SM33E-07

An Information Theoretical Approach to Solar Flare Occurrence

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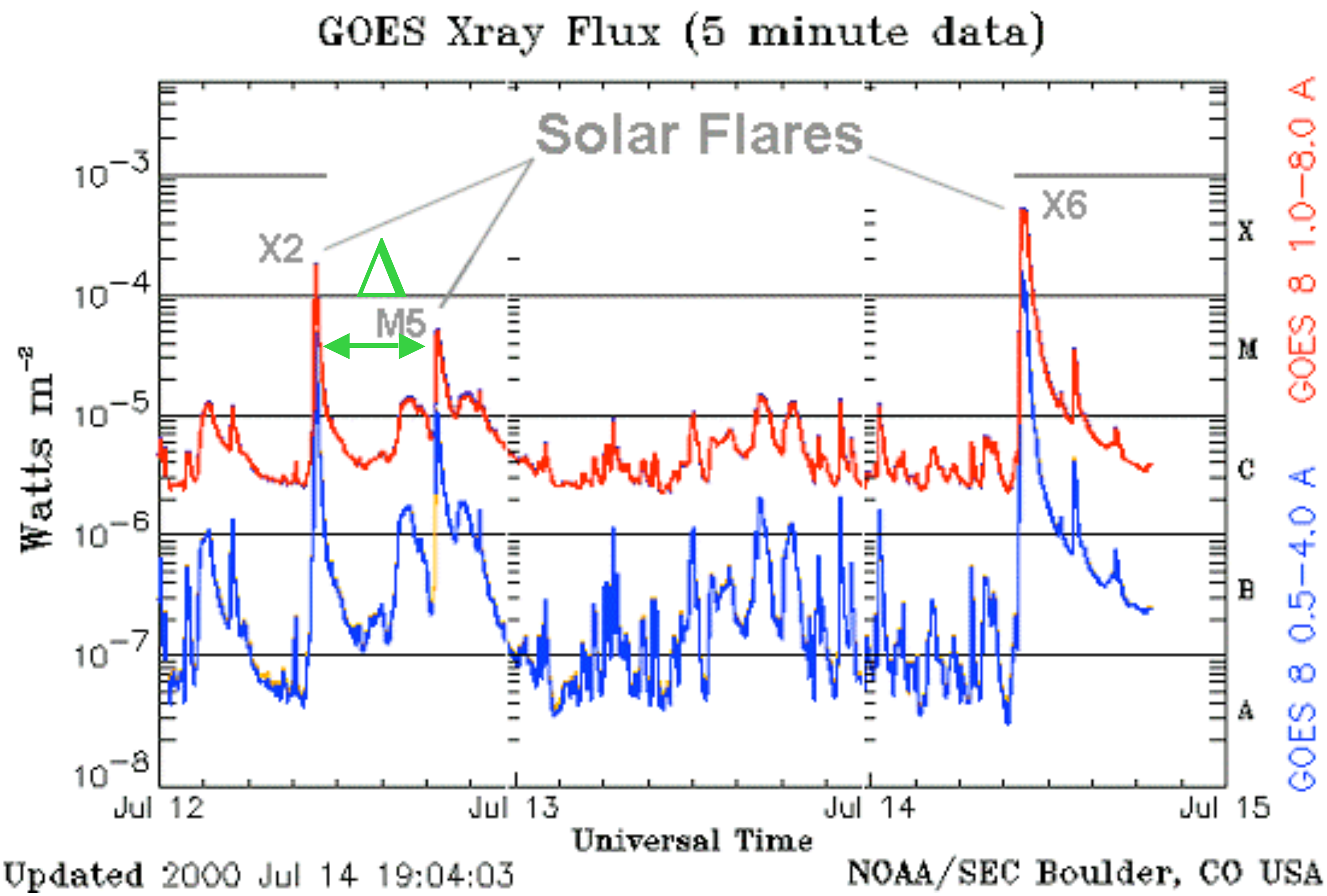
Flare Occurrence and Waiting Time

