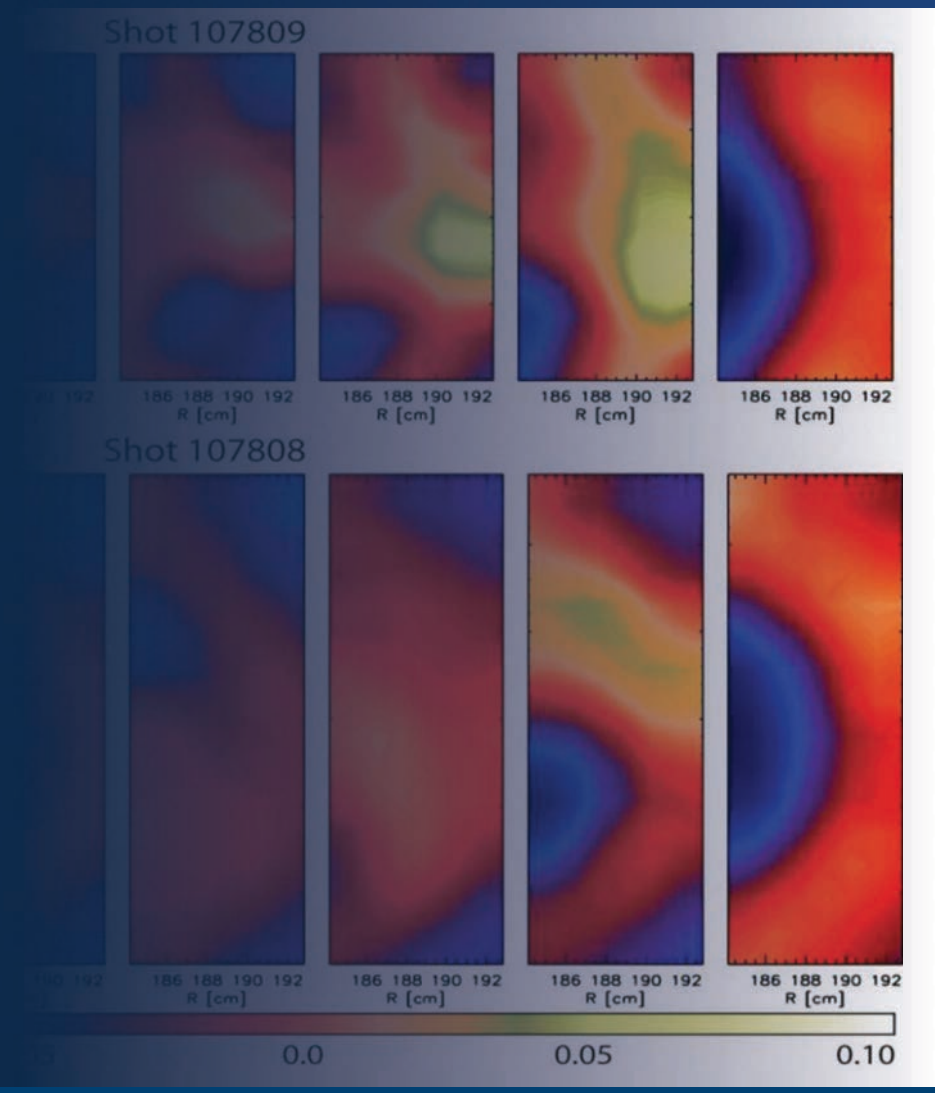
Alfvén Eigenmode studies at DIII-D

B. Tobias

Princeton Plasma Physics Laboratory

November 29th, 2011



DIII-D NATIONAL FUSION FACILITY

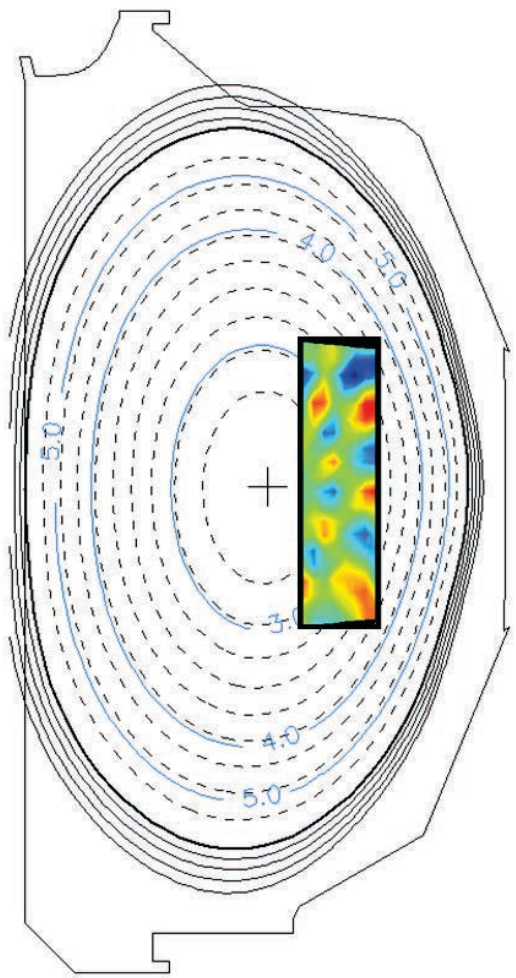
PPPL

D.M.R.C Davis MM-Wave Research Center

Outline

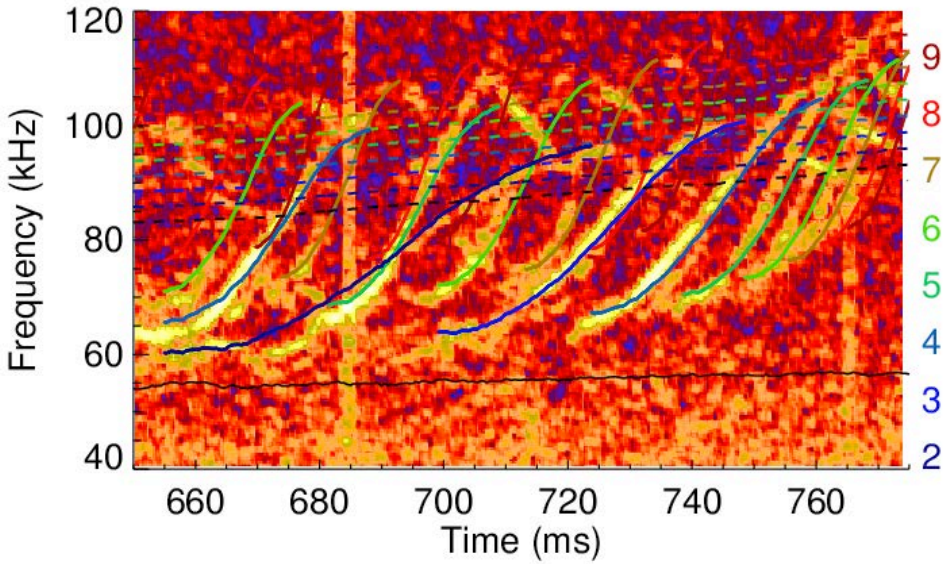
- **Famously, shot 142111**
  - Comparison to TAEFL and hypotheses posed
- **Sci-DAC initiative benchmarking of GTC, Gyro, and TAEFL**
  - Agreement and discrepancy in mode structure
  - Experimental determination of AE growth rates
  - Dual array resolution of ballooning evolution
- **Off-axis NBI experiments**
  - RSAE, TAE stability
  - Mode structure
  - BAAE frequency and mode number
- **MHD spectroscopy**

142111

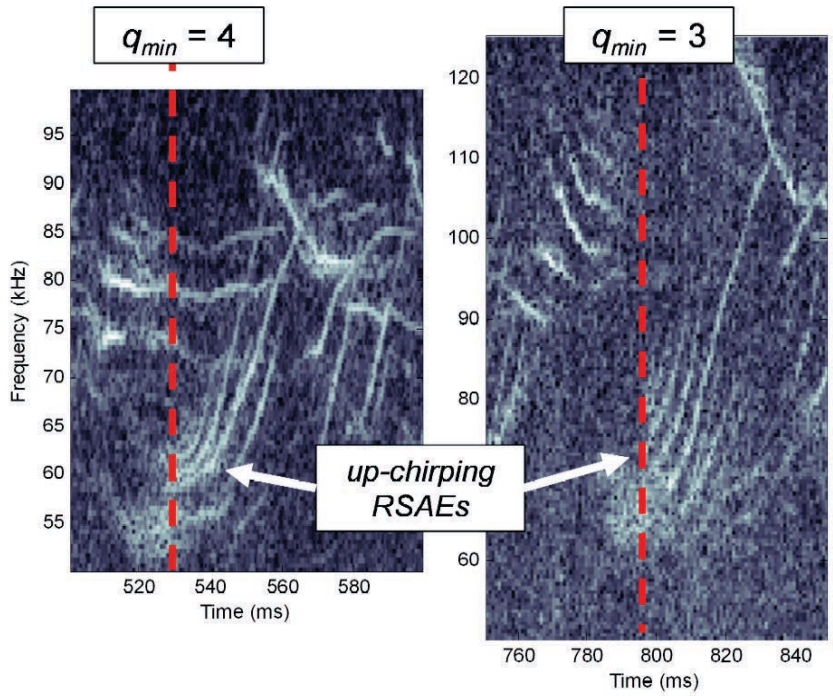


- Oval, co-NBI
- Lots and lots of analysis has been done
- Single ECEI array, LFS view of AEs, sawteeth, etc.

MHD Spectroscopy

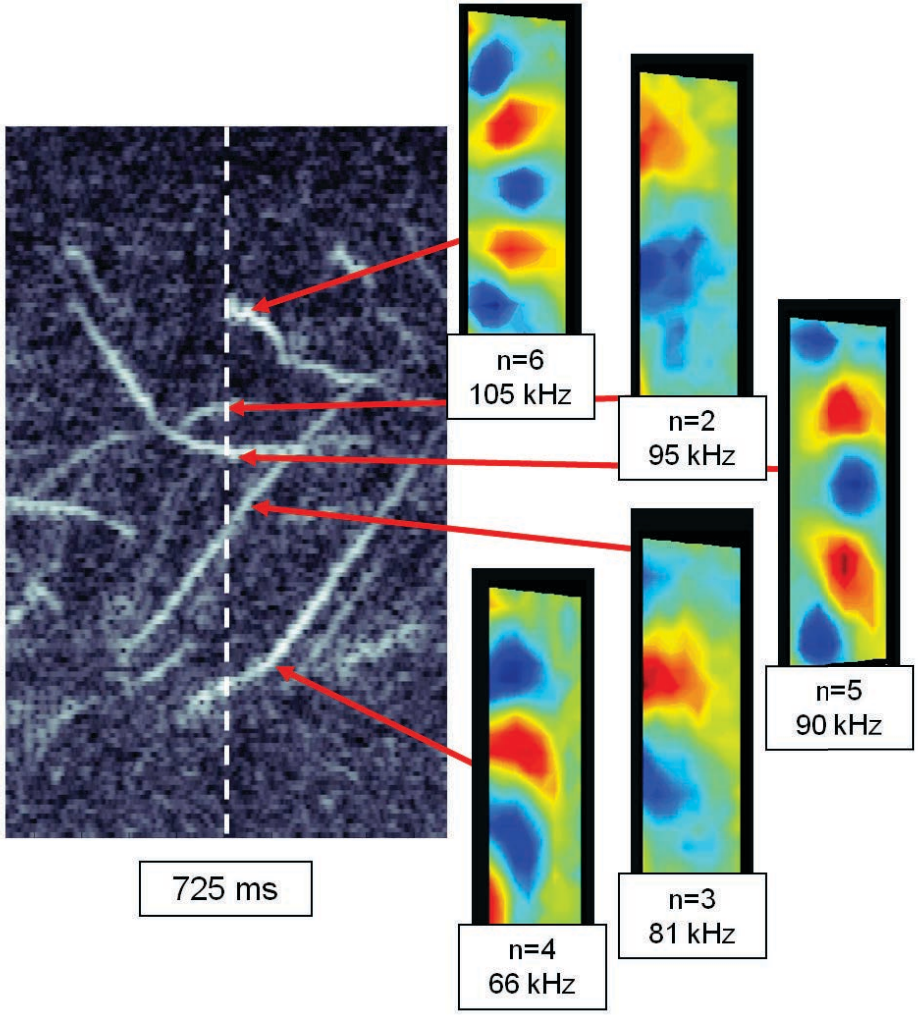


Beam Emission Spectroscopy



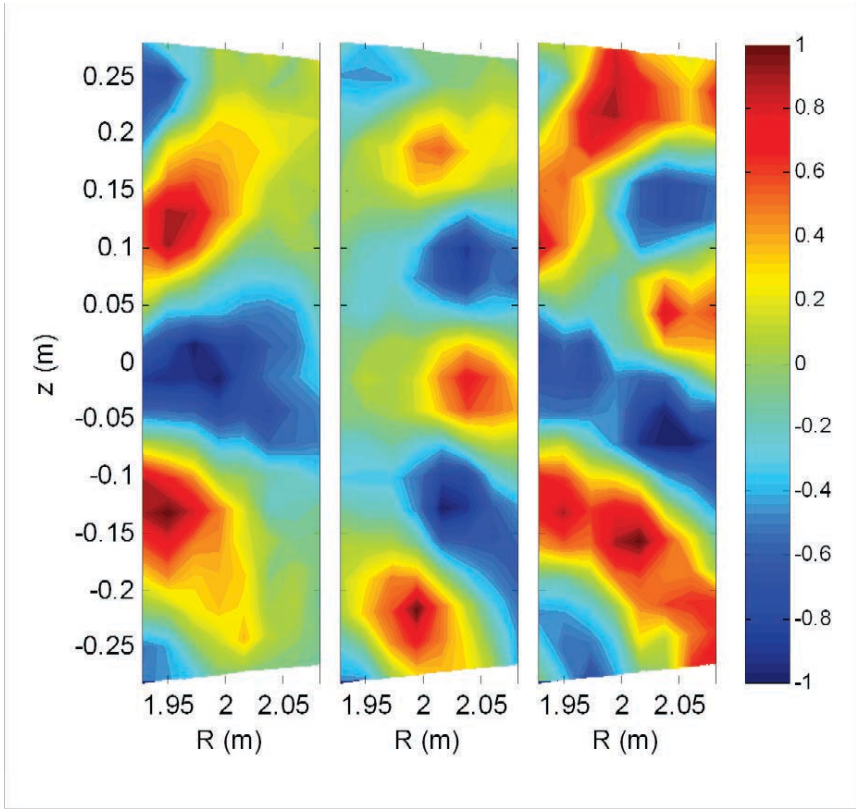
ECEI

Fourier Mode Selection



- Lots of modes identified at one time or another, particularly near q_{min}=4, 3 crossings
- Variation in mode structure, but with some clear consistencies

Linear Mode Superposition

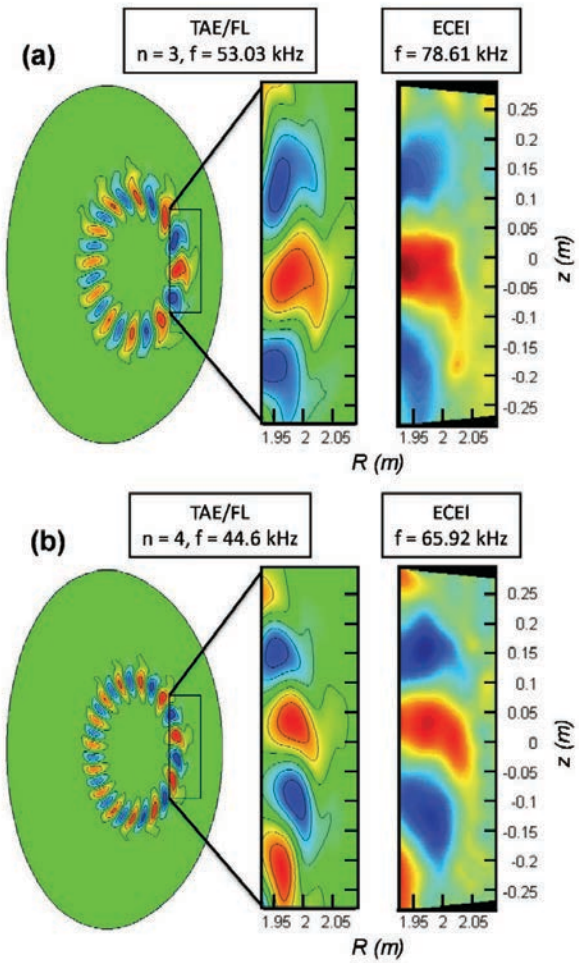


- **Modes at separate frequencies interact passively**
- **No evidence of nonlinear mixing**
  - Zero bispectrum

$$B_x(f,f') = \hat{x}(f)\hat{x}(f')\hat{x}^*(f+f')$$

$$b_x(f,f') = \frac{\left|\langle B_x(f,f')\rangle\right|}{\sqrt{\left\langle\left|\hat{x}(f)\hat{x}(f')\right|^2\right\rangle\left\langle\left|\hat{x}(f+f')\right|^2\right\rangle}} \cong 0$$

