Benchmark studies of TAE instabilities

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Acknowledgements: A. Könies, J-N. Leboeuf, V. L. Lynch
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Computational tools used for AE mode studies

- TAEFL
  - Initial value reduced MHD/gyrofluid
  - General noncircular tokamak equilibria
  - Non-perturbative
  - Recent applications to TAE/EAE/RSAE/etc. mode structure, growth rate, nonlinear evolution in DIII-D
- AE3D-K (collaboration with A. Konies, IPP)
  - Perturbative calculation of stability using stable eigenmode (AE3D) particle/wave energy transfer method
  - Applicable to both axisymmetric and 3D configurations
- Linear benchmarks: TAEFL and AE3D-K
- Nonlinear benchmark: TAEFL

TAEFL uses the gyrofluid closure technique to efficiently trigger Alfvén instabilities

*(*Acknowledgements: J-N. Leboeuf, V. L. Lynch*):*

- Reduces 5-dimensional phase space to 3-dimensions
- Implicit time-stepping can be used in linear regime, no noise issue
- Non-perturbative
- Continuum damping singularities avoided naturally due to complex ω
- While analogous physics can be present in gyro-fluid models, a vocabulary shift is suggested:
  - Finite orbit width stabilization <-> fast ion non-local drift propagation
  - Wave-particle trapping <-> fast ion eddy formation and trapping
  - Etc.
- Substantially faster (10^3) than equivalent hybrid fluid-particle models
  - Quick scans of profile/parameter dependencies
  - Time-dependent mode structures for fast ion confinement calculations
  - Initial states for nonlinear models
  - Pre-conditioning for more advanced methods
- Example niche: gyrofluid application to core transport studies
  - GLF23, TGLF models
    - trapped particle response, FLR effects, shaped geometries

TAEFL is a hybrid MHD-energetic particle gyrofluid model based on the reduced MHD FAR code:

$$\frac{\partial \psi}{\partial t} = \nabla_{\parallel}\phi$$

$$\frac{\partial U}{\partial t} = -\nabla_{\parallel}\left(\frac{J_{\zeta}}{B_{\zeta}}\right) + \sum_{thermal,fast} \hat{\zeta} \times \vec{\nabla}\left(\frac{R}{|B|}\right) \bullet \vec{\nabla} p_j$$

Reduced MHD Ohm's law and vorticity equation

$$\frac{\partial n_f}{\partial t} = \frac{T_{f0}}{m_f \Omega_{cf}} \Omega_d\left(n_f\right) - n_{f0}\nabla_{\parallel} \mathrm{v}_{\parallel,\mathrm{f}} + \frac{q_f n_{f0}}{m_f \Omega_{cf}} \Omega_d\left(\phi\right) - \frac{q_f}{m_f \Omega_{cf}} \frac{dn_{f0}}{dr} \frac{1}{r} \frac{\partial \phi}{\partial \theta}$$

$$\frac{\partial \mathrm{v}_{\parallel,f}}{\partial t} = \frac{T_{f0}}{m_f \Omega_{cf}} \Omega_d\left(\mathrm{v}_{\parallel,f}\right) - \left(\frac{\pi}{2}\right)^{1/2} \mathrm{v}_{\mathrm{th,f}} \left|\nabla_{\parallel}\right| \mathrm{v}_{\parallel,f} - \frac{\mathrm{q_f v_{th,f}^2}}{m_f \Omega_{cf} R n_{f0}} \frac{dn_{f0}}{dr} \frac{1}{r} \frac{\partial \psi}{\partial \theta}$$

Gyrofluid energetic particle closure moments

Initial model development: C. L. Hedrick, J.-N. Leboeuf, D. A. Spong, Phys. Fl. B 4 (1992) 3869
Linear: D. A. Spong, B. A. Carreras, C. L. Hedrick, Phys. Fl. **B4** (1992) 3316.
Nonlinear: D. A. Spong, B. A. Carreras, C. L. Hedrick, Phys. Plasmas **1** (1994) 1503.

TAEFL model (continued)

- Kinetic effects (ω_*, wave-particle resonance) break usual MHD symmetry
  - Conventional MHD:

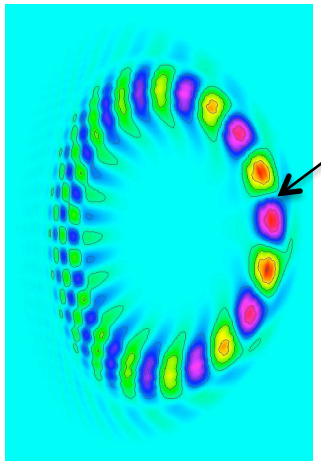
$$\psi = \sum_{m,n} \psi_{mn} \cos(m\theta - n\zeta); \quad \phi = \sum_{m,n} \phi_{mn} \sin(m\theta - n\zeta)$$

  - MHD + kinetic requires:

$$\psi = \sum_{m,n} \left[\psi_{mn,c} \cos(m\theta - n\zeta) + \psi_{mn,s} \sin(m\theta - n\zeta) \right]$$

$$\phi = \sum_{m,n} \left[\phi_{mn,c} \cos(m\theta - n\zeta) + \phi_{mn,s} \sin(m\theta - n\zeta) \right]$$

    + similar expansions for n_{fast} and $v_{||,fast}$
  - Allows mode rotation, up-down asymmetry in ζ = constant planes

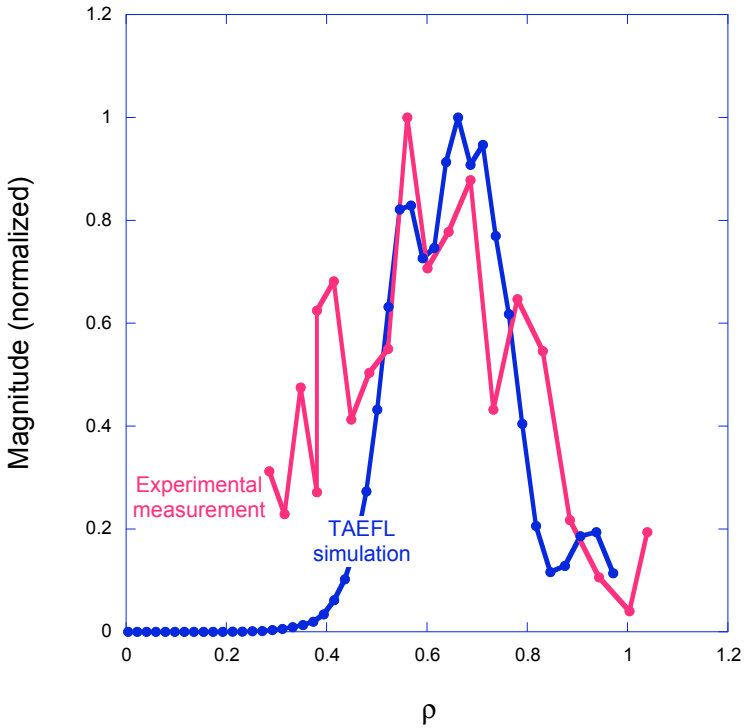


"twisted" form of mode structure (TAEFL perturbative effect) leads to observed phase variation

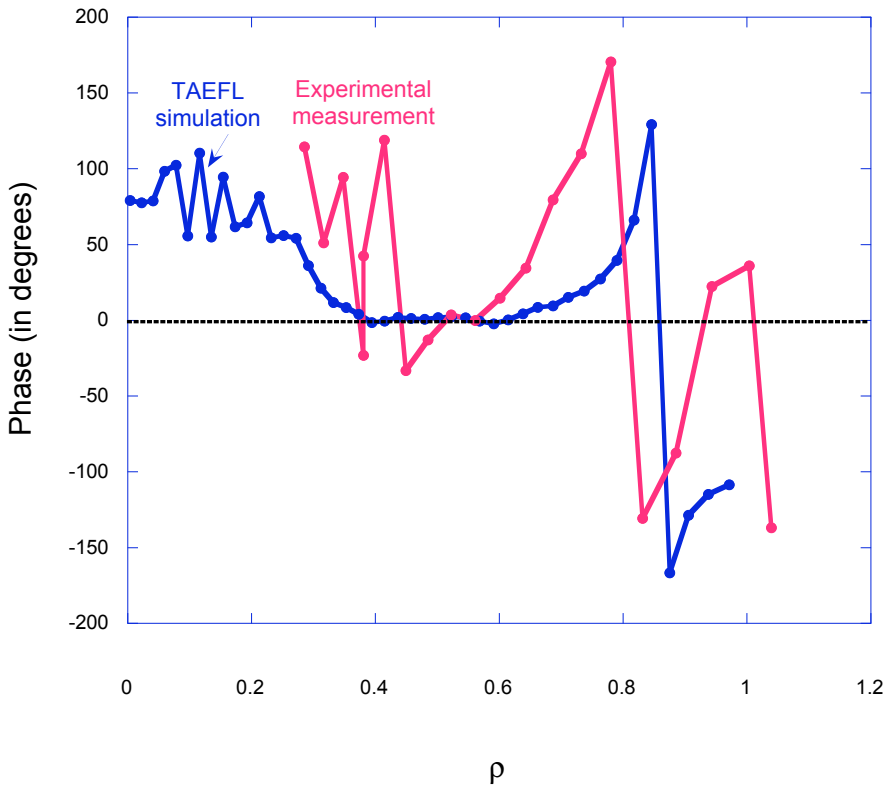
Synthetic diagnostic simulation of phase and magnitude measurements on meridional plane for above 90kHz TAE mode in DIII-D #122117 discharge

"Measurements, Modeling, and Electron Cyclotron Heating Modification of Alfv´en Eigenmode Activity in DIII-D,"
M A Van Zeeland, et al., submitted to Nuclear Fusion (2009)

