Free boundary and profile effects on the n=1,m=1 MHD instability

J Manickam

Princeton Plasma Physics Laboratory

Analysis of the ideal MHD n=1,m=1, instability where n and m refer to the toroidal and poloidal mode number, is usually based on fixed boundary conditions. This effectively suppresses a source of free energy, the finite boundary perturbations. Inclusion of this effect reduces the threshold for instability. We report on details of this effect. Another aspect of the internal kink which requires detailed numerical studies is the details of the plasma profiles in the core region. Commonly accepted criteria are based only on the pressure gradients inside the q=1 surface. Here we report on the importance of the shear, q', as well as p'. An empirical criterion for stability is shown.

This work was supported by DoE Contract No. DE-AC02-76-CHO-3073

Motivation

- The n=1 m=1instability is ubiquitous in tokamaks
- Linear stability analysis usually focuses on the internal kink
- Fixed boundary conditions are not realistic
- Free boundary conditions introduces a new source of free energy which lowers the stability threshold
- Analytical methods to include internal and external perturbations are not available ➔ numerical approach
- Must also consider profile effects, $q(\Psi)$, $p'(\Psi)$
- Note that plasma cross-section effects are not included

Profile issues

- The stability properties for the internal n=m=1 mode depends on details
  - q-axis
  - radius of q=1surface
  - q’ at the q=1 surface
  - p’ within the q=1 surface – In this study p’ is constant
- The stability of the external component is determined by
  - Boundary conditions
  - q-edge – i.e. relation to integer-q
  - Edge current density

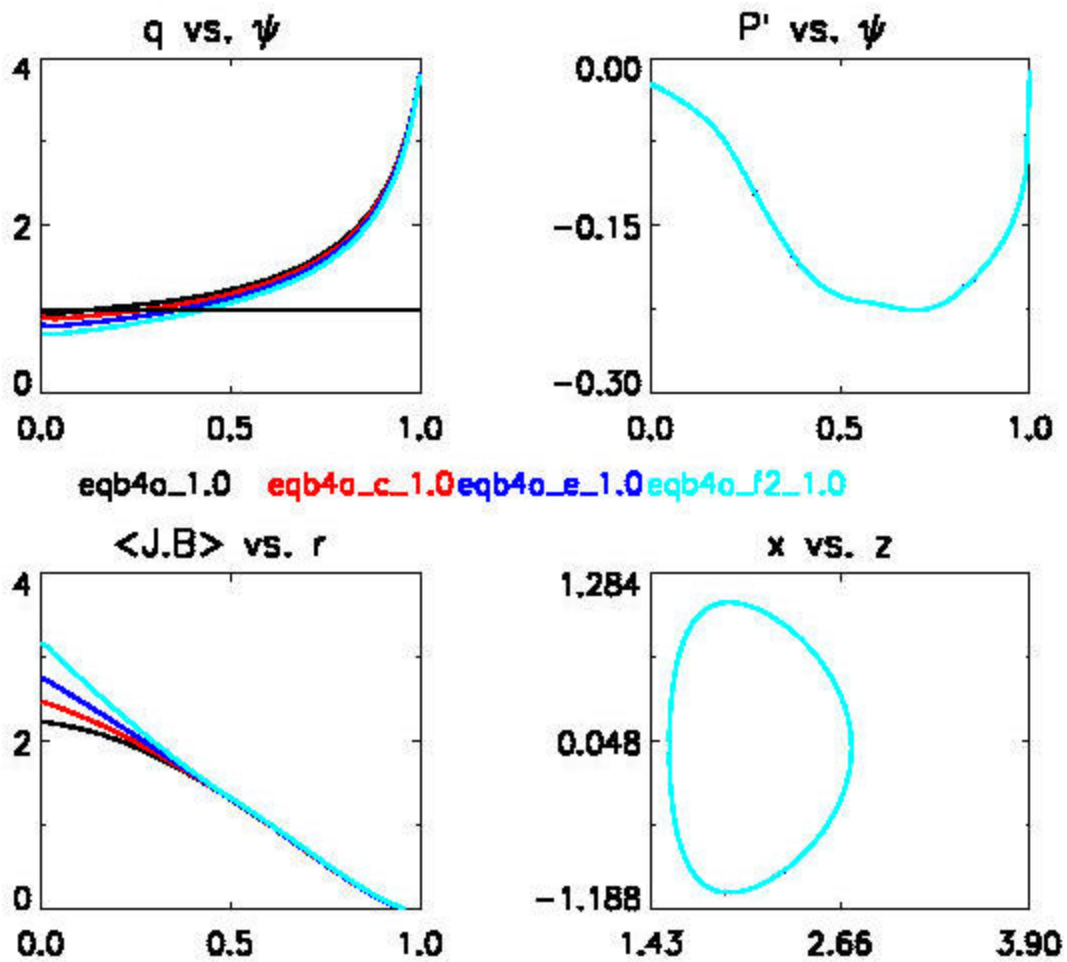
Numerical procedure-I

- Specify a fixed plasma shape
- Specify plasma profiles: $p'(\Psi)$, $\langle J.B \rangle(\Psi)$
  - Fix current density near plasma edge
  - Vary current density in core region to vary q_axis
- Equilibrium series with varying β-poloidal
- n=1 stability analysis with fixed and free boundary

Major radius R=	2.140	Minor radius a =	0.595
Aspect ratio =	3.597	Magnetic axis =	2.185
Ellipticity =	1.799	Triangularity =	0.313

Profiles of p’ and <J.B>

Note that current density near the edge is fixed as is p’ and the boundary shape

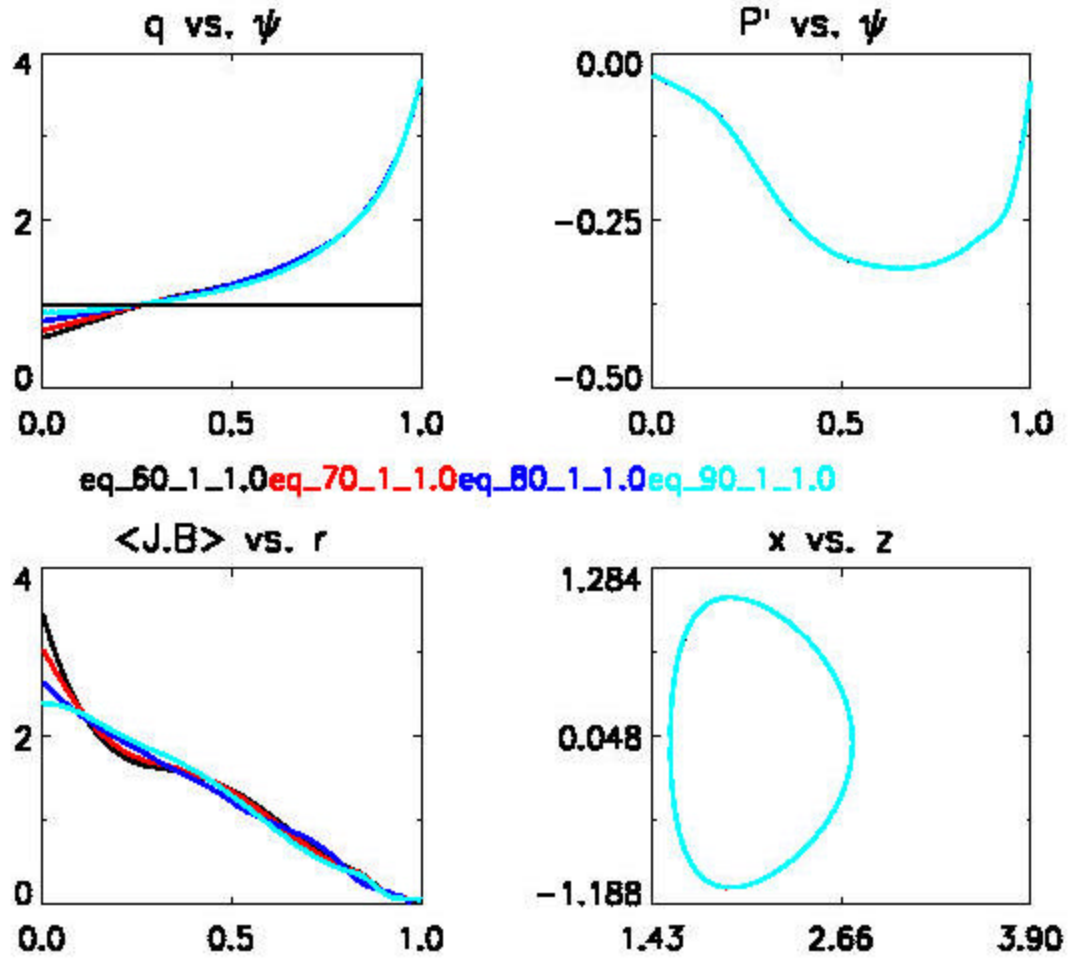


Numerical procedure-II

- Specify a fixed plasma shape
- Specify plasma profiles
  - $p'(\Psi)$, $q(\Psi)$ – specify and control details of q
  - $p'(\Psi)$, $\langle J.B \rangle(\Psi)$ – specify and control details of current-density
- Equilibrium series with varying β-poloidal

