Energetic Particle Quasi-linear Code

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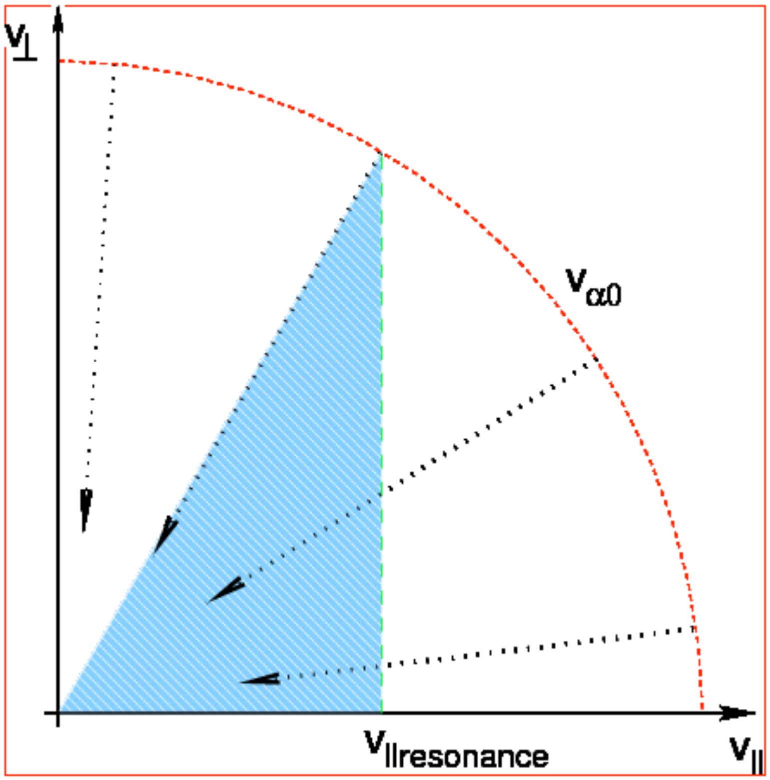
Quasi-linear Theory of Energetic Particle Transport

1. Use quasi-linear theory of energetic particles to assess how ‘alphas’ respond to the instabilities they produce and how the transport of the background plasma is affected
2. Will enable an ‘integration’ package for that will assess both short (instability times perhaps leading to rapid diffusion) and long time (transport time scale) behavior
3. Theory can be constructed that is energy and momentum conserving (energy and momentum emerging from the particles goes to the waves) and background dissipation mechanisms can be used to feed back to the plasma (determining heating, torques,channeling, etc.)
4. Theory is self-consistent. Is it right? If it essentially is, there should be a priority to develop it.

Kolesnichenko's Estimate

Quaslinear α-particle transport model: How much of alphas population affected?

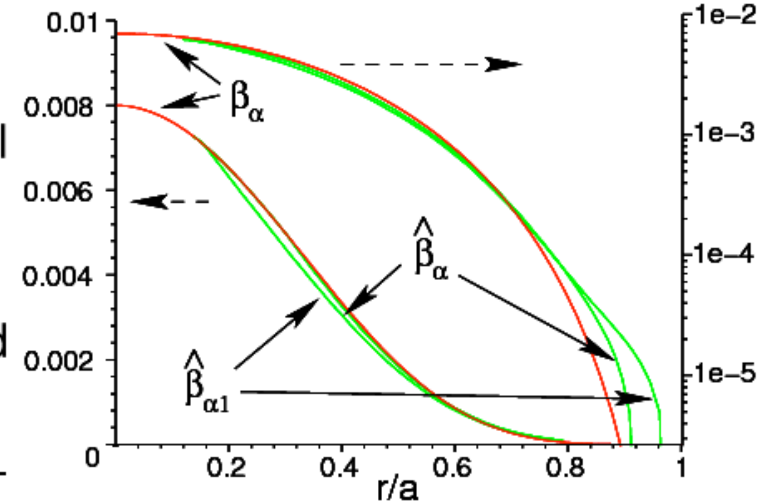


- Maximum effect from instabilities with $v_\|$ resonance
- Fraction of effected alpha power (Kolesnichenko '80) $P_{\alpha res} = P_\alpha \left(v_{\alpha 0} - v_\|\right) v_\| / v_{\alpha 0}^2 \leq 25\%$
- Other particles are not interacting with such instabilities

