Saturation of alpha particle driven instability in Tokamak Fusion Test Reactor

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A nonlinear theory of kinetic instabilities near threshold [Berk *et al.*, Plasma Phys. Rep. **23**, 842 (1997)] is applied to calculate the saturation level of toroidicity-induced Alfvén eigenmodes (TAE), and to be compared with the predictions of δf method calculations (Y. Chen, Ph.D. thesis, Princeton University, 1998). Good agreement is observed between the predictions of both methods and the predicted saturation levels are comparable to experimentally measured amplitudes of the TAE oscillations in Tokamak Fusion Test Reactor [D. J. Grove and D. M. Meade, Nucl. Fusion **25**, 1167 (1985)]. © *1999 American Institute of Physics.* [S1070-664X(99)02102-3]

Toroidicity-induced Alfvén eigenmodes[1,2] (TAE) have attracted much attention in recent years in connection with deuterium–tritium (DT) experiments on the Tokamak Fusion Test Reactor (TFTR) and Joint Europian Torus (JET) tokamaks,[3] where α particles from fusion reactions were generated and their confinement and effects on magnetohydrodynamic (MHD) stability were studied. Both Mirnov coils and reflectometer diagnostics[4] during TFTR. DT experiments have shown low level TAE signals that are driven by the fusion produced alpha particles.

In this paper we apply a recently developed nonlinear theory of kinetic instabilities[5] to calculate the saturation level of TAEs in TFTR and compare it with the results of numerical simulations, where a δf algorithm was utilized[6,7] taking collisions into account through a Monte Carlo method. A newly extended version of NOVA-K, which now includes the finite orbit width (FOW) and Larmor radius (FLR) effects,[8] allows the expression for the growth rate and saturation level to be integrated accurately over the phase space for the ensemble of α particles. Such an integration is important for alpha particles which have orbit widths comparable to the mode width and even to the radius and when the ensemble of particles has a broad energy distribution. Thus all the particles have different collisional frequencies and quantitatively have different types of interaction strengths with the TAE mode.

First we give a physical picture for the mechanisms involved in mode saturation and outline the theory of kinetic instabilities near threshold. In TFTR DT experiments, where TAEs were observed, the population of α particles had relatively low beta $\beta_\alpha \leqslant 0.1\%$, which means a weak TAE drive. At the same time Neutral Beam Injected (NBI) ions contributed to Landau damping, a dominant damping mechanism. Only 100–150 ms after the NBI was turned off and the NBI ions had slowed down, did the TAE become destabilized by fusion α particles.[4] The mode excitation, at least during the initial excitation, should be close to the near threshold regime, where $\gamma \equiv \gamma_L - \gamma_d \ll \gamma_L$, and $\omega_b / \nu_{\rm eff} \ll 1$.[9] During the peak of the activity γ_L might be appreciably bigger than γ_d. Here γ_L is the linear TAE growth rate, γ_d is the TAE damping rate, ω_b is the frequency of α-particle trapping by the perturbed field of the mode, and $\nu_{\rm eff}$ is the effective scattering frequency for particle decorrelating the wave resonance.

To clarify how the scalings for saturation arise we present the following dimensional arguments. We note that for weak kinetically driven instabilities the evolution of the wave energy, W_E (for Alfvén-like waves, the wave energy is nearly equally divided between perturbed magnetic energy and wave kinetic energy so that $W_E = \int d^3\mathbf{r}\,\delta\mathbf{B}\cdot\delta\mathbf{B}/4\pi$, where $\delta\mathbf{B}$ is the perturbed magnetic field) can be written as

$$\frac{\partial W_E}{\partial t} = P_T - 2\gamma_d W_E, \tag{1}$$

with γ_d the rate of background dissipation and P_T the power transfer from resonant alpha particles to the wave, which in general is written as

$$P_T = \int d^3\mathbf{r}\mathbf{j}_r\cdot\mathbf{E} = \int d\Gamma e_\alpha \mathbf{v}\cdot\mathbf{E} f, \tag{2}$$

where $\mathbf{j}_r$ is the perturbed current from the resonant α particles which have a charge e_α and a velocity $\mathbf{v}$, f is the alpha particle distribution near the resonance region, Γ is six-dimensional phase space, and $\mathbf{E} = \mathbf{E}(\mathbf{r},t)$ is the perturbed electric field. For each particle the resonance condition is determined by

$$\Omega = \omega + n\langle\omega_d\rangle + l\Omega_b \tag{3}$$

with $\langle\omega_d\rangle$ the particle mean toroidal transit frequency, $\langle\cdots\rangle$ denotes time averaging over unperturbed orbits, Ω_b is the unperturbed bounce frequency, and n and l are integers.