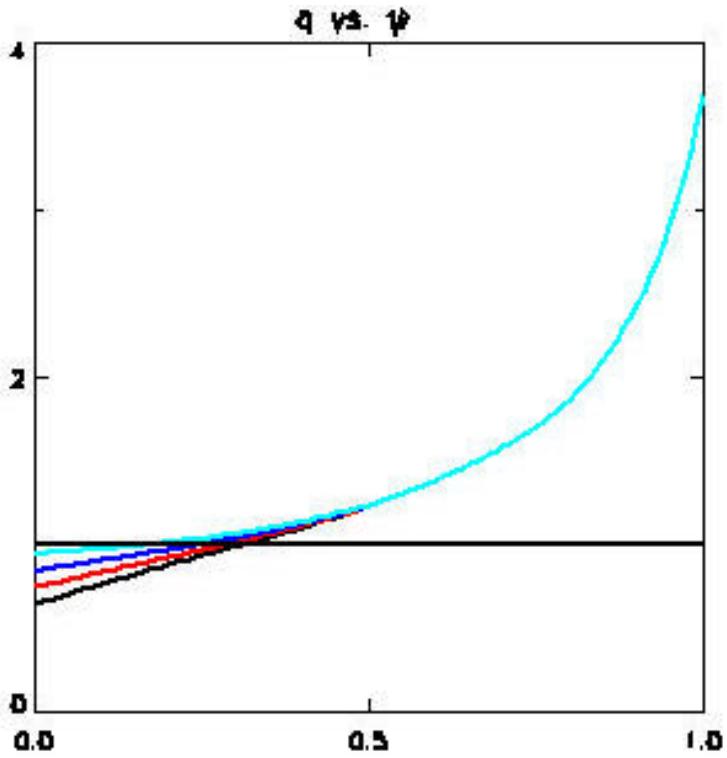
Profiles of p' and q

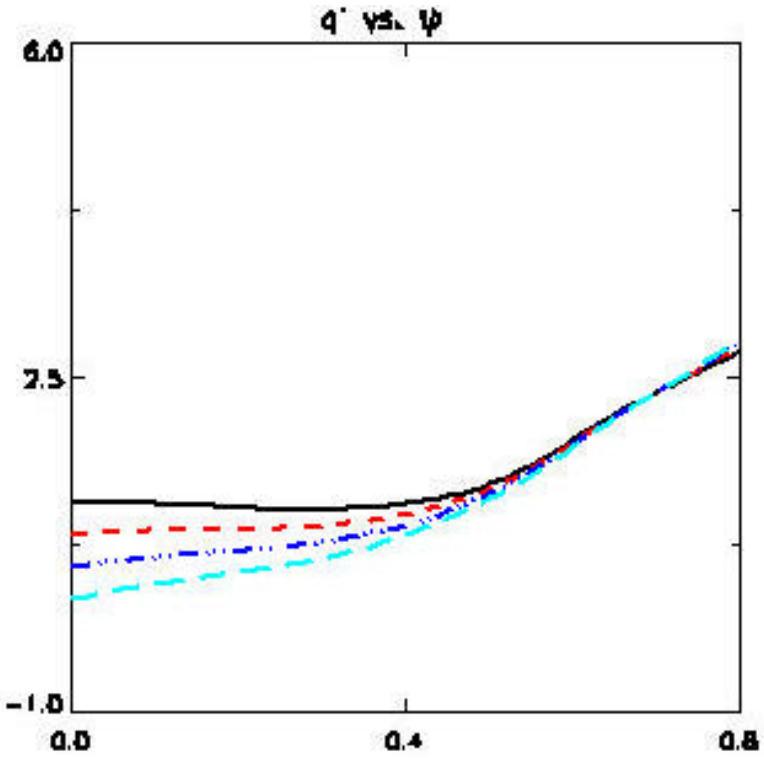
Note that the radius of q=1 and qedge is fixed as is p' and the boundary shape