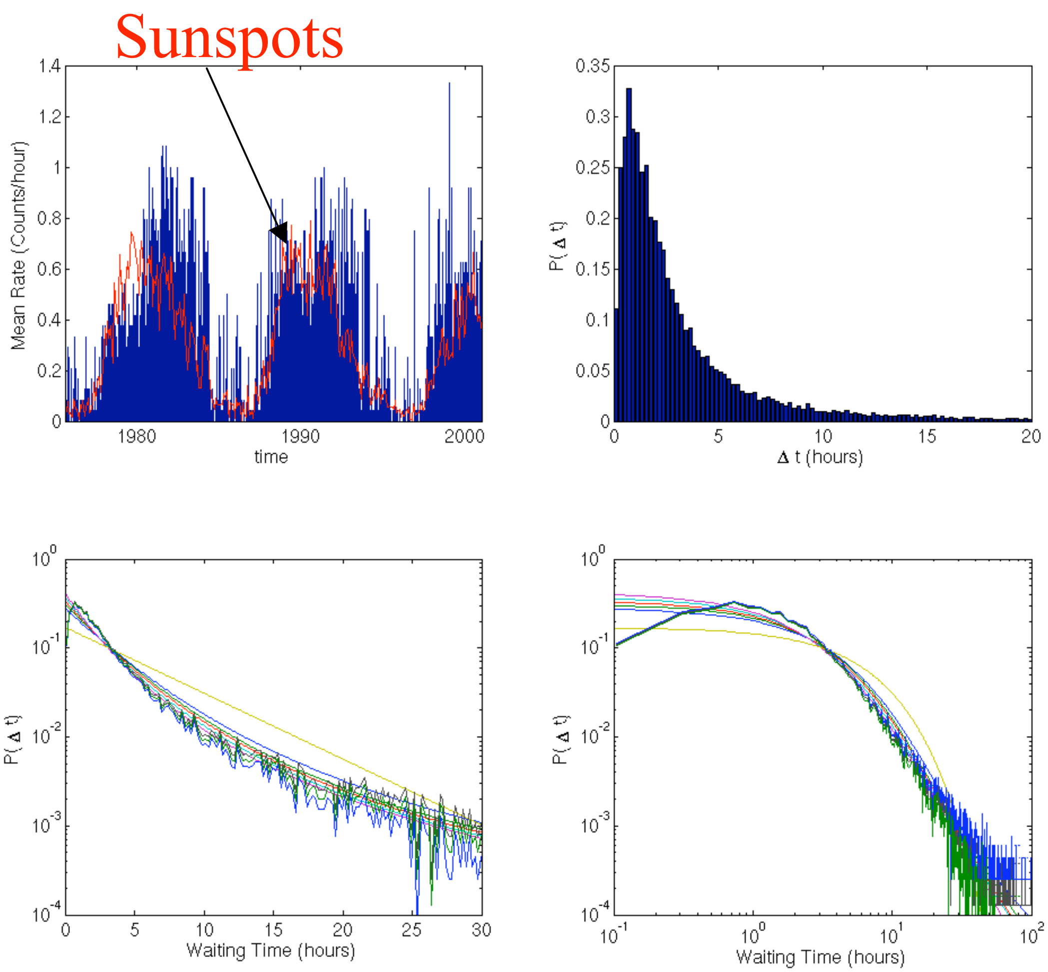


The Underlying Dynamics of Flares

Two paradigms:

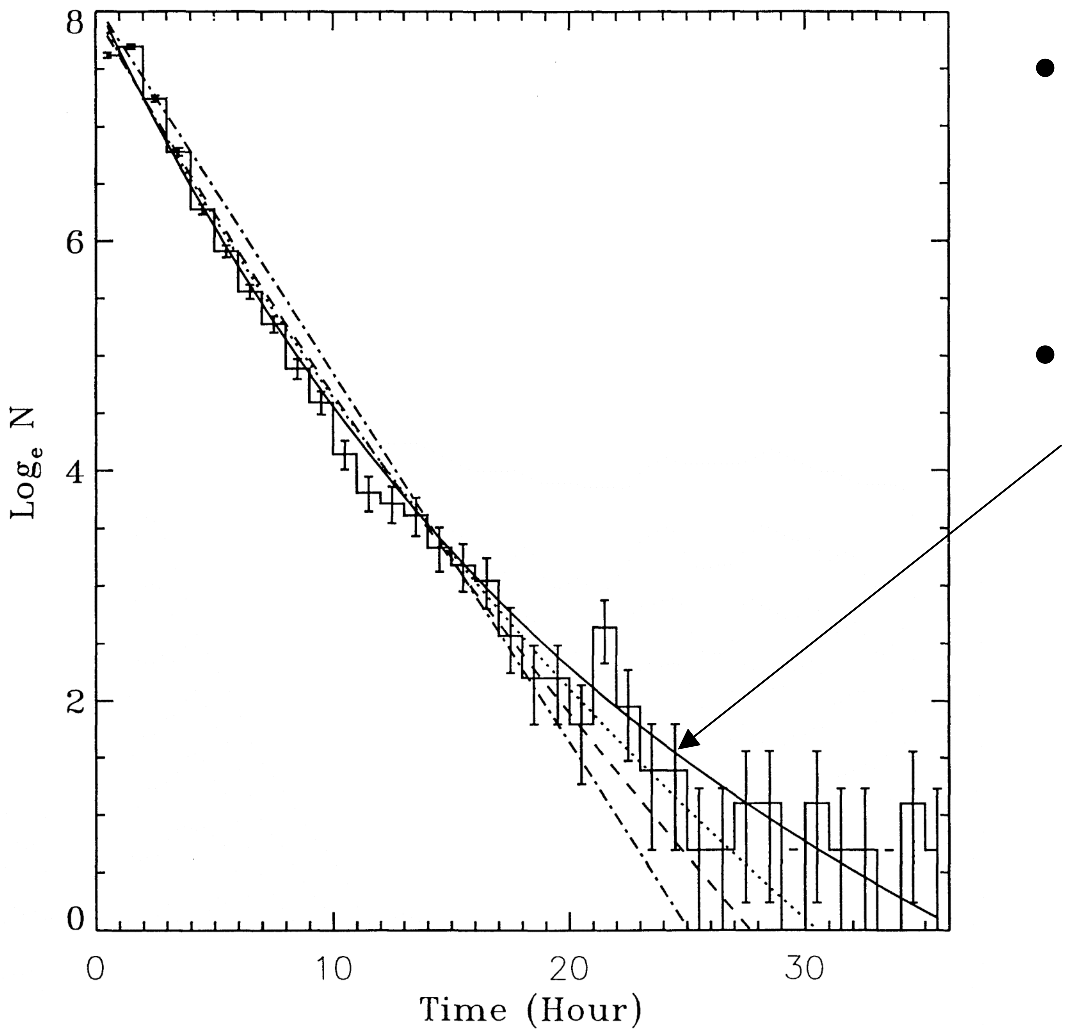
- Multiplicative cascade near marginal stability (SOC)
- Integrate and release catastrophic process

