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Reconstruction of Fast Ion Distribution in Constants-of-Motion Space from NUBEAM Output

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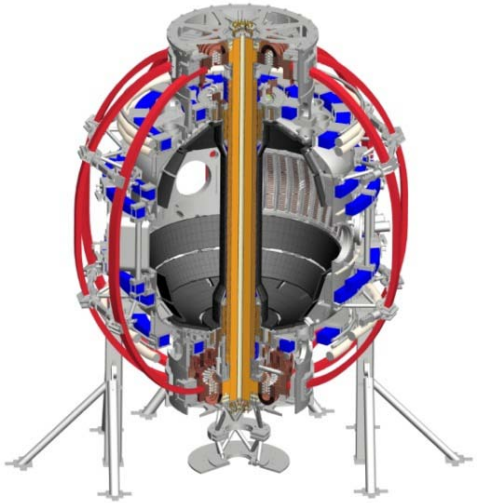
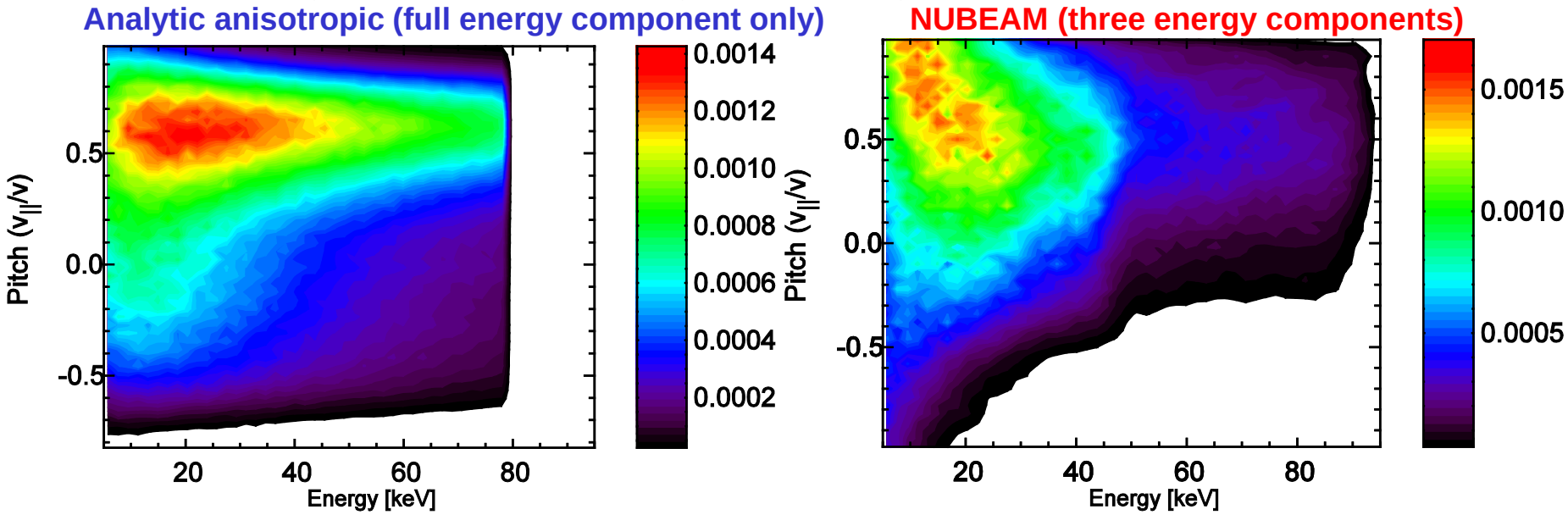


Initialization with Realistic Fast Ion Distributions could Improve Accuracy of Hybrid simulations

- Fluid/kinetic hybrid simulation codes typically initialize fast ion distribution with an analytic form, but it may not be a good fit to the actual distribution.