Comparison to TAEFL modeling

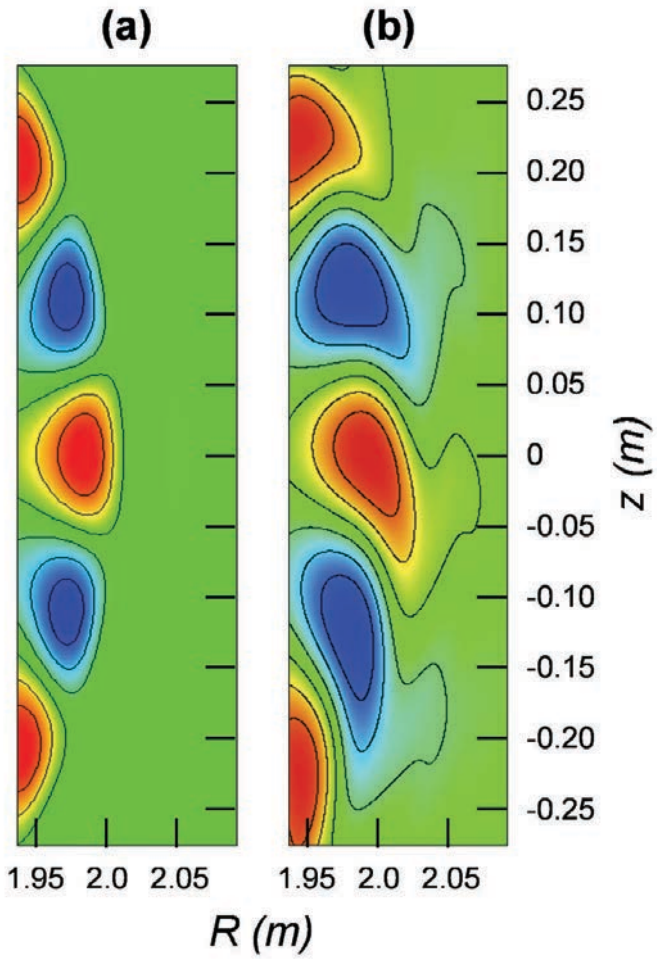


*Tobias, et. al., PRL **106**, 075003 (2011)*

- **Particularly well suited to imaging coherent eigenmodes**
  - FFT and SVD techniques powerful for these modes
- **Excellent agreement with gyrofluid-MHD predictions**
  - TAEFL (left) provides near exact match in mode structure
  - Discrepancies in *f* have been resolved by acoustic coupling without further modification of eigenmode structure

Deviations from ideal MHD that ECEI sees are present in gyrofluid and kinetic models

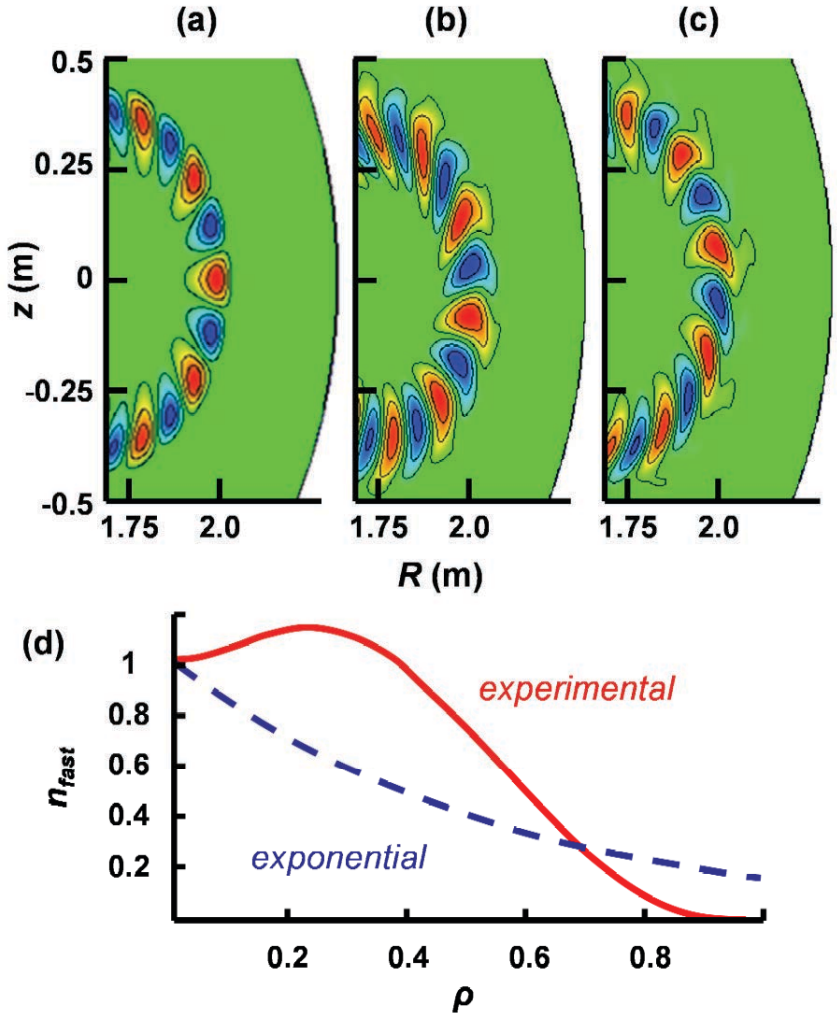
- **Ideal MHD (a) naturally conserves mode symmetry**
  - no coupling of cosine and sine terms
- **Radial phase shear ubiquitous in gyrofluid and kinetic simulations**
  - TAEFL (b), GYRO (see E. Bass, P1.1), etc.



*Tobias, et. al., PRL **106**, 075003 (2011)*

In TAEFL, β_{fast} profiles and diamagnetic flows drive twist (?, will return to this)

- **TAEFL predicts changes to eigenmode structure with variation of n_{fast} distribution**
  - relative phase coupled to diamagnetic flow
  - hope for a fast ion profile diagnostic



TAEFL is a hybrid MHD-energetic particle model based on a reduced MHD model + Landau closure:

$$\frac{\partial \psi}{\partial t} = \nabla_{\parallel}\phi + \eta J_{\zeta} + \rho_i^2 \left(\frac{\pi}{2}\right)^{1/2} \left(\frac{\mathrm{v}_A^2}{\mathrm{v}_{Te}}\right) \left|\nabla_{\parallel}\right| \nabla_{\perp}^2 \psi$$

Ion FLR

e/i Landau damping terms

$$\frac{\partial U}{\partial t} = -\nabla_{\parallel}\left(\frac{J_{\zeta}}{B_{\zeta}}\right) + \sum_{thermal,fast} \hat{\xi} \times \vec{\nabla}\left(\frac{R}{|B|}\right) \bullet \vec{\nabla} n_f T_{f0} + \omega_r \rho_i^2 \nabla_{\perp}^2 U - c_0 \frac{(\beta_e + \beta_i)}{\omega_r} \mathrm{Im}\left[X_e'' + X_i'' - \frac{(X_i' - X_e')^2}{2 + X_e + X_i}\right] \Omega_d^2(\phi)$$

Reduced MHD Ohm's law and vorticity equation

$$\frac{\partial n_f}{\partial t} = \frac{T_{f0}}{m_f \Omega_{cf}} \Omega_d(n_f) - n_{f0} \nabla_{\parallel} \mathrm{v}_{\parallel,\mathrm{f}} + \frac{q_f n_{f0}}{m_f \Omega_{cf}} \Omega_d(\phi) - \frac{q_f}{m_f \Omega_{cf} n_{f0}} \frac{dn_{f0}}{dr} \left(\frac{1}{r}\frac{\partial \phi}{\partial \theta} - \frac{\partial \psi_0 / \partial r}{R^2 B} \frac{\partial \phi}{\partial \zeta}\right)$$

EP Landau closure

EP ω_* drive

$$\frac{\partial \mathrm{v}_{\parallel,f}}{\partial t} = \frac{T_{f0}}{m_f \Omega_{cf}} \Omega_d(\mathrm{v}_{\parallel,f}) - \left(\frac{\pi}{2}\right)^{1/2} \mathrm{v}_{\mathrm{th,f}} \left|\nabla_{\parallel}\right| \mathrm{v}_{\parallel,f} - \frac{\mathrm{q_f v_{th,f}^2}}{m_f \Omega_{cf} R n_{f0}} \frac{dn_{f0}}{dr} \frac{1}{r} \frac{\partial \phi}{\partial \theta}$$

Landau-fluid energetic particle closure moment equations

where $\Omega_d \propto \left(\hat{b} \times \vec{\nabla} B\right) \bullet \vec{\nabla}, \quad X_{e,i} = \xi_{e,i} Z\left(\xi_{e,i}\right), \quad \xi_{e,i} = \omega_r / \sqrt{2} \left|\nabla_{\parallel}\right| \mathrm{v}_{th,e,i}$

Kinetic-MHD via gyrokinetic simulation

- Nonlinear gyrokinetic equation, Poisson's equation and Ampère's law

$$\frac{\partial f}{\partial t} + (v_\parallel \boldsymbol{b} + \boldsymbol{v}_\mathrm{d}) \cdot \frac{\partial f}{\partial \boldsymbol{X}} + \dot{v}_\parallel \frac{\partial f}{\partial v_\parallel} = 0$$

$$\begin{aligned} \frac{Z_\mathrm{i} n_\mathrm{i}}{T_\mathrm{i}} (\phi - \tilde{\phi}) &= \sum_\alpha Z_\alpha \bar{n}_\alpha \\ -\frac{c}{4\pi} \nabla_\perp^2 A_\parallel &= \sum_\alpha Z_\alpha \bar{n}_\alpha u_\alpha \end{aligned}$$

- In fluid limit, gyrokinetic system recovers MHD modes including Alfvén wave, interchange mode, kink mode, KBM

$$\begin{aligned} &\frac{\omega(\omega - \overbrace{\omega_{*\mathrm{P}}}^{\text{KBM}})}{v_\mathrm{A}^2} \nabla_\perp^2 \delta\phi + \overbrace{i \boldsymbol{B}_0 \cdot \nabla \left[\frac{\nabla_\perp^2 (k_\parallel \delta\phi)}{B_0} \right]}^{\text{SAW}} \\ &-i \nabla (k_\parallel \delta\phi) \times \boldsymbol{b}_0 \cdot \nabla \left(\frac{\boldsymbol{b}_0 \cdot \nabla \times \boldsymbol{B}_0}{B_0} \right) \qquad \longleftarrow \text{kink drive} \\ &\underbrace{-i\omega \frac{4\pi}{c} \left[\nabla \times \boldsymbol{b}_0 \cdot \nabla \left(\frac{\delta P_\parallel}{B_0} \right) + \boldsymbol{b}_0 \times \nabla B_0 \cdot \nabla \left(\frac{\delta P_\perp}{B_0^2} \right) + \frac{\nabla \times \boldsymbol{b}_0 \cdot \nabla B_0}{B_0^2} \delta P_\perp \right]}_{\text{interchange drive}} \\ &= 0 \end{aligned}$$

[*Gyrokinetic simulation model for kinetic magnetohydrodynamic processes in magnetized plasmas*, W Deng, Z Lin and I Holod, submitted to *Nuclear Fusion*, 2011]

GYRO overview

- GYRO is a continuum, df, gyrokinetic solver (5D phase space). Thermal and beam ions are fully gyrokinetic; electrons are drift or gyrokinetic.
- The toroidal grid is spectral with each toroidal n number considered separately. All other grid dimensions are physical.
- Nonlinear interaction between modes of different n values can be turned on or off.
- The (invariant) equilibrium distribution of each species is assumed to be a Maxwellian in velocity space (isotropic thermal distributions).
- Can run electrostatically (df only), or electromagnetically (df and $dA_{\|}$, with or without $dB_{\|}$).
- Local (linearly varying equilibrium profiles) or global (real equilibrium profiles) functionality available.
- Initial-value or eigenvalue solution methods possible.

J. Candy and R.E. Waltz, Phys. Rev. Lett. **91**, 045001 (2003).

Summary of included effects

Kinetic Model Includes Many Phenomena:

- Landau damping automatically included
- Kinetic instabilities driven by radial pressure gradients (ITG,TEM,ETG,etc.)
- MHD instabilities driven by radial pressure gradients: kinetic ballooning mode (KBM), Kelvin-Helmholtz, etc.
- Other Alfvénic instabilites (TAE, BAE, etc.) including continuum damping and finite compressibility

GYRO Formulation Excludes Some MHD Effects:

- Equilibrium current absent → no kink or tearing drive
- Field aligned coordinates with "ballooning mode" approximation → only $k_{\parallel} << k_{\perp}$ valid (no multi surface-spanning "peeling" modes, ELMs for example)

GYRO equations

GYRO keeps linear and nonlinear terms of order r_* in the gyrokinetic equation. Neglecting rotation and $d\mathrm{B}_{\|}$, we get the following equation for fluctuations dh in the guiding-center distribution for each species:

$$\partial\delta h/\partial t+(\upsilon_{\|}\hat{z}+\vec{\upsilon}_E+\vec{\upsilon}_d+\vec{\upsilon}_\chi)\cdot\vec{\nabla}\delta h+C\delta h=-\vec{\upsilon}_\chi\cdot\vec{\nabla}F_0+(\upsilon_{\|}\hat{z}+\vec{\upsilon}_d)\cdot\vec{\nabla}\langle\delta\chi\rangle[e\partial F_0/\partial\varepsilon]$$

nonlinearity

Mode-induced phase-space flux

$$\vec{\upsilon}_\chi=(c/B)\hat{z}\times\vec{\nabla}\langle\delta\chi\rangle$$

Equilibrium distribution:

$$F_0(\vec{x},\varepsilon,\mu)=n_0(m/2\pi T_0)^{3/2}\exp[-m/2T_0(\upsilon_\perp^2+\upsilon_{\|}^2)]$$

$$\varepsilon=mv^2/2+e\Phi \qquad \mu=m\upsilon_\perp^2/2B$$

Equilibrium drifts:

$$\upsilon_E=(c/B)\hat{z}\times\vec{\nabla}\Phi$$

$$\vec{\upsilon}_d=(c/B)\hat{z}\times(m\upsilon_{\|}^2\hat{z}\cdot\nabla\hat{z}+\mu\vec{\nabla}B)$$

Gyro-averaged generalized potential

$$\langle\delta\chi\rangle=J_0(k_\perp\upsilon_\perp/\Omega)[\delta\phi-\upsilon_{\|}/c\,\delta A_{\|}]$$

$$\nabla_\perp^2\delta\phi=4\pi\sum_s\int d^3\upsilon e_s\left[J_0\delta h-F_0(e_s/T)\{(1-J_0^2)(\delta\phi-\delta A_{\|}u_{\|}/c)\}\right]$$