magnitude comparison

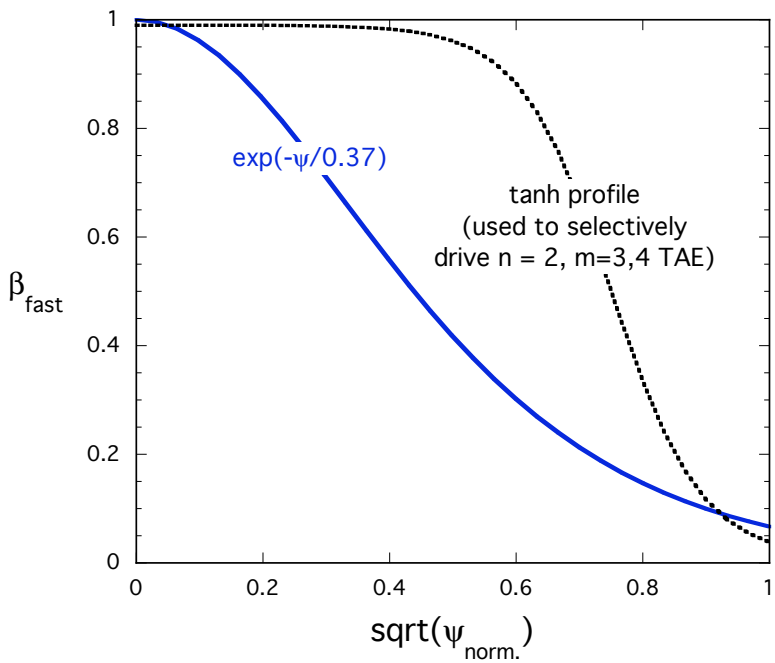


ECE phase comparison

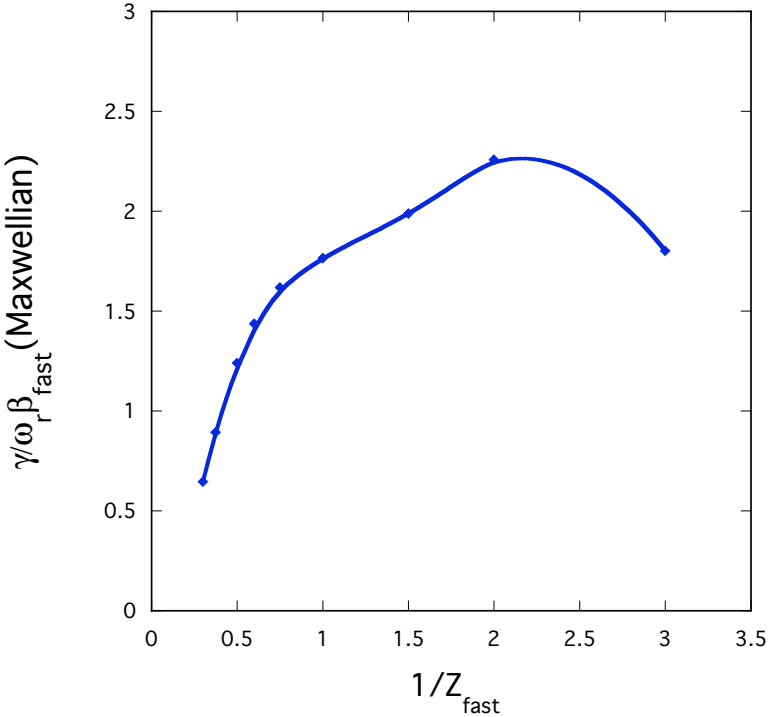


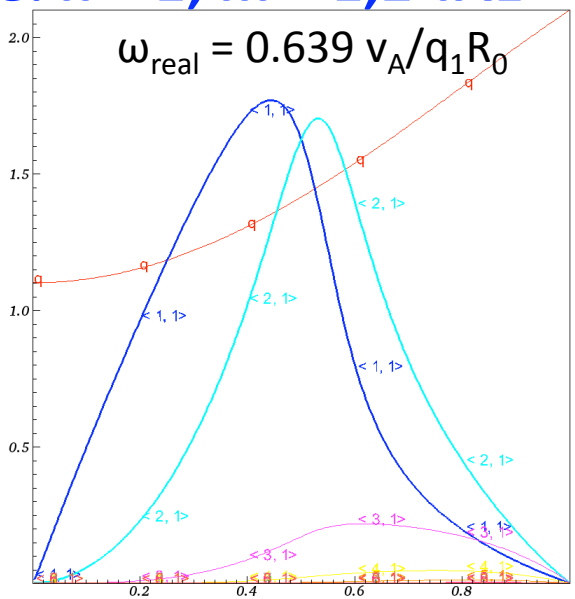
Application to NOVA-K/ITPA linear benchmark cases:

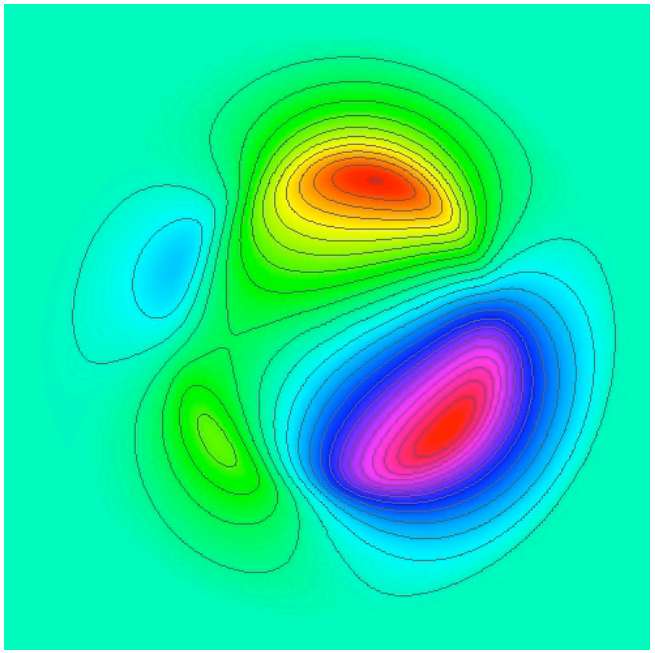
- TAEFL found three unstable modes
  - n = 1, m = 1,2 TAE
  - n = 2, m = 2,3 TAE (fastest growing n = 2 TAE)
  - n = 2, m = 3,4 TAE (subject to continuum damping)
- In order to see the n = 2, m = 3,4 TAE, it was necessary to use a different fast ion profile:

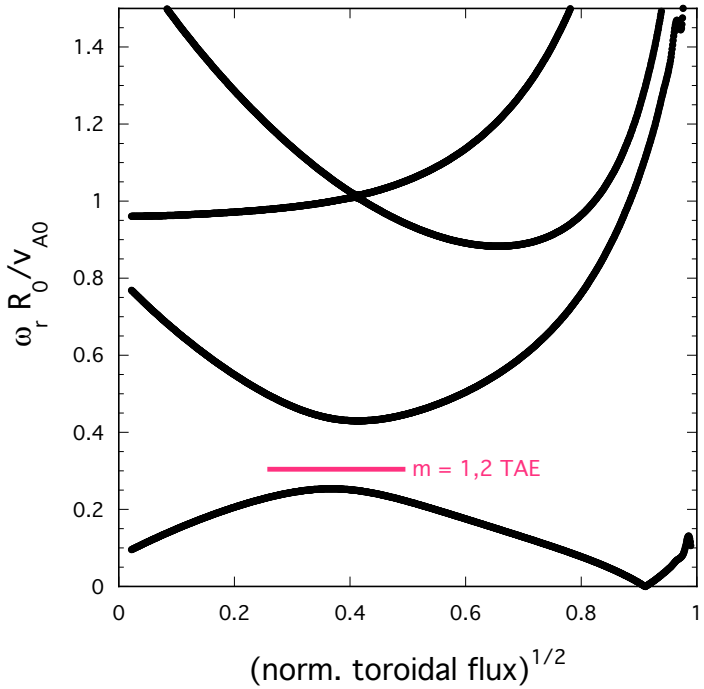


ITPA-EP Benchmark cases TAEFL code: n = 1; m = 1,2 TAE

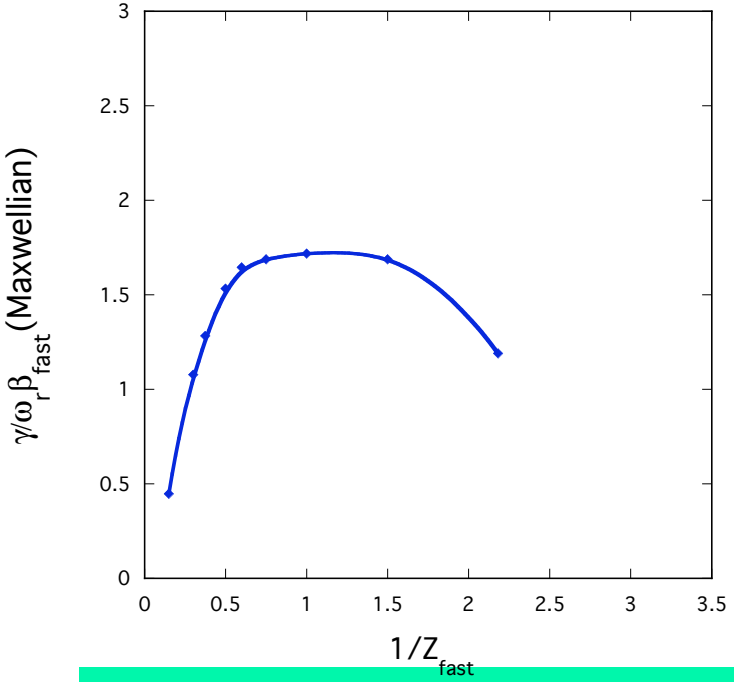


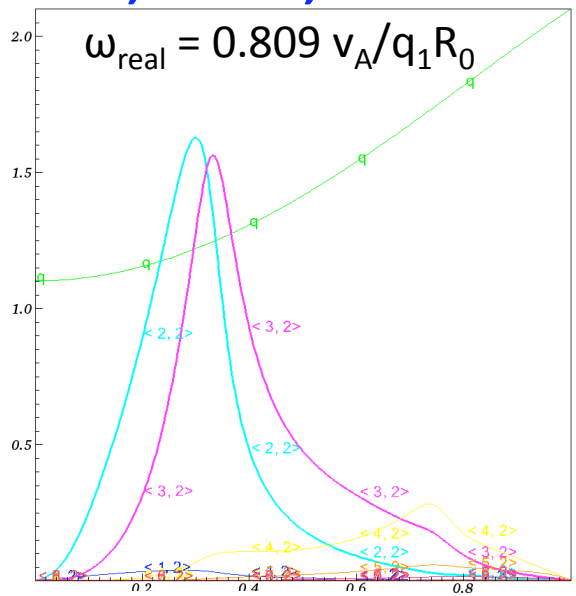


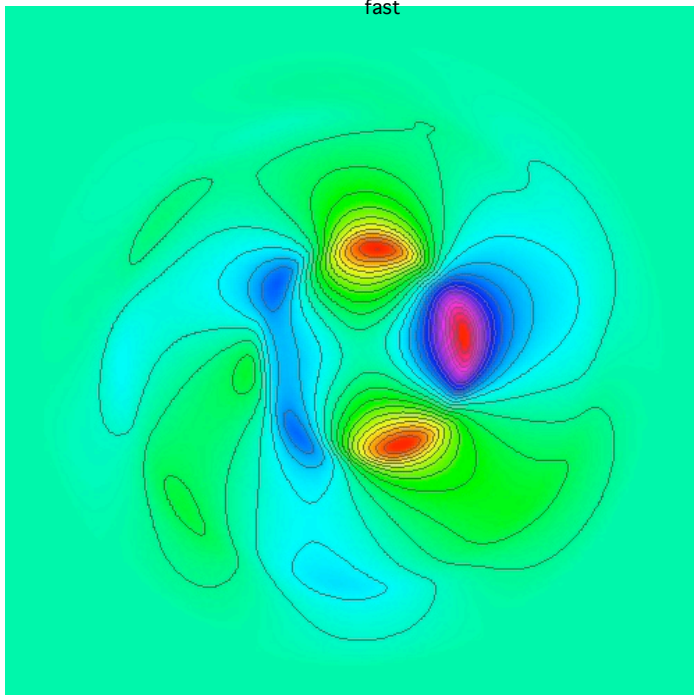


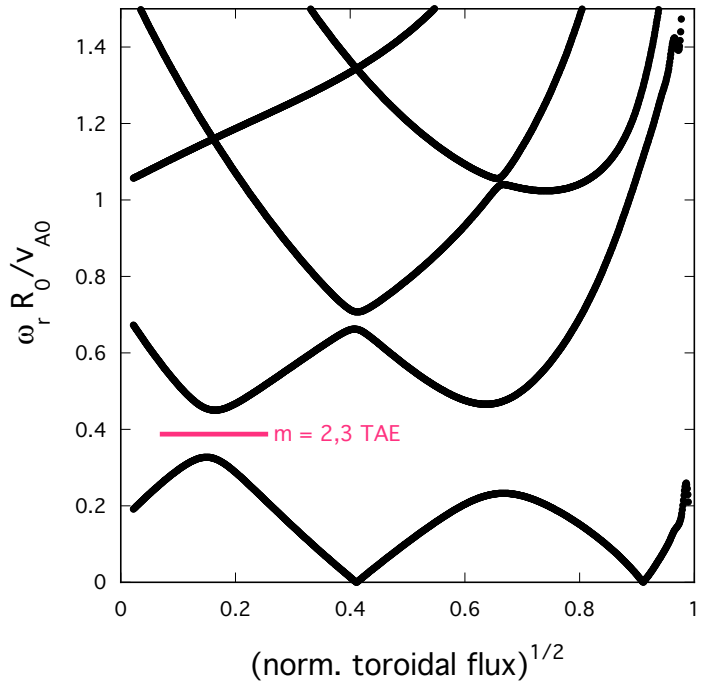


ITPA-EP Benchmark cases TAEFL code: n = 2; m = 2,3 TAE

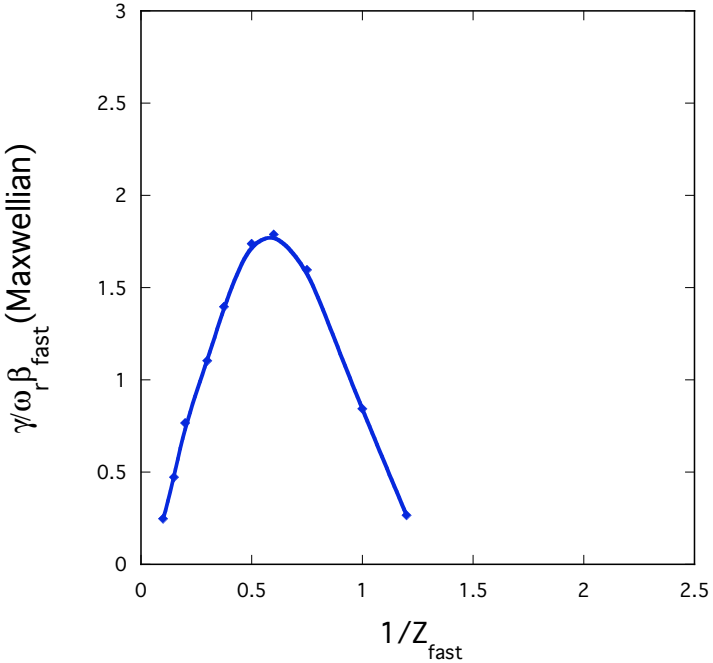




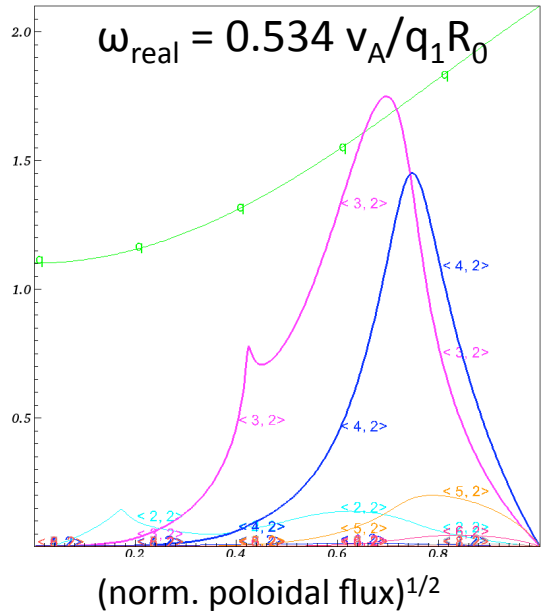


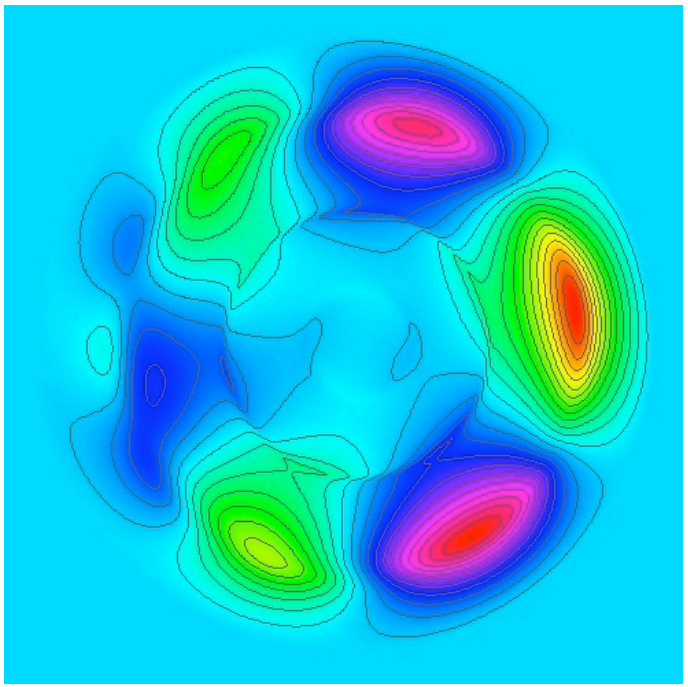


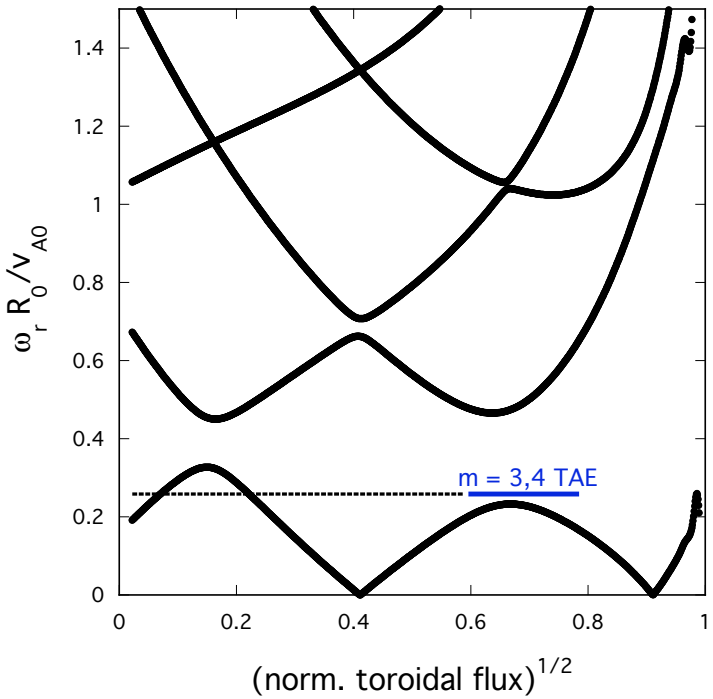
ITPA-EP Benchmark cases TAEFL code: n = 2; m = 3,4 TAE



Note: the growth rates here have been scaled by the ratio of the tanh to exp profile scale lengths near the dominant mode structure.

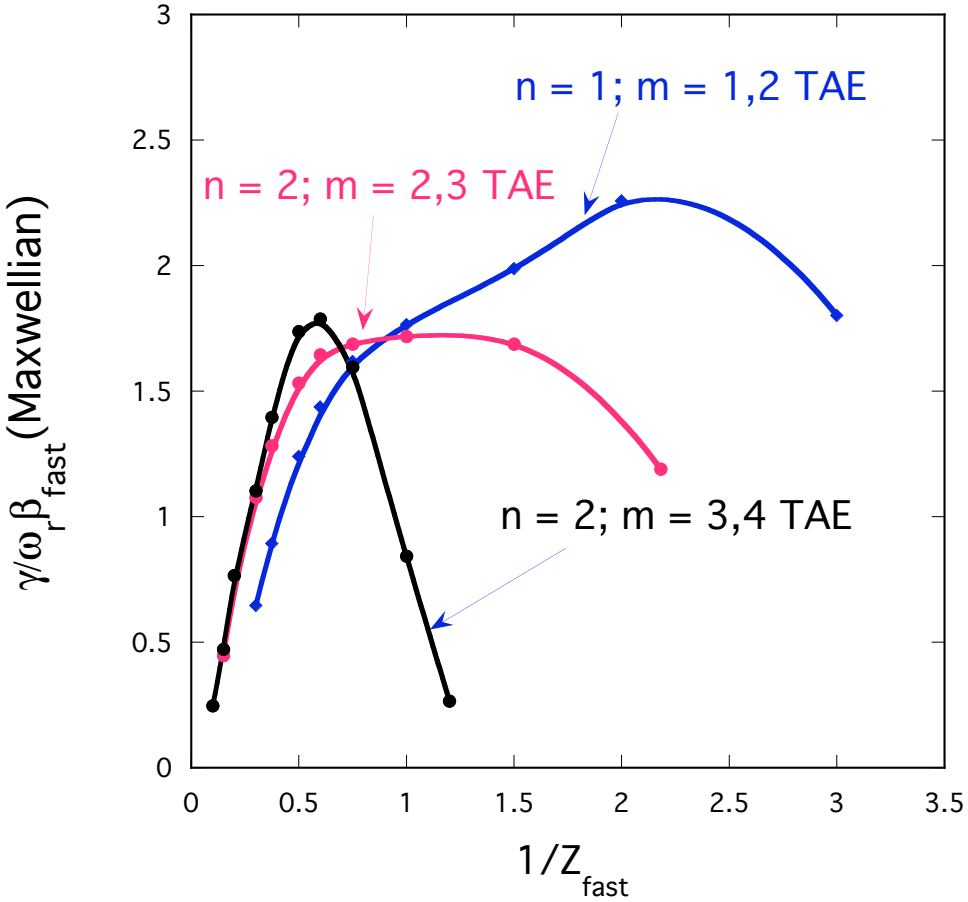




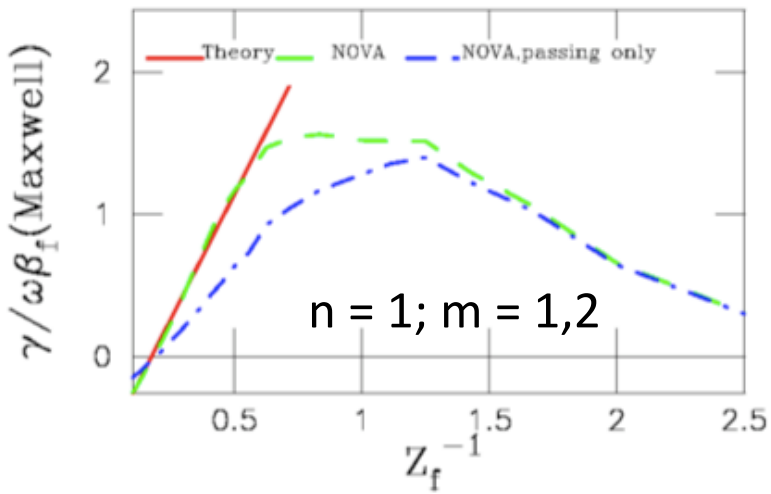


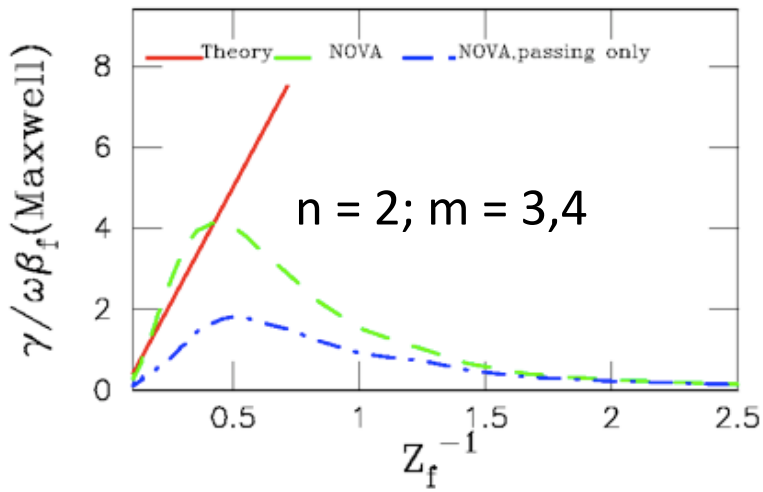
Summary of TAEFL benchmark results for $1/Z_{fast}$ variation

TAEFL



NOVA-K





Perturbative orbit–based AE3D-K model of Alfvén instabilities in 3D devices (collaboration with A. Konies):

$$\omega^2 \delta W_{kinetic} = \delta W_{magnetic} + \delta W_{hot} \qquad \Rightarrow \qquad \left[\omega_r + i\gamma\right]^2 \delta W_{kinetic} = \delta W_{magnetic} + \delta W_{hot}\left(\omega_r + i\gamma\right)$$