Flare Occurrence (>B class)



- Variable Rate
- Redactory Time
- Scaling Law

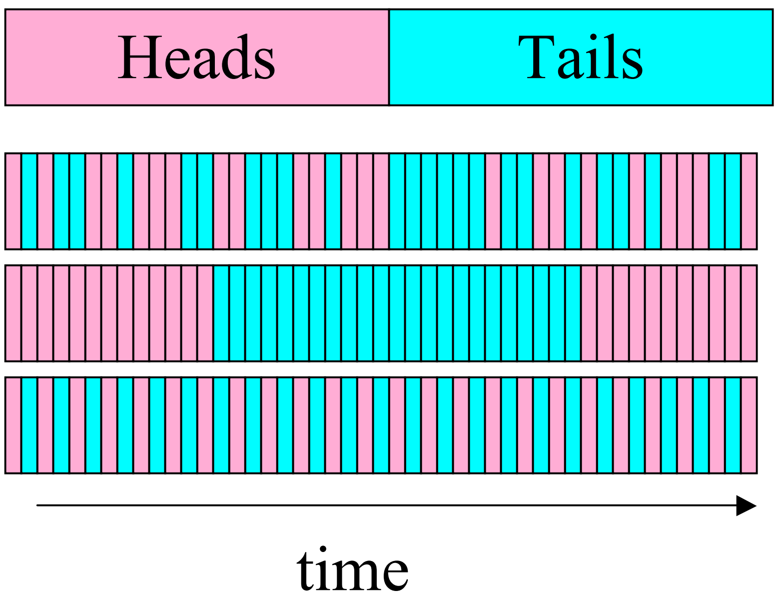
Distribution of Waiting Times



- Nearly exponential distribution suggests a Poisson process
- Distribution is fitted using a nonstationary Poisson process

[Moon et al., 2001]

Evaluating the Dynamics



- Fit of the distribution does not characterize the dynamics
- Example: Coin Flip
- Useful to also consider temporal variations (relations between occurrences and not just occurrences)

Temporal Correlations

- Discriminating Statistic that can be used to determine whether successive flare events are related or if the result from a nonstationary Poisson process

- $P(\Delta_j,\Delta_{j+1})=P(\Delta_j)\ P(\Delta_{j+1})$?

- Mutual Information

Entropy and Mutual Information

$$H(x) = -\sum_{\aleph_1} p(x)\log p(x)$$

$$x \in \{1,\ldots,N\} \equiv \aleph_1$$

$$H(y) = -\sum_{\aleph_2} p(y)\log p(y)$$

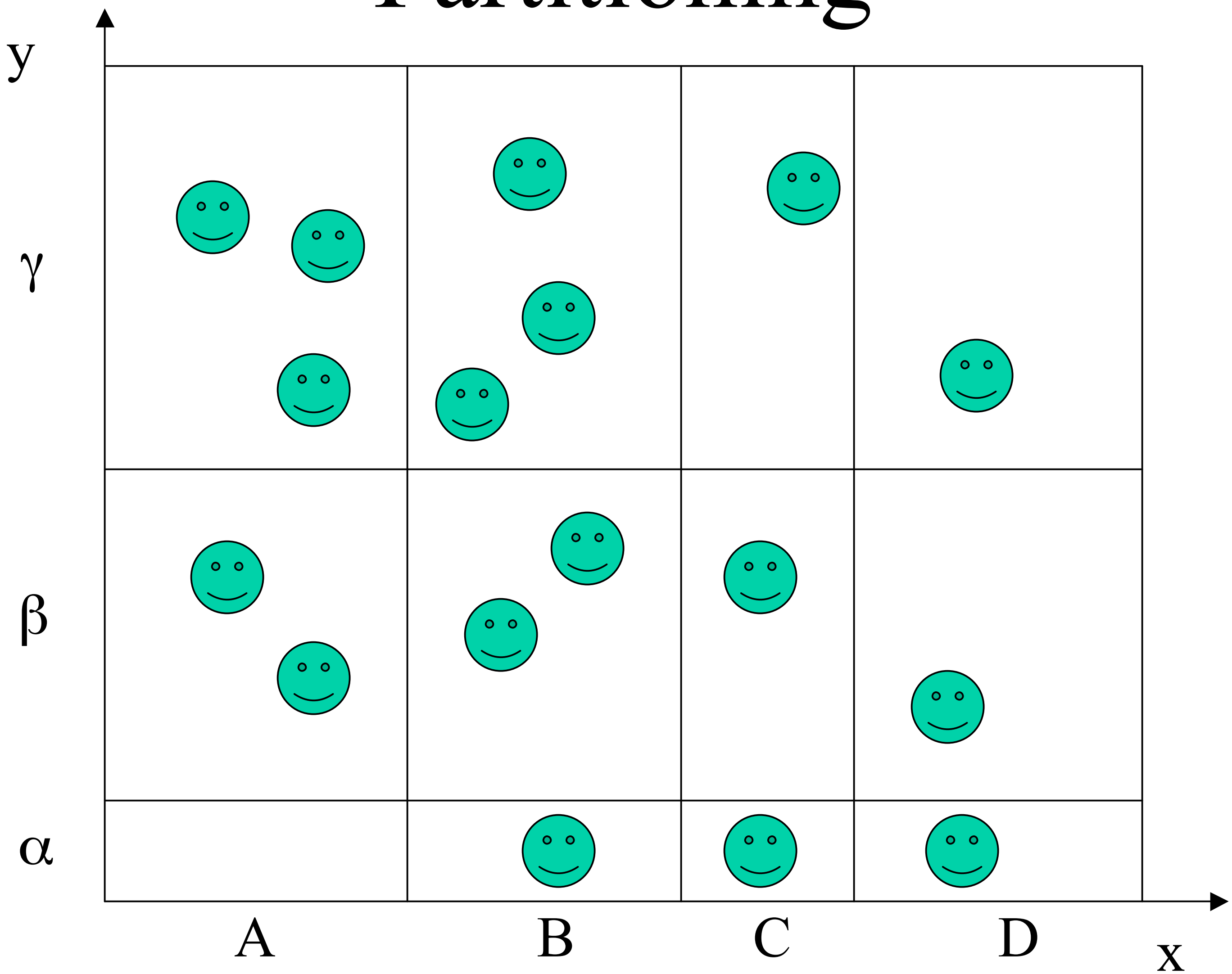
$$y \in \{1,\ldots,M\} \equiv \aleph_2$$