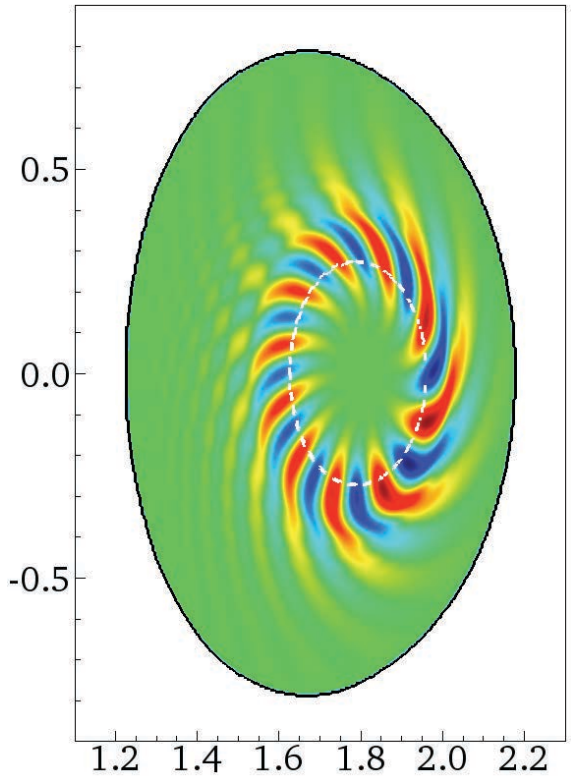
$$\nabla_\perp^2\delta A_{\|}=-4\pi/c\sum_s\int d^3\upsilon e_s\left[J_0\delta h\upsilon_{\|}-F_0(e_s/T_0)\{\upsilon_{\|}^2J_0{}^2\delta A_{\|}/c\right]$$

Poisson-Ampere field equations

GENERAL ATOMICS

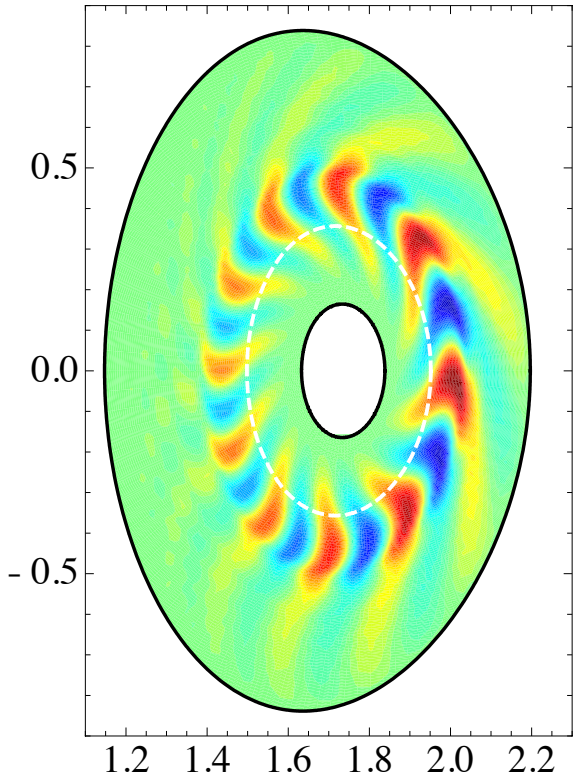
TAEFL, GTC, and Gyro code comparison

TAEFL



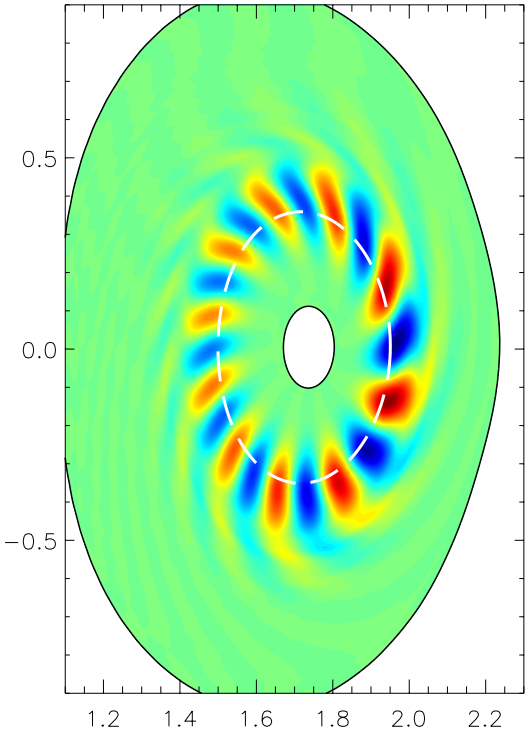
Don Spong

Gyro



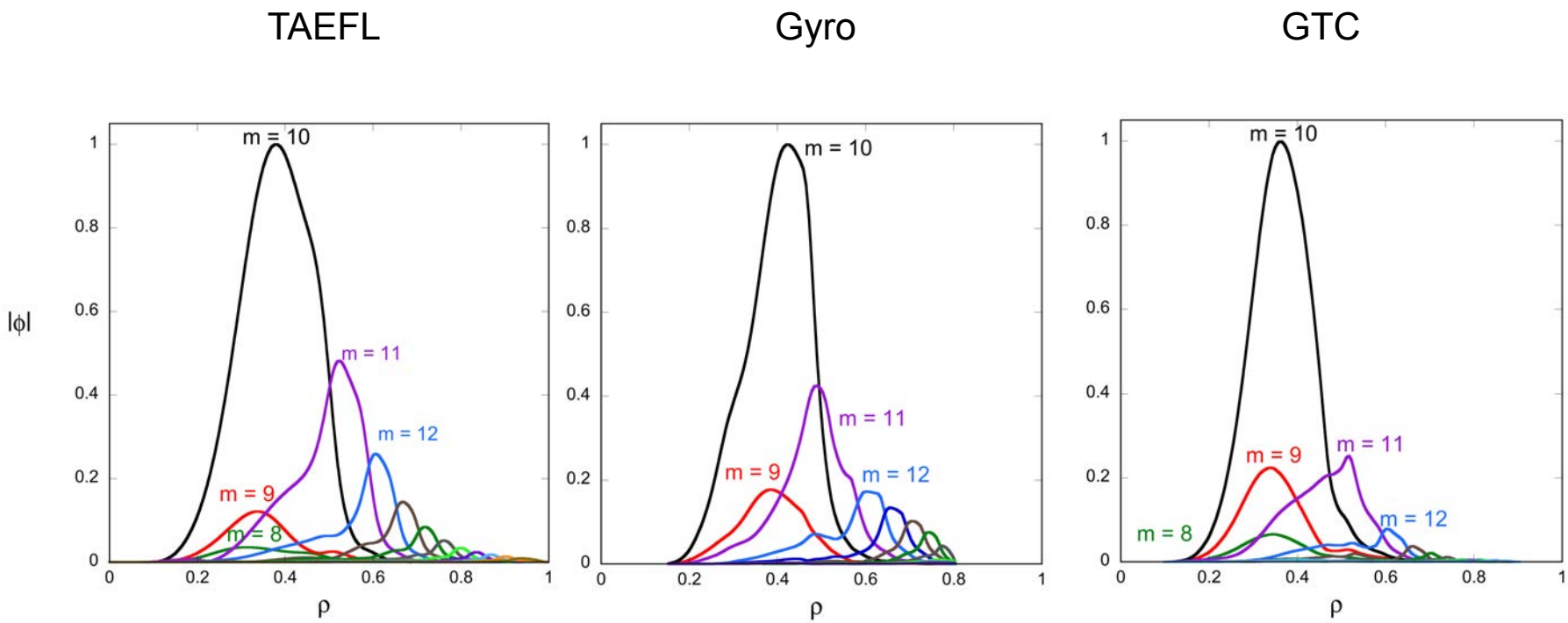
Eric Bass

GTC

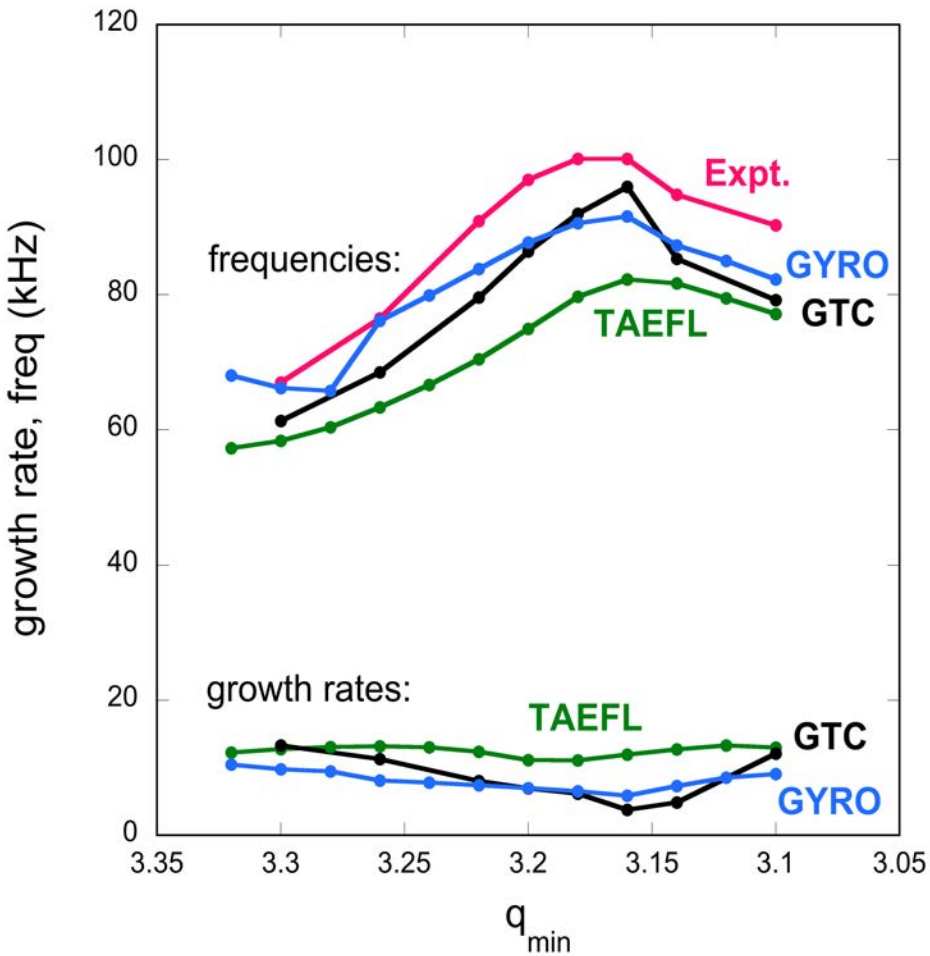


Wenjun Deng, Zhihong Lin

TAEFL, GTC, and Gyro code comparison



RSAE frequency and growth linear growth rates



- First points of experimental validation
- Frequency is relatively easy, growth rates not so much
- Next slides for discussion

Zhihong's idea, and my preliminary results

- **Treat the AE as a simple damped oscillator, lumping everything basically into a linear growth rate balanced by gross damping**
- **Evaluate the Q of the resonance, i.e. spectrogram line width**

$$\gamma = \frac{1}{2Q}$$

- **Easier said than done!**

Zhihong's idea, and my preliminary results

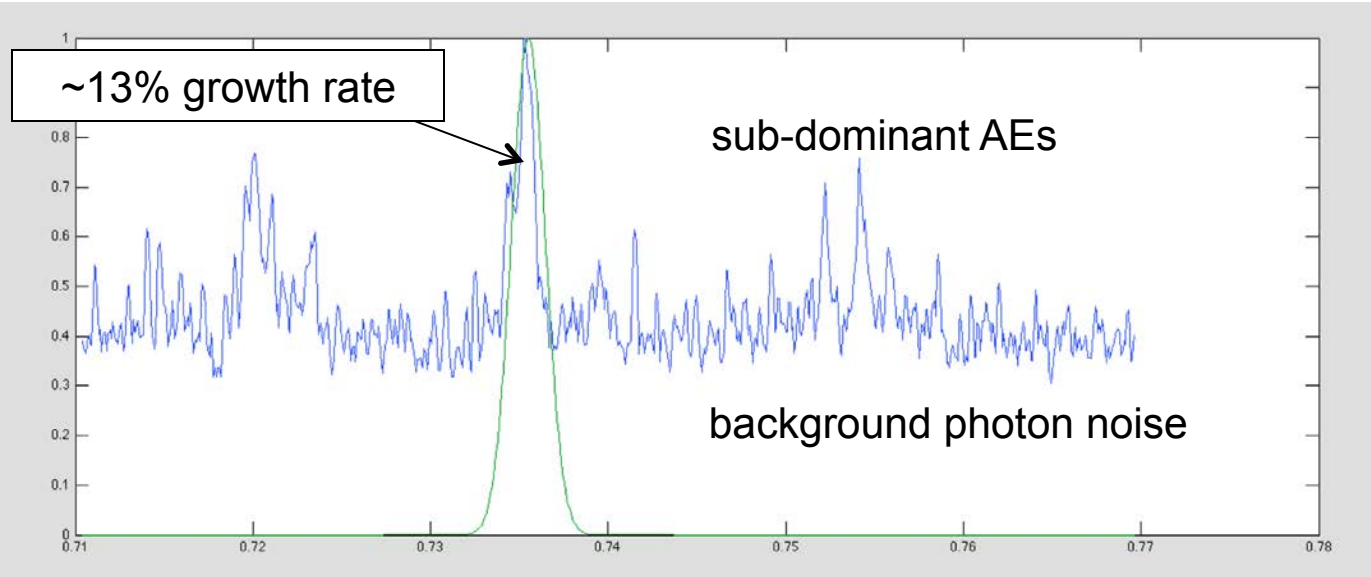
- **Constant frequency modes: you'd think these would be the best candidates (TAEs)**
- **Fit the line to a Gaussian or Lorentzian:**

$$FWHM_{FFT} = 2c\sqrt{2\ln 2} \qquad FWHM_{PSD} = 2c\sqrt{\ln 2}$$

- **Turns out that while the frequency domain is rather restrictive, FFT overlapping allows a better resolution in time, hence sweeping modes are easier to evaluate (RSAEs)**

$$A(t_1) = \int_{f_1}^{f_1+\Delta f} \int_{t_1}^{t_1+\Delta t} a e^{-\left(f - f_{00} + \nabla_t f \cdot (t - t_{00})\right)^2 / 2c^2} \, \delta t \, \delta f$$