- More realistic fast ion distribution can be obtained from NUBEAM.
  - NUBEAM: Monte Carlo; fast ions behave classically → $F(R, Z, E, \lambda=v_{||}/v)$
- Hybrid simulation codes need the distribution function expressed in terms of constants of motion $f(P_\varphi, \mu, E)$ and the function must be smooth enough to allow derivatives to be taken with E and P_φ

Jacobian of the Transformation from Velocity Space to Constants-of-Motion Space Strongly Depends on Particle Orbit Topology

- Transformation from velocity space to constants-of-motion space.

$$\int G(x,y,z,v_x,v_y,v_z)d^3\mathbf{x}d^3\mathbf{v}$$
$$=\int F(R,Z,\lambda,E)R\sqrt{E}dRdZd\lambda dE$$
$$=\sum_{v_{sign}}\int f_{v_{sign}}(P_\phi,\mu,E)\Im_{v_{sign}}(P_\phi,\mu,E)dP_\phi d\mu dE$$

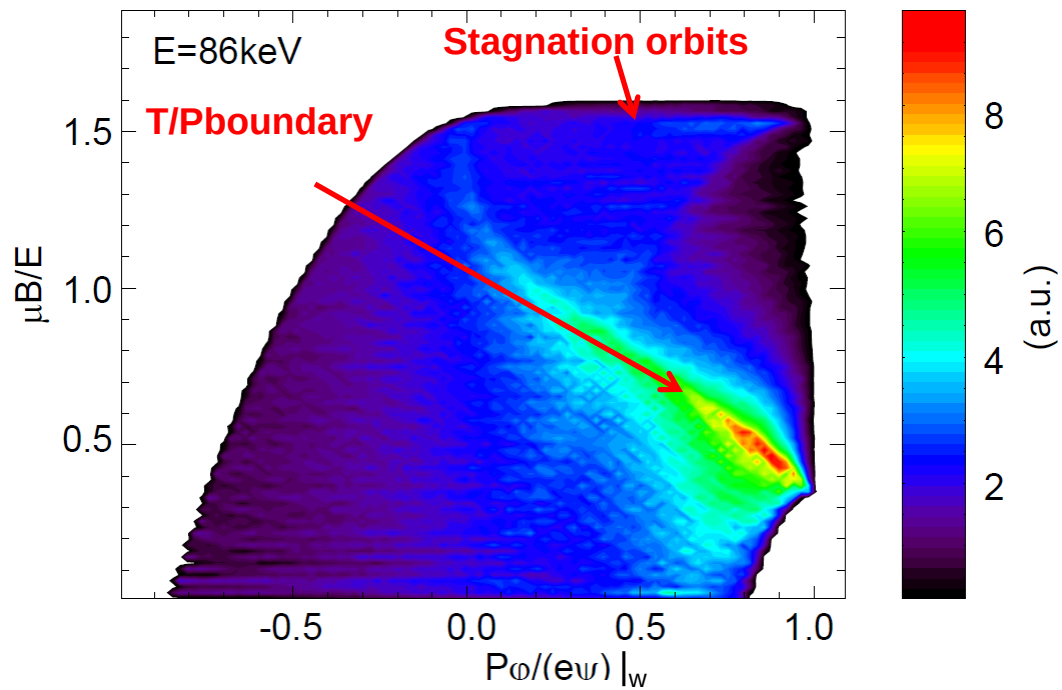
$$P_\varphi=(eA_\varphi+mv_\varphi)R\approx e\psi(R,Z)+mv_\parallel R\frac{B_\varphi(R,Z)}{B(R,Z)}$$

$$\mu=\frac{\frac{1}{2}mv_\perp^2}{B}=\frac{[1-(v_\parallel/v)^2]E}{B(R,Z)} \qquad \lambda=v_\parallel/v$$

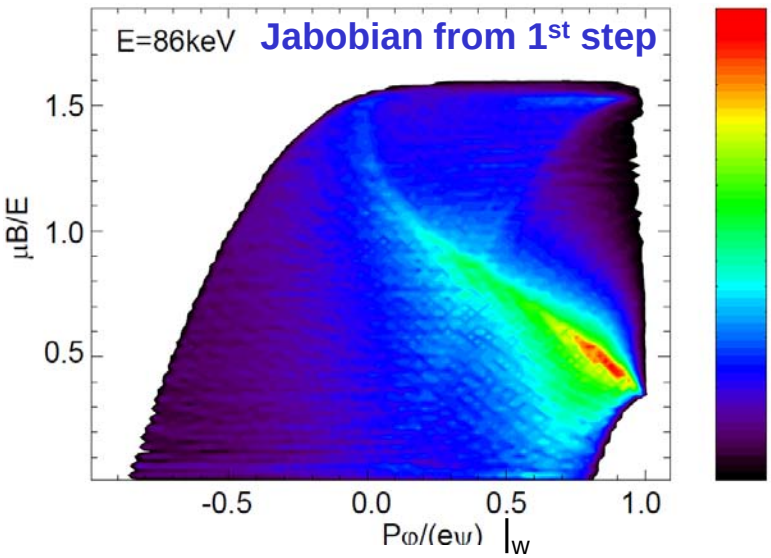
- An exact analytic $\Im_{v_{sign}}(P_\varphi,\mu,E)$ cannot be easily calculated because it requires integration over phase space orbits.