Variations of q-profiles - I

Note that q' and q=1 radius are changing

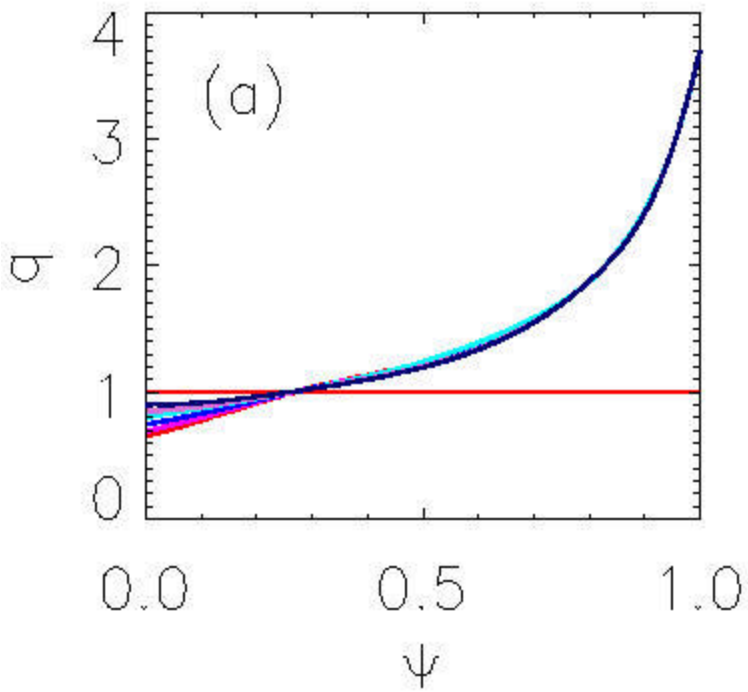


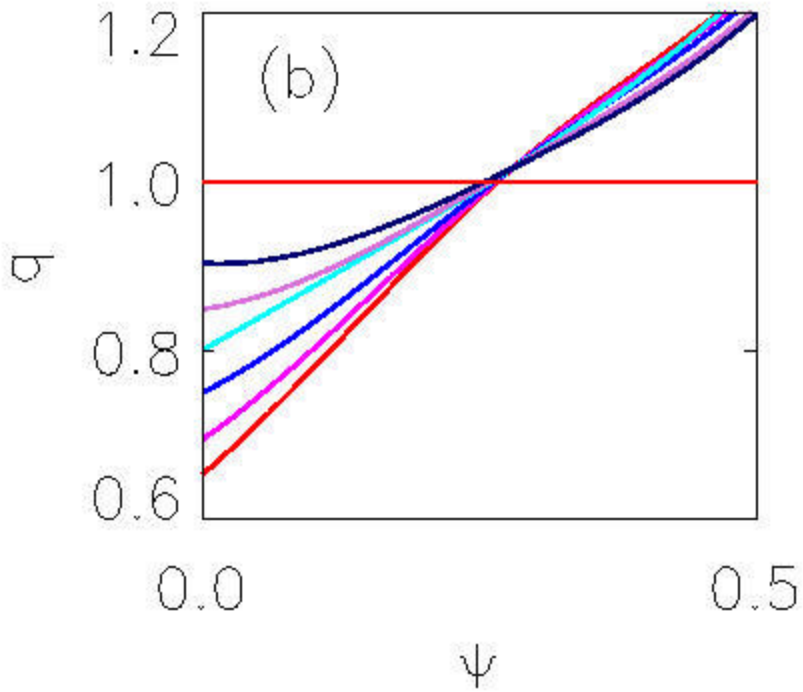


Q-1

Variations of q-profiles - II

q-profiles with fixed q=1 radius

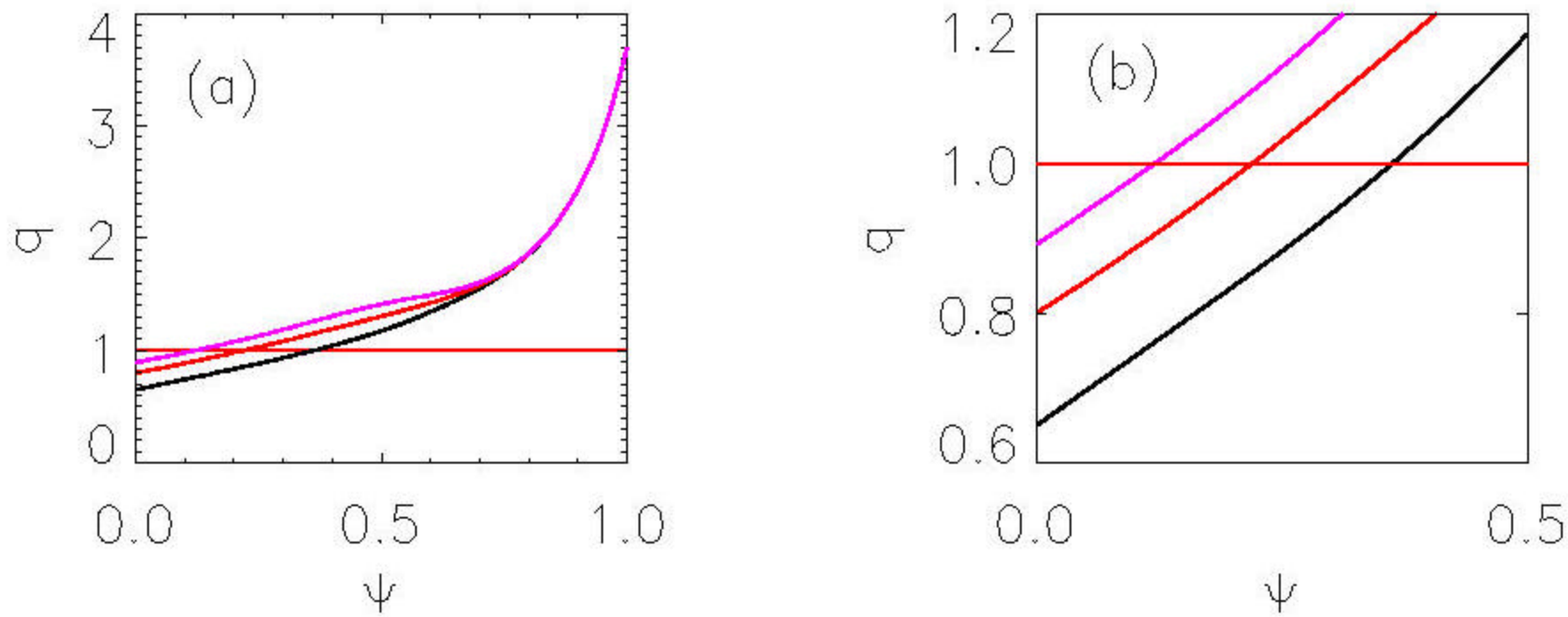




Q-2

Variations of q-profiles - III

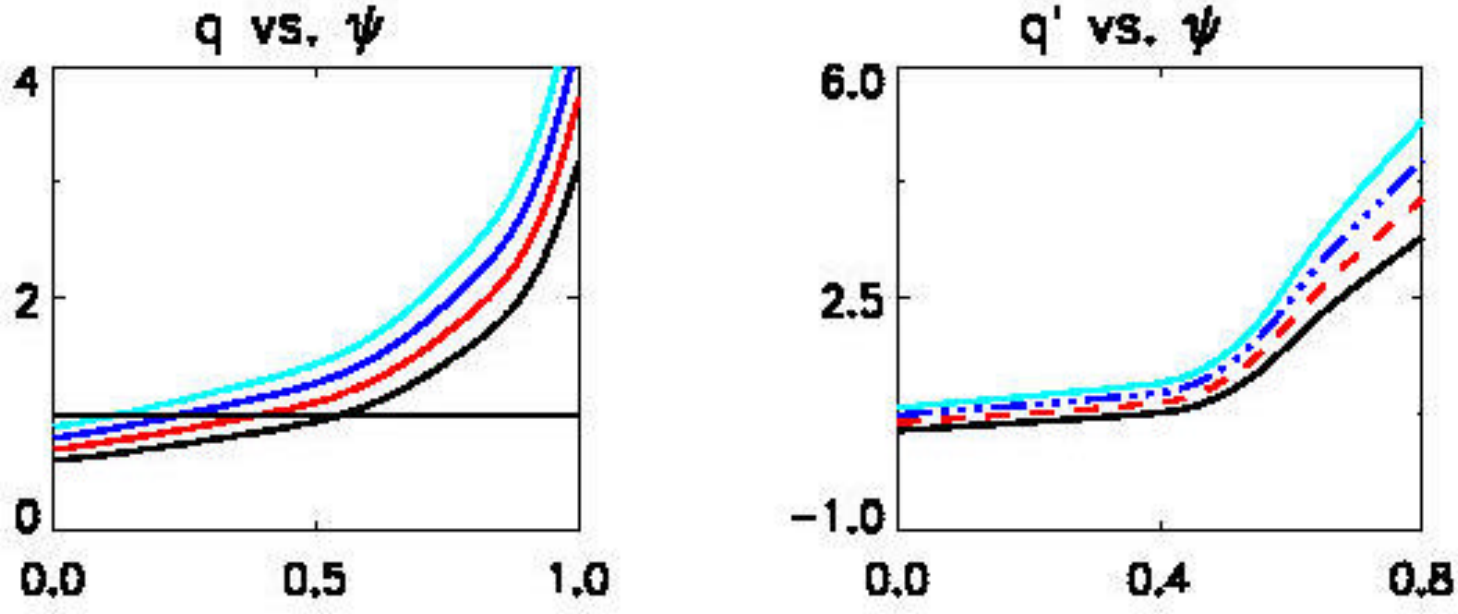
q-profiles with constant q' and varying radius of q=1



Q-3

Variations of q-profiles - IV

q-profiles from scaling B-tor



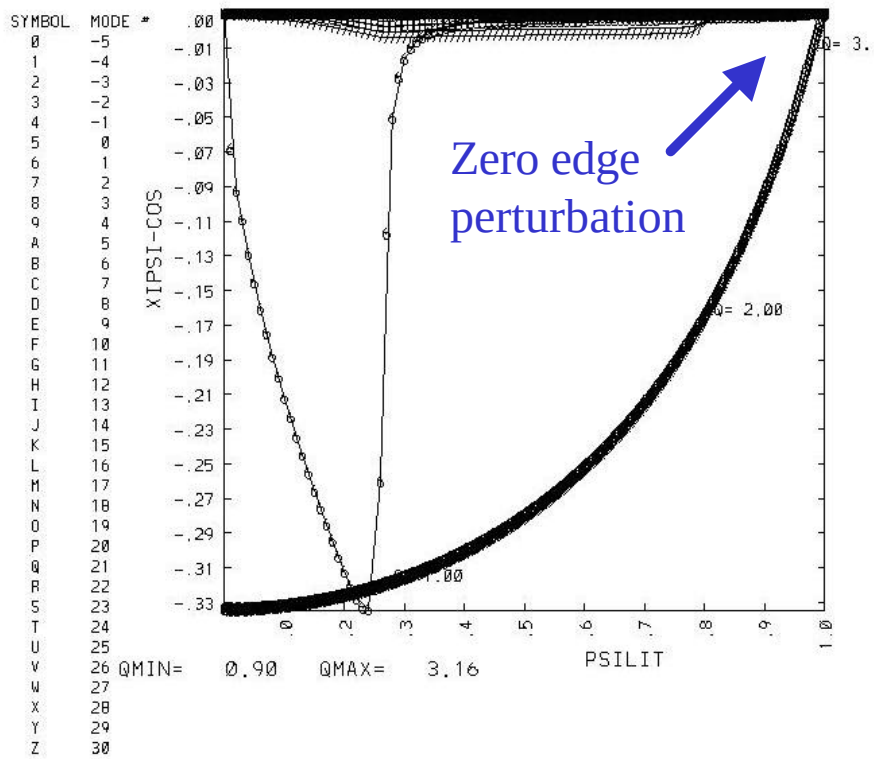
q-profiles obtained by scaling the toroidal field