$$H(x,y) = -\sum_{\aleph_1,\aleph_2} p(x,y)\log p(x,y)$$

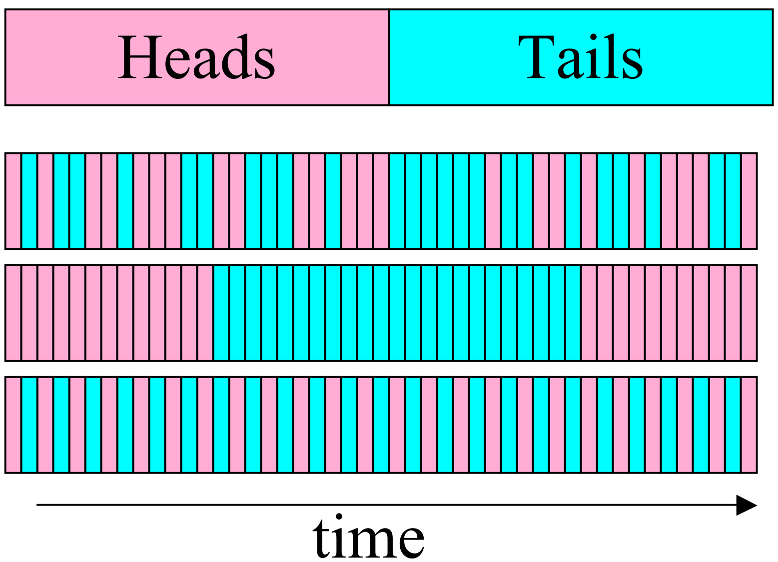
$$I(x,y) = H(x) + H(y) - H(x,y)$$

Mutual information is commonly used as an alternative to correlation functions which have limitations for nonlinear systems. Generalization to higher dimensions is called redundancy.

Partitioning



Evaluating the Dynamics Using Mutual Information



P_h	P_t	P_{hh}	P_{ht}	P_{th}	P_{tt}	I
1/2	1/2	10/46	12/46	12/46	12/46	.004
1/2	1/2	22/46	1/46	1/46	22/46	0.82
1/2	1/2	0	1/2	1/2	0	1.0

Surrogate Data

- Null Hypothesis: Assume a nonstationary Poisson process (Wheatland, 2004; Moon, 2005)

- Surrogate events are constructed based on that hypothesis

Comparison with Surrogate Data

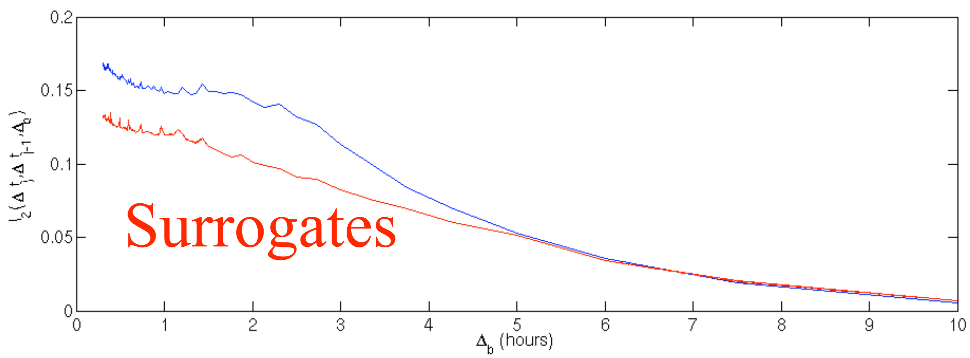
$$S = \frac{| D_0 - \mu_s |}{\sigma_s}$$

