Following an unstable mode into the stable regime using PEST-1

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This note describes the evolution of the ideal MHD spectrum as the equilibrium parameters are changed, in this instance β . It focuses primarily on following an unstable mode into the stable regime, using the PEST-1 code. The difficulties associated with such a study are due to the presence of continuum modes, which makes it difficult to distinguish the mode of interest. One solution is to look at all the eigenvectors in the regime of interest and identify the mode by inspection. The problem is that, this requires solving a very large matrix equation for each equilibrium, where the rank may be 5,000 to 30,000. However when the mode is stable, and the expected eigenvalue is a small positive number, it falls within the shear Alfven continuum and it is very difficult to distinguish the specific eigenfunction corresponding to the 'stable' kink. We use a truncated expansion set, to minimize the matrix rank. We then solve for all the eigenvalues and plot the corresponding eigenvectors close to zero, to track the unstable mode. To make this tractable, we restrict the rank to be about 2500 and examine the eigenvectors corresponding to a limited range of eigenvalues. This is done by using a less than optimal set of expansion functions for solving the energy principle.

We generate a sequence of equilibria with a fixed geometry, a dee-shape with R/a=5, ellipticity=1.8 and triangularity=0.4. The plasma profiles are given by: q-profile, $q = 1.1+5.3\Psi^{4.5}$, and p-profile, $p = p_0(1-\Psi)^2, 0 \leq \Psi \leq 1$. To demonstrate this approach, we have generated a sequence of equilibria, k1,k2,...k6 with increasing β, of which some are unstable and some stable, see Table 1. The table includes two sets of growth-rates, the first corresponds to a large expansion set used for solving the energy pinciple equations,

$$\xi = \sum_{m=1}^{200} \sum_{l=-10}^{30} \xi_{m,l} e^{il\theta - n\phi}$$

with, $n = 1$. The second set refers to a reduced set with,

$$\xi = \sum_{m=1}^{50} \sum_{l=-1}^{0} \xi_{m,l} e^{il\theta - n\phi}$$

The corresponding matrix ranks are, 24723 and 1836 respectively. The reduced set results in shifting the eigenvalue upwards, this is illustrtaed in Fig. 1. With the larger matrix, we can only determine the eigenvalue if it is unstable. With the smaller rank matrix, all the eigenvalues and eigenvectors can be determined. The positive eigenvalues corresponding to an eigenfunction, similar to the unstable vector, Fig. 2, are used in the table and plot. Note however, that as β_N is reduced and the eignvalue becomes more positive, mode identification becomes ambiguous due to coupling with Alfven continuum modes. This is illustrated in figures 4 through 10. In each figure modes, which are candidates to evolve into the unstable mode are circled. Figure 3 shows the progression of three sets of eigenvalues which have similar structure.

Eq. ID	β_N	ω^2	${\omega_s}^2$
$k1$	1.336	stable	stable
$k2$	1.948	stable	stable
$k2bc$	2.096	stable	.002220
$k2bb$	2.120	-.000103	.002196
$k2ba$	2.144	-.000267	.002152
$k2b$	2.193	-.000651	.001939
$k2c$	2.314	-.001784	.000754
$k2d$	2.434	-.003111	.000223
$k3$	2.553	-.00461	-.000480
$k4$	3.167	-.01444	-.004595
$k5$	3.724	-.02810	-.008767
$k6$	4.257	-.04507	-.012533

Table 1. β_N vs. PEST-1 eigenvalue, for an external kink using ω^2, 200 radial finite elements and $-10 \leq l \leq 30$ fourier modes, and ${\omega_s}^2$ using 50 radial elements and $-1 \leq l \leq 10$.