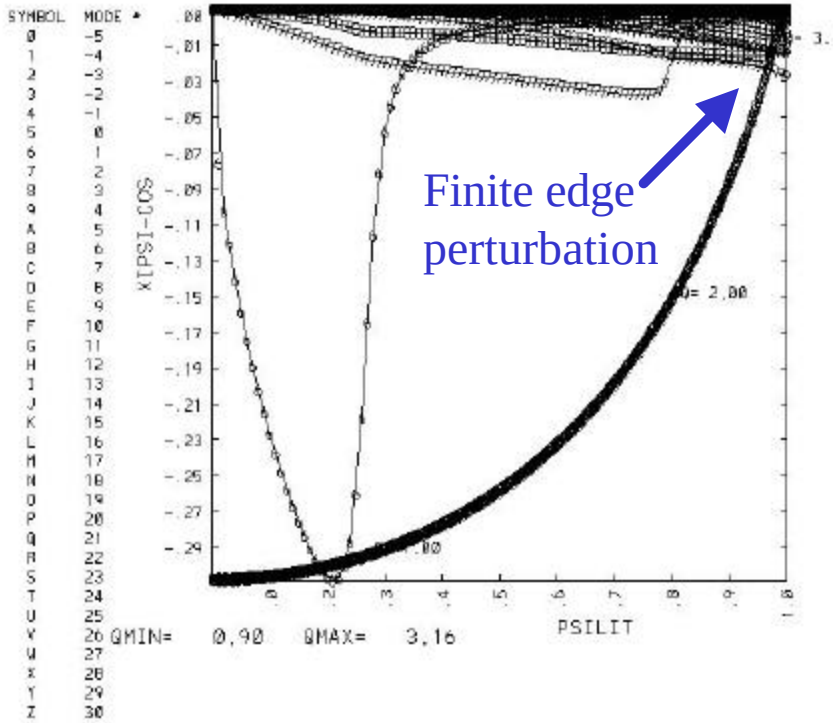
$B^2 = B^2 + \text{const}$

Grad-Shafranov equation remains the same

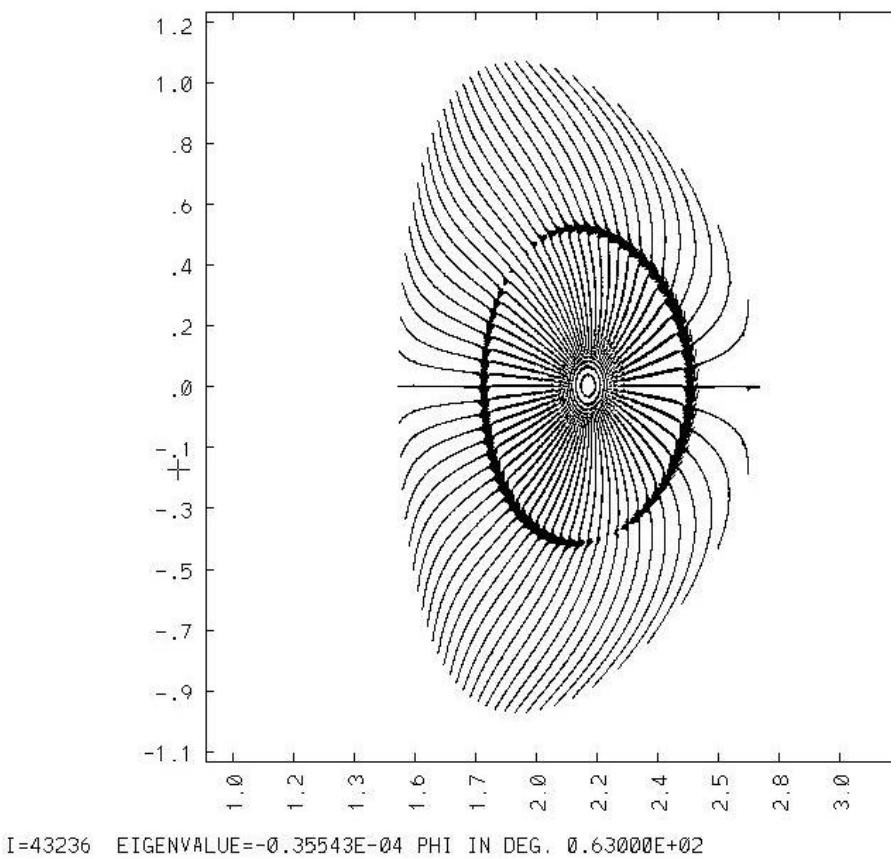
q scales with B and p scales likes $1/B^2$