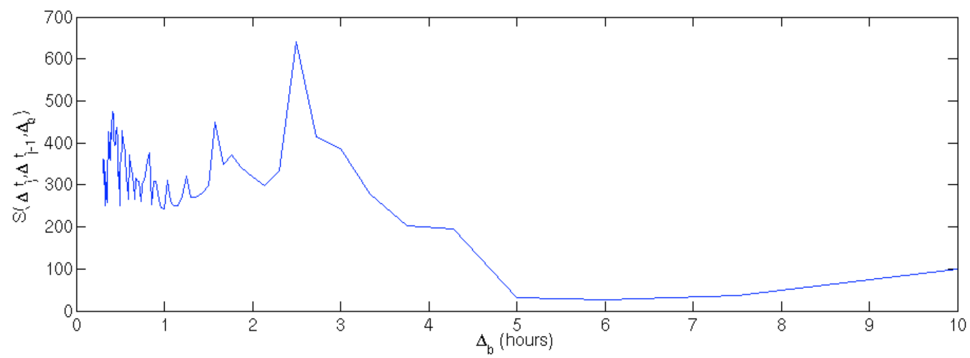
Significance Measured
Relative to a Null Hypothesis

$$\mu_s = \frac{1}{N} \sum_{i=1}^{N} D_{S_i}$$

$$\sigma_s^2 = \frac{1}{N-1} \sum_{i=1}^{N} (D_{S_i} - \mu_s)^2$$

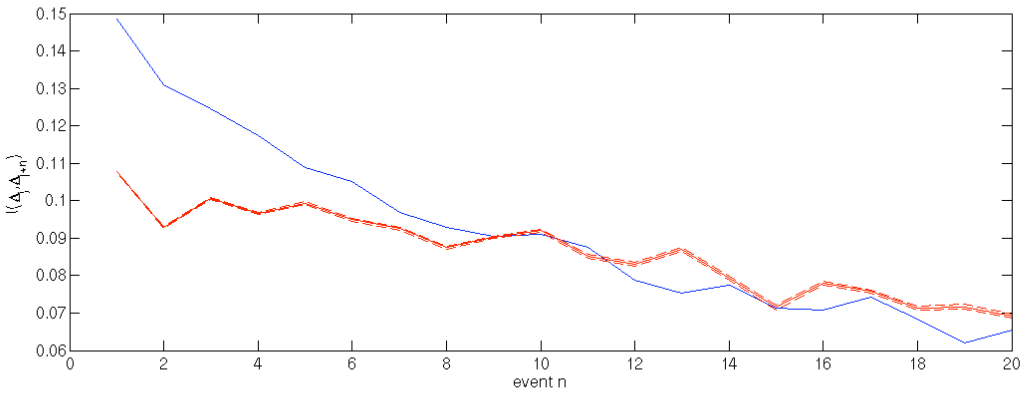
Significant Difference From Nonstationary Poisson Process



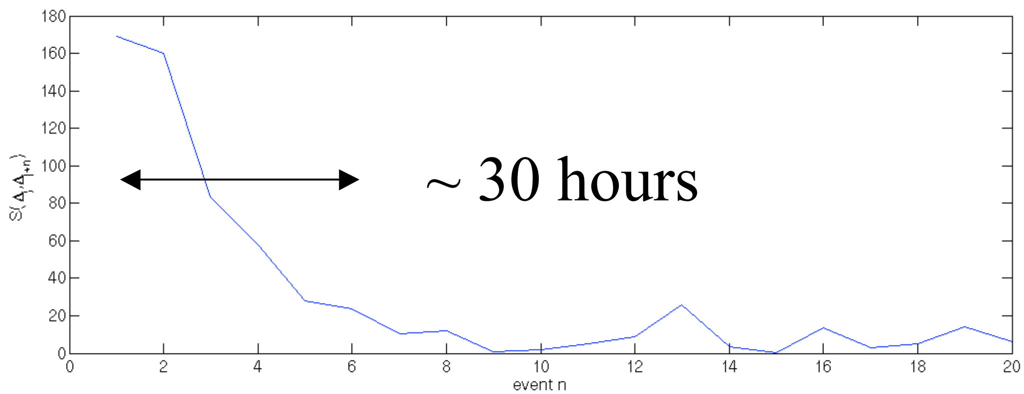


- Mutual Information and Significance as a function of bin width, $\Delta_{b.}$
- Implication is that the waiting time depends on the prior waiting time (not Poisson).

