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# EQUATORIALLY GENERATED ULF WAVES AS A SOURCE FOR THE TURBULENCE ASSOCIATED WITH ION CONICS

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*Abstract.* Low frequency turbulence present on closed field lines in the central plasma sheet has been used to explain ion heating and conic formation with remarkable success. However, the source of the turbulence has yet to be established, and there are no obvious local sources which could power such a broadband spectrum. Alternatively, observations reveal that ion distributions in the equatorial region are often anisotropic, and such distributions excite both ion cyclotron waves below the proton gyrofrequency and waves near the harmonics of the hydrogen cyclotron frequency. As these waves propagate to lower altitudes where the magnetic field is stronger, their left-hand circularly polarized component resonates with heavy ions. The presence of a parallel gradient in the magnetic field complicates the details of wave propagation so that downcoming right-hand circularly polarized waves, which acquire a left-hand circularly polarized component at the crossover frequency, may tunnel through a "stop zone" to altitudes where they contribute to the observed ion heating.

## 1. Introduction

Ion conics at auroral altitudes ( < a few $R_E$) in the central plasma sheet (CPS) are often accompanied by a broadband (turbulent) electric field spectrum which can account for their existence [*Chang et al.*, 1986; *Retterer et al.*, 1987; *Crew and Chang*, 1988]. In this theory of conic formation, oxygen ions are energized as the result of resonant cyclotron heating in the mirror geometry of the Earth's magnetic field. Although the turbulence and conic form are well correlated for the events studied to date [*Crew et al.*, 1989], the origin and nature of the turbulence remains uncertain.

Naturally, consideration should be given to possible local sources for the spectrum, but as yet the lack of any meaningful correspondence between the conics and the sparsely observed currents and/or electron beams suggests that a more global approach is required [*Gurnett et al.*, 1984; *Chang et al.*, 1986]. Although much free energy exists in various regions of the magnetosphere, in this paper we consider, in particular, the possibility of an equatorial source.

## 2. Equatorially Generated Waves

In the equatorial region near geosynchronous orbit on the nightside, anisotropic ion distributions are often observed [*Perraut et al.*, 1982]. As ions are injected earthward from the magnetotail, they drift adiabatically about the earth along drift shells. Because the shell on which a particle is constrained to move depends on the equatorial pitch angle, initially isotropic distribution functions tend to develop thermal anisotropies ($T_\perp > T_\parallel$) and acquire anomalous loss cones (enlarged over the 3° atmospheric loss cone) [*Ashour-Abdalla and Cowley*, 1974; *Sibeck et al.*, 1987]. In addition, the observations reveal that these distributions often contain a large energetic population with 90° pitch angle [*Perraut et al.*, 1982]. Below we consider how anisotropic ion distributions can generate waves with frequencies near $f_{cH^+}^{eq}$ in the equatorial region and how these waves may reach the CPS where their frequency matches the local oxygen gyrofrequency ($f_{cO^+}$) over altitudes ranging from the upper ionosphere to several $R_E$ .

*Waves with $f < f_{cH^+}^{eq}$.* Ion distributions with loss-cones or temperature anisotropies can excite waves below the proton gyrofrequency. Indeed, this instability has been studied comprehensively in the equatorial region in connection with Pc1 ($0.2 < f < 5$ Hz) wave generation and helium heating [*Young et al.*, 1981; *Roux et al.*, 1982; *Oscarsson and André*, 1986]. Waves generated with frequencies below the helium gyrofrequency (typically < 1 Hz) are guided along field lines to the ground [*Rauch and Roux*, 1982; *Perraut et al.*, 1984]. On the other hand, it is believed that waves generated above the helium gyrofrequency but below the hydrogen cyclotron frequency ($1 < f < 5$ Hz), rather than penetrating directly to the ground, travel between geomagnetic conjugate points near the magnetic equator which correspond to the ion-ion or Buchsbaum frequency. Although the growth rates for these waves are smaller than those for the waves near the helium gyrofrequency, the power in the waves may be comparable because they amplify as they bounce back and forth through the equatorial plane [*Rauch and Roux*, 1982]. Furthermore, when the concentration of helium is low enough, these waves tunnel through the reflection layer to lower altitudes in the right-hand circularly polarized (RHCP) mode (see Figure 6 of *Perraut et al.*, [1984]). In fact, ground based instruments often detect both types of waves—an observation which strongly suggests that they contribute to the auroral turbulence [*Perraut et al.*, 1984].