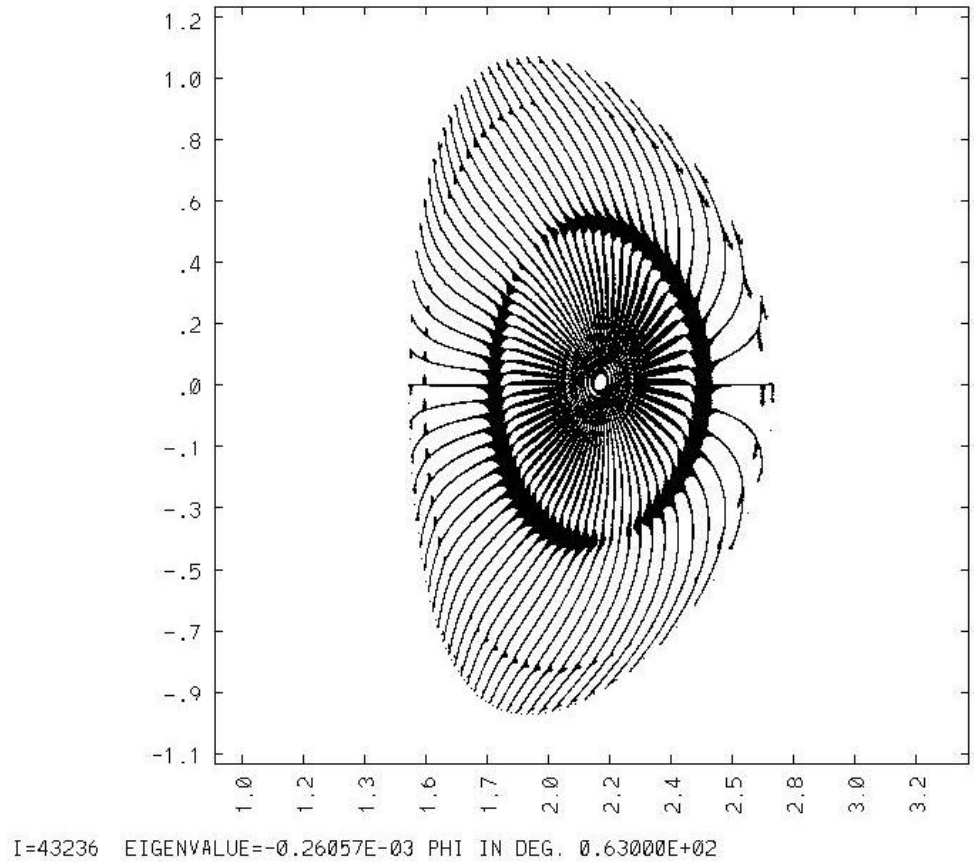
Free boundary effects -I





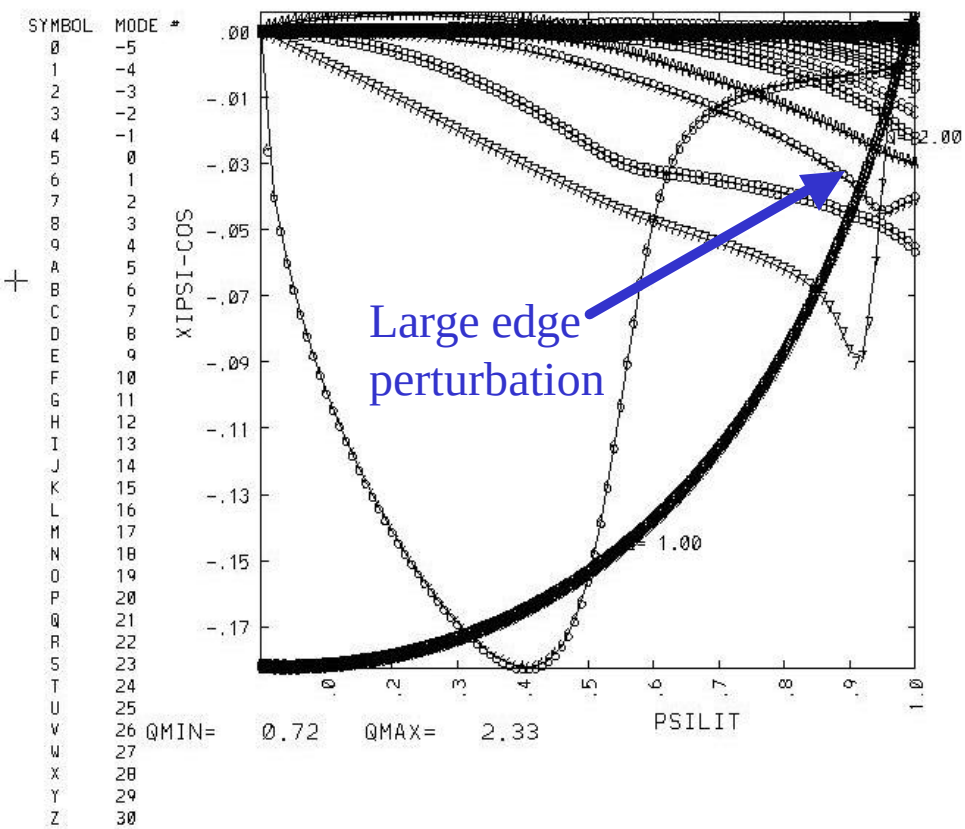
Free boundary effects -II

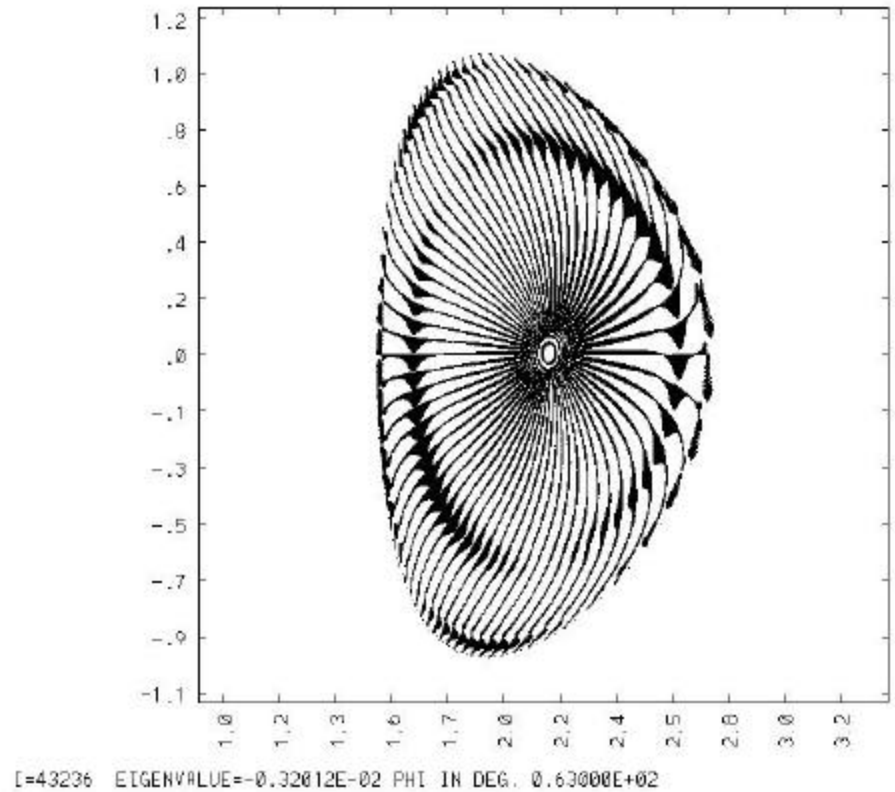




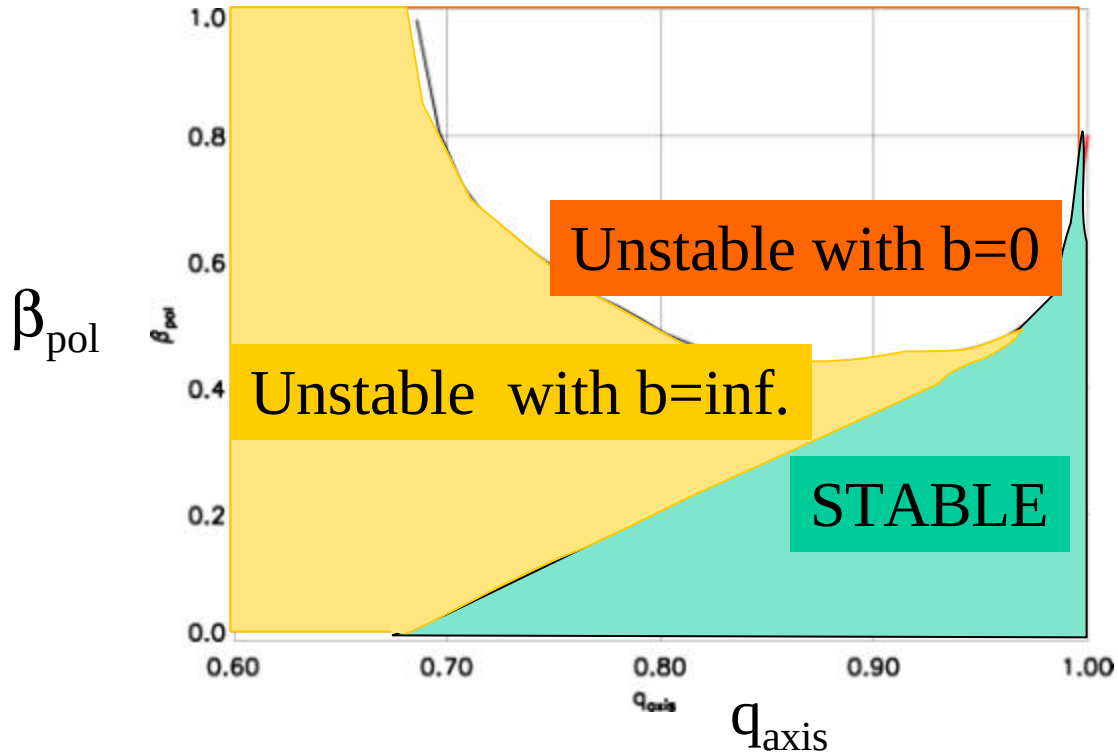
Modest edge perturbation – enhanced coupling of internal and external modes