$$\Im(P_\varphi,\mu,E)=\frac{4\pi^2\tau_{bounce}}{qm^2}$$

J. Egedal Nucl. Fusion 2005



Jocabian Can be Calculated Accurately with Iterative Monte Carlo Method

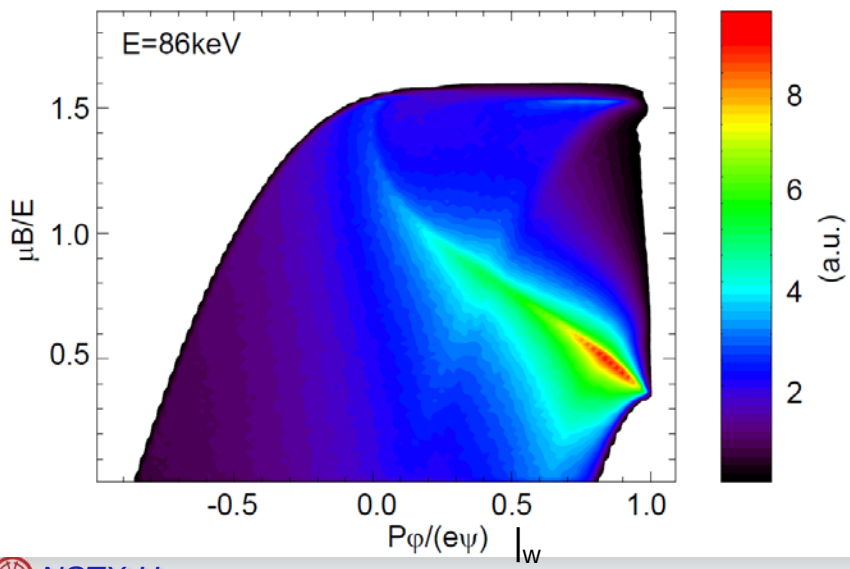


- Jacobian can be considered as the ratio of infinitesimal volume in (P_φ, μ ,E) space to the infinitesimal volume in (R, Z, λ, E) space

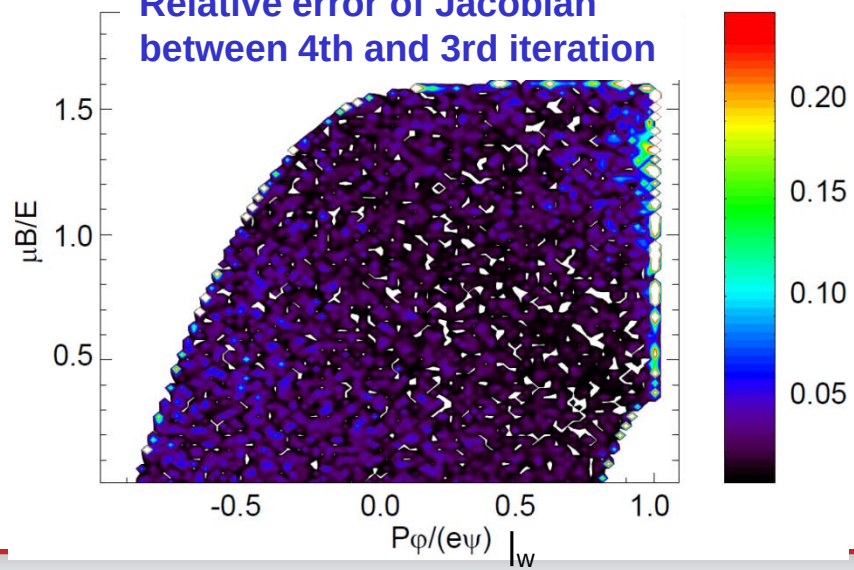
$$\int F(R,Z,\lambda,E)R\sqrt{E}\,dRdZd\lambda dE = \sum_{v_{sign}}\int f_{v_{sign}}(P_\phi,\mu,E)\Im_{v_{sign}}(P_\phi,\mu,E)dP_\phi d\mu dE$$