*Waves with $f > f_{cH^+}^{eq}$.* Although propagating waves generated below $f_{cH^+}^{eq}$ may explain a significant part of the wave power observed in the ion heating region, there is considerable wave power above 5 Hz. Because some of the heating takes place at lower altitudes where the local oxygen cyclotron frequency is larger than Pc1 frequencies it is important to consider the generation and propagation of waves with $f > f_{cH^+}^{eq}$ in the equatorial region.

Observations indicate that ion distributions with loss-cone character ($\partial F/\partial v_\perp > 0$) can generate waves at multiples of

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$f_{cH^+}^{eq}$ [*Perraut et al.*, 1982]. Moreover, the calculations of *Perraut et al.* [1982], which modeled the hot ion population with a ring distribution and only examined the solutions of the dispersion relation with $k_\parallel = 0$, were used to explain the general characteristics of the observations. Their results, however, predict a group velocity perpendicular to the magnetic field which would imply that the waves remain in the equatorial region.

On the other hand, we have solved the dispersion relation of linear waves using the somewhat more realistic model used by the WHAMP code [*Rönnmark*, 1982, 1983]. In particular, we have included a thermal spread in the hot ion distribution function which we have modeled with subtracted Maxwellians using the parameters listed in Table 1. The distribution consists of a cold background plasma with a small energetic proton population which peaks in the perpendicular direction at 7 keV.

We find that in addition to the electrostatic modes with $k_\parallel = 0$ there exists a broadband of unstable electromagnetic magnetosonic waves with $k_\parallel \neq 0$ and $v_{g\parallel}/v_{g\perp} \sim 1$ (Figure 1). Although the growth rates for the $k_\parallel = 0$ modes are somewhat larger, the waves with $k_\parallel \neq 0$ excite a much larger region of k-space and may compose much of the noise which accompanies the harmonic spectrum. Moreover, the fact that $v_{g\parallel}/v_{g\perp} \sim 1$ implies that these waves initially propagate out of the equatorial region. Increasing the temperature and density of the energetic population increases the growth rates as one would expect, but for sufficiently warm background electrons or ions, the $k_\parallel \neq 0$ modes become damped.

If we compare the Poynting fluxes in the equatorial and auroral waves using the observed wave amplitudes (assuming that the auroral waves are Alfvén waves and that the equatorial waves obey the dispersion relation obtained from WHAMP) we find that they are comparable. Although detailed ray tracing calculations will be required to confirm the notion that the auroral turbulence originates in the equatorial plane, this simple comparison elicits some confidence.

Table 1. Plasma Model ($B$ = 0.001 G)

| Species | $n$ (cm$^{-3}$) | $T_\parallel$(keV) | $T_\perp$(keV) |
|---|---|---|---|
| cold $e^-$ | 66.6 | 0.001 | 0.001 |
| cold $H^+$ | 65.0 | 0.001 | 0.001 |
| hot $H^+$ | 2.3 | 10.0 | 10.0 |
| | 0.7 | 10.0 | 3.0 |