In Ref. 5 it was shown that the resonant nonlinear trapping frequency of a particle near an O point of a finite amplitude is given by

$$\omega_b^2 = \left\langle e_\alpha \mathbf{E}\cdot\mathbf{v}\frac{n}{\omega}\frac{\partial\Omega}{\partial P_\varphi}\bigg|_{H'}\right\rangle, \tag{4}$$

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where $H'=H+\omega P_\varphi/n$, with P_φ and H the particle canonical momentum and energy (note that H' is conserved during the particle motion in a steady wave, whose toroidal angle φ and time dependence varies only as $n\varphi+\omega t$) and the time average is taken for an exactly resonant particle. Further, it was found that at fixed H' and magnetic moment, the steady distribution function satisfies the equation

$$\Omega\frac{\partial f}{\partial\psi}-\omega_b^2\sin\psi\frac{\partial f}{\partial\Omega}-\nu_{\rm eff}^3\frac{\partial^2 f}{\partial\Omega^2}=-\nu_{\rm eff}^3\frac{\partial^2 F}{\partial\Omega^2}, \tag{5}$$

where $\psi=-n\varphi-\omega t-l\Omega_b t$, and $\nu_{\rm eff}^3=\nu_c<|\partial P_\varphi/\partial(\mathbf{v}/v)|^2>(\partial\Omega/\partial P_\varphi|_{H'})^2$ with ν_c the 90° pitch-angle scattering rate, and F is the equilibrium distribution. The two extreme limits of interest where analytic calculations are straightforward are (a) $\omega_b^2/\nu_{\rm eff}^2\ll 1$ and (b) $\omega_b^2/\nu_{\rm eff}^2\gg 1$.

For case (a) we note that the lowest order estimate of P_T should reproduce linear theory, and we are required to iterate to higher order in powers of $\omega_b^2/\nu_{\rm eff}^2$ if we wish to determine the saturation amplitude for which $\partial W_E/\partial t=0$. Several iterations are required as symmetry properties cause some iterants of the P_T integrals to vanish. An expansion in powers of $\omega_b^2/\nu_{\rm eff}^2$, allows us to write $f=\Sigma_p f_p$, with

$$\Omega\frac{\partial f_p}{\partial\psi}-\nu_{\rm eff}^3\frac{\partial^2 f_p}{\partial\Omega^2}=\omega_b^2\sin\psi\frac{\partial f_{p-1}}{\partial\Omega},\quad p>0, \tag{6}$$

and $f_0=F$. To estimate f_p we note that it is 2π periodic in ψ and thus, $\partial f_p/\partial\psi\sim f_p$. Further, the important resonance dependence of f_p on Ω is determined when the two terms on the left-hand side of Eq. (6) are comparable, leading to $\Omega f_p\sim\nu_{\rm eff}^3\partial^2 f_p/\partial^2\Omega\simeq\nu_{\rm eff}^3 f_p/\Omega^2$. Hence the effective value and range of Ω is $\Omega\sim\nu_{\rm eff}$, which then implies the following ordering:

$$f_1\sim\frac{\omega_b^2}{\nu_{\rm eff}}\frac{\partial F}{\partial\Omega},\quad f_p\sim\frac{\omega_b^2}{\nu_{\rm eff}^2}f_{p-1},\quad \int d\Omega f_p\sim\nu_{\rm eff}f_p. \tag{7}$$

Now we substitute our estimates for f_p in the expression for P_T, using that $\mathbf{v}\cdot\mathbf{E}$ is proportional to ω_b^2. We find,

$$P_T\simeq\int d\Gamma_\perp d\Omega d\psi\sum_l \alpha_l\frac{\omega_b^4}{\nu_{\rm eff}}\frac{\partial F}{\partial\Omega}\left[1-\sum_{p=1}^{\infty}\lambda_p\left(\frac{\omega_b}{\nu_{\rm eff}}\right)^{2p}\right], \tag{8}$$

where $d^3\mathbf{r}d^3\mathbf{v}=d\Gamma_\perp d\Omega d\psi$, α_l is the appropriate proportionality factor, and $\lambda_p=O(1)$. To lowest order we recovered the scaling of linear theory, including that the drive is independent of $\nu_{\rm eff}$. To obtain the next order correction we need to note that the ψ integral in P_T vanishes for all odd values of p and therefore the leading correction term is for $p=2$. The scaling for the stationary solution to $P_T=2\gamma_d W_E$ is then found to be the same as in Ref. 9,

$$\omega_b^2\simeq\nu_{\rm eff}^2\sqrt{\frac{\gamma_L}{\gamma_d}-1}. \tag{9}$$

In the opposite limit (b), rigorously treated in Ref. 10, we assume $\nu_{\rm eff}/\omega_b\ll 1$. In this case the $\nu_{\rm eff}^3$ term is small and we need to solve the kinetic equation for $f=F+\Sigma_p f_p$ in the following iterative manner:

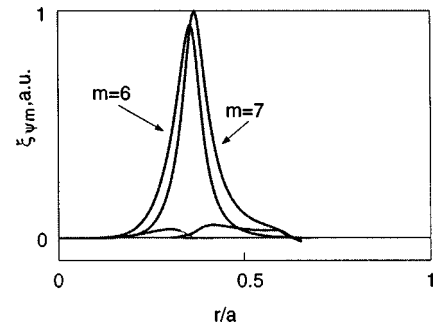

FIG. 1. Radial structure of poloidal harmonics of $n=4$ core localized TAE mode radial displacement obtained by NOVA for TFTR shot 103 101 at 2.92 s.

$$\Omega\frac{\partial f_p}{\partial\psi}-\omega_b^2\sin\psi\frac{\partial f_p}{\partial\Omega}=\nu_{\rm eff}^3\frac{\partial^2 f_{p-1}}{\partial\Omega^2}. \tag{10}$$

The lowest order solution ($p=0$, where the right-hand side vanishes) shows that f_0 varies on a scale $\Omega\sim\omega_b$, and hence $f_0\simeq\omega_b\partial F/\partial\Omega$ for $\Omega\sim\omega_b$. However, in this order P_T vanishes, so we need to estimate f_1 to find P_T. Thus, as $\Omega\sim\omega_b$, we have

$$f_1\simeq\frac{\nu_{\rm eff}^3}{\omega_b^3}f_0\sim\frac{\nu_{\rm eff}^3}{\omega_b^2}\frac{\partial F}{\partial\Omega}. \tag{11}$$

Substituting this estimate for f_1 in Eq. (2) gives for P_T,

$$P_T\sim\int d\Gamma\sum_l \alpha_l\omega_b\nu_{\rm eff}^3\frac{\partial F}{\partial\Omega}\simeq\frac{\nu_{\rm eff}^3}{\omega_b^3}\gamma_L W_E. \tag{12}$$

As, at saturation, $P_T=2\gamma_d W_E$, we find

$$\omega_b^2\simeq\nu_{\rm eff}^2\left(\frac{\gamma_L}{\gamma_d}\right)^{2/3}. \tag{13}$$

In experiments the distribution of fast particles may be in both regimes of collisionality. In Ref. 5 a simple interpolation formula, which combines the two regimes mentioned above, was proposed and shown to be accurate in numerical simulations. The interpolation formula is given by

$$\frac{\gamma_d}{\gamma_L}=\overline{1+0.57U(\Gamma)/(1+1.45/U(\Gamma))^{1/3}},$$
$$\overline{(\cdots)}=\int d\Gamma Q(\cdots)\Big/\int d\Gamma Q, \tag{14}$$

where $U(\Gamma)=(\omega_b(\Gamma)/\nu_{\rm eff}(\Gamma))^3$, $Q=\Sigma_l\langle e\mathbf{E}\cdot\mathbf{v}\rangle^2 F'_{H'}\delta(\Omega-\omega)$. Equation (14) is used to calculate the amplitude of TAEs using a recently developed version of NOVA-K,[8] which includes FOW and FLR effects. The results are compared with the results of a direct simulation of the nonlinear evolution of TAEs carried out based on a collisional δf algorithm[6] and a Hamiltonian formulation of the guiding center motion.[11] The pitch-angle scattering operator for Coulomb collisions is implemented using the Monte Carlo technique, and thus the collisional effect is correctly accounted for without explicitly calculating $\nu_{\rm eff}$.

TAE amplitude saturation in TFTR. The observed plasma oscillations in the presence of fusion alpha particles in TFTR DT experiments[4] have been attributed to the exci-

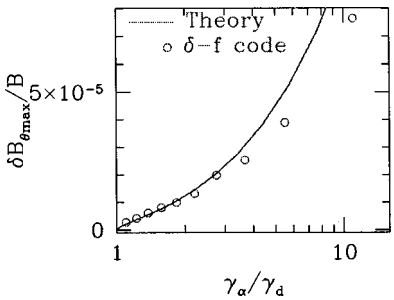

FIG. 2. Theory vs δf method calculations.

tation of core localized TAE modes. The amplitude of TAEs in those measurements are low with $\delta B_\theta/B \simeq 10^{-5}$. For the purpose of comparison of the two methods outlined above and the experiment we choose equilibrium of TFTR shot 103 101 at 2.92 s where $n=4$ was clearly seen for about $\sim$100 ms. The safety factor profile was flat at the center and had $q(r=0)=1.6$, and $q(r=a)=5$. The central Alfvén velocity was $v_A=1.2\times10^9$ cm/s. The TAE poloidal harmonics radial structure is calculated from the NOVA[12] code and the mode structure is shown in Fig. 1 at the eigenfrequency $f=214$ kHz which matches the measured frequency at 2.92 s. At that time, alpha particles born at the end of NBI termination had slowed down to an energy of $\mathscr{E}_c \simeq 1.5$ MeV. Hence we take for alpha particle distribution function a slowing down distribution with the upper cutoff at energy $\mathscr{E}=\mathscr{E}_c$.