- Iterative MC method *(*Improved from Breslau's work)*
  - Keep f uniformly in (R, Z, λ, E) space
  - Launch more particles in the region where the Jocabian has large relative error, adjust their weight

Jabobian from 4th iteration



Relative error of Jacobian between 4th and 3rd iteration



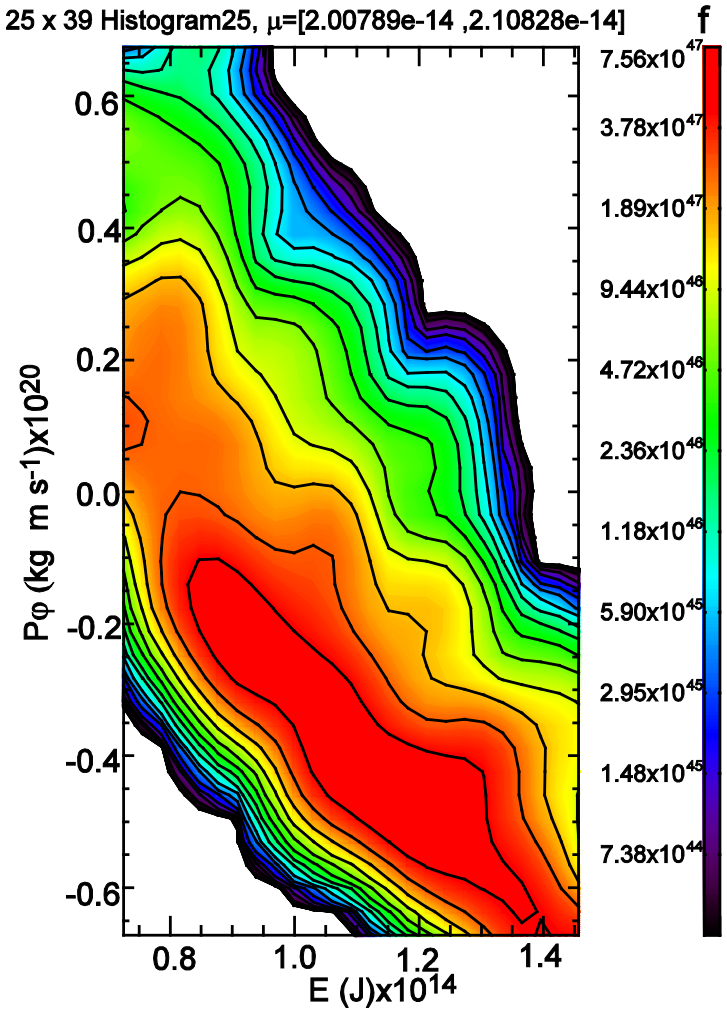
Reconstruction of Fast Ion Density Function in Constants-of-Motion Space

- Divide NUBEAM output particle data into two subsets based on $v_{||}$ sign
- For each subset, sort particles by μ and divide them into several subpopulations of equal width in μ.
- Divide each subpopulation into a number of bins in the P_φ and E directions. The bin width in P_φ and E direction is the same for each subpopulation.
- Multiply by the numerically calculated Jacobian.
- Apply Gaussian smoothing in both P_φ and E directions.
- Fit 2D cubic B-spline to the smoothed data using GSL routines, with uniform knots and a number of coefficients in each direction approximately 5/8 the number of bins.