What are expected effects on alphas profiles (normal shear)

Let's assume:

- Quasilinear theory is applicable
- α's are redistributed, but keep critical beta profile
- β_α is equilibrium profile
- $\hat{\beta}_\alpha$ is final profile with near threshold gradient
- $\hat{\beta}_{\alpha 1}$ is at critical beta at 0.7 of theoretical beta critical



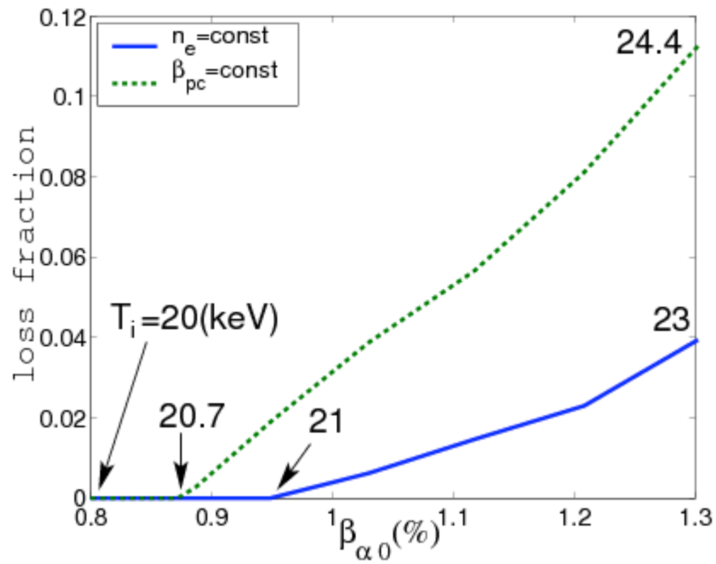
How it was done:

$$\frac{\partial \beta_{\alpha cr}}{\partial r} = -\frac{\gamma_{iL} + \gamma_{ecoll}}{\gamma\prime_\alpha},\ \gamma_\alpha\prime = \gamma_\alpha / (\partial \beta_\alpha / \partial r)$$

Use the phase space particle conservation law $\int_0^a r \left(\beta_\alpha - \hat{\beta}_\alpha \right) dr = 0$

(Gorelenkov, Berk, Budny, NF'05)

This model can also predict losses of alphas



1% alpha's central beta is threshold

5% of all alphas is tolerable for ITER (Putvinskii, NF '99)

TAE effects will be benign in nominal regular shear scenario according to the quasilinear diffusion model.

Losses are calculated at fixed beta with ion temperature within $20 < T_{i0}(keV) < 24$ ($\beta_{\alpha 0} \sim T_{i0}^{5/2}$) and density $20 < T_{i0}(keV) < 23$ ($\beta_{\alpha 0} \sim T_{i0}^{7/2}$).

Quasi-Linear Equation with Resonance Discreteness

$$\frac{\partial f}{\partial t}=\frac{\pi}{2}\sum_{n,l}\frac{\partial}{\partial\Omega_l}|\,\omega_{b;n,l}^2\,|^2\,\Delta\left(\omega_n-\Omega_l,\gamma_l\right)\frac{\partial f}{\partial\Omega_l}-\hat{\nu}\left(f-f_0\right);\quad \Omega_l=k_l\upsilon$$

$$\Delta(x,\varepsilon)=\frac{\varepsilon}{x^2+\varepsilon^2}\cong\delta(x)\cong G(x),\ \text{with}\int_{-\infty}^{\infty}dxG(x)=1;\ \omega_{b;n,l}^2=\frac{ek_l}{m}E_{n,l}$$

Refine resonance width $\gamma^2\to\gamma^2+\nu_{eff}^2+\omega_b^2+\ldots$

Wave energy and momentum conservation built in!

rate of wave momentum change

$$k_l\frac{G_\omega}{8\pi}\frac{\partial|E|^2}{\partial t}\propto\sum_{n,l}\int d\Omega_l\left|\omega_{b;n,l}^2\right|^2\Delta\left(\omega_n-\Omega_l\right)\frac{\partial f}{\partial\Omega_l}$$

Diffusion does not go beyond ‘KAM’ surface unless there is mode overlap

Then interesting dynamics arise: global diffusion, avalanches…

Overlap releases more free energy (negligible collisions)