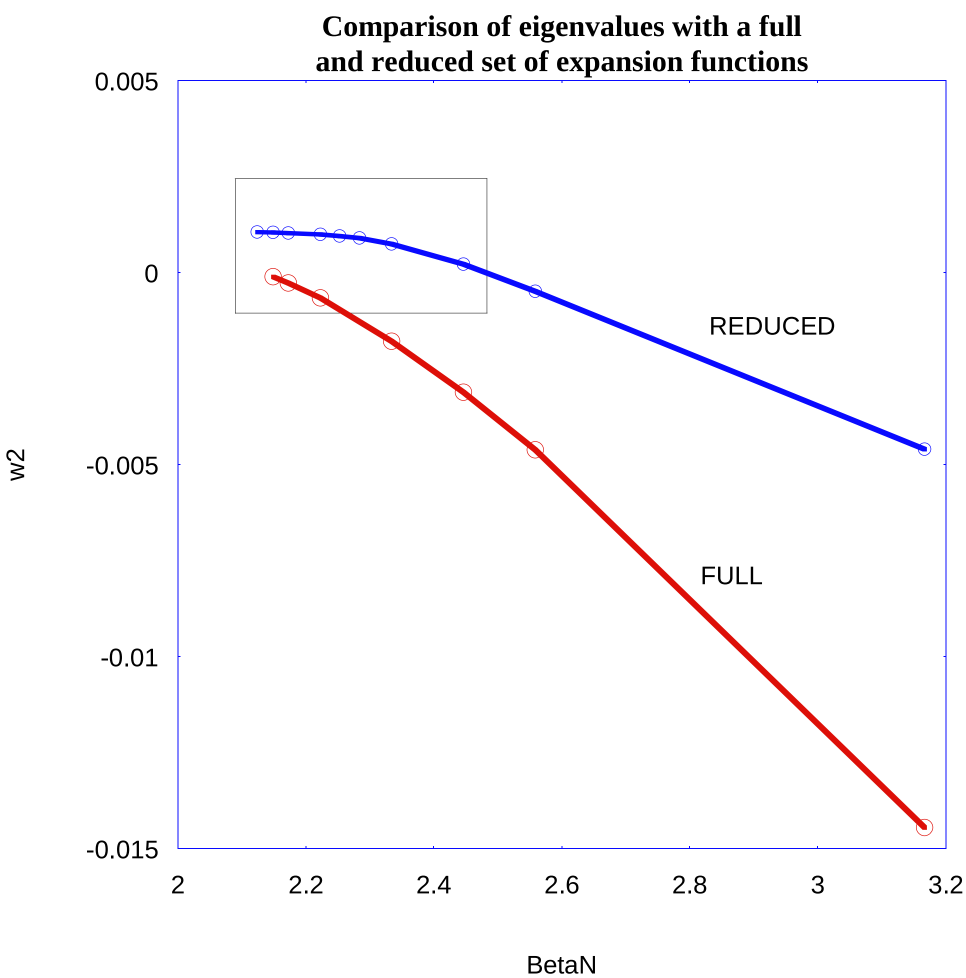

Figure 1: Eigenvalues corresponding to the equilibria in Table 1. The curve marked 'FULL' corresponds to the optimal expansion set, $-10 \leq l \leq 30$, and the curve marked 'REDUCED' uses $-1 \leq l \leq 10$. The inset box indicates the regime studied in greater detail in Fig. 3.

EXTERNAL KINK EIGENVECTOR CLOSE TO MARGINAL STABILITY-K2CA

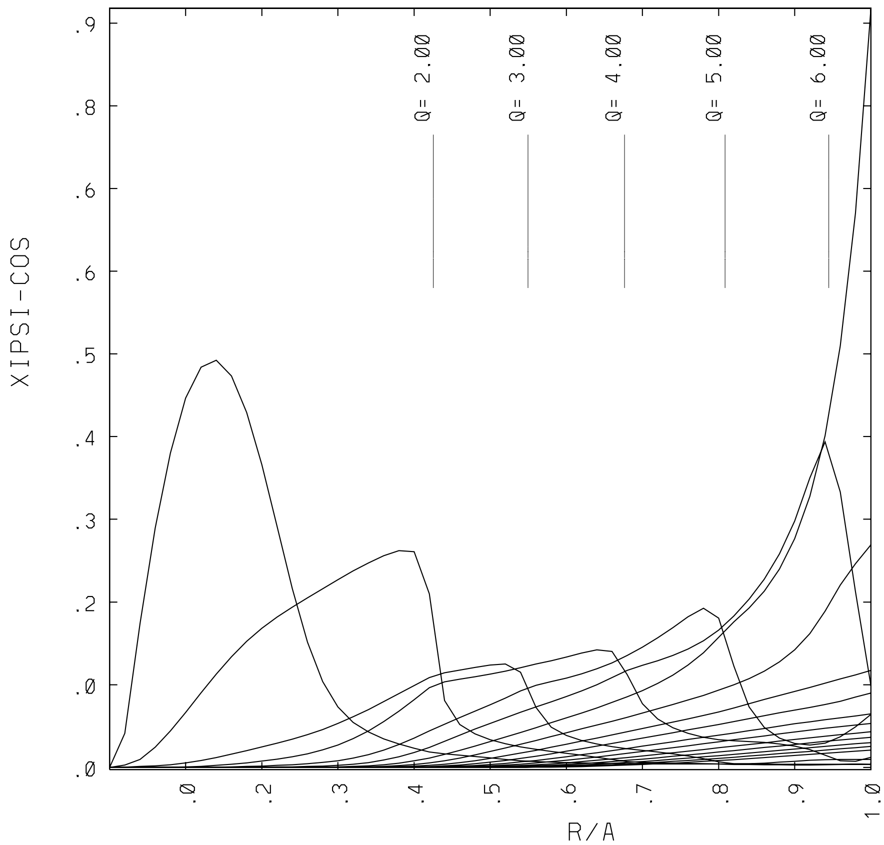

Figure 2: Eigenvector corresponding to an unstable equilibria in Table 1, corresponding to the optimal expansion set, $-10 \leq l \leq 30$.