Zhihong's idea, and my preliminary results



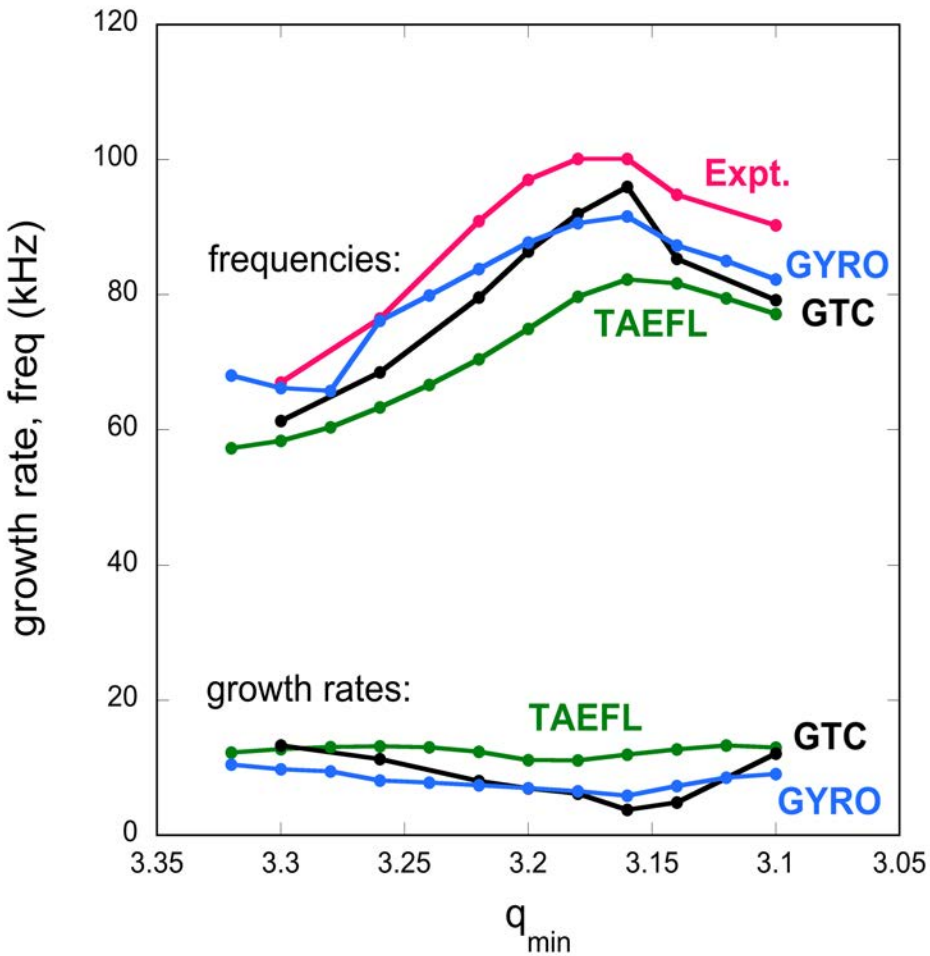
FFT window:
1.302 kHz
0.768 ms

90% overlap:
0.078 ms dt

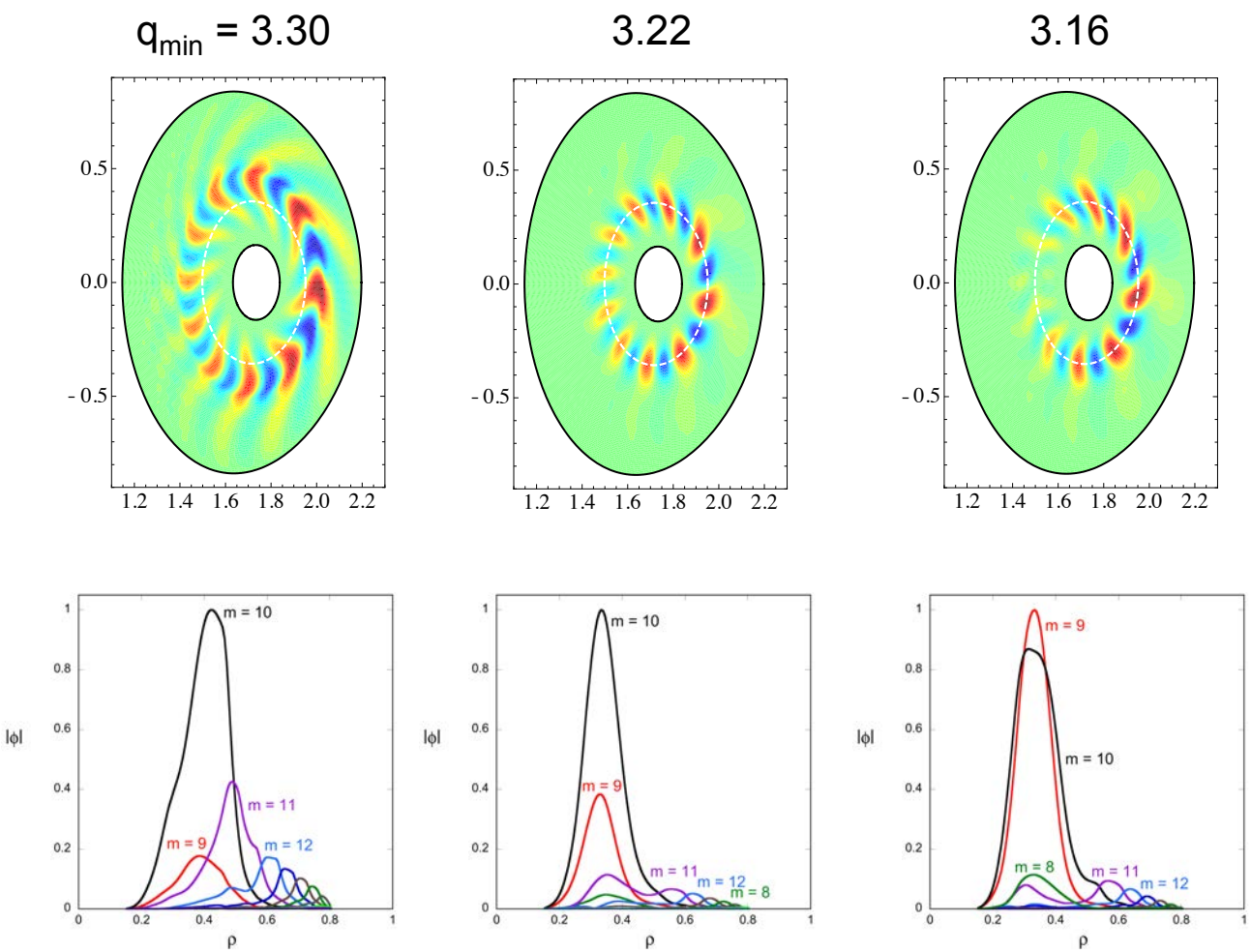
mode sweep:
~1.3 kHz/ms

- **Function shown is not the original lineshape, but an integral**
- **For small growth rates, this function gets rather square and makes a poor fit above the noise**

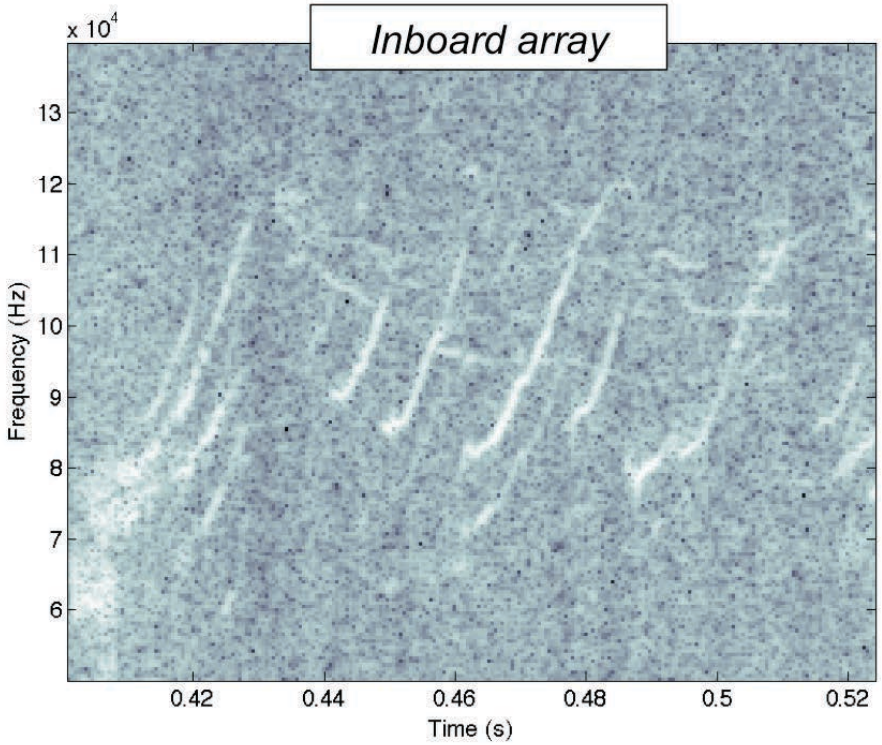
RSAE frequency and growth linear growth rates

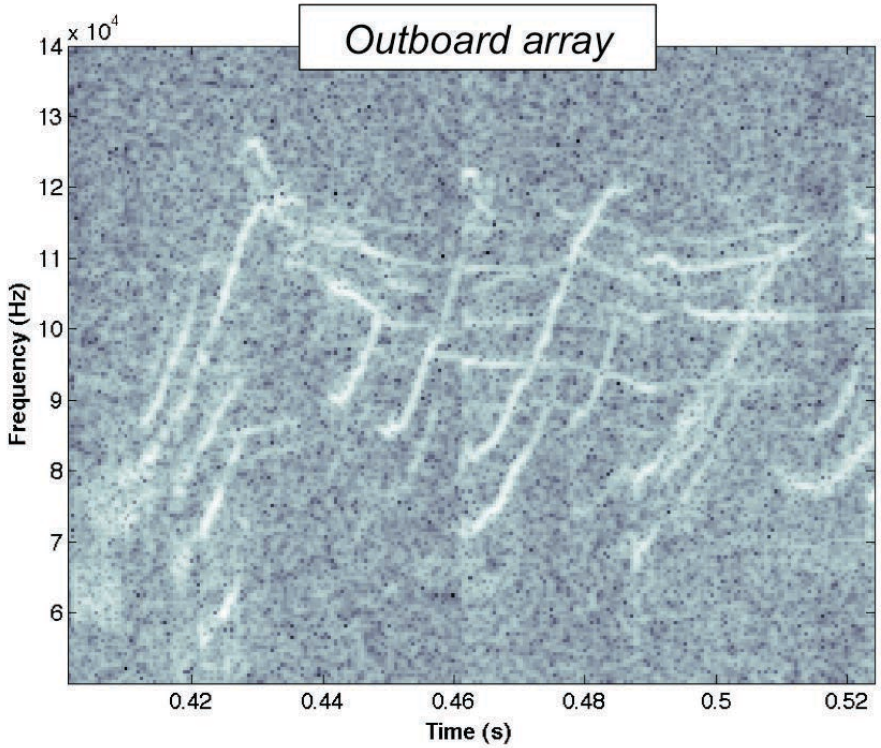


Ballooning character of the RSAE sweep

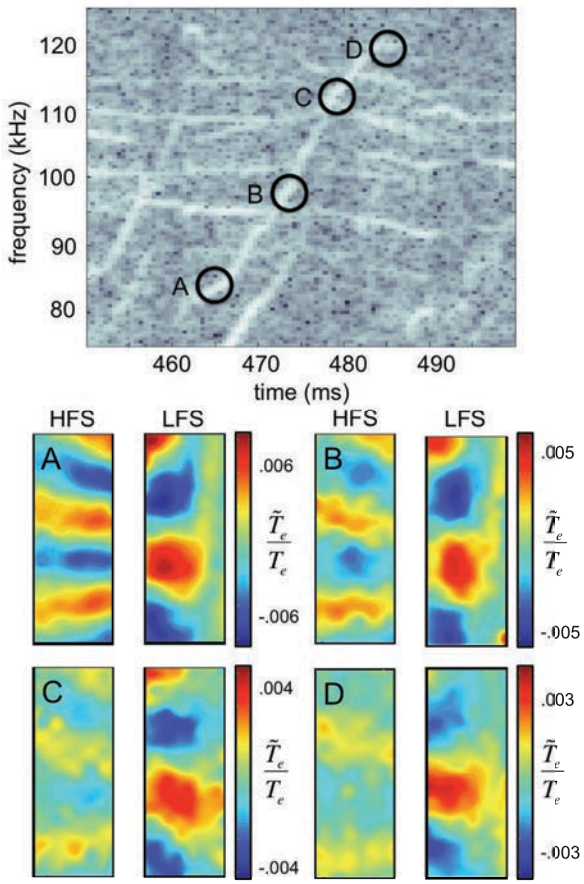


144256, dual-array data in a LSN discharge



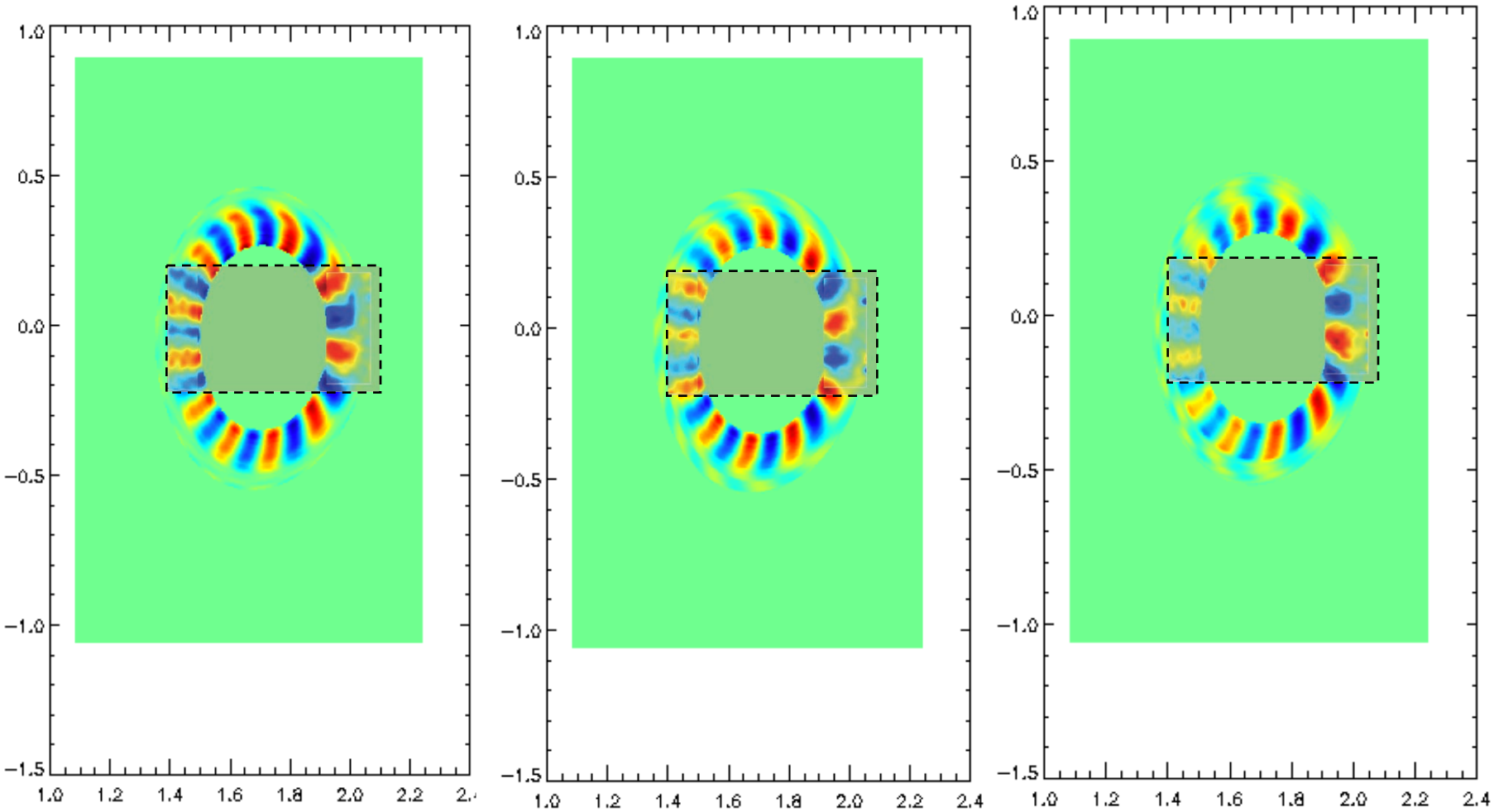


Sweeping RSAEs take on a ballooning nature

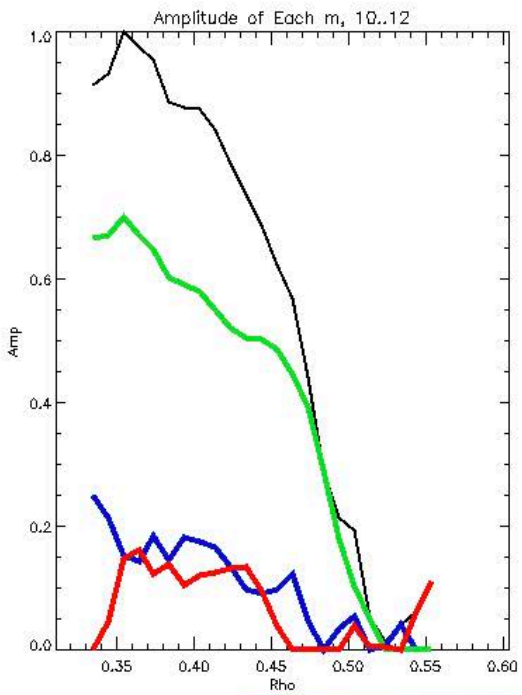


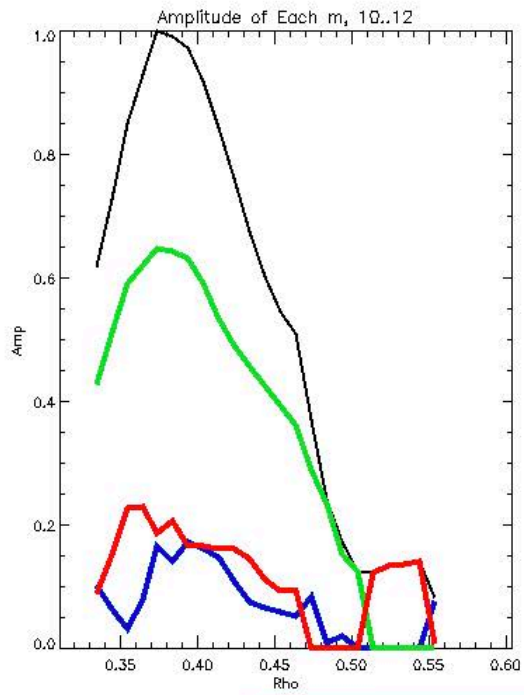
- **Even without a decomposition into poloidal mode numbers, the progression toward a more TAE-like mode is clear in the imaging data**
- **This behavior is ubiquitous and naturally connected to the evolution of the continuum**