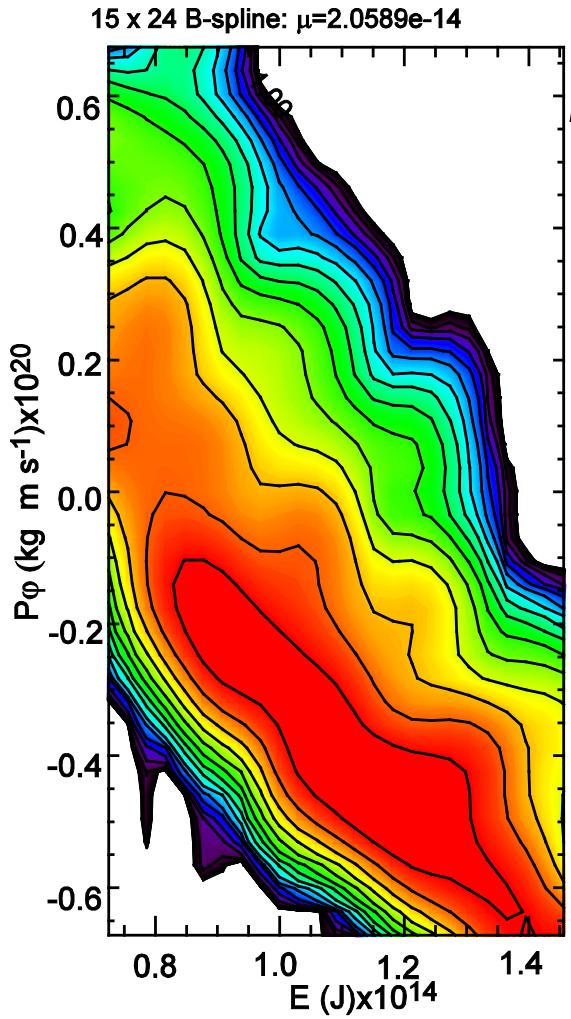
**Improved from the work of Breslau et al. Sherwood Meeting 2011*

Good Agreement between Raw Fast Ion Density Function and Spline Fit in Constants-of-Motion Space