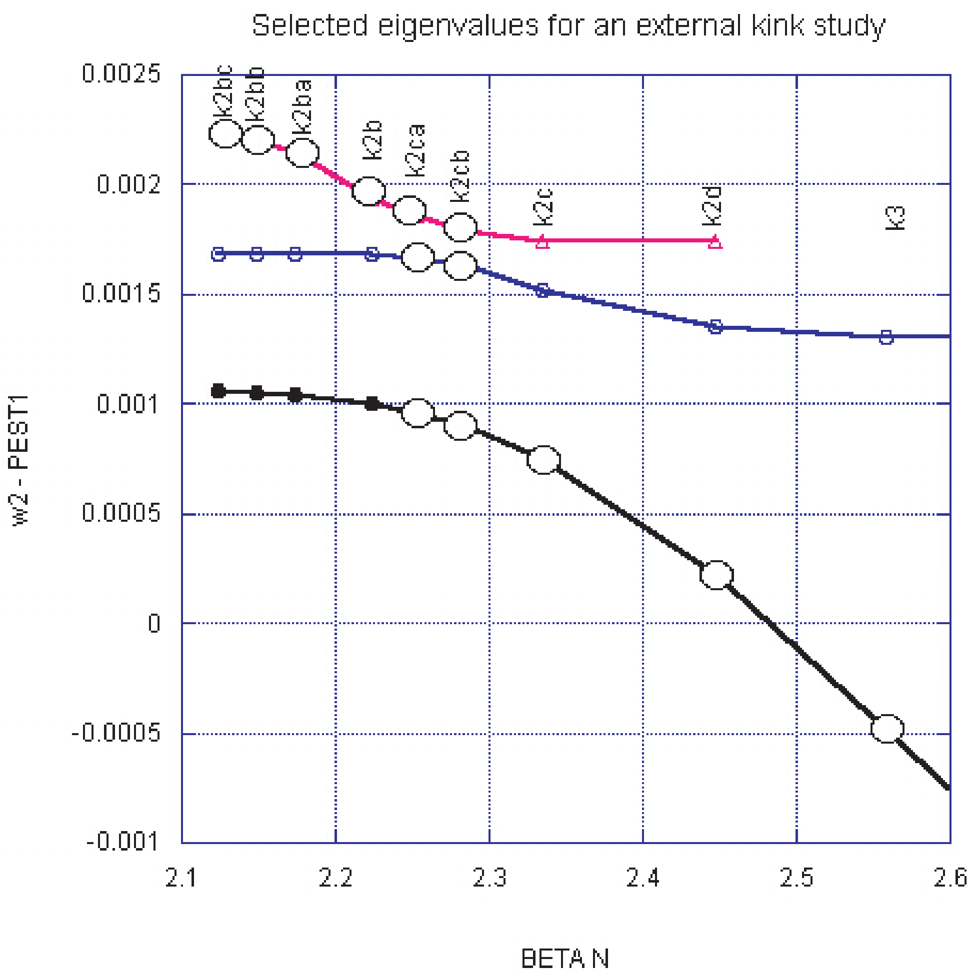

Figure 3: Selected eigenvalues, obtained from the reduced set. The points marked by large circles have eigenfunctions similar to the unstable mode in Fig. 1. A subset of the eigenvectors for each equilibrium is shown in the following figures, with a broken circle identifying the relevant modes.