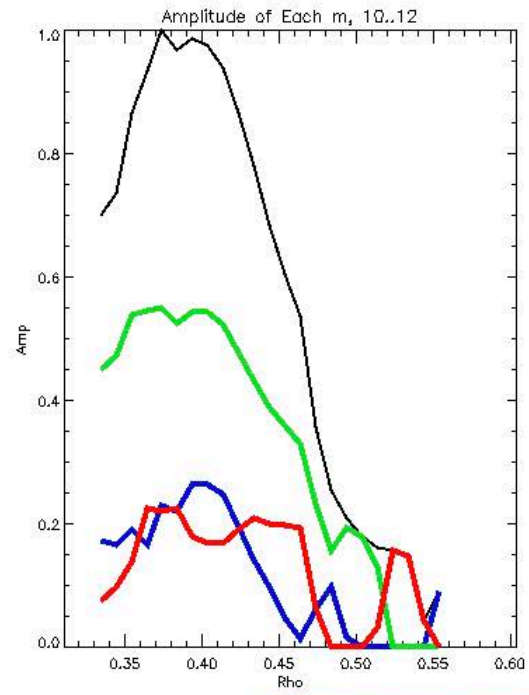
ECEI data and mode reconstruction



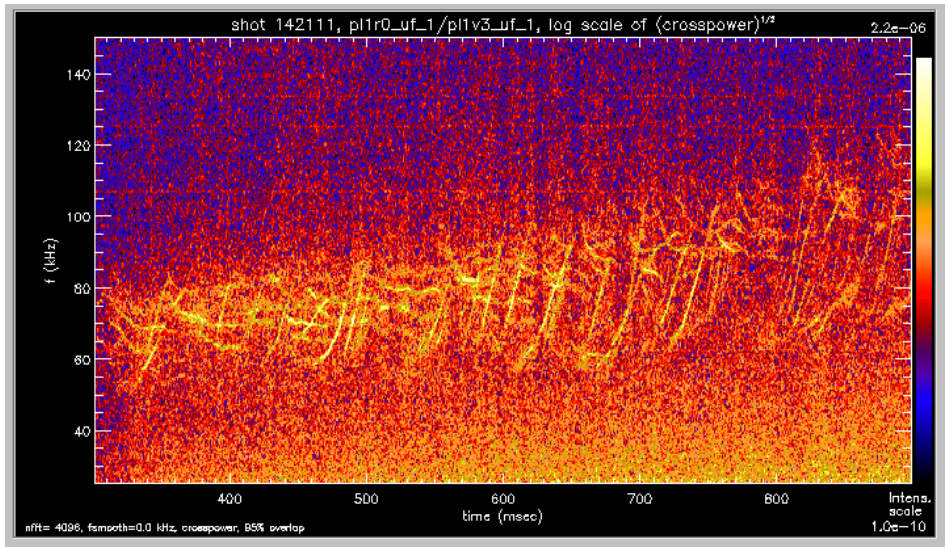
Nearest sideband decomposition

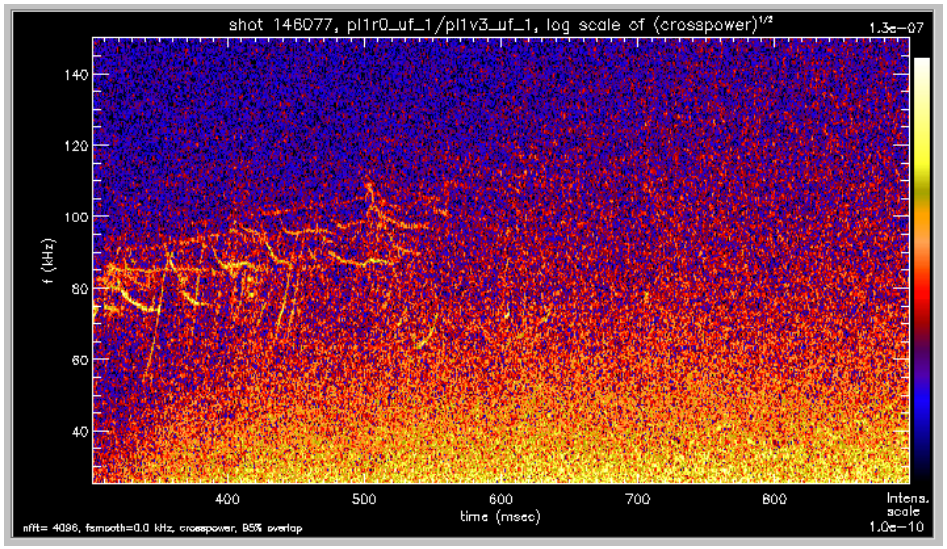






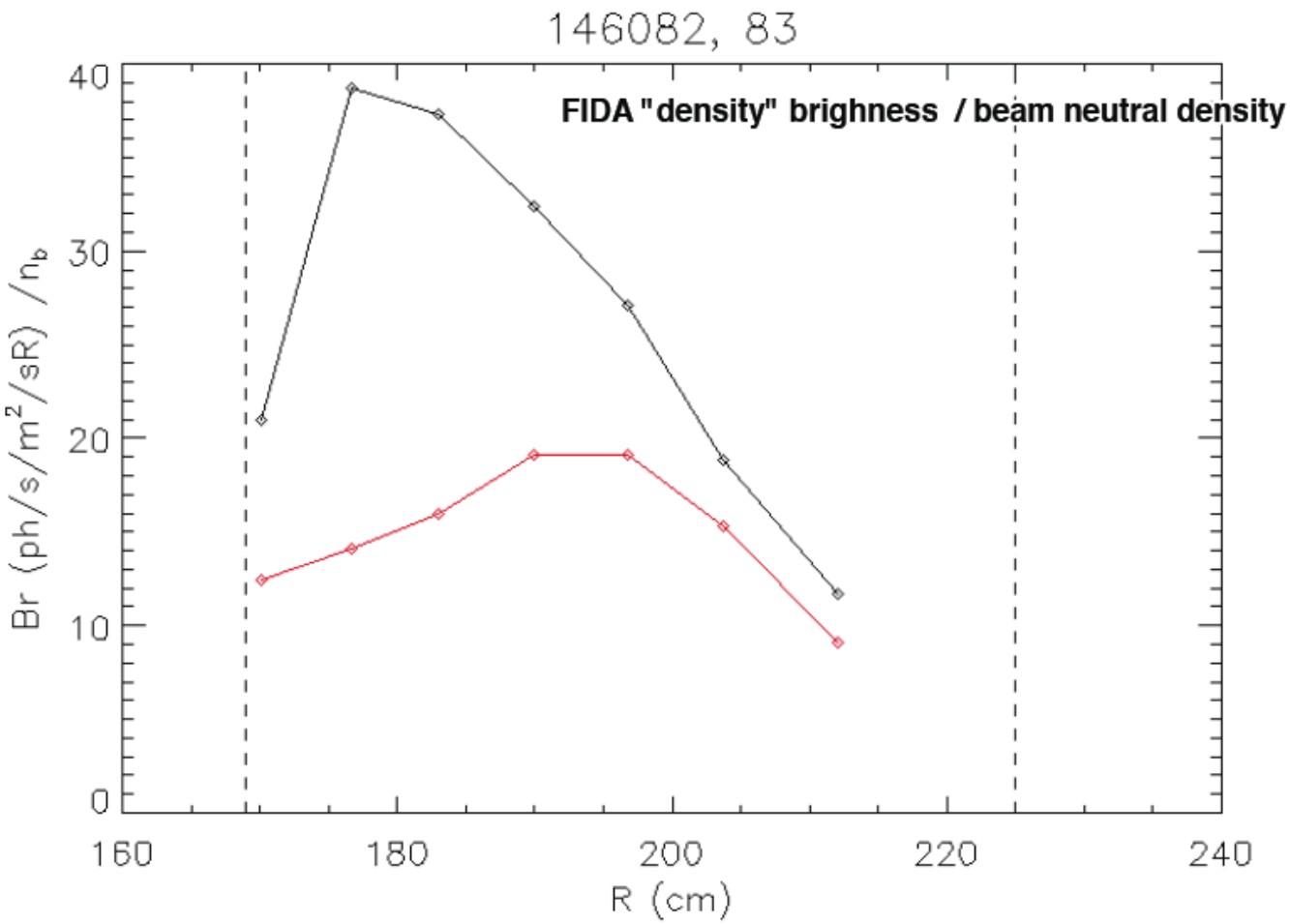
Off-axis injection modifies Alfvén eigenmode stability and MHD spectra





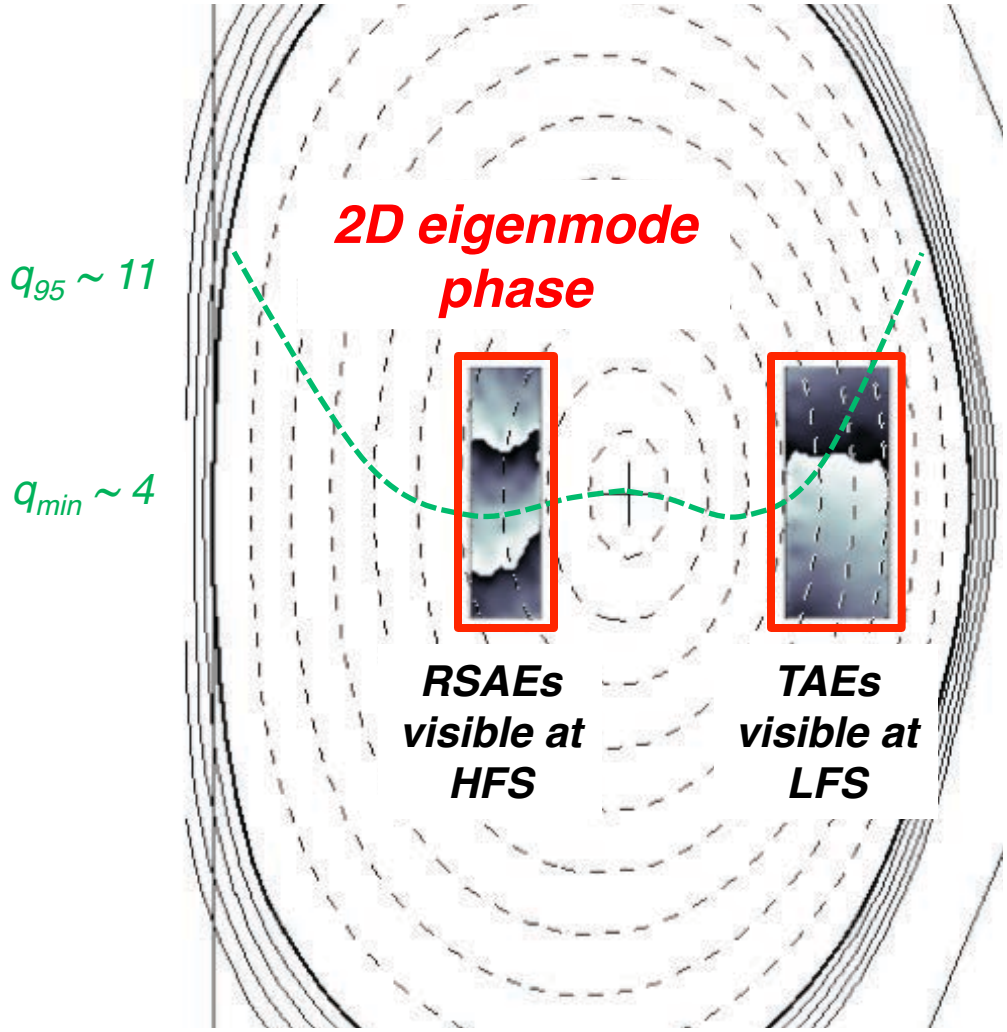
- **TAEs and strong RSAEs excited during on-axis NBI (left)**
- **Only a few weak RSAEs arise in off-axis phases (right)**
- **Global TAEs remain in both discharges**
- **Local Alfvén eigenmode drive is associated with fast gradients—remain negative at TAE radii**

A peek at the fast ion profile



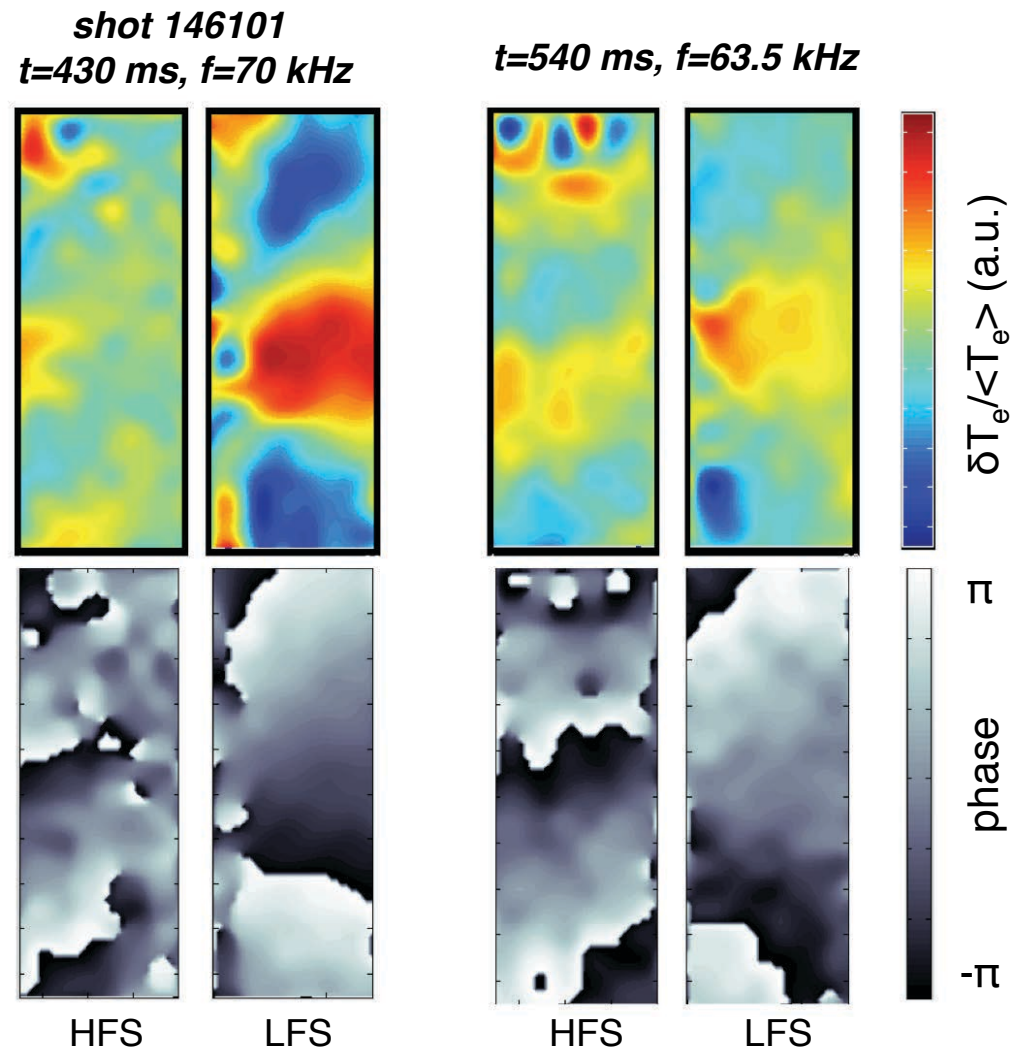
Dual-array operation allows simultaneous observation of RSAEs and TAEs in OANBI