Information Horizon

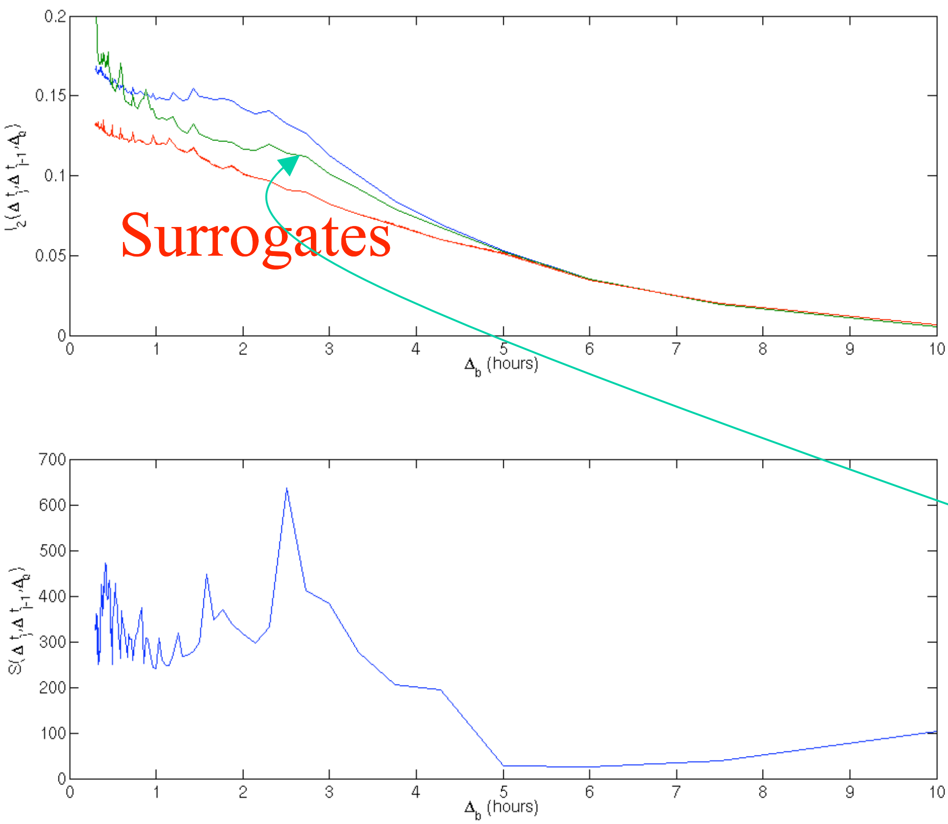


Mutual Information between successive flares $I(\Delta_j, \Delta_{j+n})$.



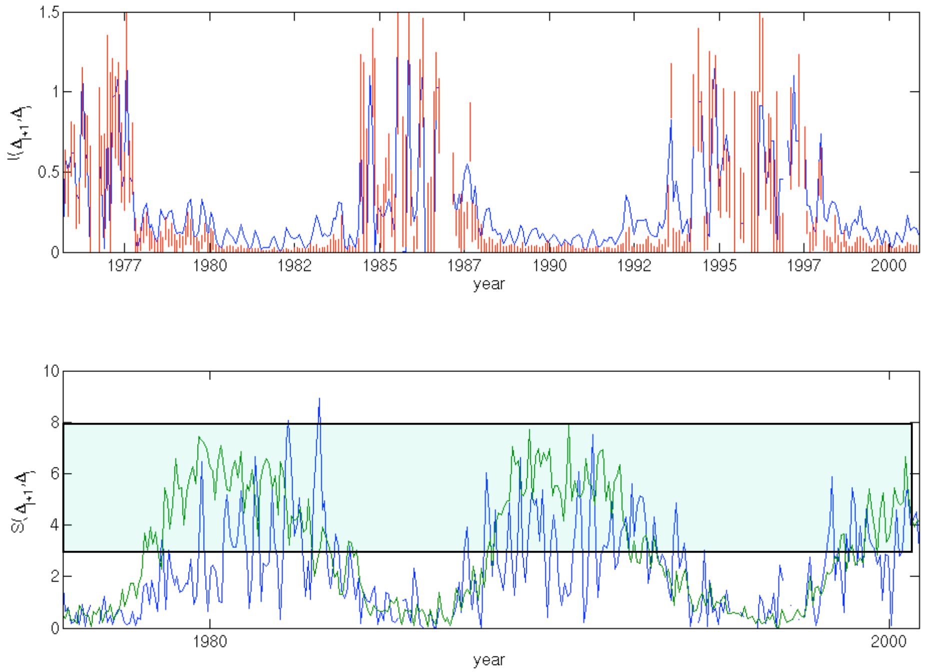
Implication is that the information drops rapidly after 6 flares (~30 hours).

Is the Dependency Periodic?



- Mutual Information and Significance as a function of bin width, $\Delta_{b.}$
- Implication is that the waiting time depends on the prior waiting time (not Poisson).
- Residual Information after periodicity removed

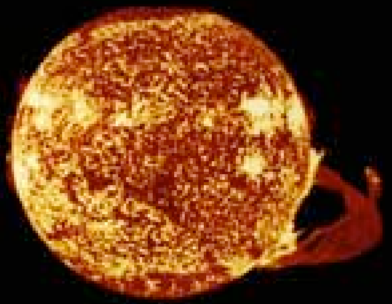
Solar Cycle Dependence



sunspot number

- Nonstationary dynamics
- Significant dependencies during solar maximum
- Statistics not very good near solar minimum