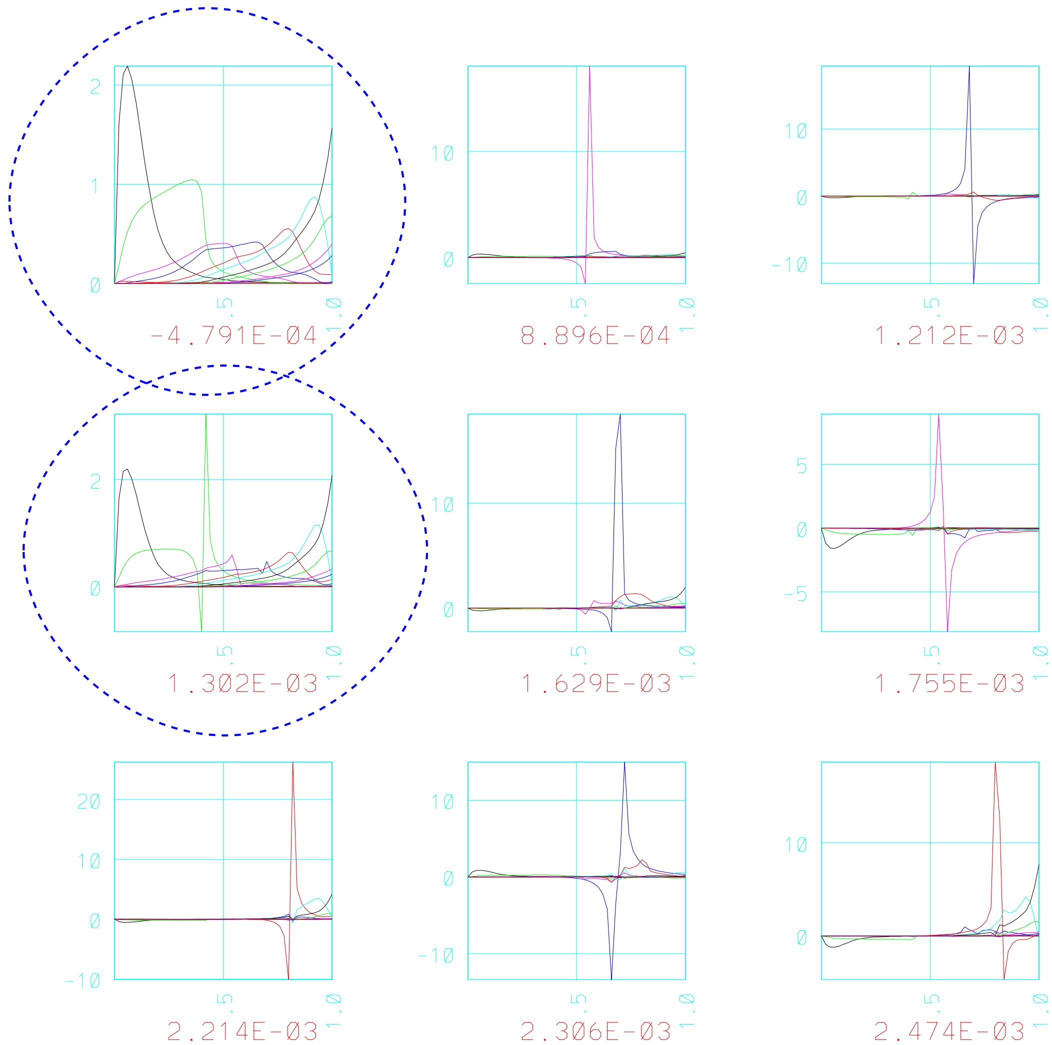

Figure 4: Eigenvectors corresponding to the lowest nine eigenvalues for $k3$

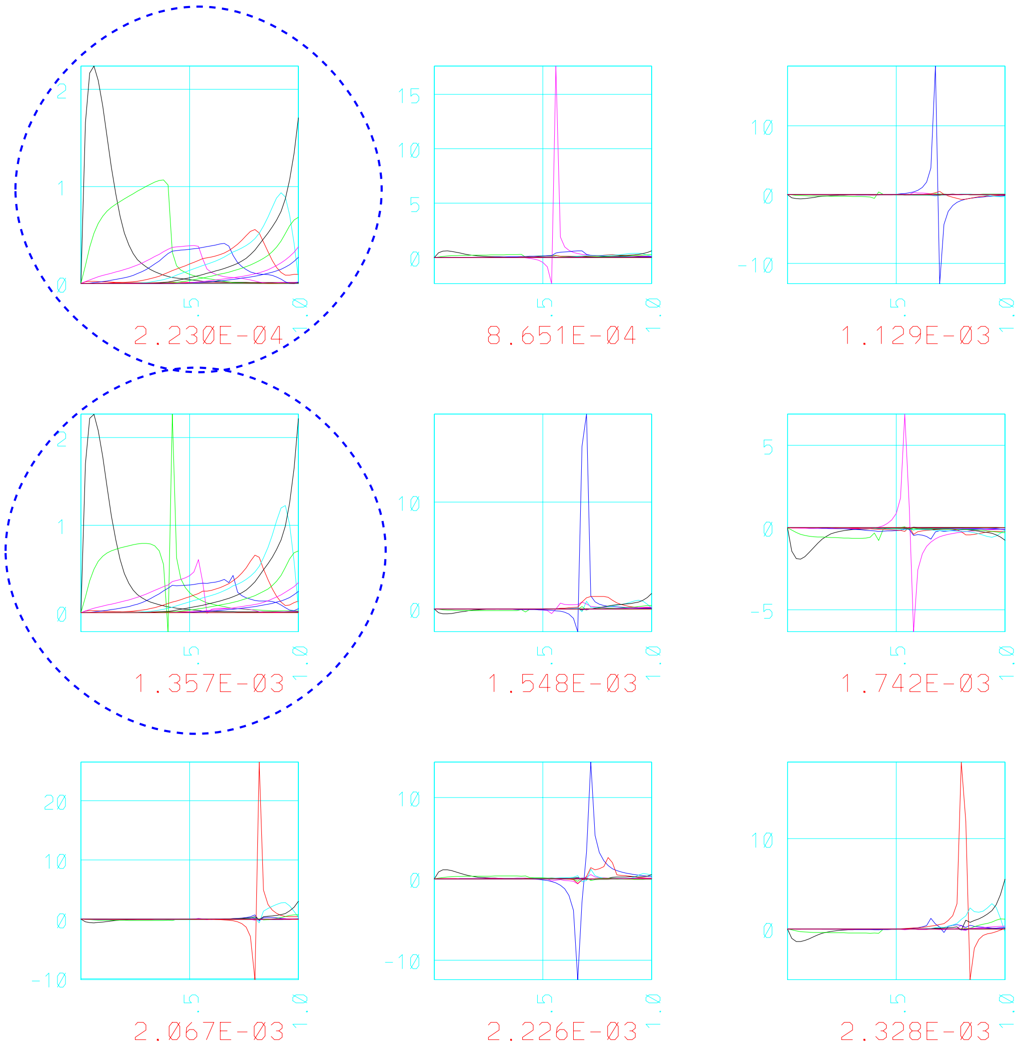