As already noted, NOVA-K calculates the growth rate induced by alpha particles including FOW and FLR effects[8] and the integral in Eq. (14). For the case under consideration the contribution of alpha particles with $\beta(0)=0.083\%$ to the growth rate is $\gamma_\alpha/\omega=1.1\%$ without FLR corrections and $\gamma_\alpha/\omega=0.74\%$ with FLR corrections. The results of using the formula given by Eq. (14) are shown in Fig. 2 along with the results of the δf calculation implemented in ORBIT code[7] (both without FLR effects). The two results were compared for the same alpha particle contribution to the growth rate, which required choosing a slightly lower α-particle beta, $\beta(0)=0.07\%$, in the ORBIT code than in the NOVA-K calculations. The discrepancy is due to some technical simplifications employed in the ORBIT calculations, such as using a Shafranov shifted equilibrium with circular surfaces and an approximate representation of MHD displacement vector, etc.[7] With this synchronization of the linear growth rates we see that the saturation levels predicted by the two codes agree quite well with each other if $\gamma_L/\gamma_d \leqslant 10$. Note that the agreement is best if the saturation level or equivalently γ_L/γ_d, is not too large. It has been noted in Ref. 13 that such discrepancy is expected at larger saturation levels, since then the analytic approximation, that assumes the nonlinear displacement of an alpha particle is small compared to the spatial scale of a mode, begins to fail. Note that the experimental value for the saturated amplitude, $\delta B_\theta/B \simeq 10^{-5}$, agrees with these calculations, when we assume that $\gamma_L/\gamma_d \simeq 2$, a value compatible with predictions of the NOVA-K code for linear TAE stability studies.[14]

One issue in the comparison of the two calculations is the assumption that the saturation is at a steady level, rather than in the form of pulsations, that can be regular or pulsating. The simulations show a pulsating response. We need to clarify whether such a response is intrinsic physics or numerical noise. Theory predicts that if $\nu_{\rm eff}$ is too small, the wave amplitude will oscillate in time, and perhaps even show explosive behavior. A detailed criteria has been obtained when the linear instability is due to a single resonance in phase space, and the condition for a steady state response has been found to be $\nu_{\rm eff}/(\gamma_L-\gamma_d)>2$, when $(\gamma_L-\gamma_d) \ll \gamma_d$. In the opposite regime, $\gamma_L \gg \gamma_d$, physical energy arguments,[15] as well as supporting simulations,[16] indicate that $\nu_{\rm eff}/\gamma_d \gtrsim 1$ is required for achieving a steady state saturation level. An interpolation formula for the two regimes gives $\nu_{\rm eff}/[\gamma_d(1-\gamma_d^2/\gamma_L^2)] \gtrsim 1$ to have a steady response. These criteria have an additional imprecision as we are dealing with a continuum of resonance interactions, so that a precise criterion for the transition to a pulsating response is not easily determined. Nonetheless, the rough criteria to achieve a steady response is expected to be the same, within the numerical factor, with substitution $\nu_{\rm eff} \to \overline{\nu_{\rm eff}}$ with the ''bar'' denoting an appropriate average over all particle resonances. The averaging used in NOVA-K gives $\overline{\nu_{\rm eff}} \simeq 6\times10^3\ {\rm s}^{-1}$, and for $\gamma_L=10\times10^3\ {\rm s}^{-1}$ we find $\overline{\nu_{\rm eff}}/\gamma_d(1-\gamma_d^2/\gamma_L^2) \geqslant 1.56$. This value is sufficiently large that it is plausible that the TAE signals in the experimental data are quasistationary.

As the ORBIT code shows considerable pulsations, the results plotted in Fig. 2 are time averaged with enough particles chosen to obtain convergent results. The noise does not appear to have a regular frequency pattern, which might be expected from pulsations arising from dynamical noise-free behavior and long runs with four times the normal number of particles show a reduced fluctuation level. Thus we interpret the numerical data to be statistically stationary. Nonetheless, further work is needed to definitively clarify whether oscillations due to nonlinear dynamics can arise.

In summary, good agreement has been demonstrated between two different approaches for determining the saturation amplitude of alpha particle driven TAE modes: a phase space integral evaluation based on an interpolation formula between two nonlinear analytic limits and a particle simulation based on a combined δf and Monte Carlo algorithm. The agreement makes both models credible and allows for the rapid evaluation of the saturation amplitude in the NOVA-K code.

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