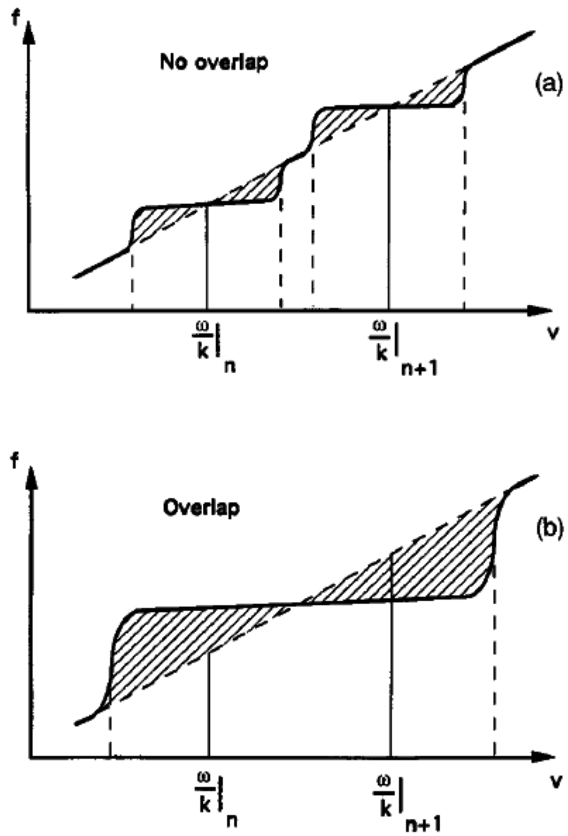


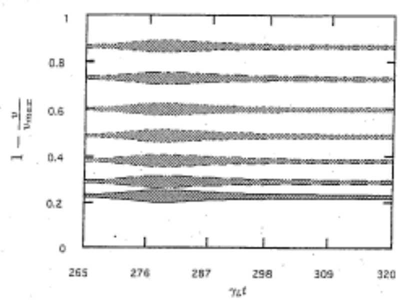
FIG. 7. Schematic diagram of plateau formation with two adjacent modes. (a) No overlap. (b) Overlap.

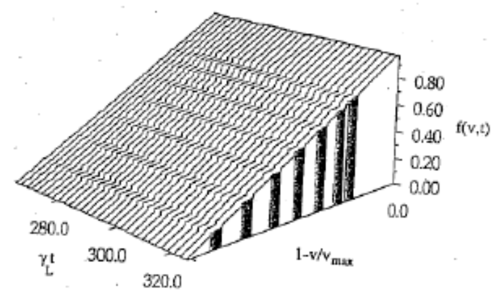
Overlap leads to:

$4 \times$ the phase space interchange;
$16 \times$ the energy release

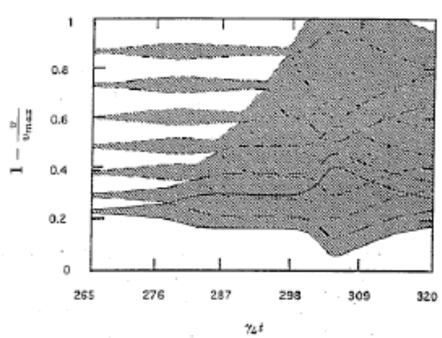
Avalanche

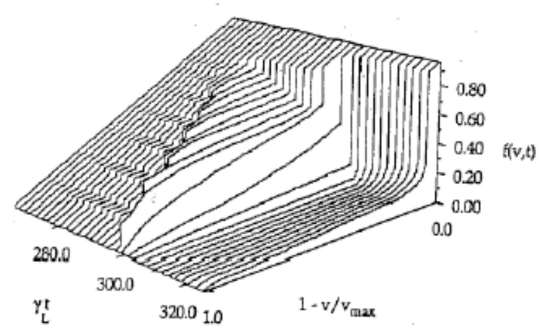
Benign Excitation of Discrete Modes





Trigger of Avalanche





Quasi-linear equation in tokamak ($\omega << \omega_{ci}$)

$$\frac{\partial f}{\partial t} = \frac{\pi}{2}\sum_{n,l}\frac{\partial}{\partial \Omega_l} | \omega_{b;n,l}^2 |^2 \Delta\left(\omega_n - \Omega_l, \gamma_l\right)\frac{\partial f}{\partial \Omega_l} - \hat{\nu}\left(f - f_0\right);$$