Figure 5: Eigenvectors corresponding to the lowest nine eigenvalues for $k2d$

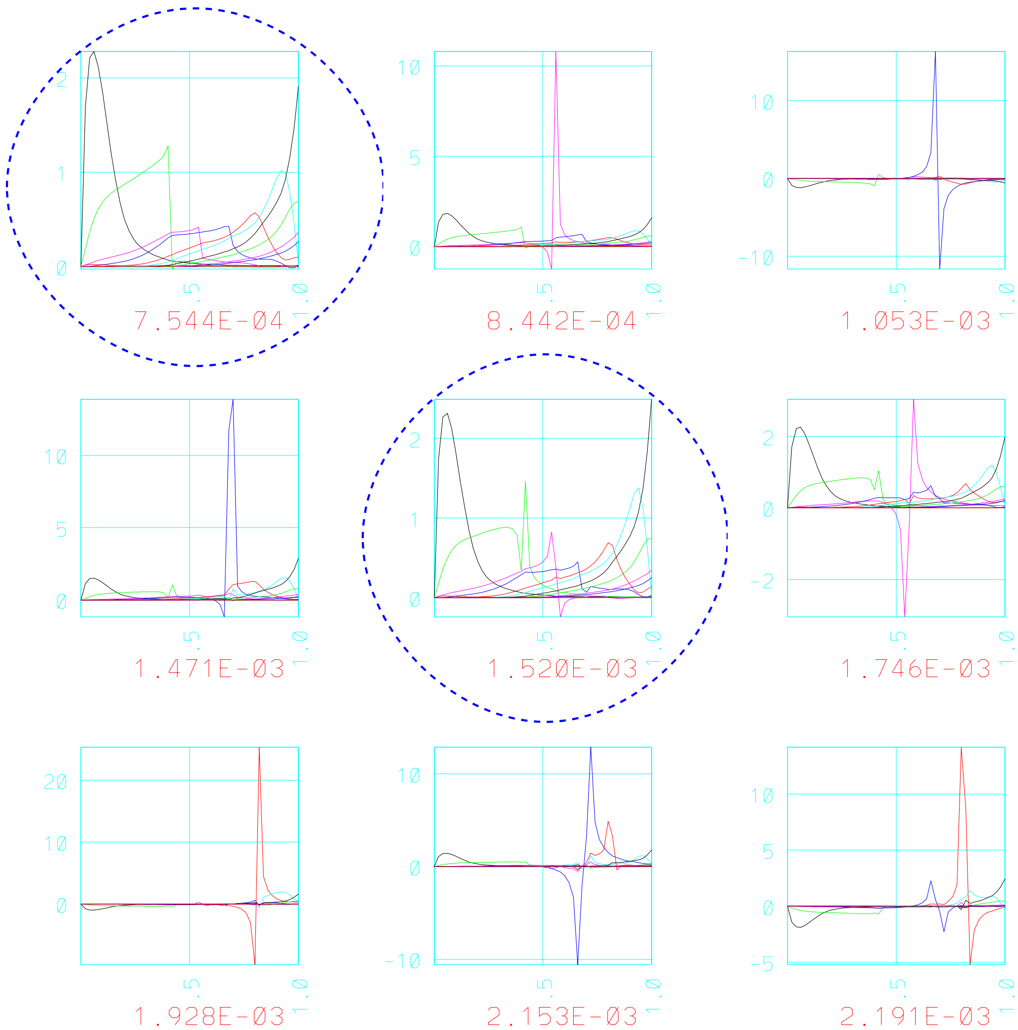

Figure 6: Eigenvectors corresponding to the lowest nine eigenvalues for $k2c$