- **Simultaneous dual-array capability is new for 2011**
- **LFS: outside q_{min}**
  - Predominantly views TAEs
- **HFS: near/inside q_{min}**
  - Viewing RSAEs
- **Reference shots reproducing 142111 with additional diagnostic views**
  - subsequent shots view q_{min} at both HFS/LFS



Though more weakly driven, RSAE mode structure is unchanged during OANBI

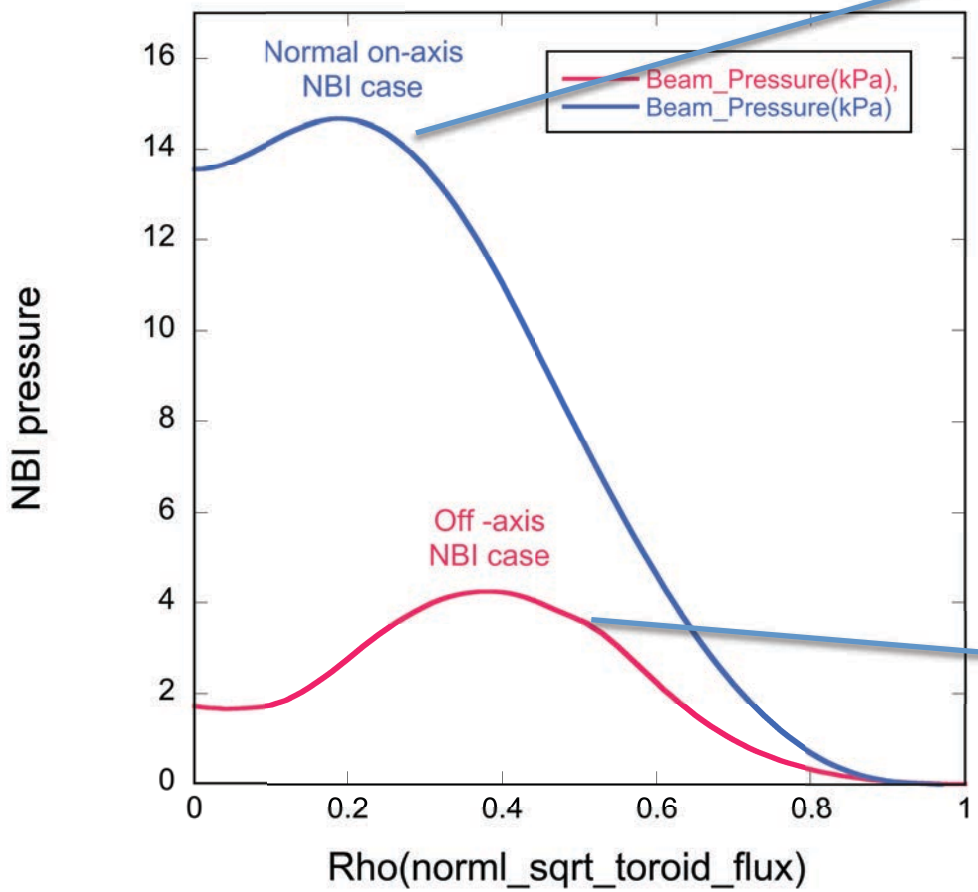
- At < 0.2% mode amplitude, diagnostic response often corrupted by systematic and statistical noise, but phase structure is clear and consistent
- A major surprise, significant changes in beam deposition have no qualitative effect on mode structure
- ω_{RSAE} (or n) has not changed sign

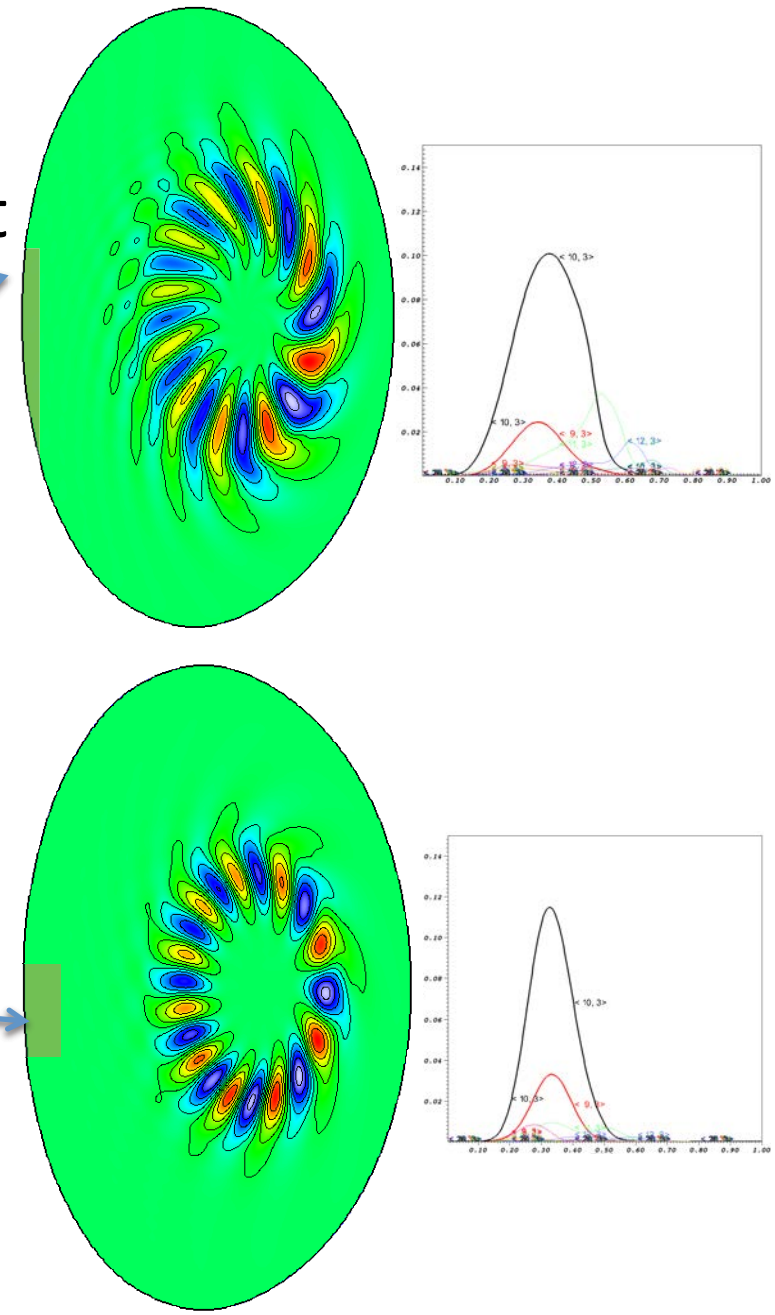


So here's the conundrum

- **At first glance, the fast ion pressure profile certainly looks like it is (as it should be), inverted, with reversed diamagnetic flow**
  - No "reversed" RSAEs are found
- **A merely flattened profile would explain the change in stability**
  - Still presents a serious change in diamagnetic flow, but there is no change in mode "twist"
- **Is the twist really a "thermal" property?**

TAEFL predicts that off-axis NBI profile drives an RSAE off of the positive gradient region of the profile and has opposite mode twist from the on-axis profile case





What about transport effects that might be localized in phase space?

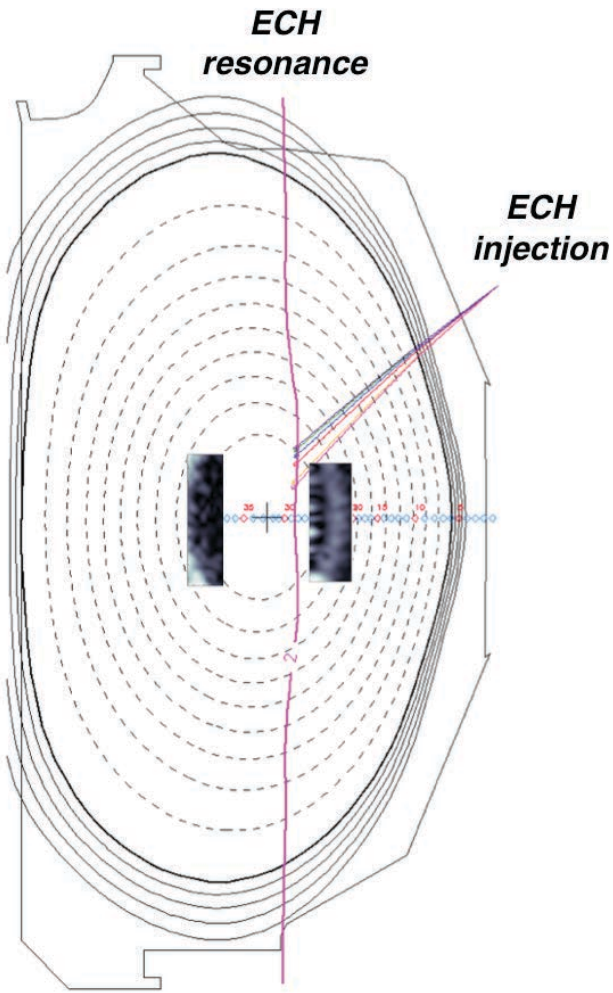
- **TRANSP is predicting a relatively subtle change in the total fast ion profile compared to the FIDA data**
- **FIDA is naturally sensitive to a select region on phase space**
- **Different FIDA systems are saying different things**
  - yet to be fully explored
- **Does the region of phase space that drives our RSAEs really have an altered profile?**

BAAE behavior is a whole different story

- Modes are strong in both types of discharges
- Mode structure doesn't change at all, but…
- Frequency is dramatically altered

Enhancement of diagnostic response with ECH

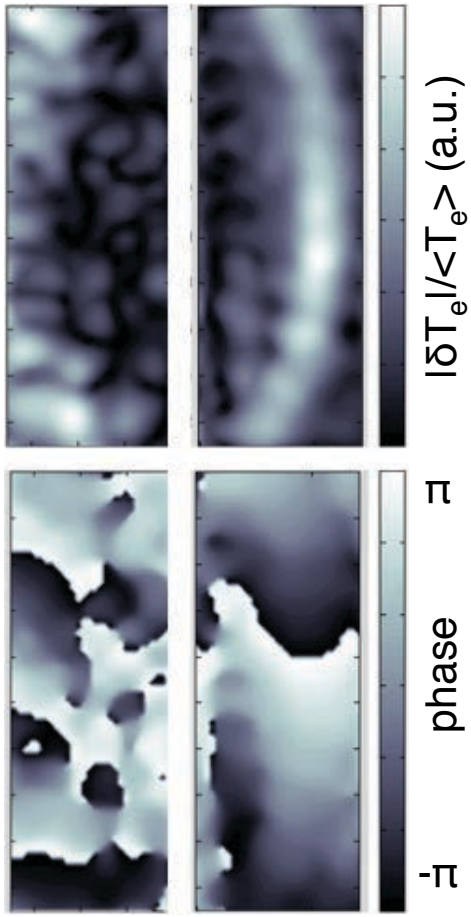
- **Much of the BAAE's energy is in an electrostatic polarization and hence temperature perturbations are often weak**
- **ECH injected on axis aids in enhancing δT_e response**
  - NBI remains dominant current drive
- **On and off axis modes have been observed at both HFS and LFS**



BAAE: on-axis NBI

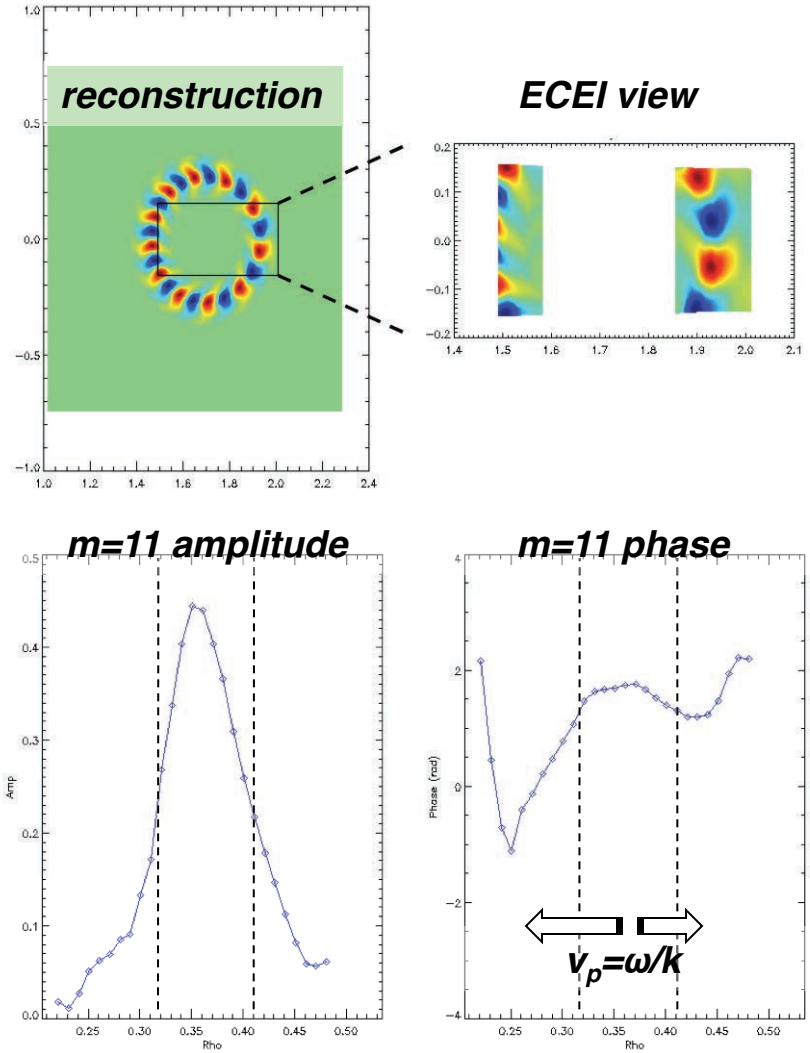
- **ECEI has provided the first 2D images of the BAAE**
  - Highly localized radially about ρ =0.2 ($\leq q_{min}$)
  - Modes rotate in same sense as RSAEs/TAEs (E×B and normal $\omega_{*i,thermal}$ direction)
  - Phase shear curvature is reversed w.r.t. RSAEs/TAEs

shot 146080
BAAE mode structure



Overview of prominent BAAE features

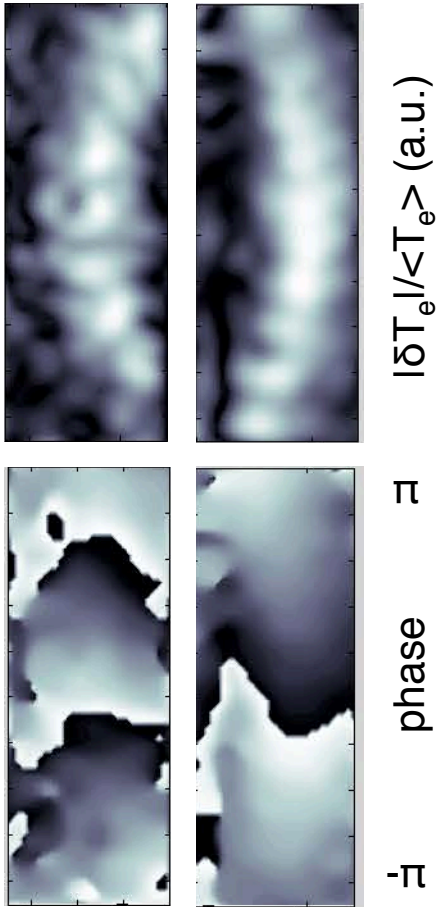
- **A best fit to the dominant poloidal mode number of a single mode reveals a singularly peaked, narrow, radial eigenmode**
- **Apparent phase velocity due to radial variation is *away* from mode peak**
  - reversed w.r.t. RSAE/TAE