Relation of Waiting Time and Flare Strength



Discriminating Statistic Mutual Information

$$\lambda(\mathbf{X},\mathbf{Y}) \equiv \sqrt{1 - \frac{\det C(\mathbf{X},\mathbf{Y})}{\det C(\mathbf{X}) \det C(\mathbf{Y})}}$$

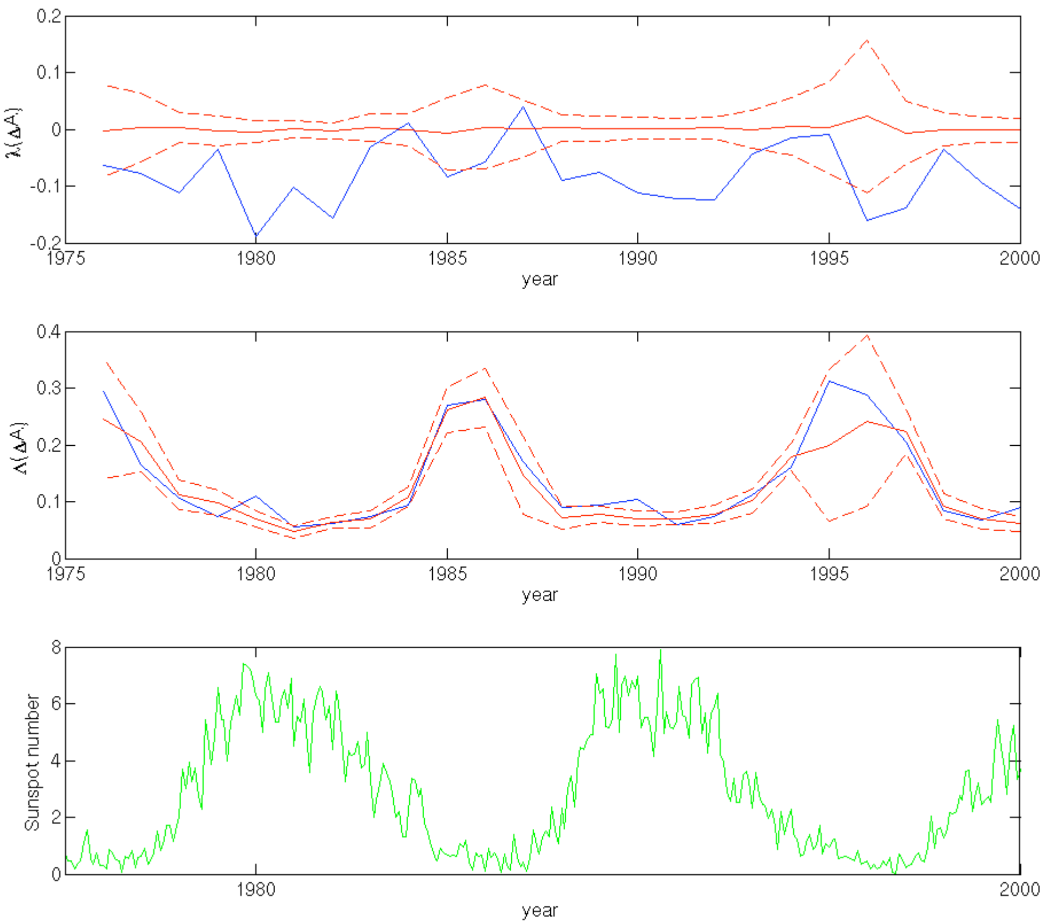
$$\Lambda(\mathbf{X},\mathbf{Y}) \equiv \sqrt{1 - \exp(-2I(\mathbf{X},\mathbf{Y}))}$$