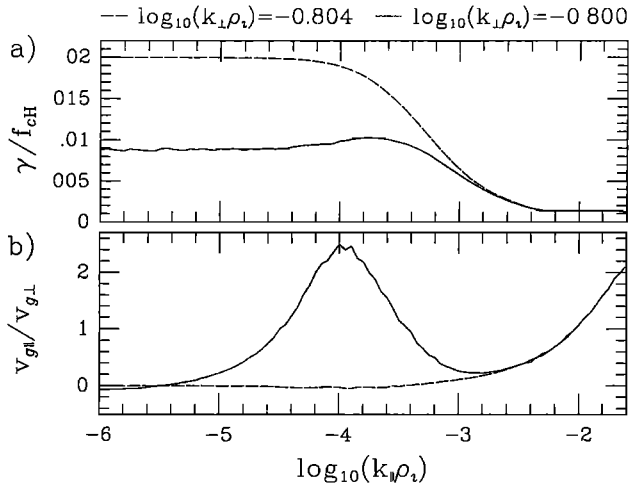


Fig. 1. Growth rates (a) and group velocity ratio $v_{g\parallel}/v_{g\perp}$ (b) for a plasma with parameters listed in Table 1 taken at constant $k_\perp\rho_i$ where $\rho_i$ is the ion gyroradius. The dashed curve corresponds to maximum growth at $k_\parallel$=0 for electrostatic modes near $f = 4f_{cH^+}$ while the solid curve corresponds to modes slightly offset from maximum growth at $k_\parallel$=0 and for larger $k_\parallel$ may be classified as magnetosonic.

## 3. Wave Propagation

Having identified a likely source for the turbulence, we now turn our attention toward the nature of the observed spectrum. Because only the left-hand circularly polarized (LHCP) component of the waves resonates with the ions, the waves must contain a non-negligible fraction of LHCP at the local oxygen cyclotron frequency for heating to occur. However, for a cold plasma the branch which is RHCP for $k_\perp = 0$ contains no LHCP at the ion gyrofrequency even for $k_\perp \neq 0$. Furthermore, waves generated with $f > f_{cH^+}$ are most likely RHCP because the LHCP branch is cut off above $f_{cH^+}$. In order to heat ions, these waves must acquire some fraction of LHCP as they propagate to lower altitudes. Such a transmutation of wave polarization may occur via mode conversion and tunneling.

In order to understand how RHCP waves might gain LHCP as they propagate to auroral altitudes, we consider linear theory for an idealized model consisting of a two ion component plasma which has the dispersion relation shown in Figure 2. Because wavelengths are very long and the density does not vary appreciably [*Moore et al.*, 1987], we assume that the plasma is cold with no variation in density. Furthermore, we assume that the magnetic field varies slowly along itself and that the wavelength is much smaller than $L_B \equiv [d\ln(B)/dz]^{-1}$. This assumption reduces the problem to one dimension and fixes $k_\perp$.

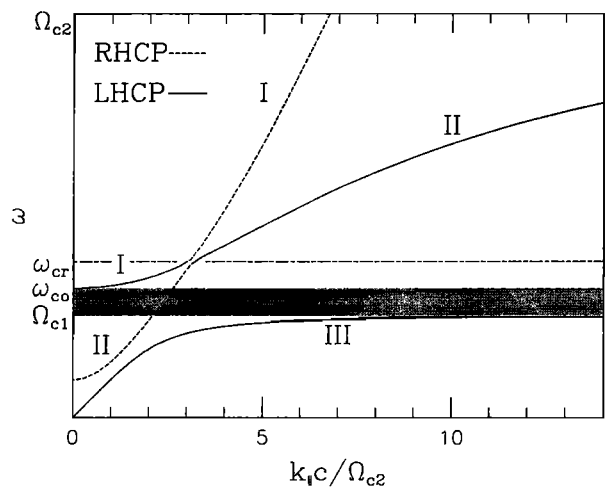


Fig. 2. Dispersion relation for a cold two ion component plasma for $k_\perp c/\Omega_{c2} = 0.1$.