To first order in γ/ω_r : $\gamma = \dfrac{\delta W_{hot}(\omega_r)}{2\omega_r \delta W_{kinetic}}$ $\qquad \delta W_{kinetic} = \dfrac{1}{2}\int d^3x \dfrac{Mn}{B^2}\left|\nabla_\perp \phi\right|^2$

Particle based AE3D-K approach

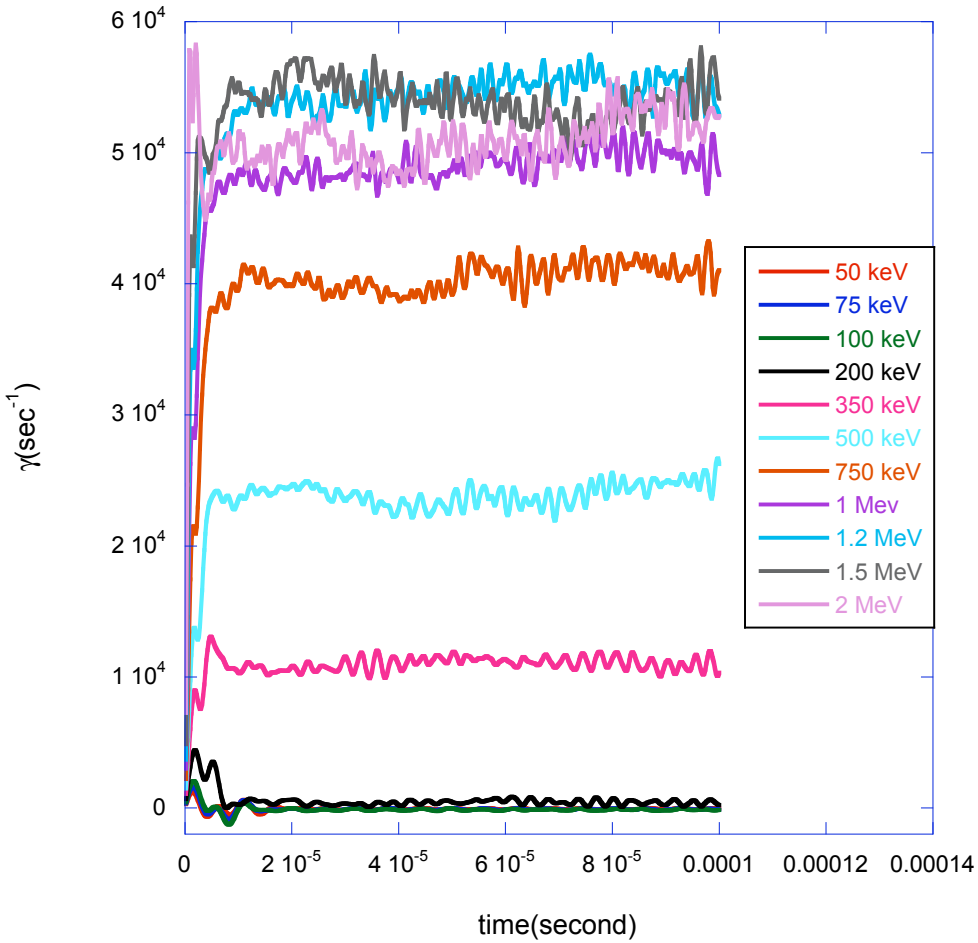
- Adaptable to complex B fields rippled tokamak, stellarator
- Finite orbit widths included
- Flat particle initialization in phase space; weight function used to select specific profile and velocity distribution function

$$\gamma = \frac{Ze\int d^6z\, B_\parallel^* f^{(1)} \dot{R}^{(0)} \bullet \nabla\left(\phi - \mathrm{v}_\parallel A_\parallel\right)}{\int d^3x \left\{\dfrac{Mn}{B^2}\left|\nabla_\perp \phi\right|^2 + \dfrac{1}{\mu_0}\left|\nabla A_\parallel\right|^2\right\}} \propto \frac{\left\langle J_{hot} \bullet E\right\rangle}{\text{wave energy}}$$

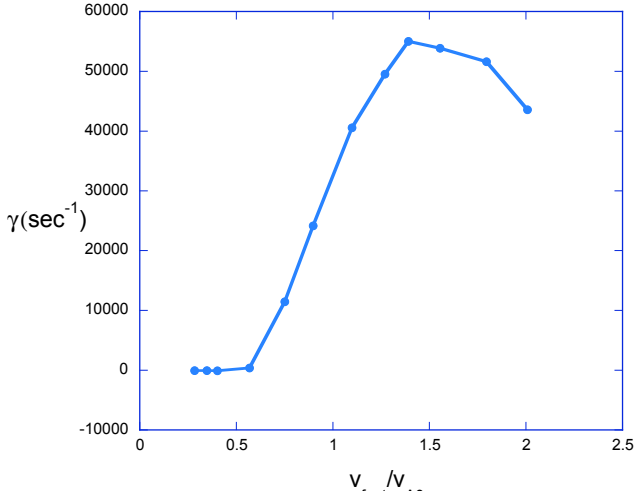
$$f^{(1)} = \sum_{particles} w_i \;\; \delta\left(\mathbf{x} - \mathbf{x_i}\right) \;\; \delta\left(\mathbf{v} - \mathbf{v_i}\right)$$

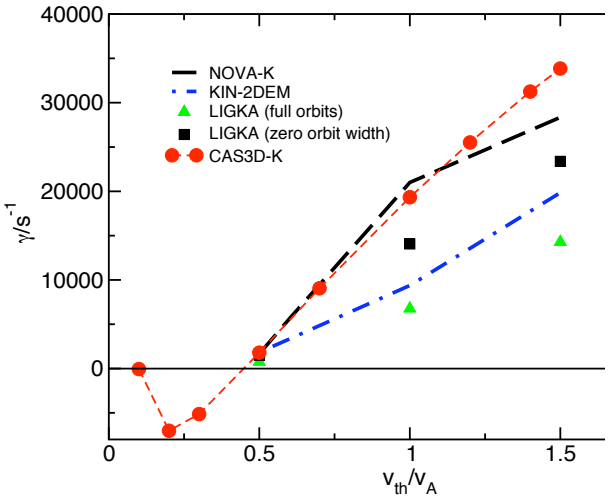
$$\dot{w}_i = -\frac{f(t=0)}{g(t=0)}\frac{1}{f_0}\left[\frac{\mathbf{b}\times\nabla f_0}{ZeB} \bullet \nabla - \left(\frac{\partial f_0}{\partial \varepsilon}\right)\frac{\partial}{\partial t}\right]\left[Ze\left(\phi - \mathrm{v}_\parallel A_\parallel\right)\right]$$

Particle-wave transfer function from AE3D-K shows resonant Alfvén destabilization using particle/time-domain approach:



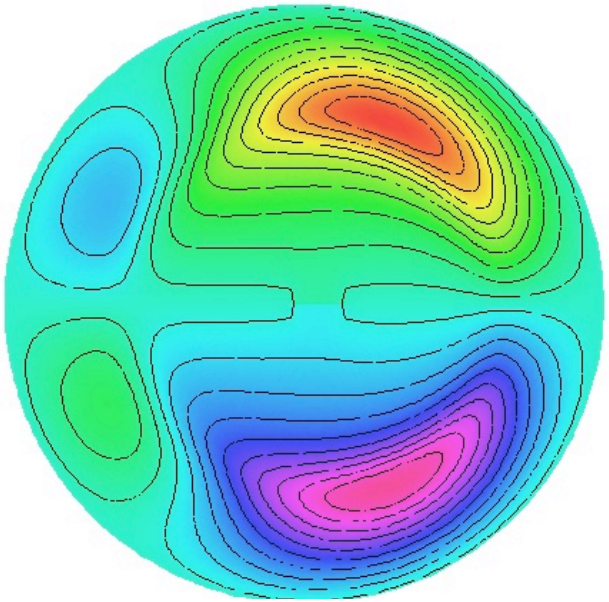
AE3D-K

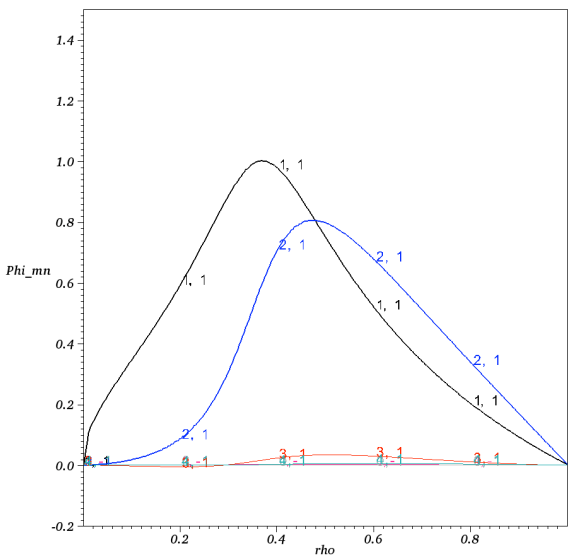


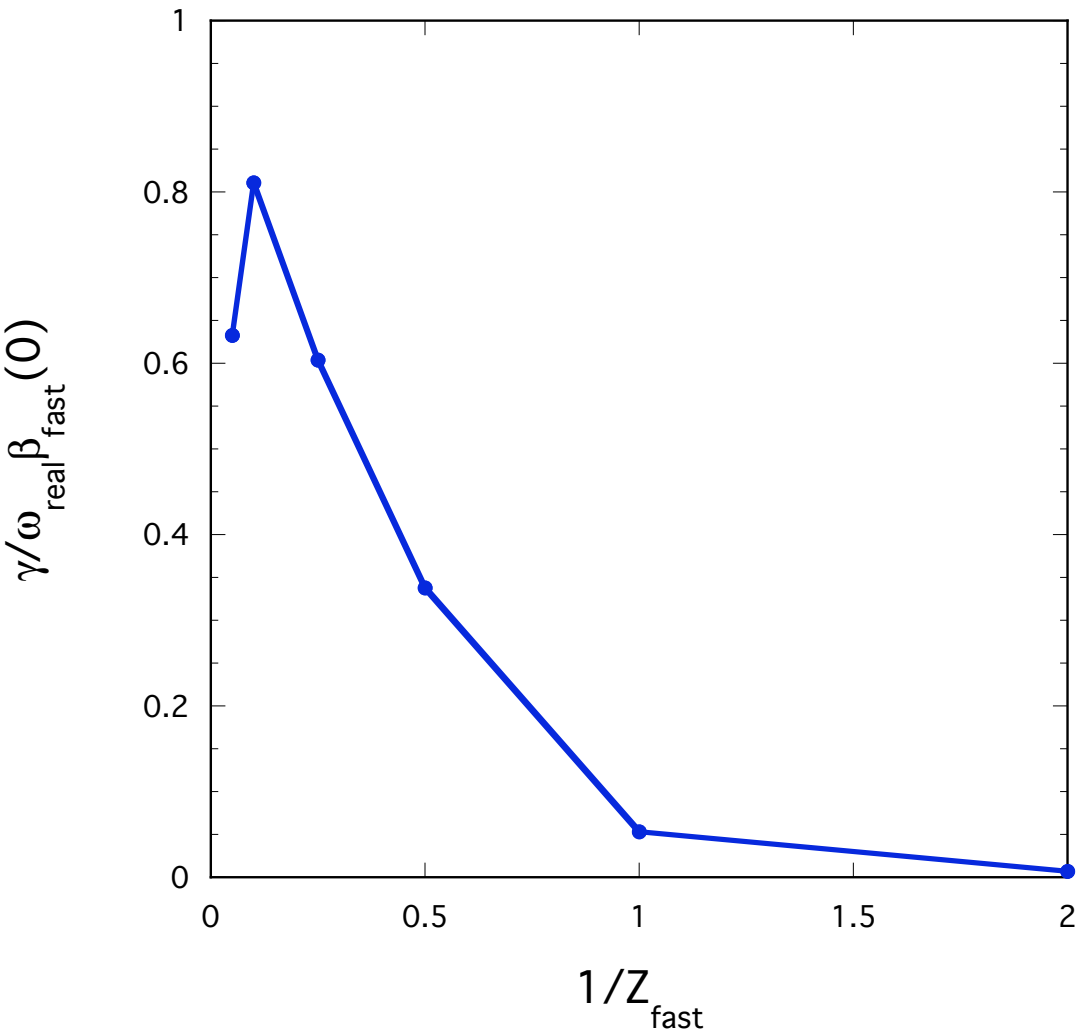


ITPA-EP Benchmark cases AE3D-K code: n = 1; m = 1,2 TAE

AE3D eigenfunction

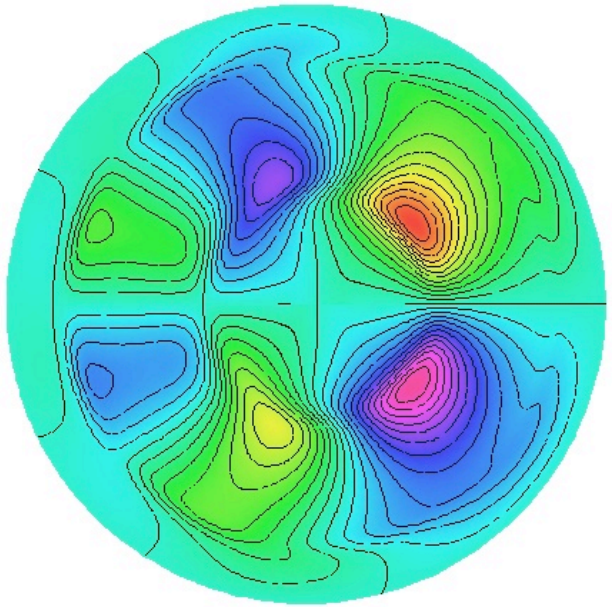


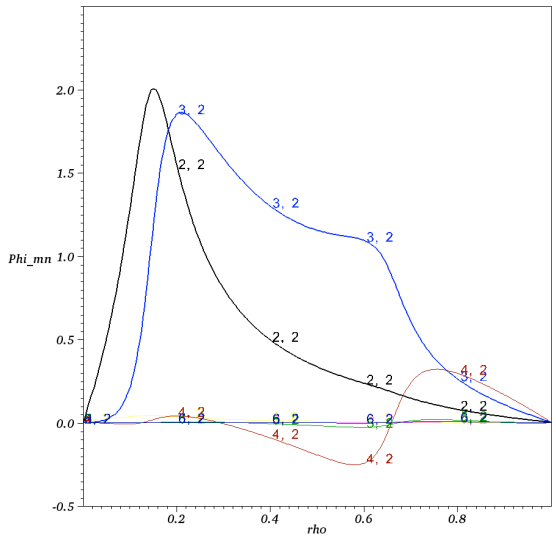


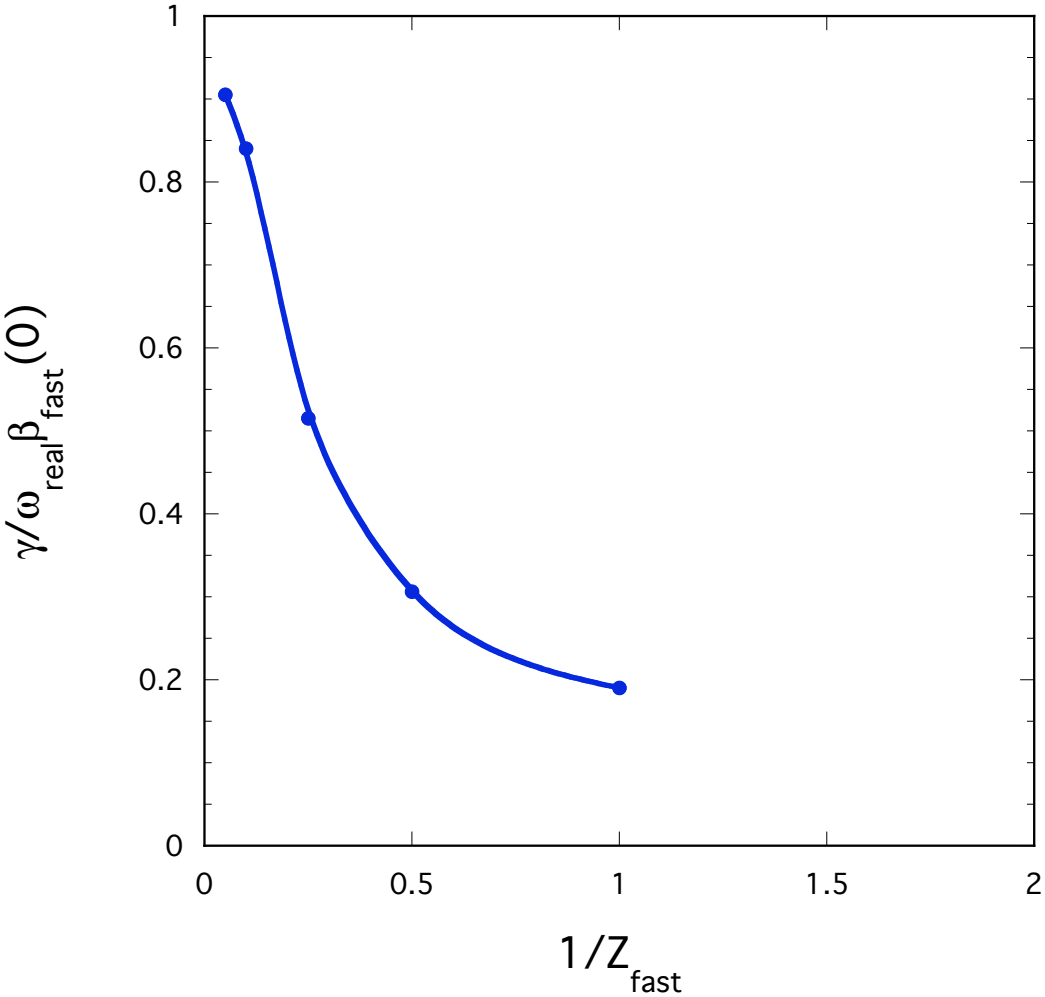


ITPA-EP Benchmark cases AE3D-K code: n = 2; m = 2,3 TAE

AE3D eigenfunction

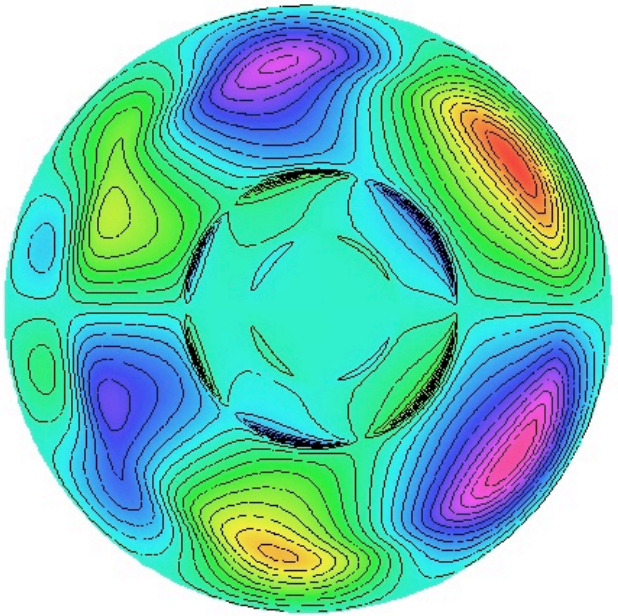


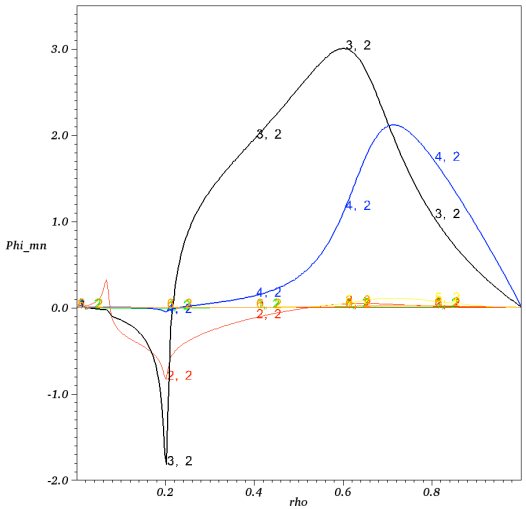


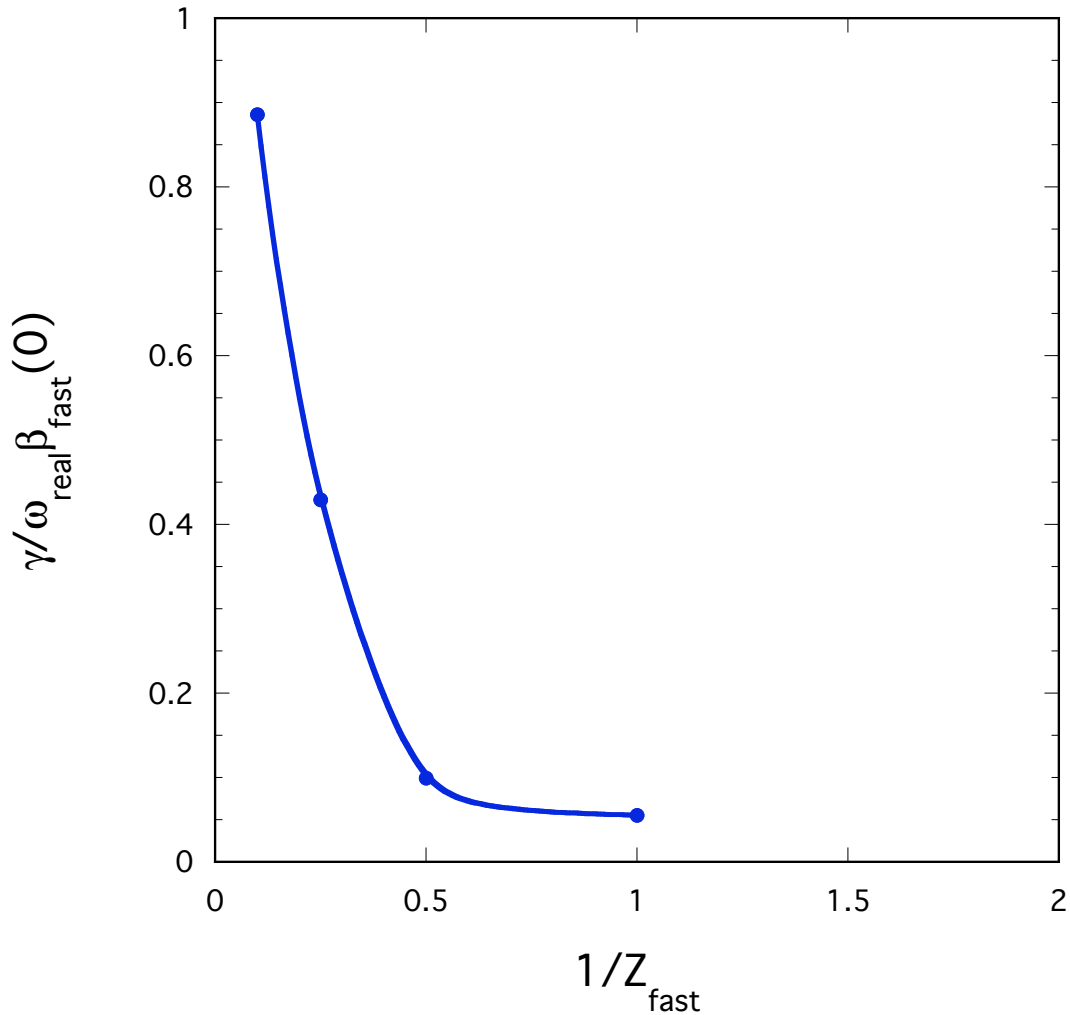


