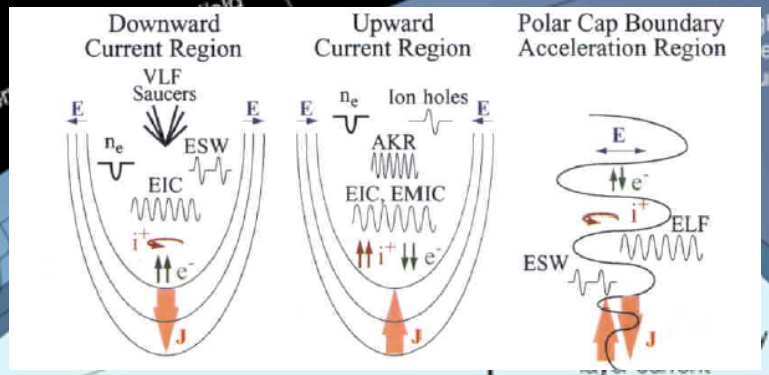

Plasma Sheet Ion Properties
pressure
density
temperature
(a) growth
(b) expansion
(c) early recovery
(d) late recovery
(e) growth
(f) expansion
(g) early recovery
(h) late recovery
(i) growth
(j) expansion
(k) early recovery
(l) late recovery
Y(R_E)
X(R_E)
nPa
cm^{-3}
keV

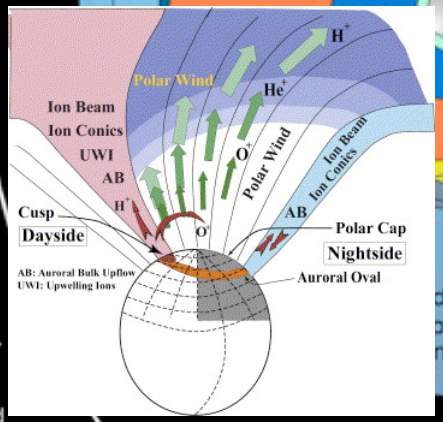

Growth
Early Recovery
ΔT
ΔT
ΔT
ΔT
Expansion
Late Recovery
AL and AU (nT)

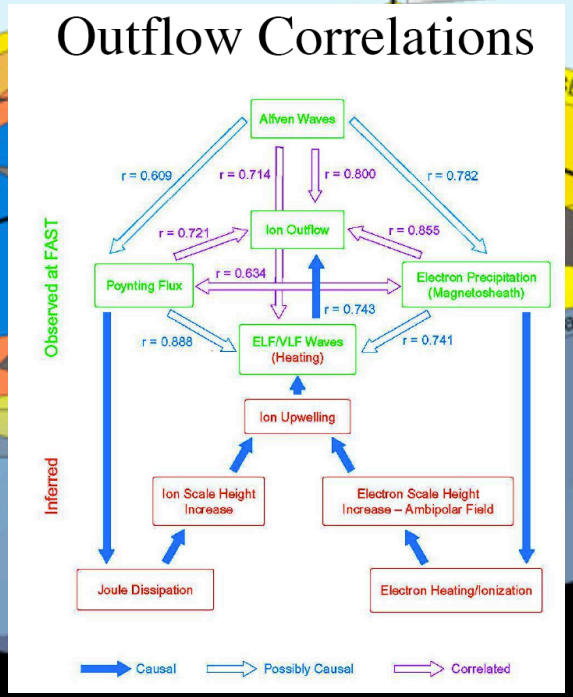
Wave–Particle Field Aligned Acceleration/Ion Outflow