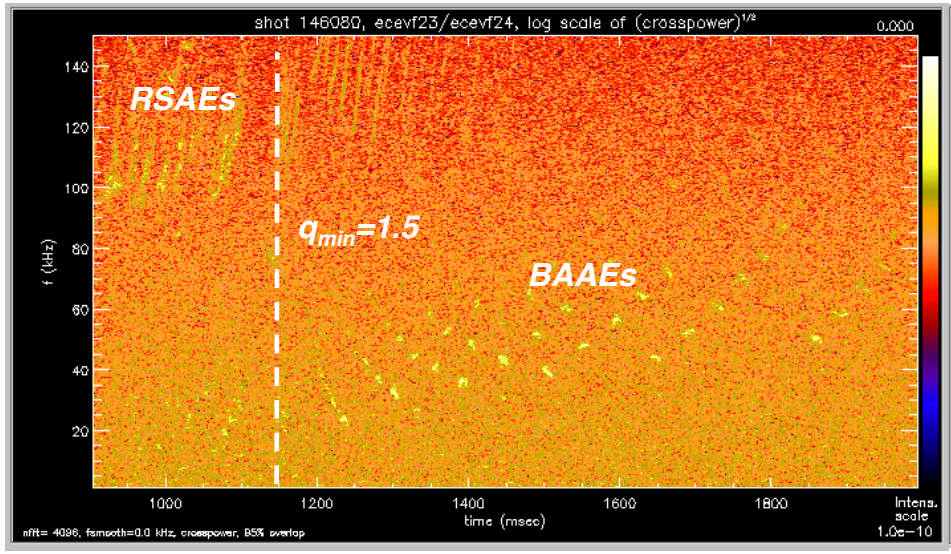
BAAE mode structure: off-axis NBI

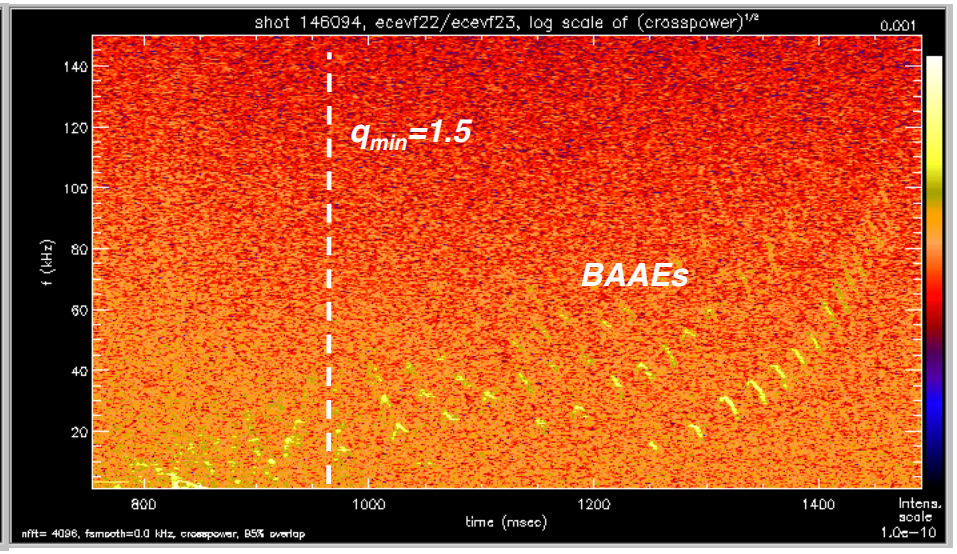
- **No qualitative differences in mode structure can be observed**
  - Again, we are surprised to find that neither mode structure nor phase orientation are modified by the inverted, positive gradient in β_{fast}

shot 146094
BAAE mode structure



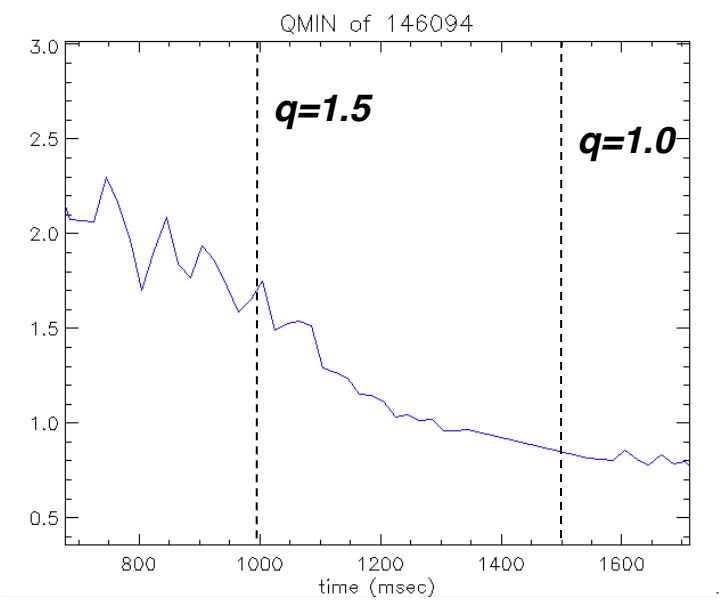
Comparison of MHD spectra obtained during on and off-axis NBI

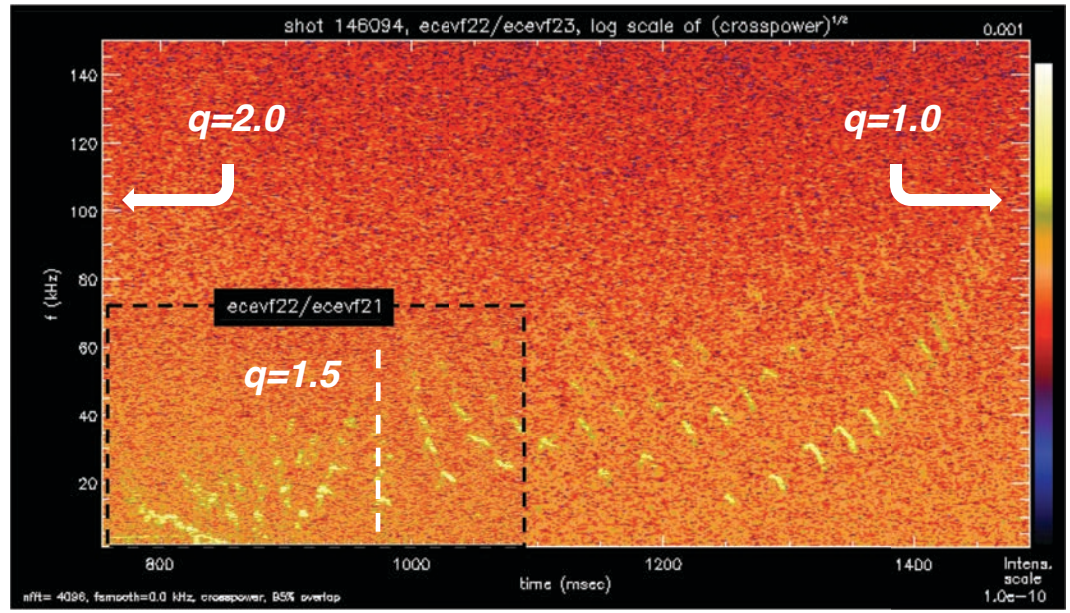




- **Upper limits of the BAAE chirp observed in both discharges**
  - "top hat" response, rolling over in time
- **RSAEs are notably absent in the OANBI MHD spectra**
- **BAAE frequencies are downshifted ~20 kHz in lab frame**
  - comparable rotation and Doppler shift
- **Modes migrate inward as *q* profile evolves, tracking q_{min}**

Features of the OANBI BAAE spectra





- **Current penetration is faster in the OANBI case shown; modes from q_{min}=2.0 to 1.0 can be observed**
  - MHD spectroscopy agrees well with EFIT01
- **BAAEs are observed with frequencies down to ~5 kHz**

Shift in observed BAAE frequency

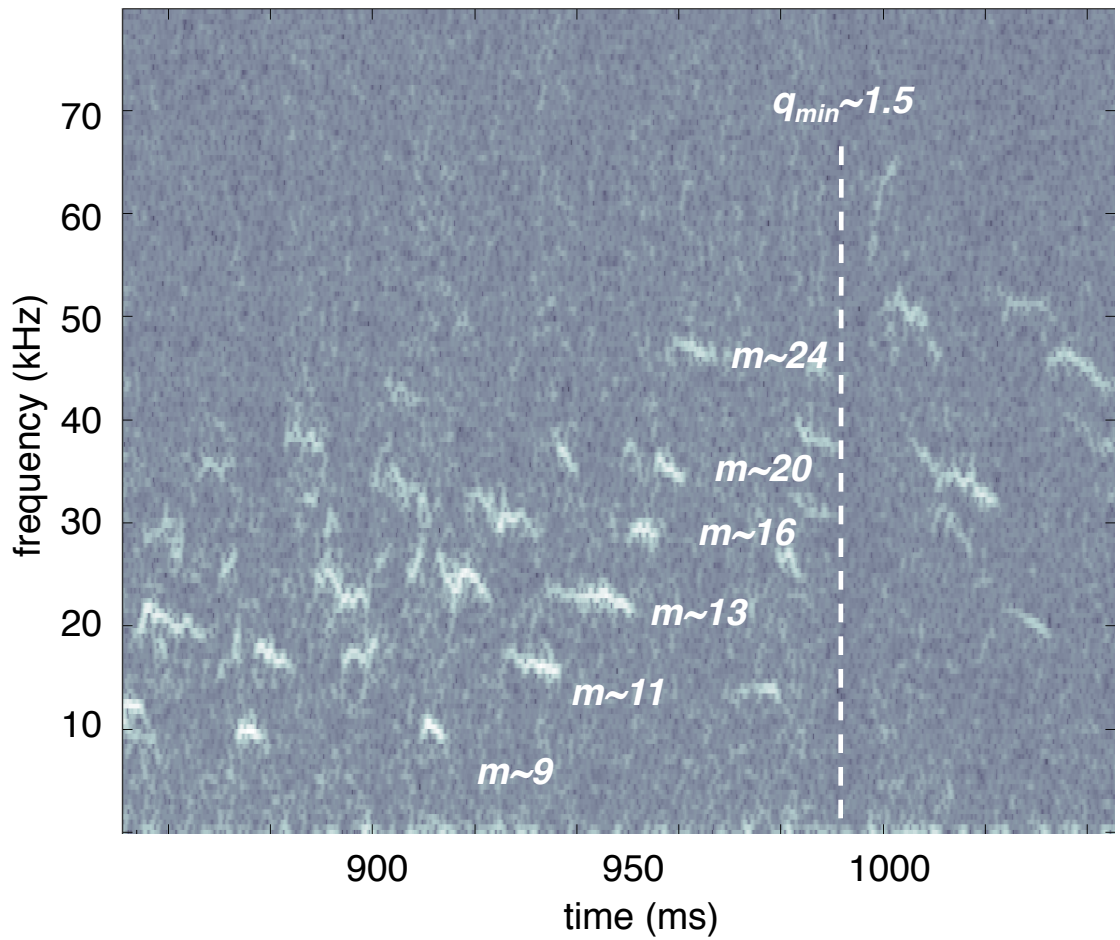
- **Doppler shift dominates for n ≈ 4**
  - observed frequency shifted in positive ω direction by $n\,\omega_{tor} \sim 5n$ kHz
  - normal spectral and eigenmode behavior expected
- **MHD frequency dominates only for lowest mode numbers**
  - counter-rotating modes?
  - counter phased eigenfunctions?
  - inverted frequency chirping?

Shift in observed BAAE frequency (continued)

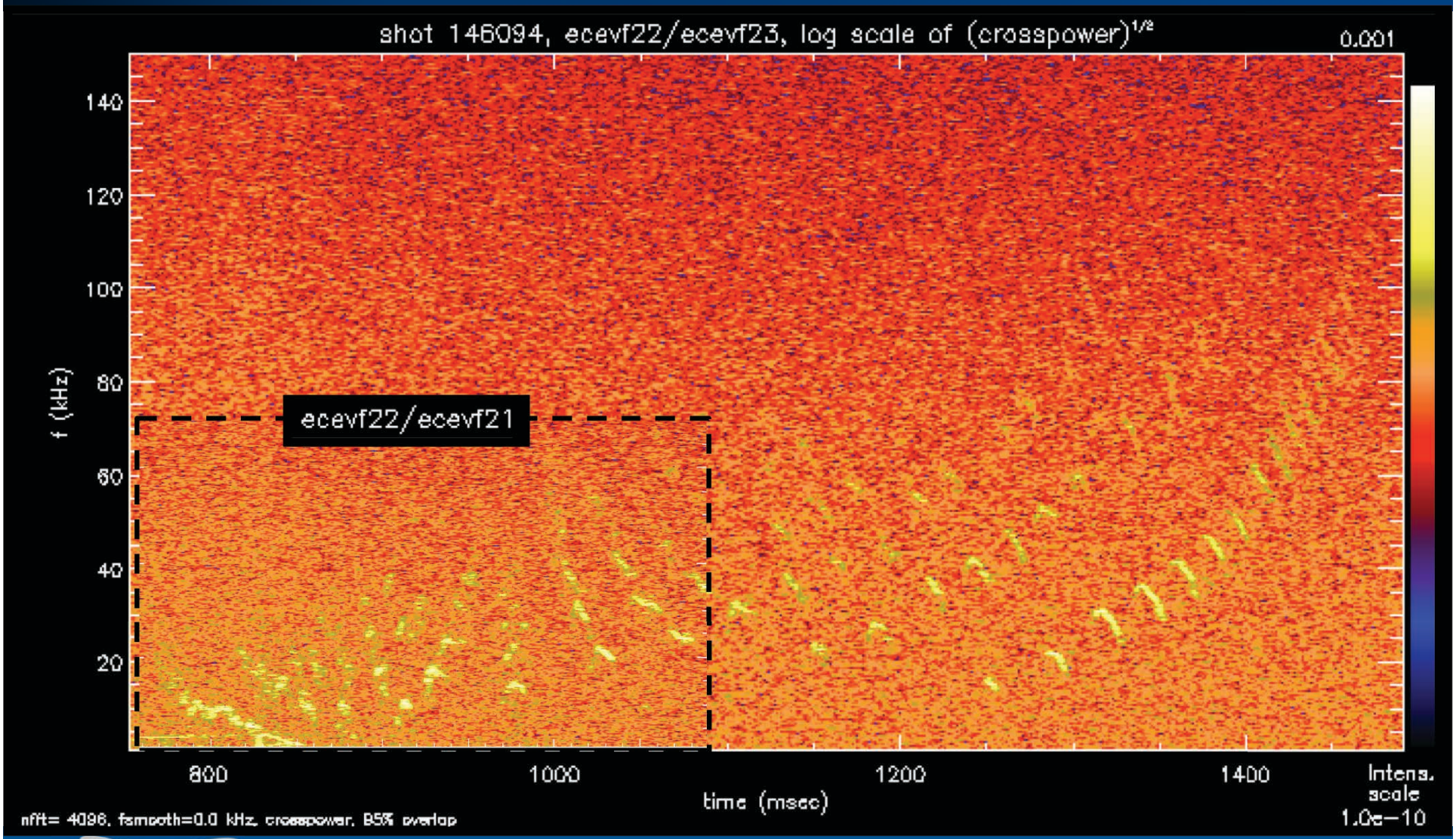
- **Low order modes are notably absent from both on- and off-axis spectra**
  - $m_{min} \sim 7$, assuming $m/n = q$ requires $n \geq 4$
- **All observed modes rotate in the normal sense**
- **All frequency chirps produce a typical "top hat" response**
  - even if MHD frequencies are negative, they sweep toward positive
  - continuum is not inverted