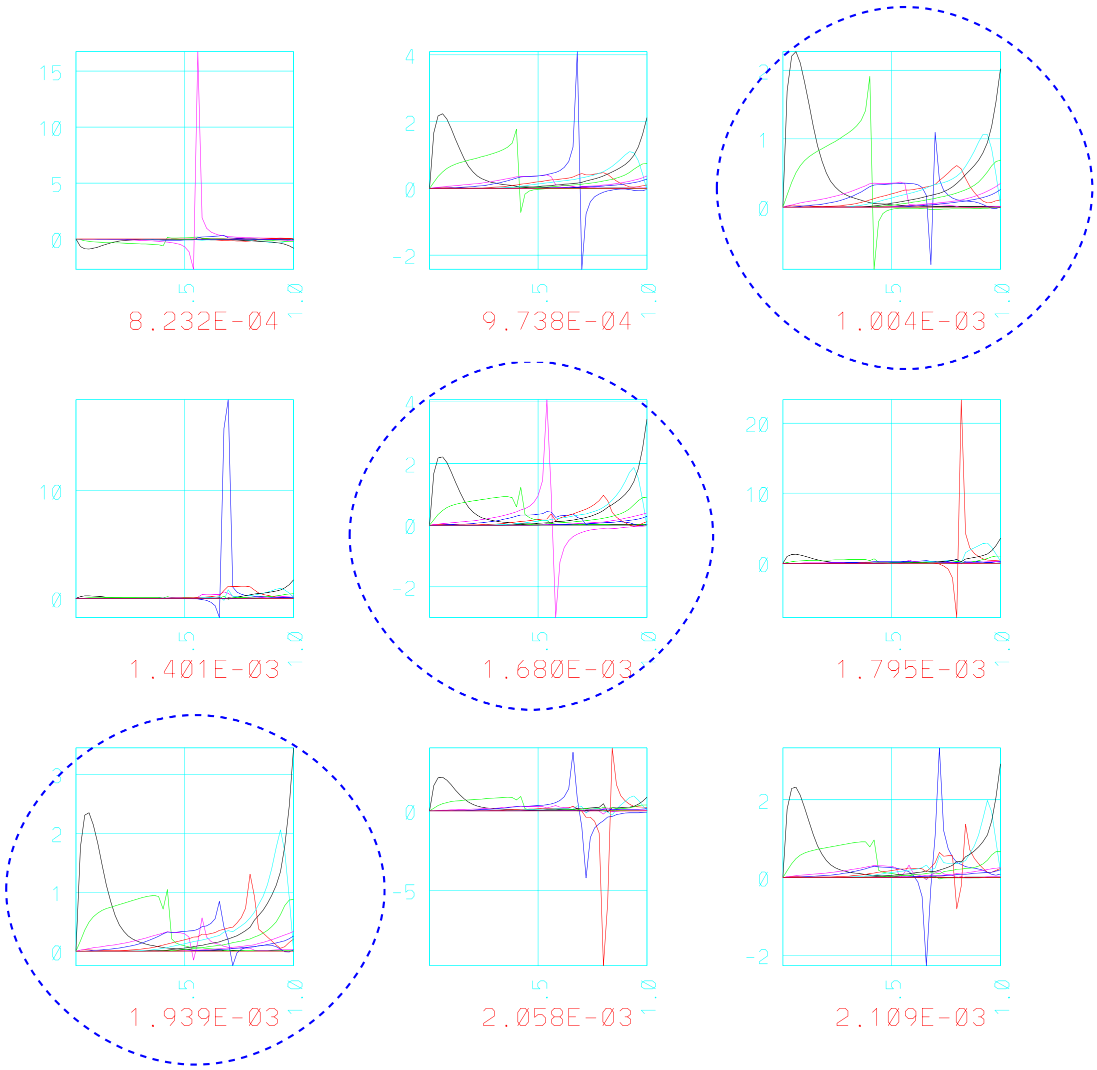

Figure 7: Eigenvectors corresponding to the lowest nine eigenvalues for $k2b$

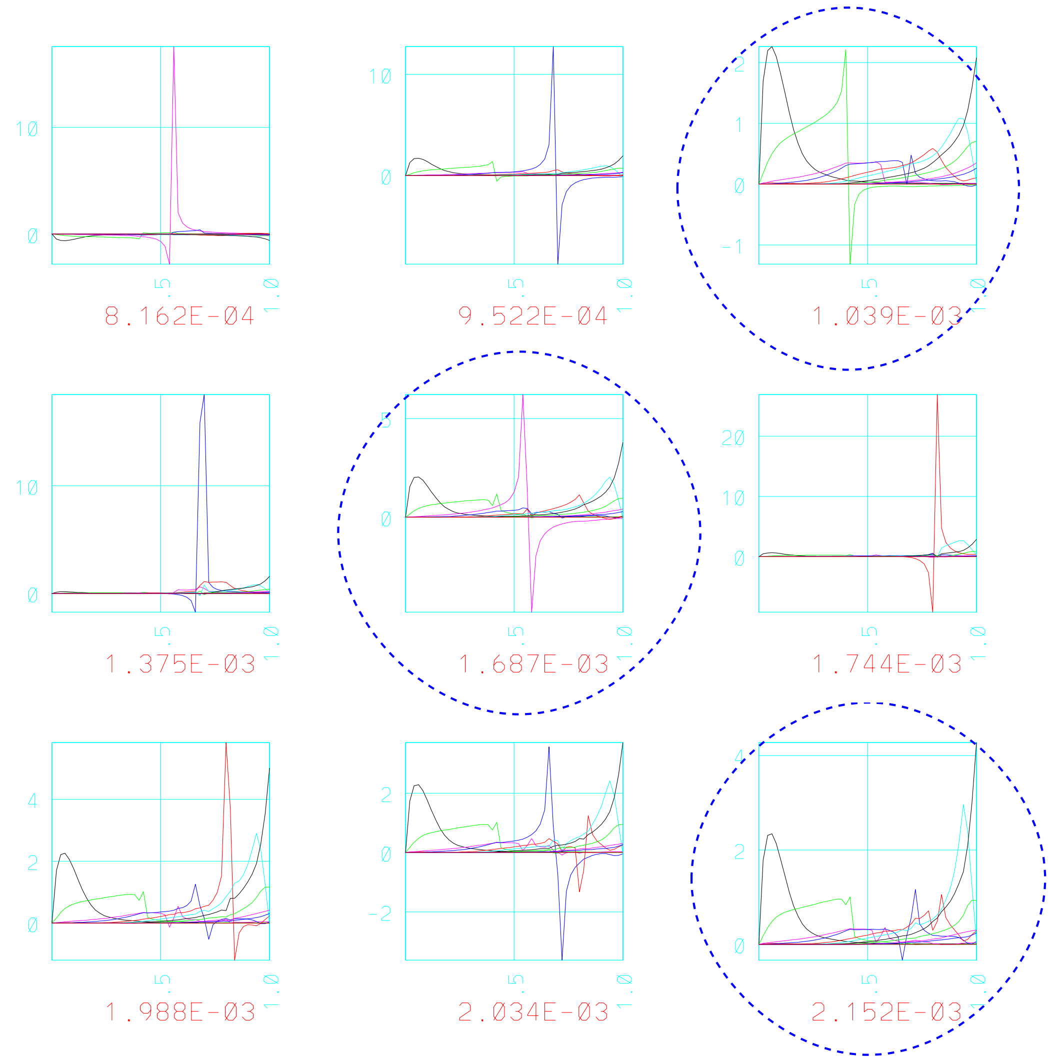