$$\Omega_l (H - \omega_l P_\phi / n, P_\phi, \mu) = n\bar{\omega}_\phi + l\bar{\omega}_\theta;$$

$$\omega_b^2 (E,\mu,P_\phi) = \frac{n}{\omega}\left| < e\vec{\upsilon}(t)\cdot\vec{E}(t) >_{n,l}\right| \frac{\partial \Omega_{n,l}(H',P_\phi,\mu)}{\partial P_\phi} |_{\mu,H'=H-\omega P_\phi / n}$$

where $< e\vec{\upsilon}(t)\cdot\vec{E}(t) >_{n,l}$ is calculated in NOVA

with this notation arising from instantaneously power transfer into its spectral components which is calculated in NOVA

$$e\vec{\upsilon}(t)\cdot\vec{E}(t) = \sum_{n,l} < e\vec{\upsilon}(t)\cdot\vec{E}(t) >_{n,l} \exp\left[in\phi - il\theta - i\omega_n t\right] + cc$$

$$< e\vec{\upsilon}(t)\cdot\vec{E}(t) >_{n,l} = \xrightarrow[T_b \to \infty]{} \frac{e}{T_b}\int_0^{T_b} dt\vec{\upsilon}(t)\cdot\vec{E}(t)\exp\left[-in\left(\phi(t) - \dot{\bar{\phi}}(t)\right)t + il\omega_\theta t - i\omega t + i\chi_l\right]$$

pendulum equation for phase χ_l; $\frac{d^2\chi_l}{dt^2} + \omega_{b;n,l}^2 \sin\chi_l = 0$

Limits to diffusion without overlap of resonances

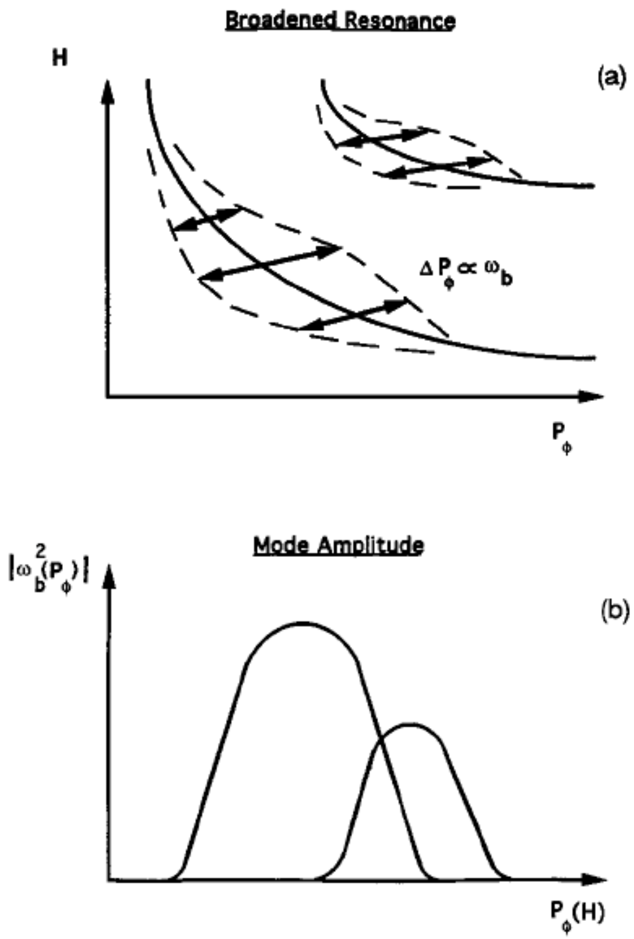


FIG. 10. Schematic diagram of the resonance broadening in phase space, is shown in (a). Equation (14) determines the center of the resonance, while the shape of the linear eigenfunction, schematically shown in (b), and the bounce frequency of particle in the wave determines the width of the resonance.

What can be gained with Q-L code

1. Keep track of details of entire phase space region which is especially important for knowing when to expect mode overlap and as an aid in the interpretation of diagnostics
2. Feed transport information to global transport codes as well as process how background transport effect energetic particles
3. Assess understanding of results of numerical simulation and if achieved, is likely to an easier tool to plan for new experiments
4. Use to plan for experiments and assess experimental results.