In the ensuing analysis we consider a particular scenario whereby waves generated with $f > f_{cH^+}^{eq}$ propagate earthward. The wave initially propagates downward in the RHCP (magnetosonic) mode (I in Figure 2). At the crossover frequency $[\omega_{cr}^2 = (n_1\Omega_{c2}^2+n_2\Omega_{c1}^2)/(n_1+n_2)]$, where linear theory predicts that the predominate polarization of modes I and II reverses, mode coupling may occur [*Gurnett et al.*, 1965]. As the modes couple, some fraction of the wave power will be transferred to mode II and will remain predominately RHCP. For an idealized, cold plasma, mode II contains no LHCP at the heavy ion gyrofrequency ($\Omega_{c1}$), and thus, this transferred wave power will not heat ions. Nevertheless, the rest of the wave power remains in mode I, which becomes predominately LHCP for $\omega < \omega_{cr}$. As the mode I wave propagates to lower altitudes, it encounters the cutoff frequency $[\omega_{co} = (n_1\Omega_{c2}+n_2\Omega_{c1})/(n_1+n_2)]$ where the wave is reflected. However, some of the wave power may tunnel through the evanescent layer to mode III where the waves can resonate with the ions. Below, we calculate the coupling ($C$), transmission ($T$), and reflection ($R$) coefficients where $C$ is the fraction of the wave power that remains in mode I as the wave passes through $\omega_{cr}$, $T$ is the fraction of wave power that tunnels from mode I to mode III at $\omega_{co}$, and $R$ is the fraction of wave power that is reflected back along mode I at $\omega_{co}$.

To proceed with the analysis, Maxwell's equations combined with the continuity and momentum transfer equations yield differential equations for the electric field which for near parallel propagation are coupled by the parameter $\varepsilon \equiv k_\perp^2 c^2/2\omega^2 \ll 1$. Because the waves that we are considering have $E_\parallel \simeq 0$ [*Stix*, 1962, §2.5], the equations for the electric field components ($E_\pm = E_x \pm iE_y$) take the form

$$\left[\frac{d^2}{dz^2} + \begin{bmatrix} R-\varepsilon & \varepsilon \\ \varepsilon & L-\varepsilon \end{bmatrix}\right]\begin{bmatrix} E_- \\ E_+ \end{bmatrix} = 0 \qquad (1)$$

where $R$ and $L$ are defined by *Stix* [1962, §1.2].

*Coupling at the Crossover Frequency.* Because the functions $R$ and $L$ vary slowly in altitude, we may apply the WKB analysis and solve for the electric field perturbatively in $\varepsilon$. The coupling coefficient involves a phase integral over an oscillating function which becomes stationary at $\omega = \omega_{cr}$ so that we may evaluate the magnitude of the coupling using the method of stationary phase. Comparing the Poynting's flux of the incident wave above the coupling region (mode I, RHCP) to that of the coupled wave below the coupling region (mode I, LHCP) we obtain the coupling coefficient

$$C \equiv \left|\frac{\vec{S}_{below}}{\vec{S}_{above}}\right| = -\frac{\pi}{2}\frac{\varepsilon^2}{\sqrt{R}D'}\bigg|_{\omega=\omega_{cr}} \sim \frac{k_\perp^4}{k_\parallel^4} L_B\, k_\parallel \qquad (2)$$

$$D'(\omega=\omega_{cr}) = -\frac{2c\,\omega_{cr}^2}{L_B}\frac{(\omega_{p1}^2/\Omega_{c1}+\omega_{p2}^2/\Omega_{c2})}{(\Omega_{c1}^2-\Omega_{c2}^2)^2}\frac{(n_1+n_2)^2}{n_1 n_2} \qquad (3)$$

Thus, although the dispersion curves appear to be continuous and separate for modes I and II, the presence of a gradient does introduce coupling. At small angles of propagation most of the wave power is transferred from mode I to II in which case the waves remain RHCP, while at larger angles of propagation much of the wave power remains in mode I, which is predominately LHCP below $\omega_{cr}$.