Figure 8: Eigenvectors corresponding to the lowest nine eigenvalues for $k2ba$

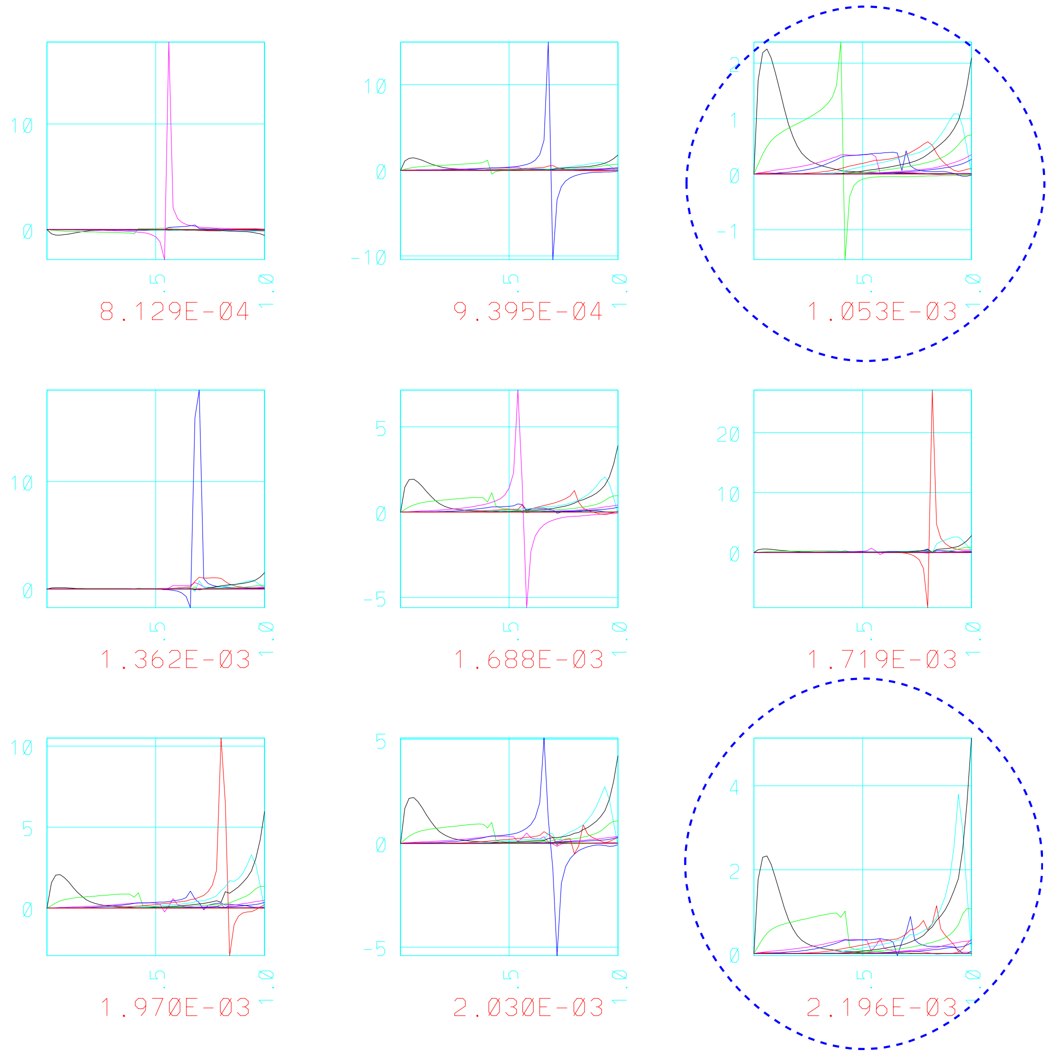

Figure 9: Eigenvectors corresponding