Raw data from NUBEAM



Reconstructed from cubic B-spline fit

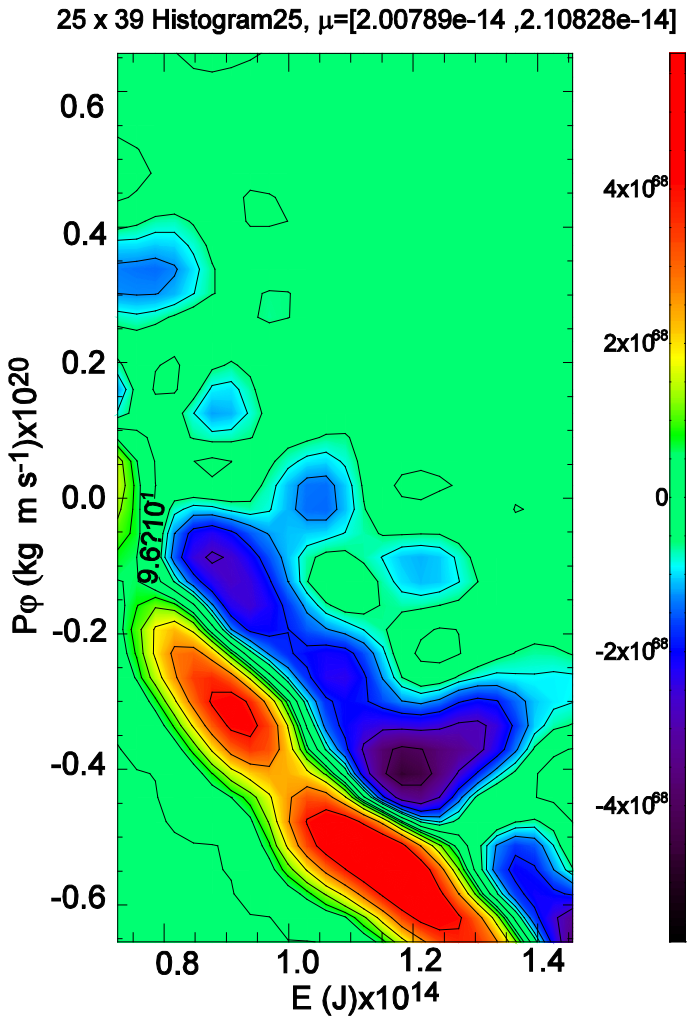


$$f_{co-current}(P_\varphi, E, \mu)|_{\mu=2.058e-14}$$

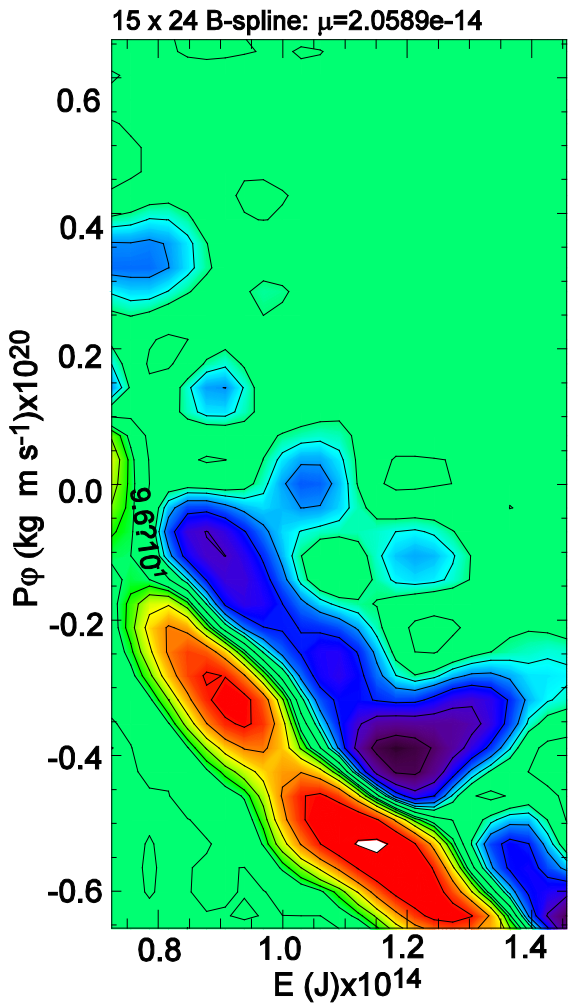
- 40 bins in μ for $v_{||}$>=0
  38 bins in μ for $v_{||}$<0
- B-spline fits for high-occupancy μ subset are good
- B-spline fits for low-occupancy μ subsets are adequate