ITPA-EP Benchmark cases AE3D-K code: n = 2; m = 3,4 TAE

AE3D eigenfunction

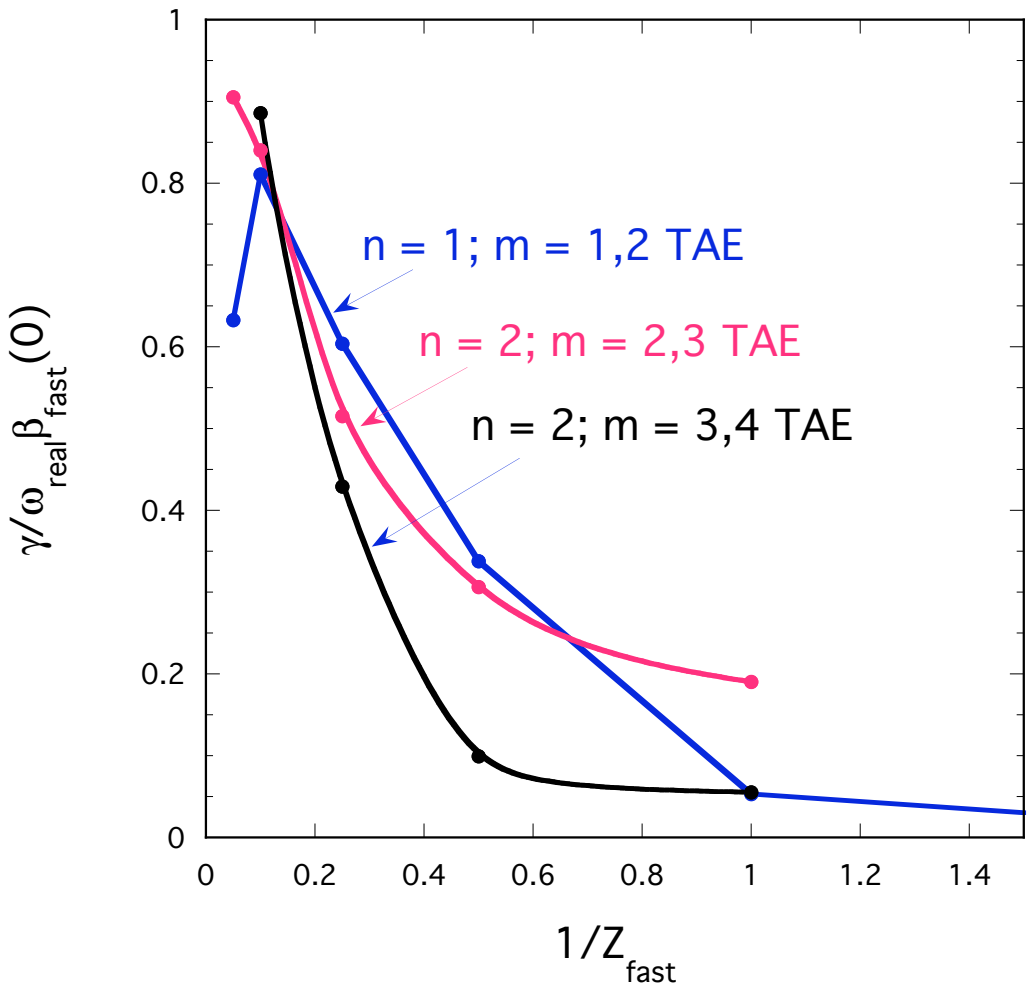




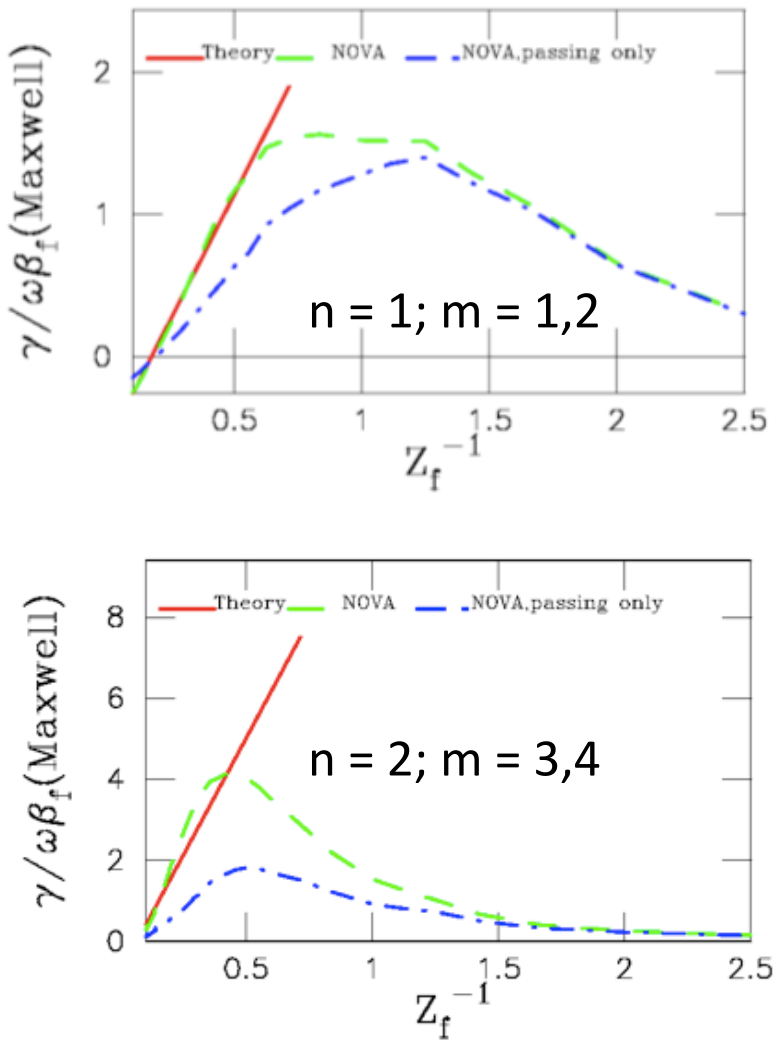


Summary of AE3D-K benchmark results for $1/Z_{fast}$ variation

AE3D-K



NOVA-K



Nonlinear TAEFL benchmark

Used profiles and equilibrium parameters given at:

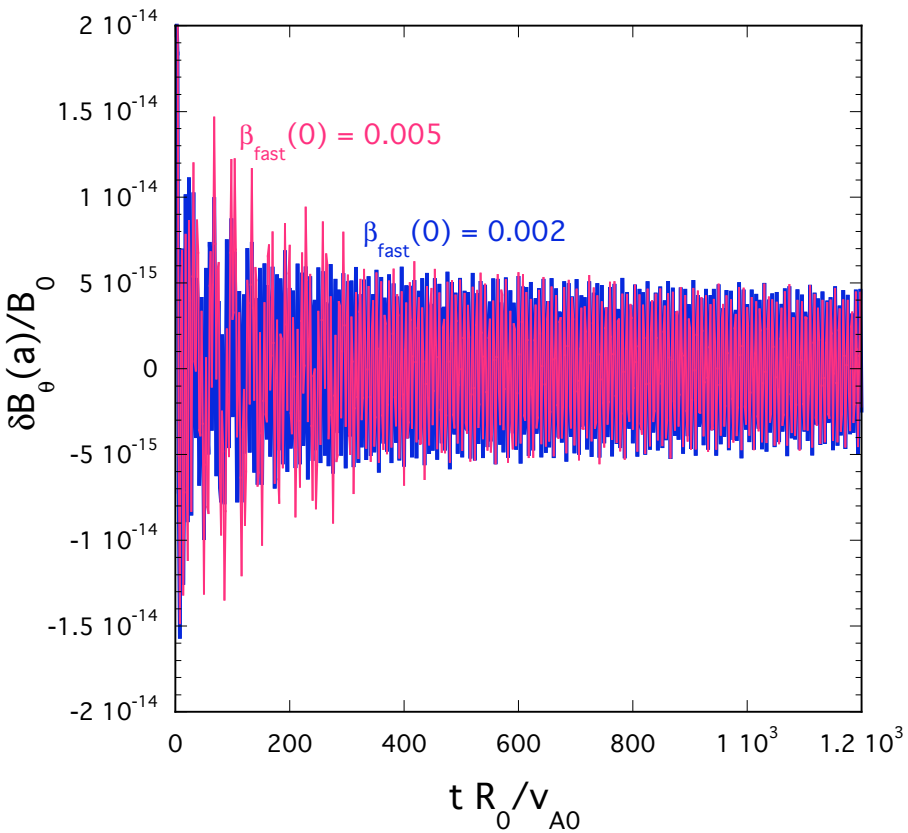
http://w3.pppl.gov/~ngorelen/TAE_nlin4.html

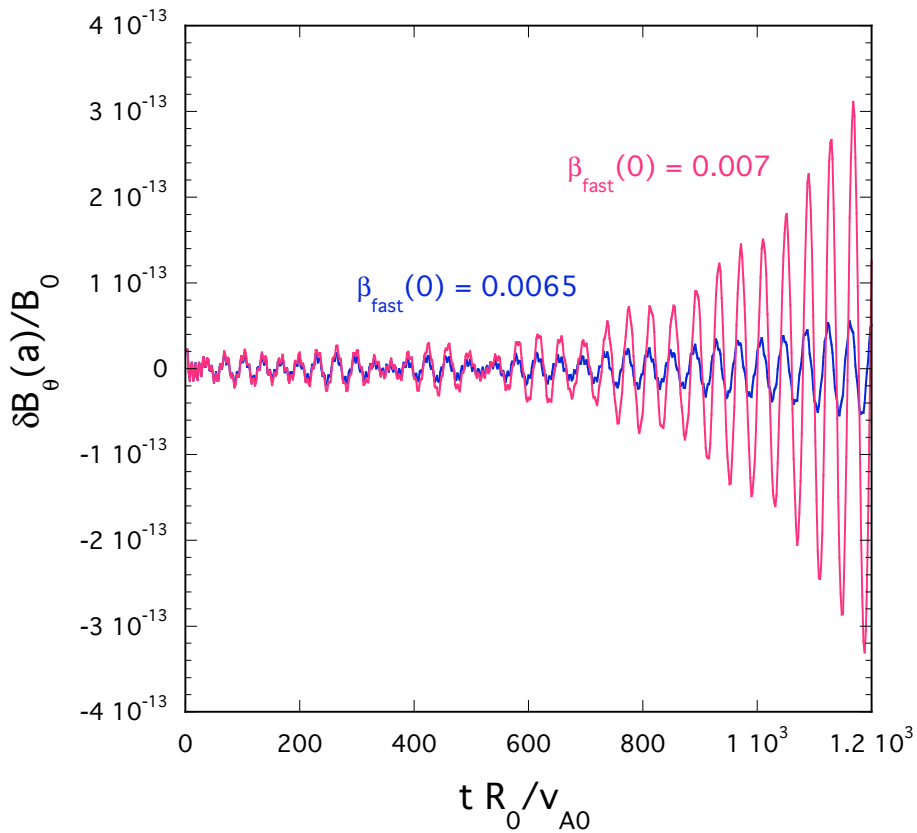
Mode selection: n = 4, m = 4 – 12, cos/sin components

n = 0, m = 0 – 4 (for nonlinear couplings)