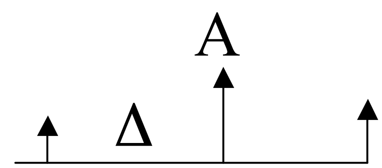
$D_{MI} = \Lambda - \lambda$

$\Lambda = \lambda$

when Gaussian distributed joint PDF

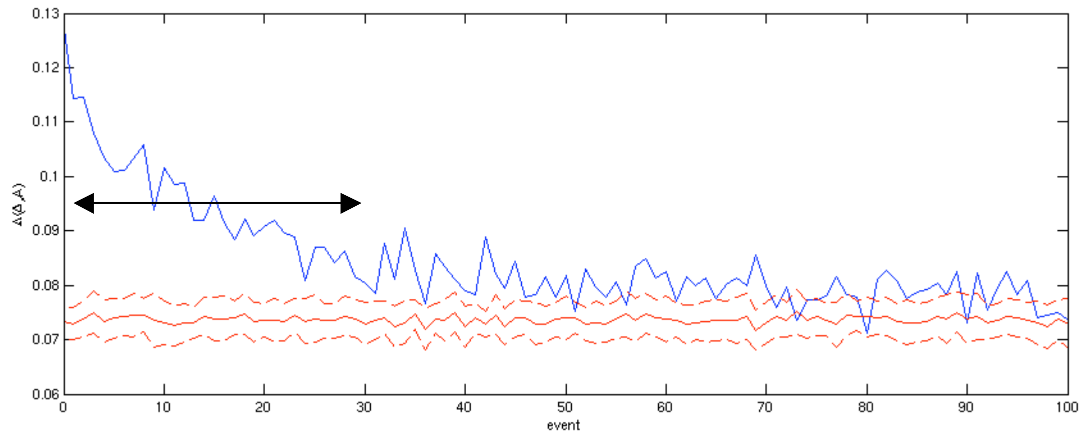
Waiting Time and Amplitude

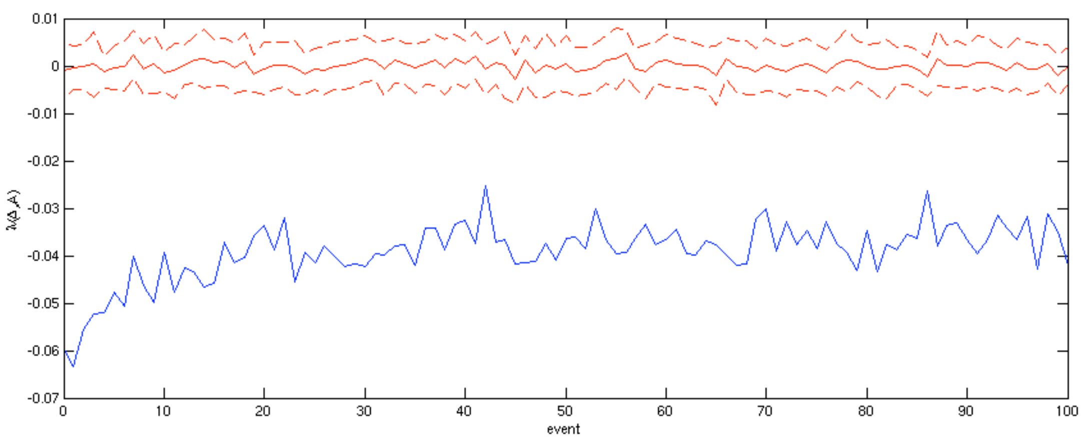




- Correlation Coefficient is:
  - Negative
  - Small
  - Significant at solar max
- Generalized correlation gives no information because of poor statistics

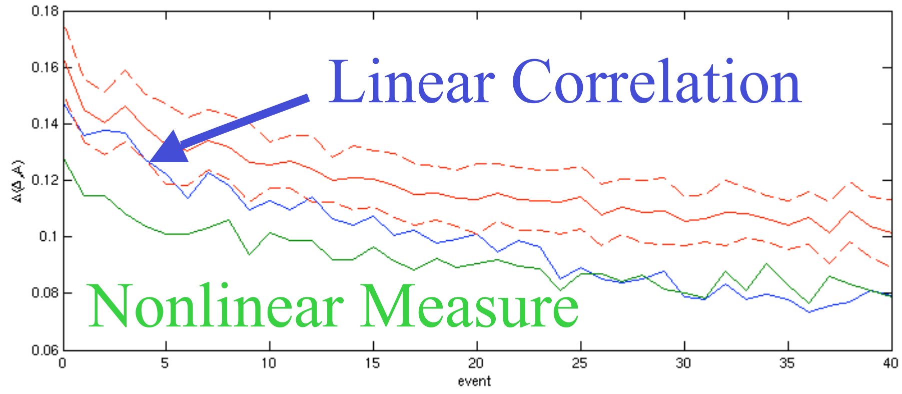
Correlation Horizon

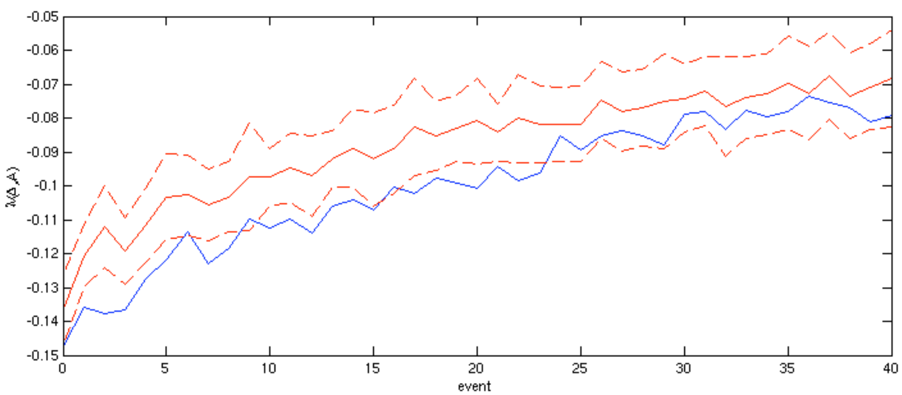




- Compare Δ_j with A_{j+n}
- Correlation horizon about a week based on mutual information
- Is the correlation nonlinear?

No Evidence of Nonlinearity





- Surrogates constructed with the same autocorrelations and cross-correlations
- Correlation function and generalized correlation are statistically indistinguishable

Conclusions

- Flare waiting times are not consistent with a nonstationary Poisson process
- Some flaring is periodic
- There are additional dependencies between flare waiting times up to roughly 30 hours
- There is a weak anticorrelation between flare strength and waiting interval with roughly a 1 week horizon but no evidence of nonlinearity (inconsistent with additive loading)

Future Considerations

- Dependencies related to homologous and/or sympathetic flares?
- Impact of a variable rate loading process?