Gradients are Smooth, Match well with NUBEAM Raw Data

Raw data from NUBEAM



Reconstructed from cubic B-spline fit

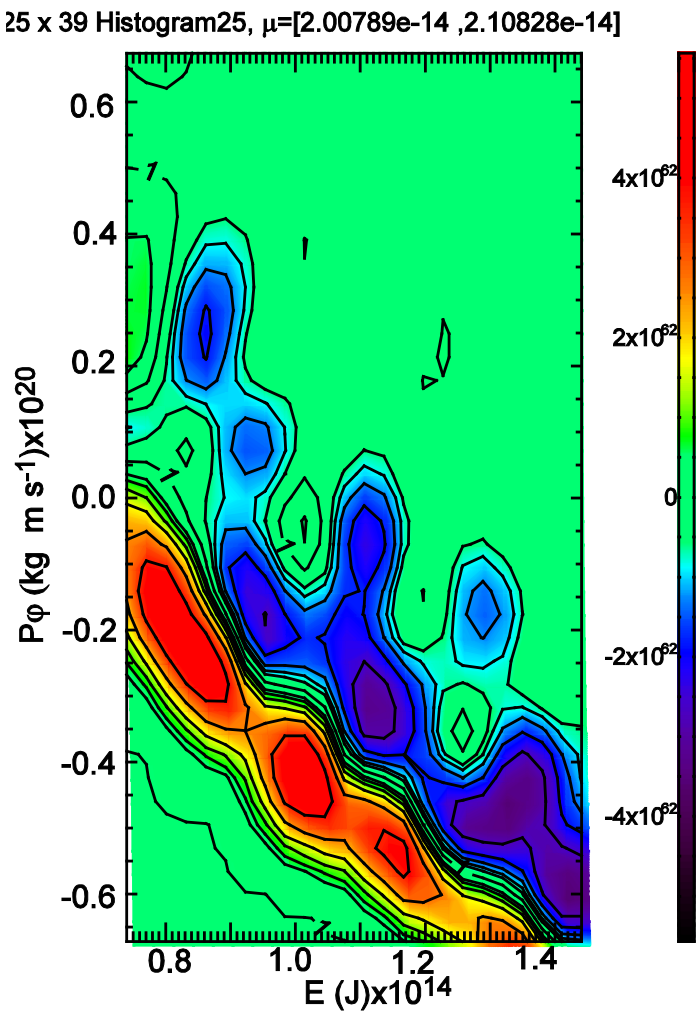


$$\frac{\partial f_{co-current}(P_\varphi, E, \mu)}{\partial P_\varphi}$$

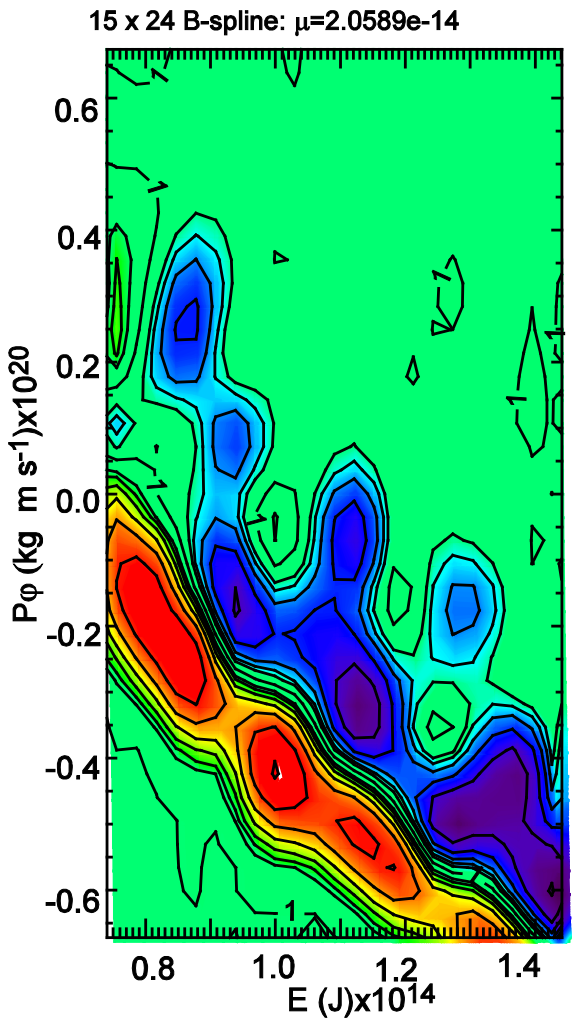
Derivative relative to P_φ

Gradients are Smooth, Match well with NUBEAM Raw Data (Cont'd)

Raw data from NUBEAM



Reconstructed from cubic B-spline fit



$$\frac{\partial f_{co-current}(P_\varphi, E, \mu)}{\partial E}$$

Derivative relative to E

Summary

- An interface has been developed to reconstruct fast ion distribution in constants-of-motion space from NUBEAM output. The spline coefficients are saved in a file, which can be used to reconstruct fast ion density function and perform quick spline and derivative interpolations at arbitrary location.

- The interface has been tested with M3D-K code and reasonable results have been obtained. This interface could be used for other simulations codes.

Backup Slides

Jocabian Can be Calculated Accurately with Iterative Monte Carlo Method

- Jacobian can be considered as the ratio of infinitesimal volume in (Pφ, μ ,E) space to the infinitesimal volume in (R,Z,λ,E) space
- Step 1: Launch 10M random particles uniformly in (R,Z,λ,E) grids, Jacobian $\Im(P_\phi, \mu, E)$ is inversely proportional to the particle number/weight in each (P_φ, μ ,E) grid

The accuracy of $\Im(P_\phi, \mu, E)$ is generally not good near T/P or C/L boundaries.