Particle acceleration



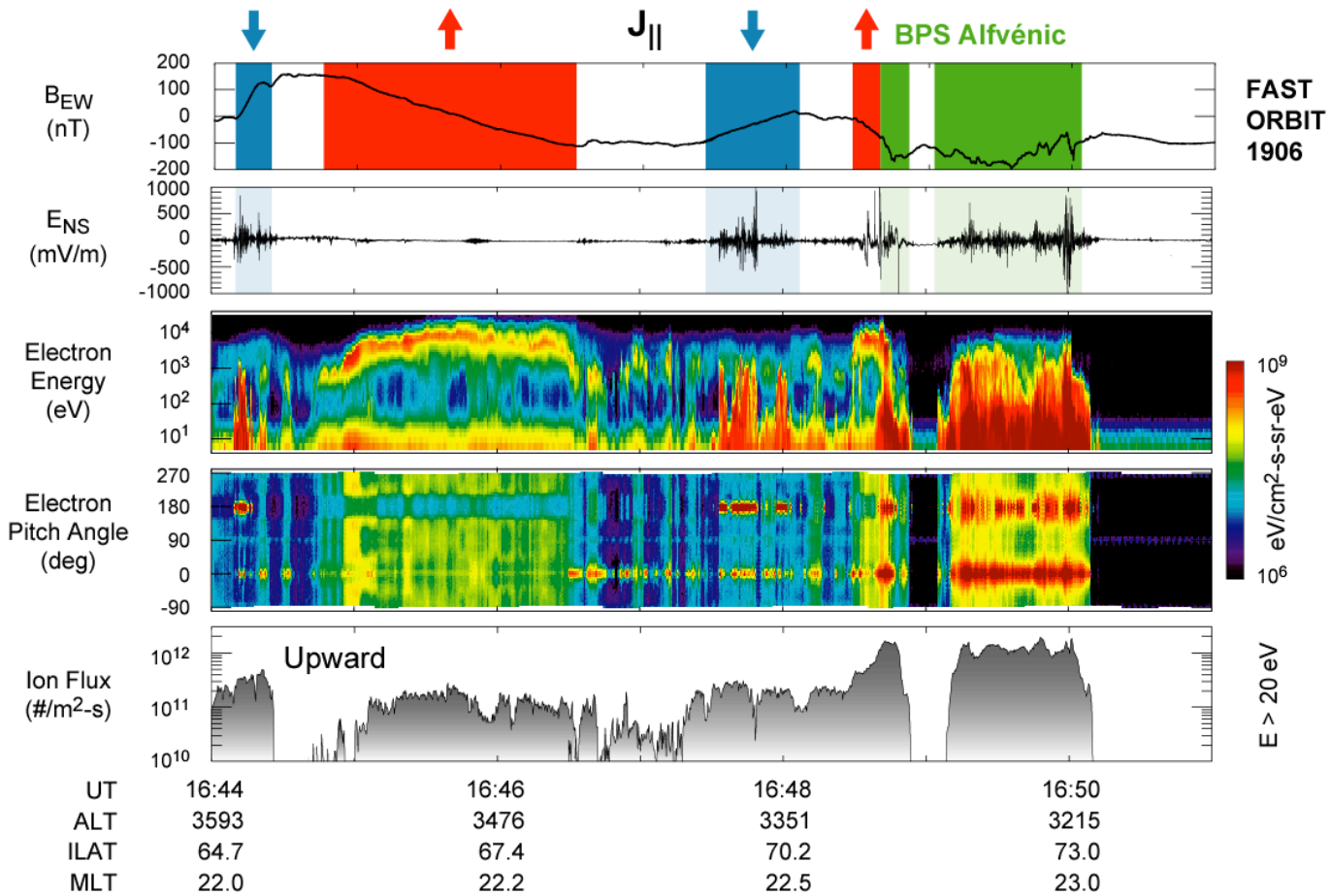






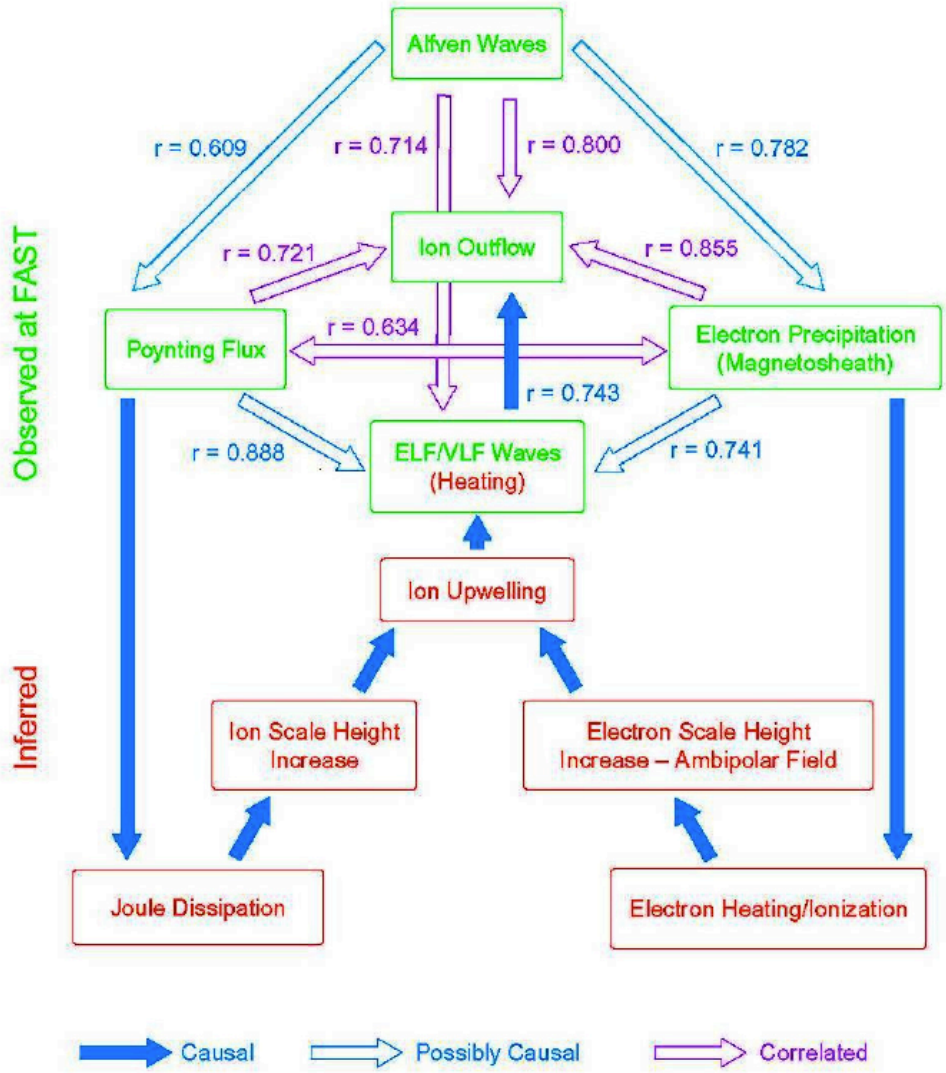
Ion Outflows

Three Regions of Auroral Acceleration



Paschmann et al. '03

Outflow Correlations



Robert Strangeway, 2005

Future Activities

- Apply MI/Cumulant analysis to
  - discover nonlinear dependencies
  - determine information horizon
  - obtain a coupling function
- Apply Transfer Entropy to Examine Causality
  - pressure/currents/outflows
  - waves/outflows
  - substorm dynamics