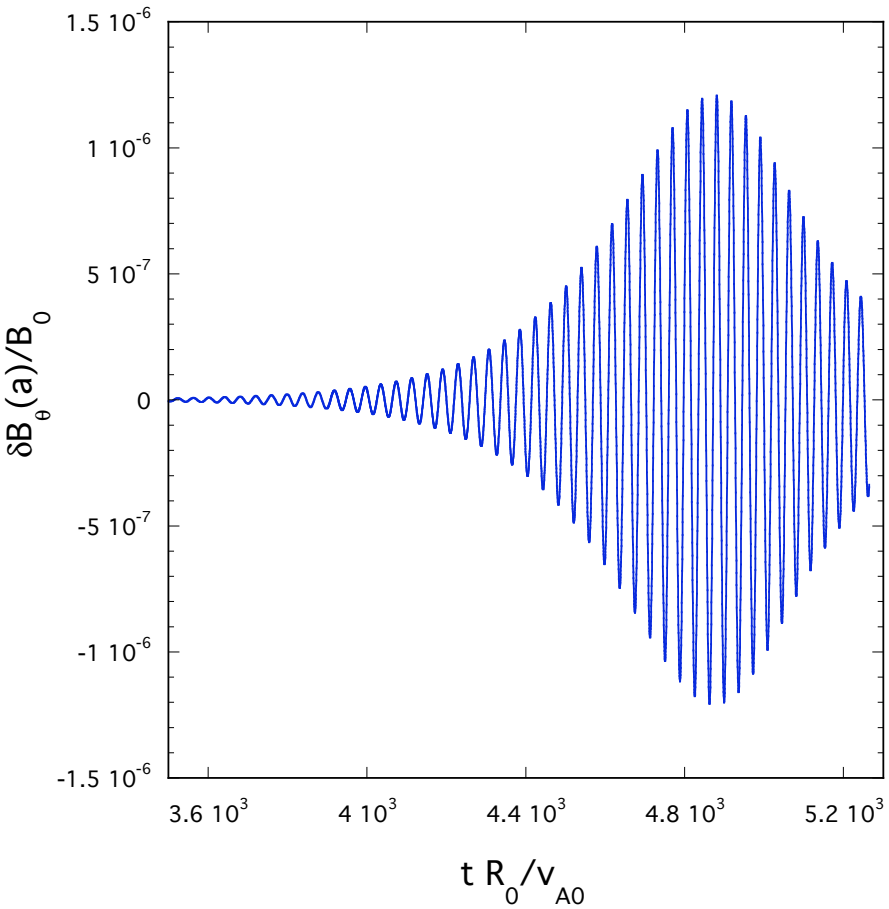
Modification: suggested $\beta_\alpha(0) = 0.00083$ did not give growing modes

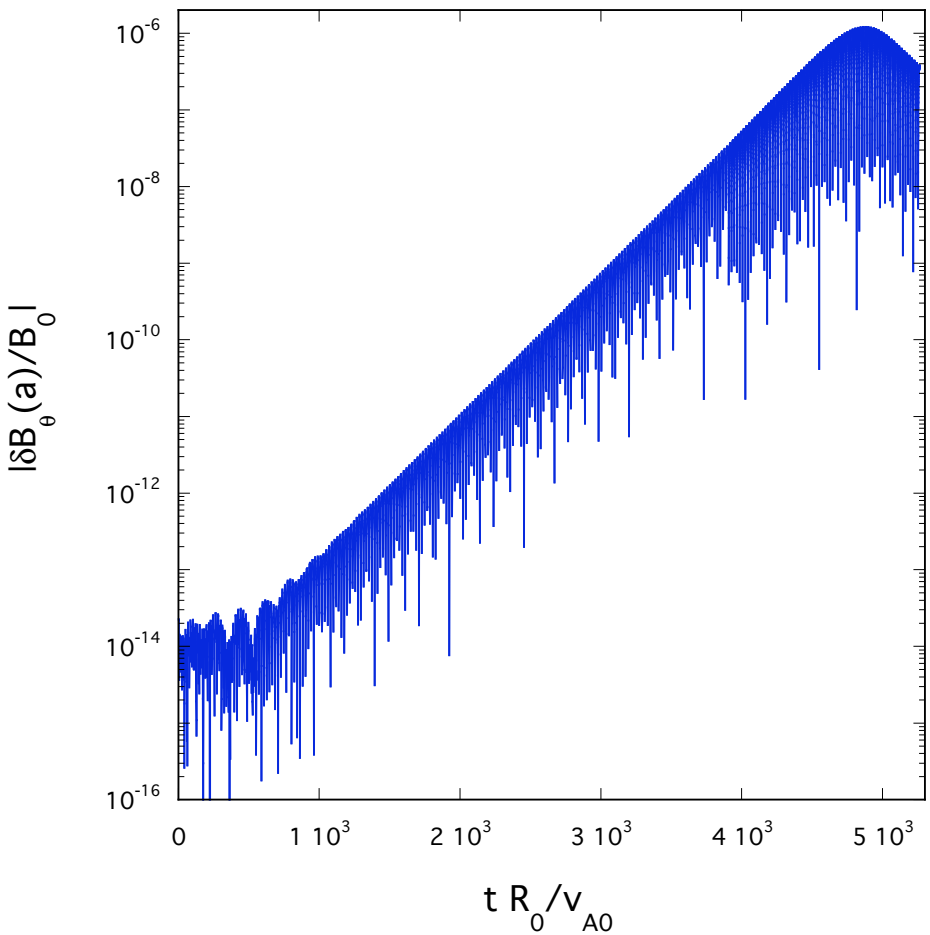
- increased to $\beta_\alpha(0) = 0.007$ to get reasonable growth rate



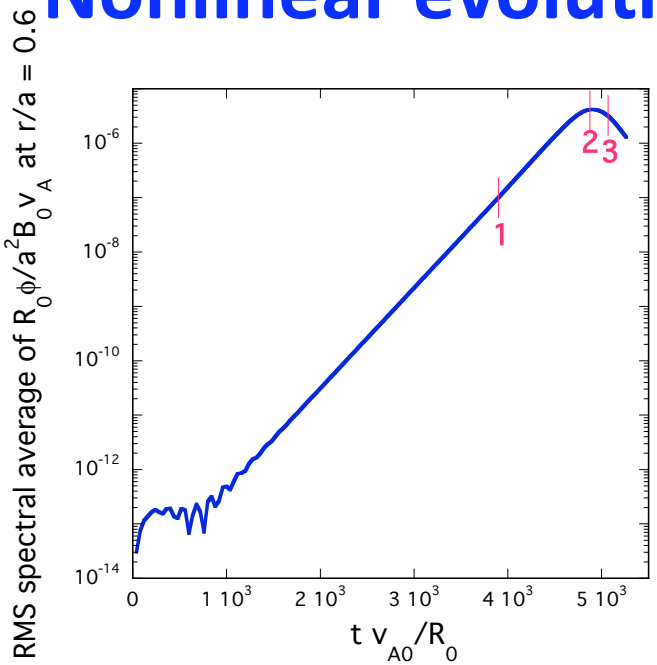


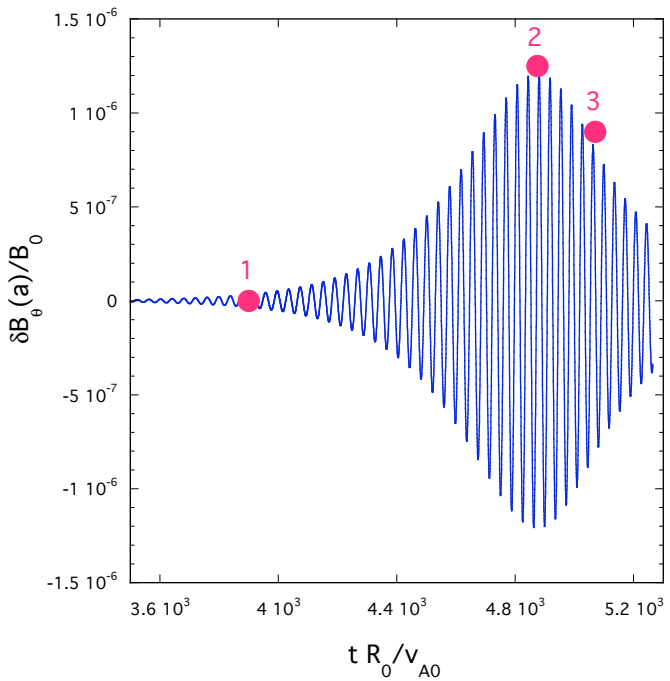
An growing Alfvén mode is found that saturates and then decays:

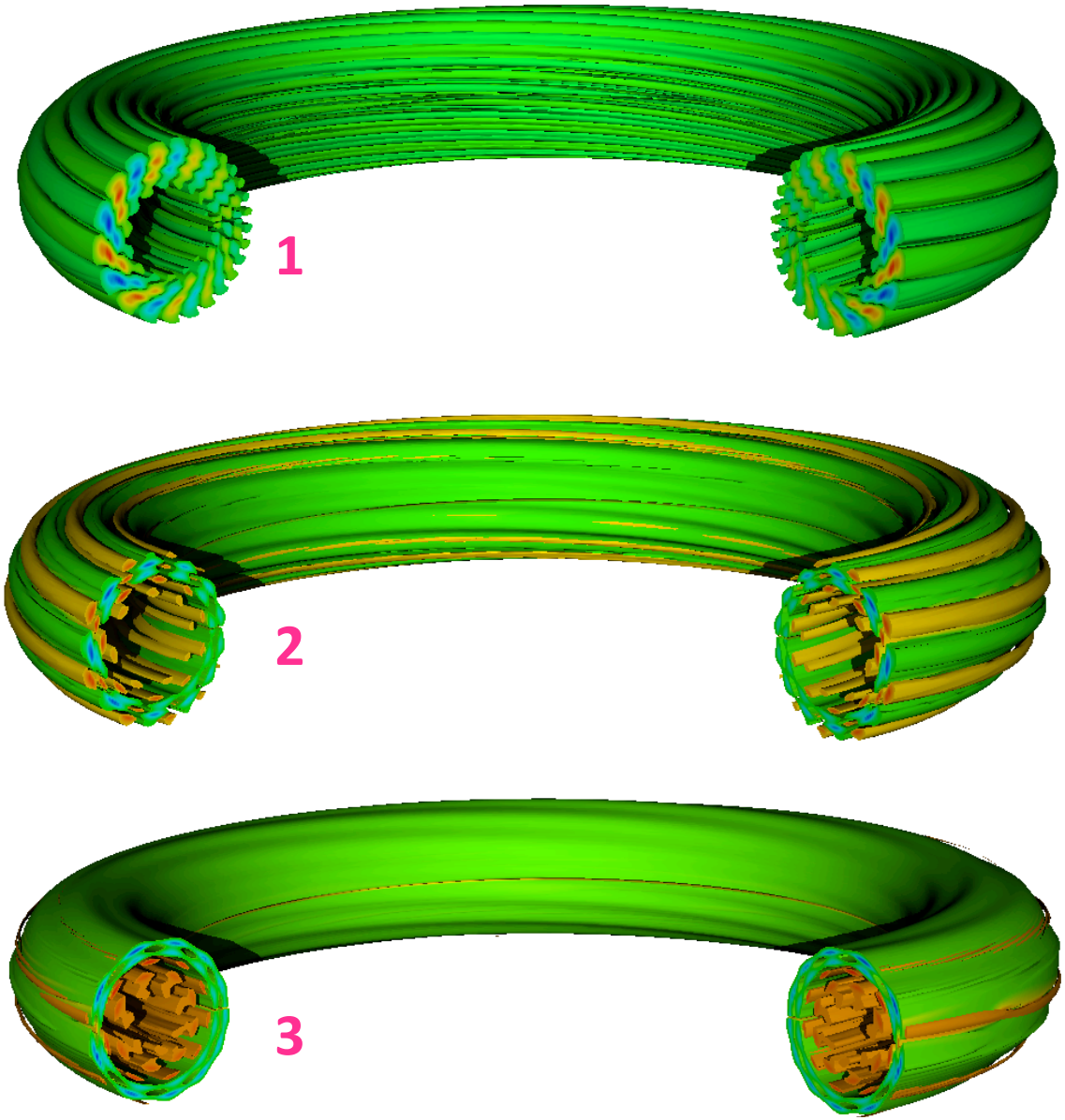




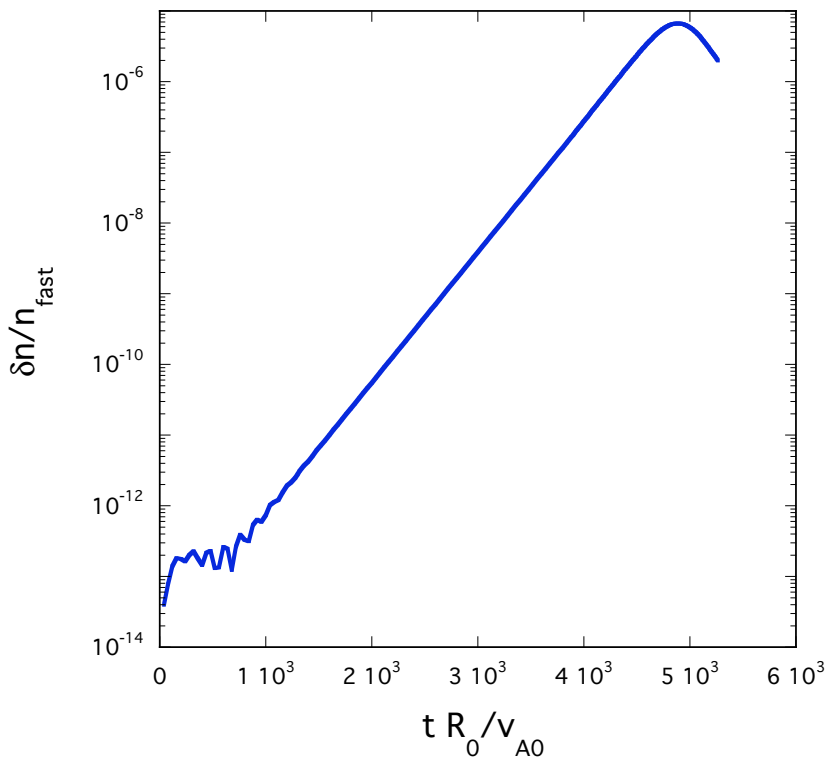
Nonlinear evolution of potential mode structure

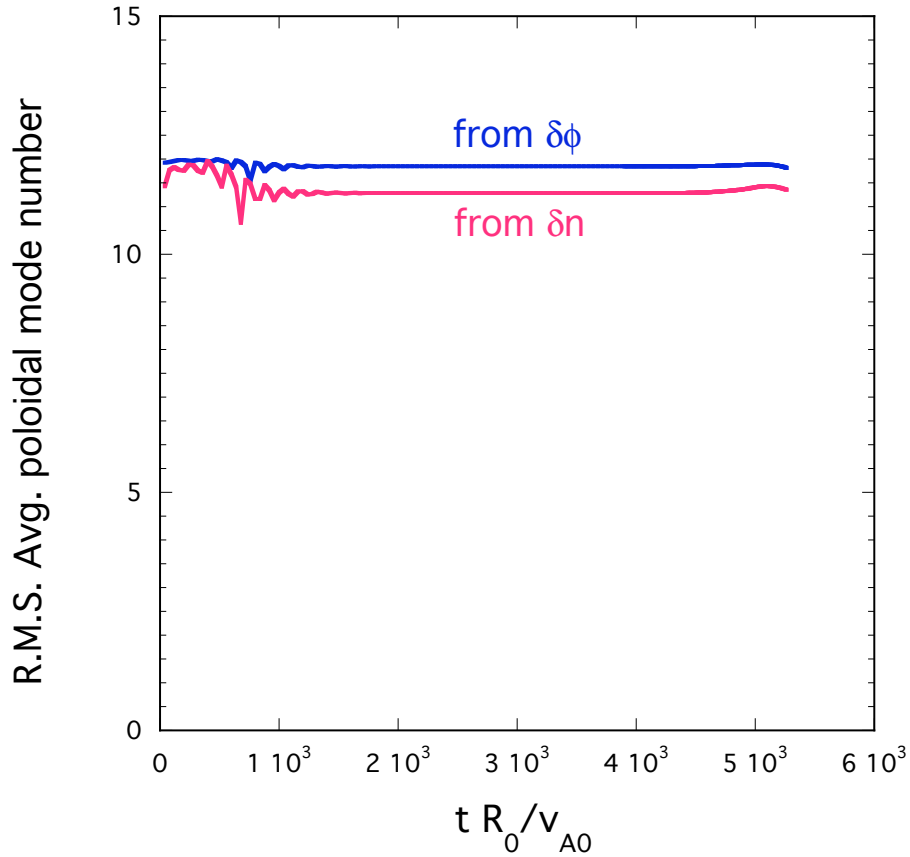






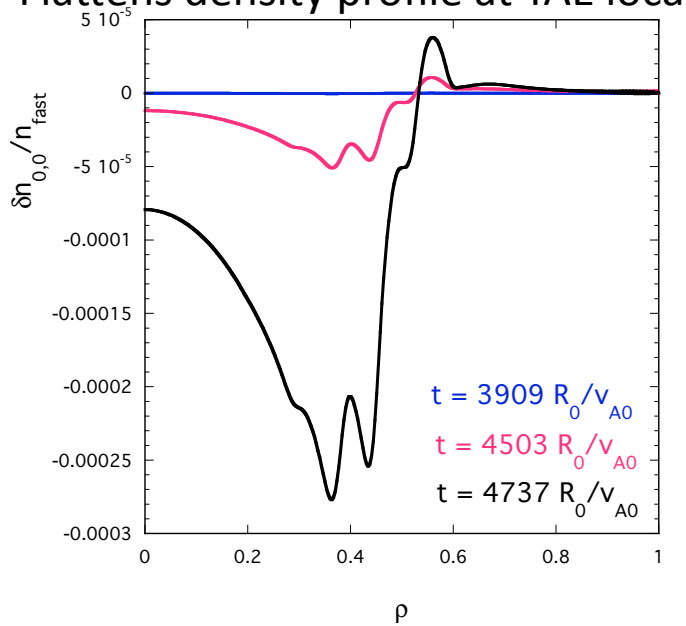
Alpha density saturation and average mode number



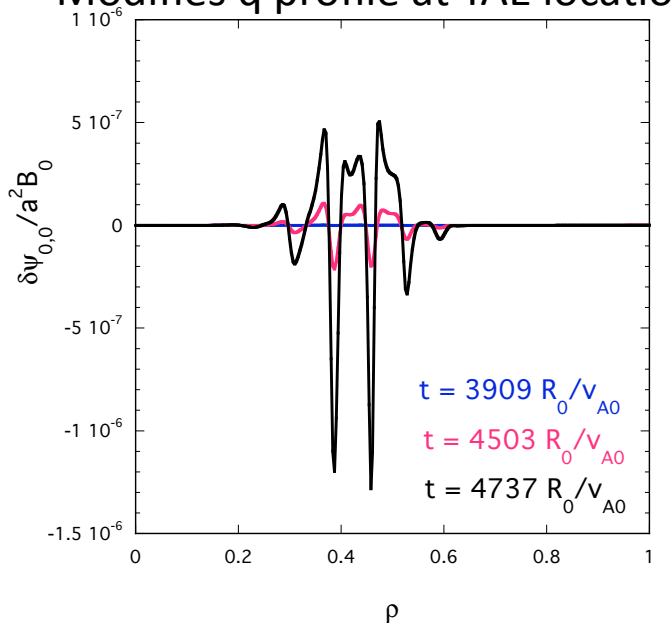


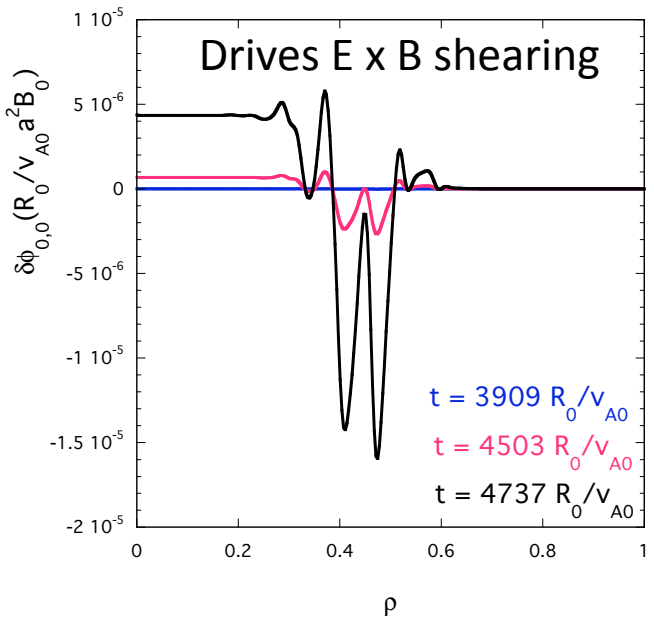
Evolution of m = 0, n = 0 components show a variety of saturation mechanisms occurring simultaneously:

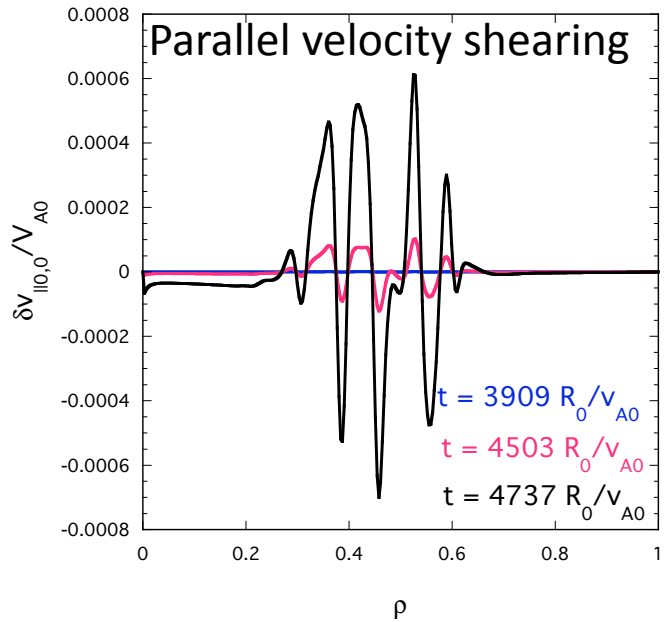
Flattens density profile at TAE location



Modifies q profile at TAE location







Conclusions

- *Linear n = 1 and 2 benchmarks*
  - TAEFL
    - Finds similar mode structure as benchmark
    - Alpha pressure profile modification needed to excite continuum damped n = 2, m = 3,4 TAE
    - Finite orbit stabilization evident
      - Somewhat weak for n = 1
      - Stronger for n = 2, m = 3,4
  - AE3D-K
    - Finds similar mode structure as benchmark
    - Strong finite orbit width stabilization seen for n = 1 and 2
- *Nonlinear benchmark*
  - Wave-packet like nonlinear evolution and decay
    - Amplitude peaks at $\delta B_\Theta/B$ (edge) = 1.4×10^{-6}
    - Similar to collisional regime of benchmark case
    - Gyrofluid model maintains Maxwellian f_α <--> rapid collisions
  - Mode structure peaked at n = 4, m ≈ 12
  - Nonlinear generation of n = 0 components
  - TAEFL required more drive than benchmark case for a growing mode