*Tunneling Through the Evanescent Layer.* Once the downcoming RHCP wave has gained some LHCP at the crossover frequency, the wave encounters a cutoff-resonance pair of frequencies ($\omega_{co}$, $\Omega_{c1}$). Because intermode coupling at this point is small, we neglect $\varepsilon$ and consider only the lower component of equations (1) which may be written

$$\frac{d^2E_+}{dz^2} = \frac{\omega^2}{v_A{}^2}\frac{\Omega_{c1}\Omega_{c2}}{\omega_{co}}\frac{\omega-\omega_{co}}{(\omega-\Omega_{c1})(\omega-\Omega_{c2})}E_+ \qquad (4)$$

where $v_A$ is the Alfvén velocity. If we linearize about $\Omega_{c1}$ we find that this equation is of the Budden form [*Budden*, 1985, §19.6], so that $T = \exp(-\pi\beta)$ and $R = [1-\exp(-\pi\beta)]^2$, where $\beta = \Omega_{c1}L_B(1-m_2/m_1)^{1/2} / v_A(1+n_2m_2/n_1m_1)$. As is characteristic of Budden Tunneling $R+T \neq 1$, and the missing wave flux is absorbed by the medium. The absorption typically occurs because wave power is converted to a thermal branch which heats the ions [*Budden*, 1985, §19.5].

*Numerical Results.* We consider an $O^+$–$H^+$ plasma in the auroral region at 2 $R_E$ along a field line with invariant latitude 60°. If we take parameters typical of this region: $n$ = 100cm$^{-3}$, $B$ = 0.05 G, $f_{cH^+}$ = 80 Hz, $f_{cO^+}$ = 5 Hz, $L_B$ = 1 $R_E$, we find that for sufficiently low concentrations of $O^+$ (less than a few percent of the total ion population) a few percent LHCP is obtained at the heavy ion gyrofrequency via mode conversion and tunneling as indicated in Table 2. These results are consistent with ion heating theory, which required only a small fraction of the waves to be LHCP [*Retterer et al.*, 1987].

Table 2. Transmission and Coupling Coefficients

| $n_{O^+}/n_{H^+}$ | $\tan^{-1}(k_\perp/k_\parallel)$ | $C$ | $T$ | $R$ |
|---|---|---|---|---|
| 0.05 | 18° | 10% | 0 | 100% |
| 0.01 | 27° | 10% | 18% | 67% |

## 4. Discussion

In this paper we have identified a plausible source region for the turbulence associated with CPS ion conics. It is evident that equatorially generated low frequency ($f < f_{cH^+}^{eq}$) waves do propagate to low altitudes where they may contain sufficient LHCP to heat the ions. In addition, we have determined that waves at higher frequencies may also be generated in the equatorial region, where they initially propagate earthward. Careful ray tracing will demonstrate whether these waves propagate to auroral altitudes.

The high frequency waves ($f > f_{cH^+}^{eq}$) are important because they may be responsible for extracting oxygen from the ionosphere where $f_{cO^+} > f_{cH^+}^{eq}$. Although the wave power at high frequencies provides a resonant mechanism whereby a small fraction of the ionospheric ions may be accelerated to magnetospheric energies, at low altitudes where the oxygen cyclotron frequency is increased, the power available to heat the ions is diminished by several orders of magnitude. On the other hand, the spectral density seems to be peaked at the lowest frequencies. This fact elicits the question of whether at some altitude, non-resonant heating, resulting

from small fluctuations in the fields, might become important [*Ichimaru*, 1975; *Lundin and Hultqvist*, 1989].

We have also demonstrated that the effects of mode conversion and tunneling become significant in the presence of parallel magnetic field gradients. In the special case that we considered, we found that at the crossover frequency, power in the RHCP branch may be transferred to the LHCP branch, and at lower altitudes, the waves tunnel through an evanescent layer and resonate with the ions. Of course, a similar analysis would apply to waves generated in the LHCP mode. Furthermore, the addition of another ion species would complicate wave propagation further and introduce another crossover frequency and resonance.

Consideration should also be given to other effects which might affect wave propagation and heating. Although the inclusion of finite temperature should not affect the crossover coupling, it will affect the resonance-cutoff tunneling by introducing higher order derivatives which become important at the singularity of the differential equation (4). In addition, nonlinear coupling of the turbulence may allow more heating to take place near the gyrofrequency.

In conclusion, the notion that the CPS turbulence arises in the equatorial region appears to be a very promising possibility, and the details of wave propagation from this region to auroral altitudes will be the subject of further study.

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