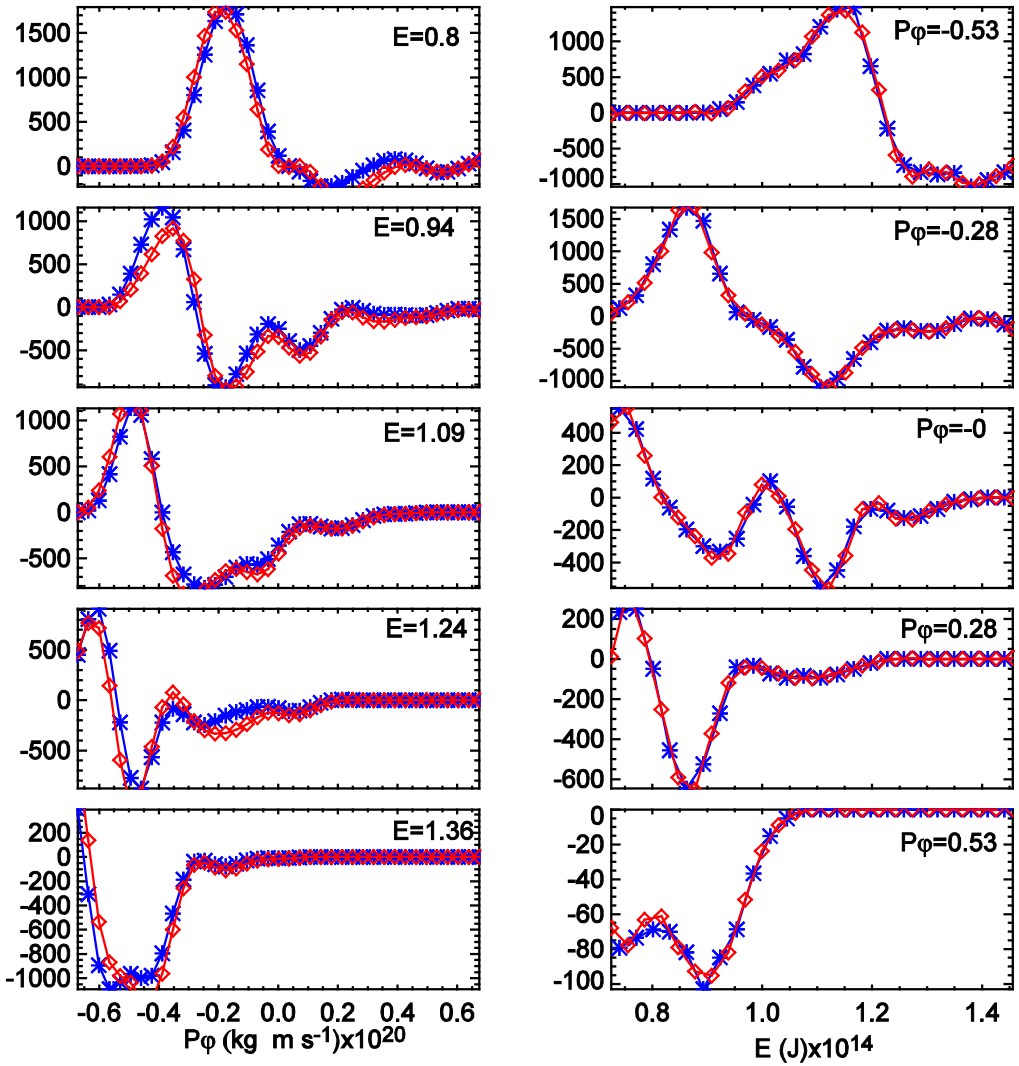
To improve the accuracy of Jacobian, set i_level=1 for all (R,Z,λ,E) grids

- 2nd iteration: Double the particle number (20M) and reduce the particle weight by half; re-launch particles uniformly in (R,Z,λ,E) grids; recalculate the Jacobian; find the grids in (R,Z,λ,E) space whose corresponding Jacobian has relative larger error and change their iteration marker i_level to i_level+1

…

- n-th Iteration: Launch ~20M particles uniformly in (R,Z,λ,E) space except the grids whose Jacobian has relative larger error. For those grids, increase the particle number by 2^(i_level), and decrease their weight by (½)^i_level. ; recalculate the Jacobian; find the grids in (R,Z,λ,E) space whose corresponding Jacobian still has relative larger error and set their iteration marker i_level to i_level+1
- Repeat n-th Iteration until relative error is acceptable.

Gradients are Smooth, Match well with Raw Data



Blue: raw data from NUBEAM
Red: reconstructed from cubic B-spline fit

$$\frac{\partial f_{co-current}(P_\varphi, E, \mu)}{\partial E}$$