Free boundary effects -III





Generic stability diagram for n=m=1

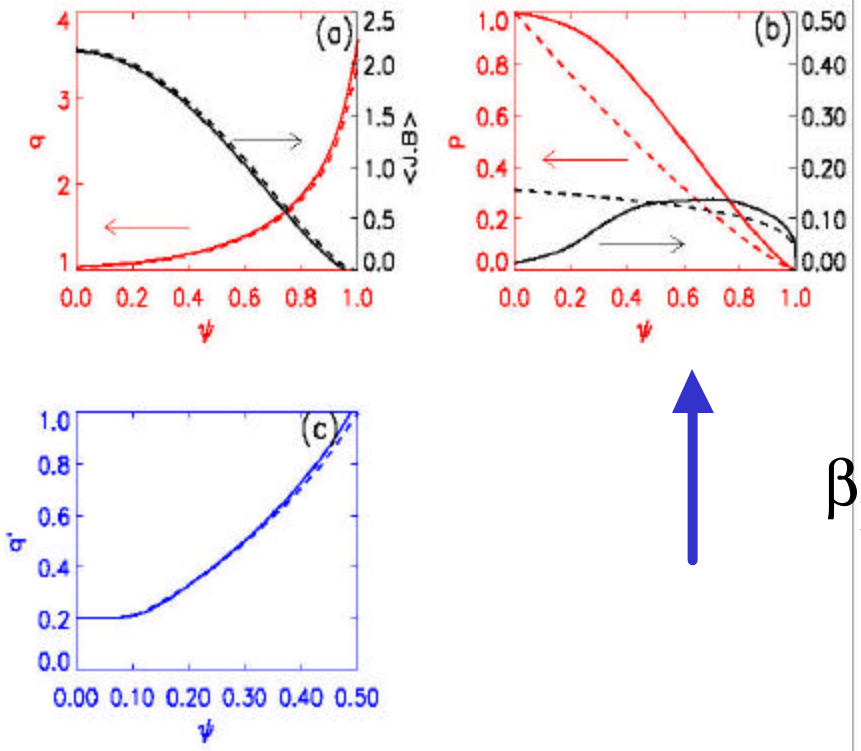


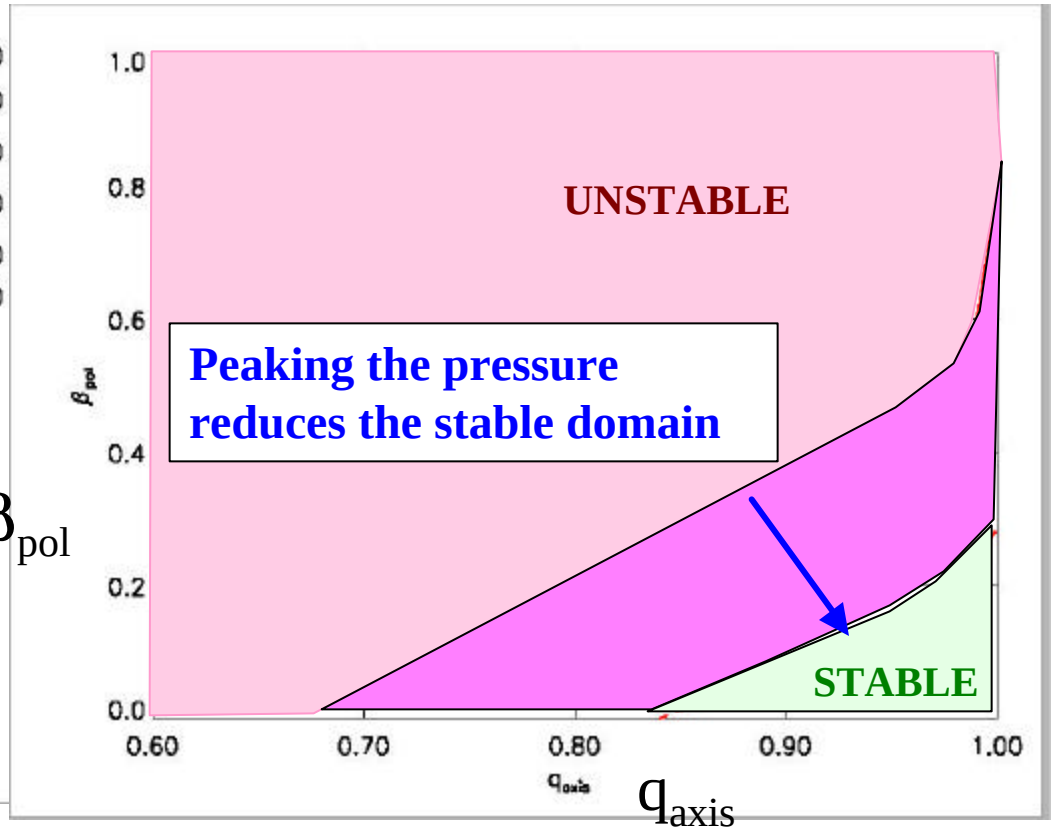
Free boundary conditions reduce stability thresholds for low-q_axis