to the lowest nine eigenvalues for *k2bb*

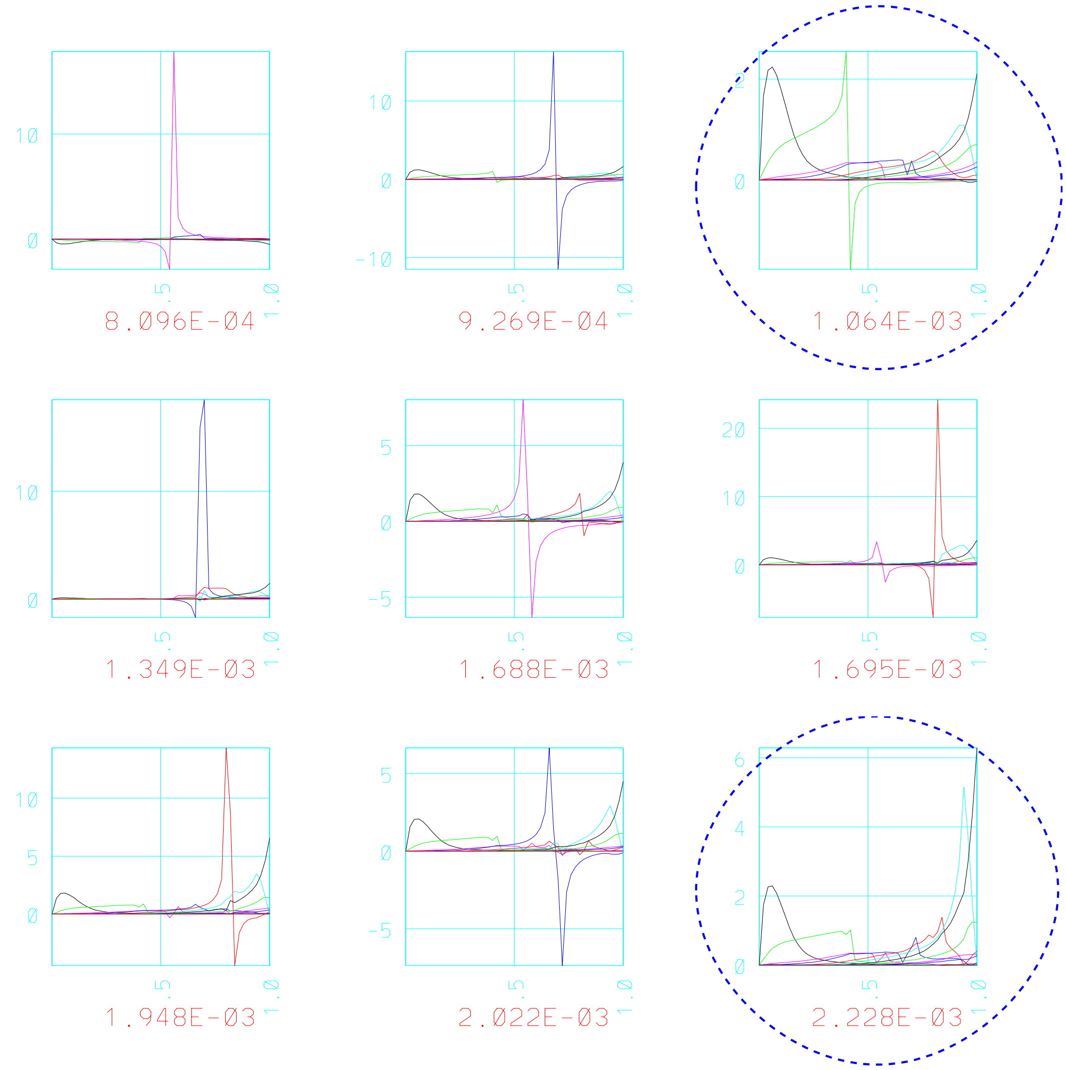

Figure 10: Eigenvectors corresponding to the lowest nine eigenvalues for $k2bc$