β-limit goes to zero

Effect of the pressure peaking factor-I

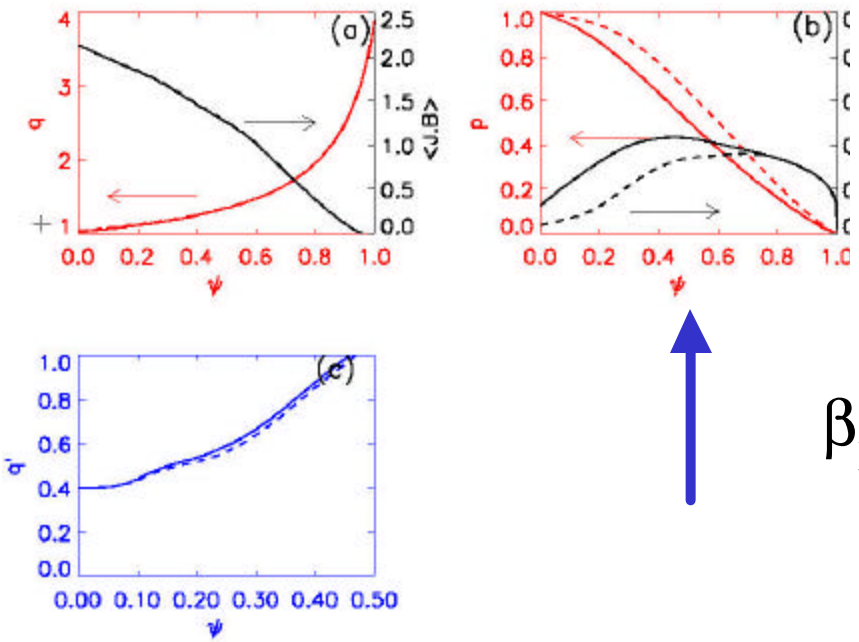
B-field scaling method

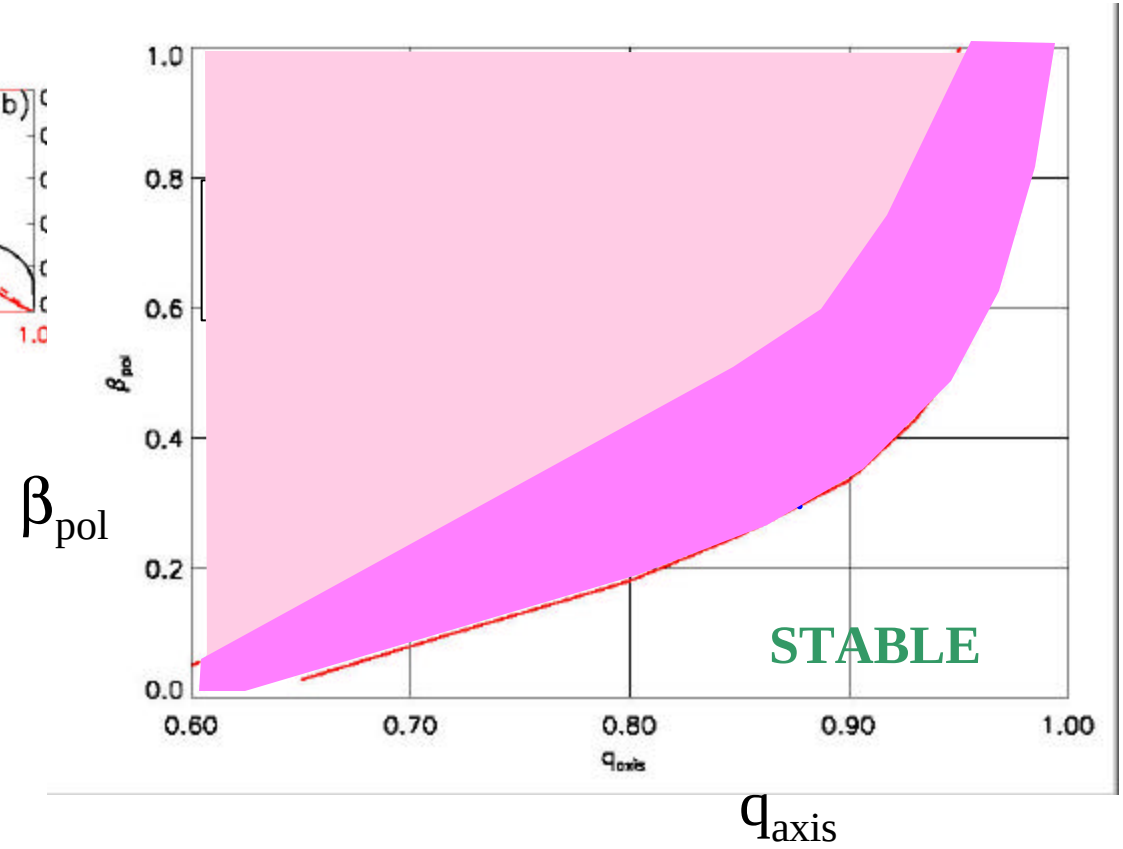




Effect of the pressure peaking factor-II

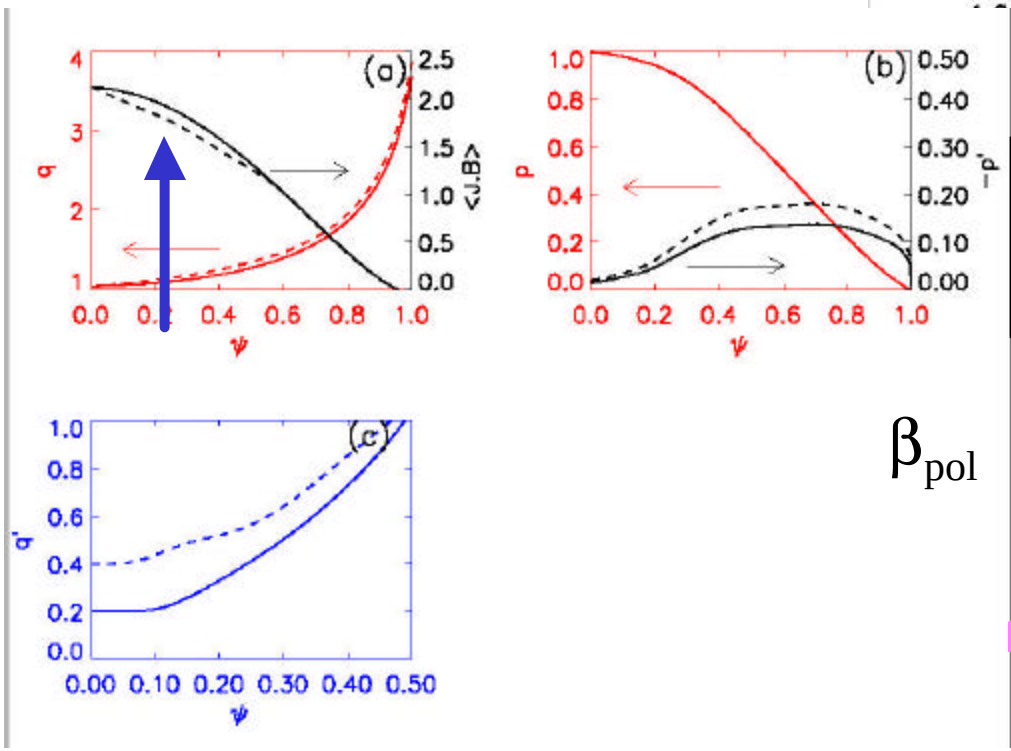
B-field scaling method

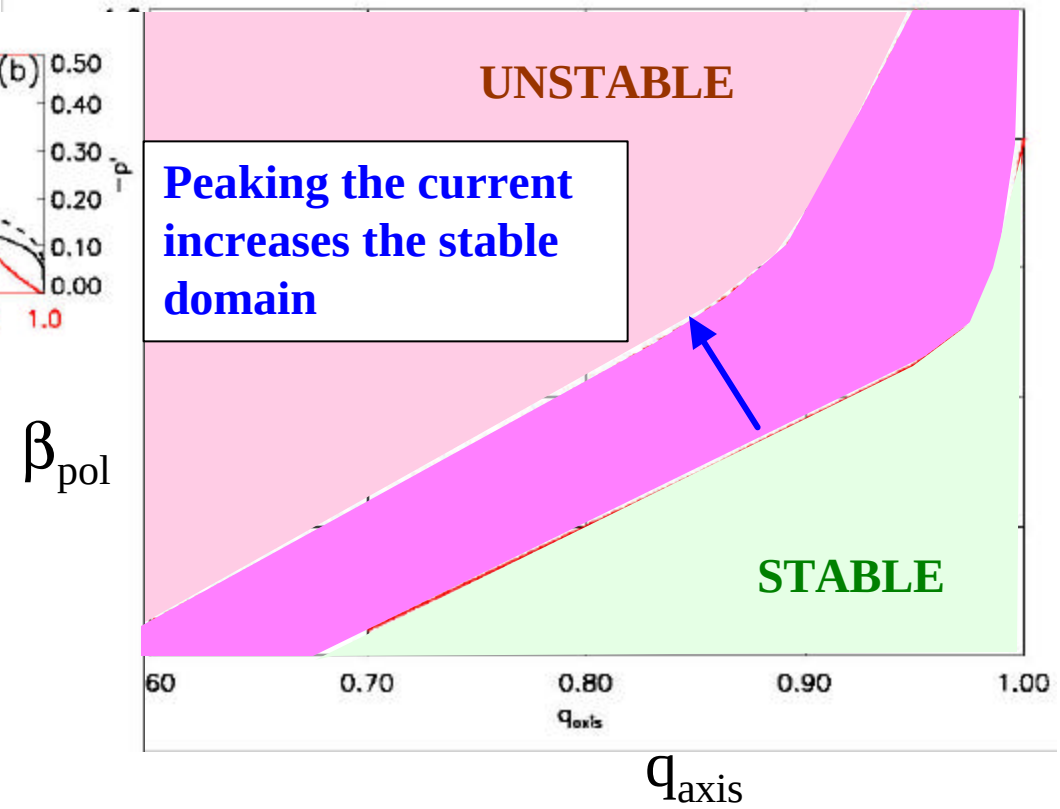




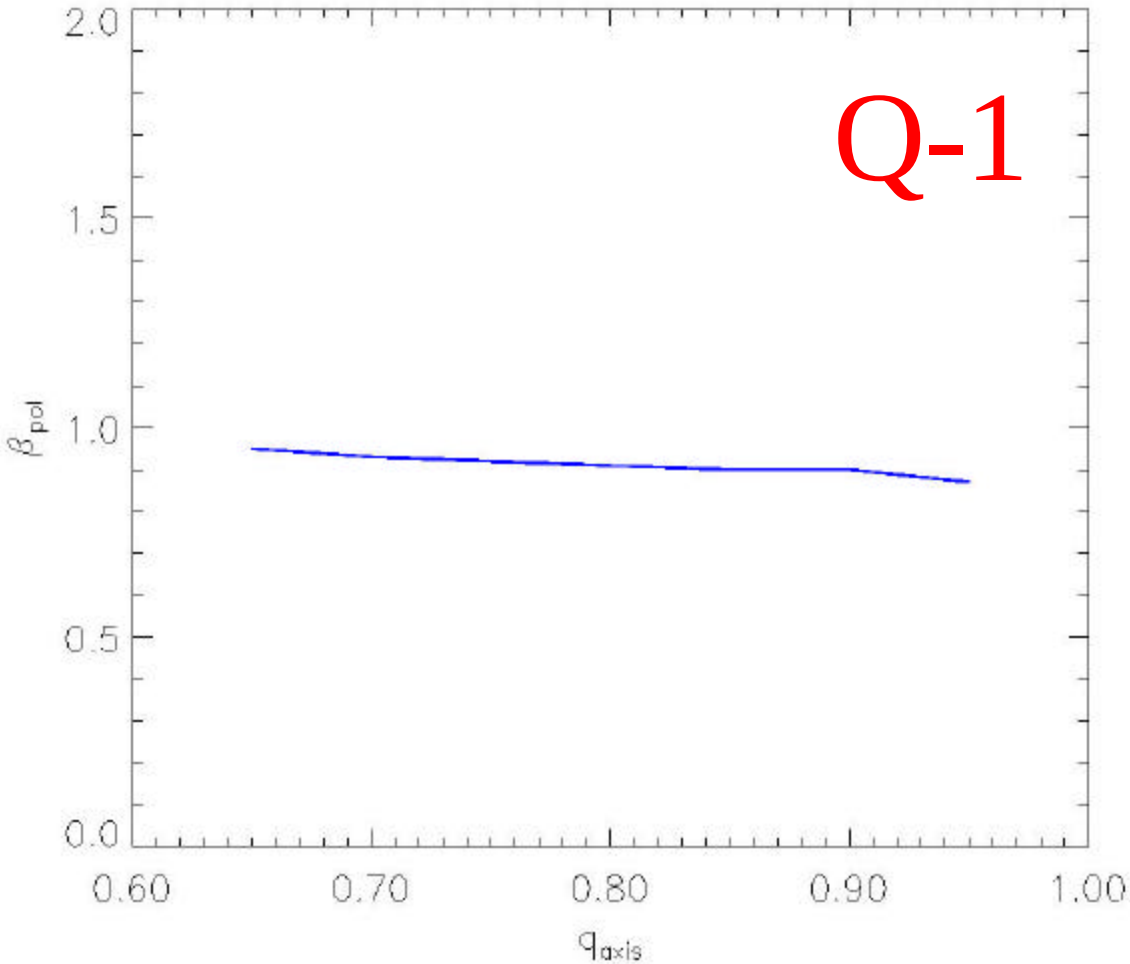
Effect of the current peaking factor

B-field scaling method

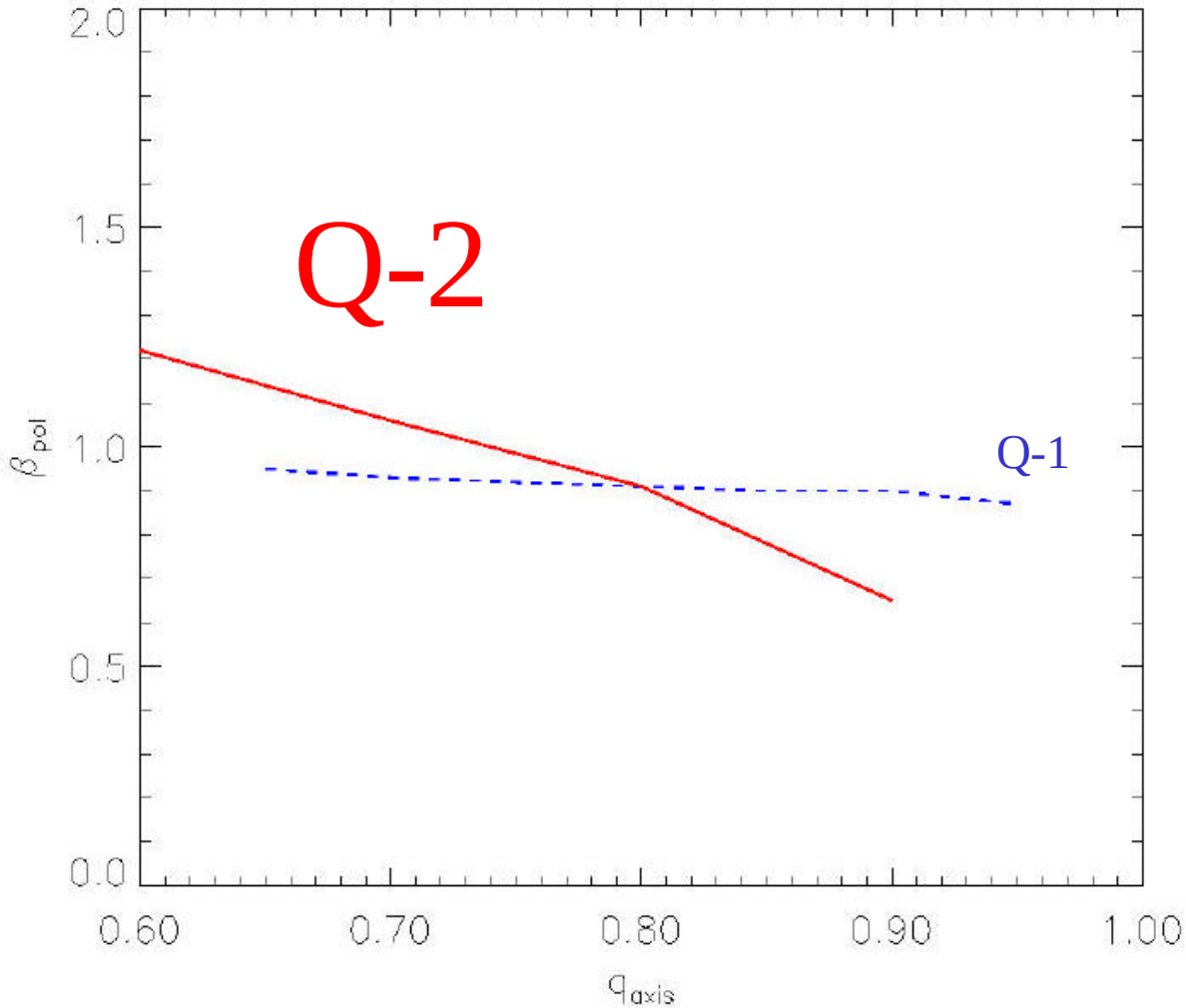




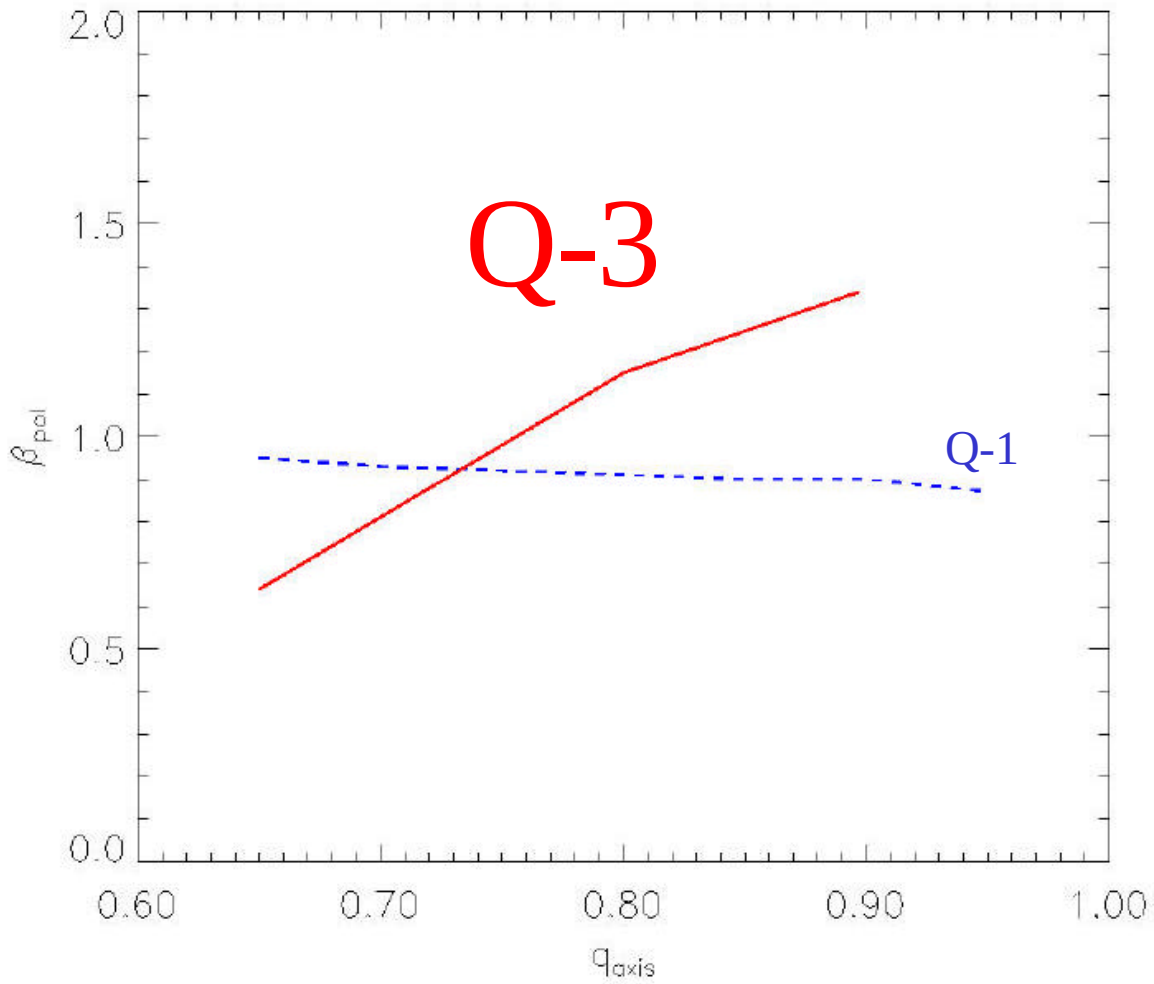
Stability threshold for varying q' and q=1 radius



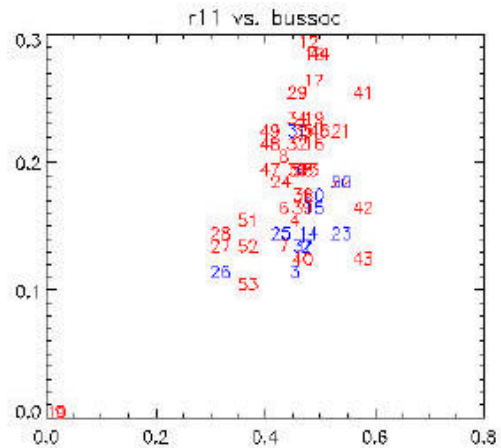
Stability threshold for fixed q=1 radius with varying q'

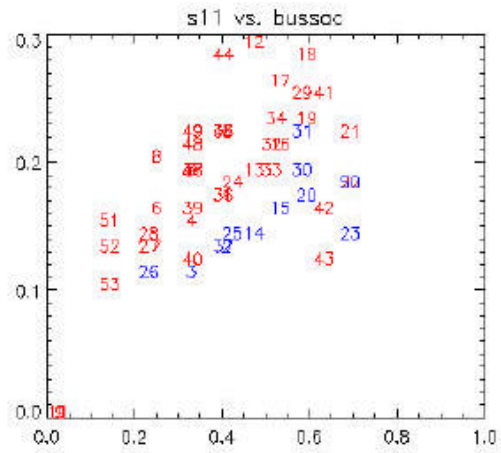


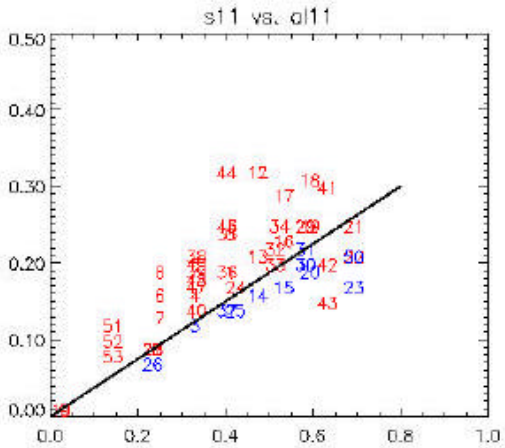
Stability threshold for fixed q' and with varying q=1 radius



An empirical scaling law for the stability threshold







$$\alpha_{crit} = 0.375\ s_{11}$$

Summary

- Free boundary effects reduce the instability threshold for the m=n=1 mode
- The stability limit is determined by details of the plasma profiles
  - Pressure gradient inside q=1 radius
  - Radius of q=1
  - Shear, q', at the q=1 radius
- An empirical scaling law relates *s* and α at the q=1 radius

$$\alpha_{crit} = 0.375\, s$$

Other issues

- Plasma shaping also has an effect – not included here
- Suggests that sawtooth activity can be modulated by driving co- or counter-current near axis to modify the local shear
- Detailed understanding of sawteeth requires simulation with transport to describe profile evolution
- This study only looked at linear stability
- Rotation effects need to be studied

This poster is on the web as

http://w3.pppl.gov/~manickam/aps2002.pdf