Fitted BAAE poloidal mode numbers

- **Dominant poloidal mode numbers do not fit naïve expectation**
- **Coupling to global modes?**
  - modes remain highly localized
  - ~2-3 cm outward migration at top of chirp



Another look at the spectrogram

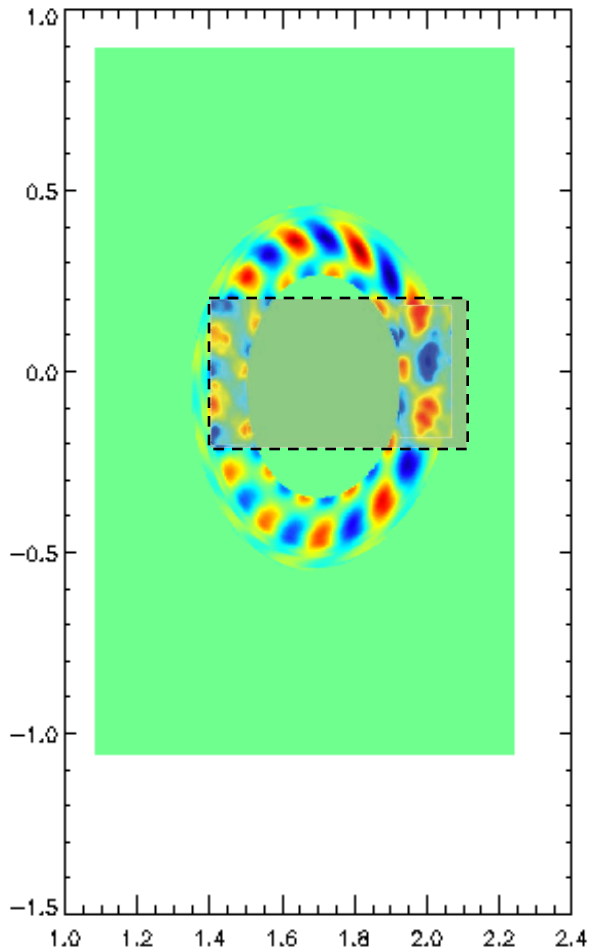


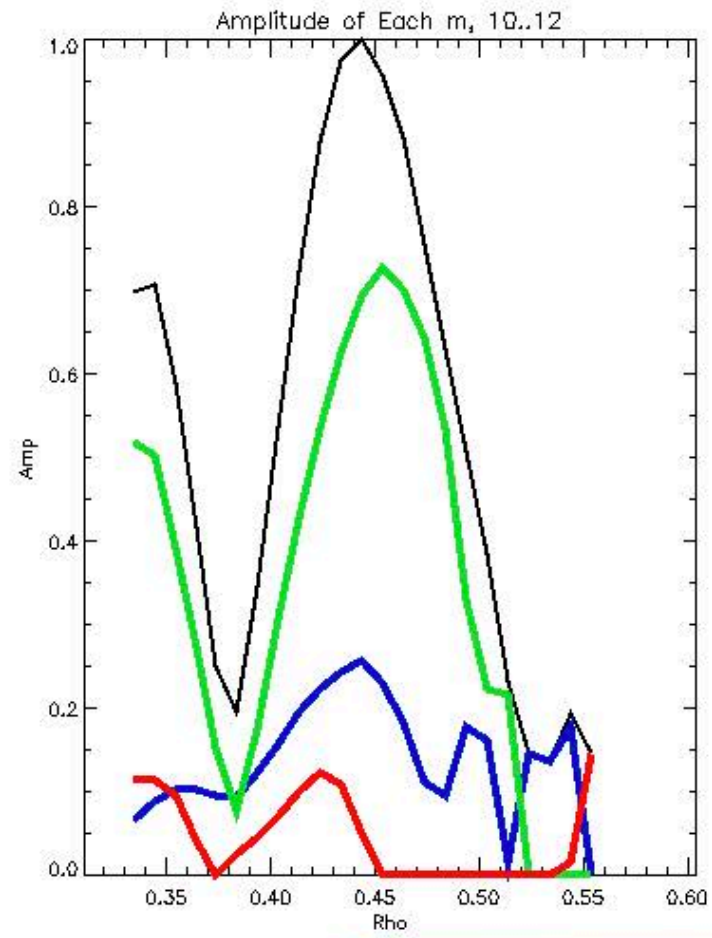
DIII-D NATIONAL FUSION FACILITY

PPPL

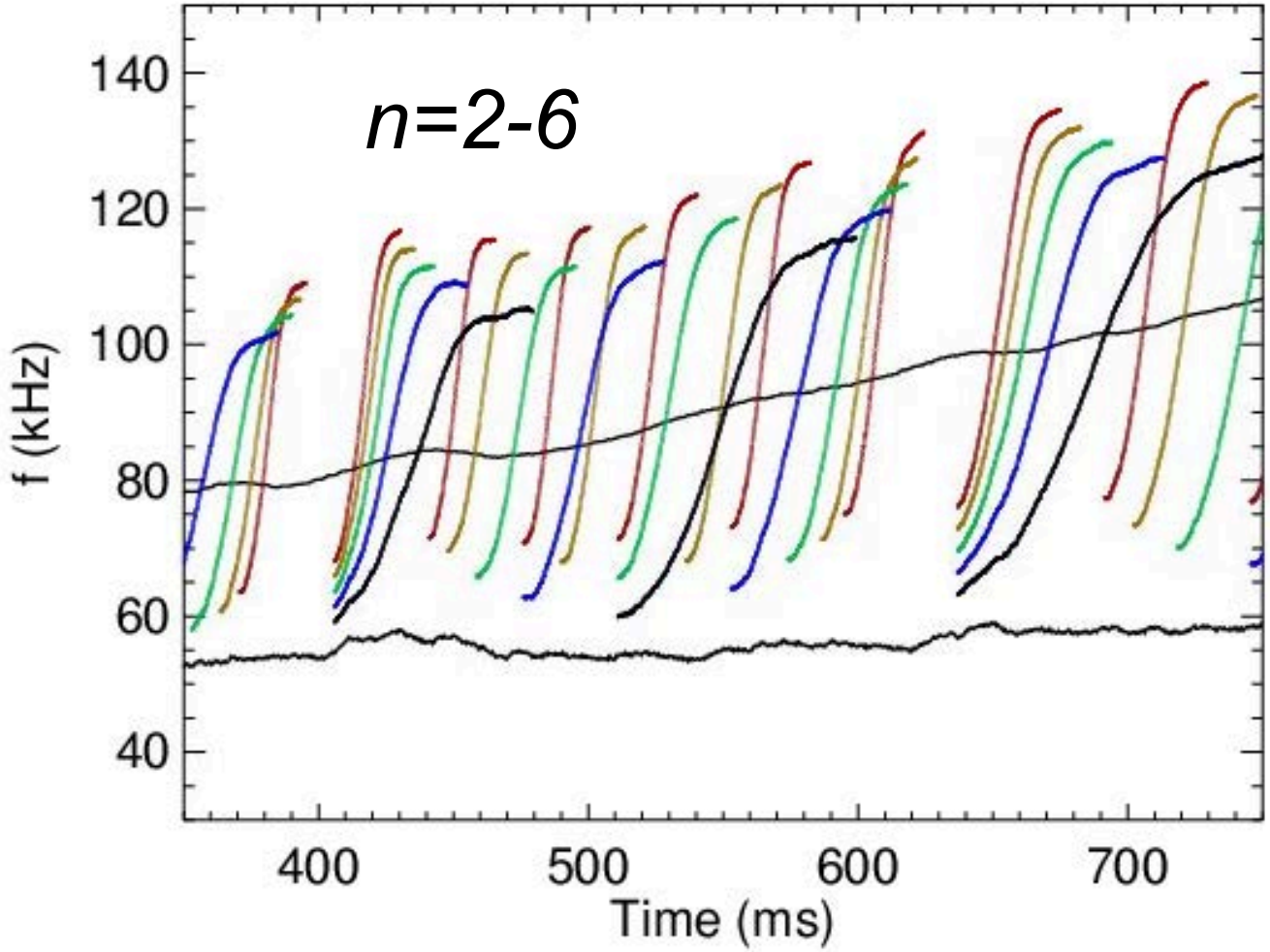
D.M.R.C Davis MM-Wave Research Center

MHD spectroscopy using RSAEs in 144256

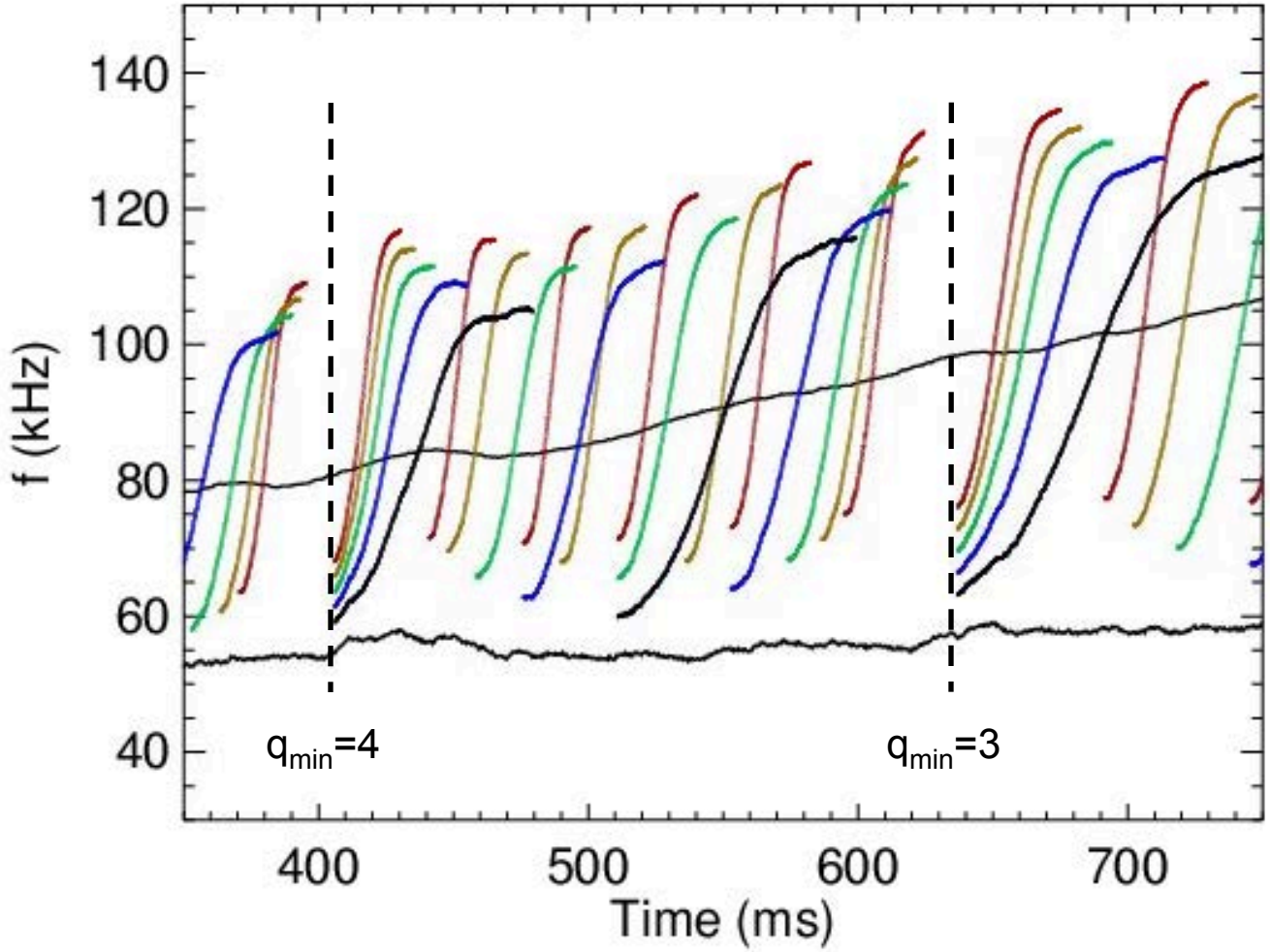




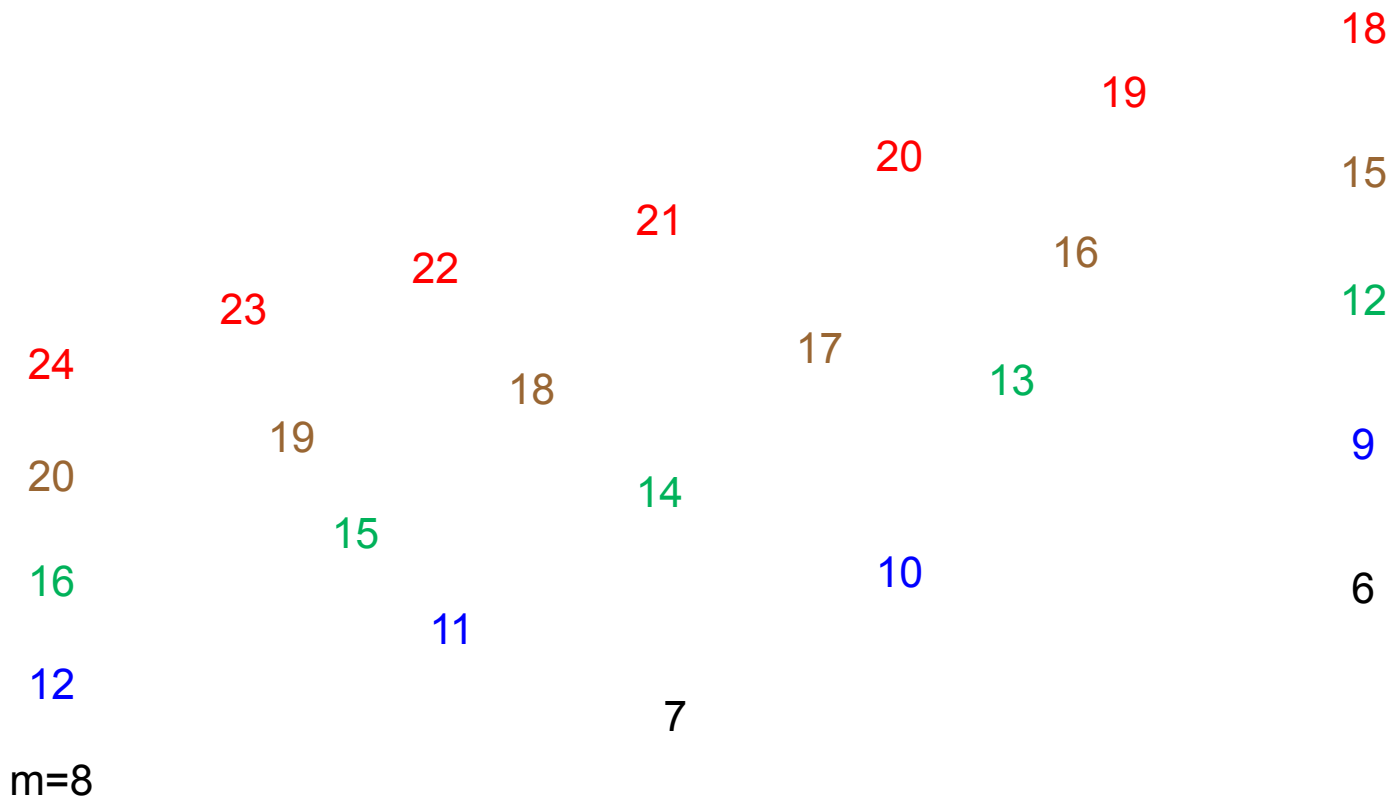
MHD spectroscopy using RSAEs in 144256



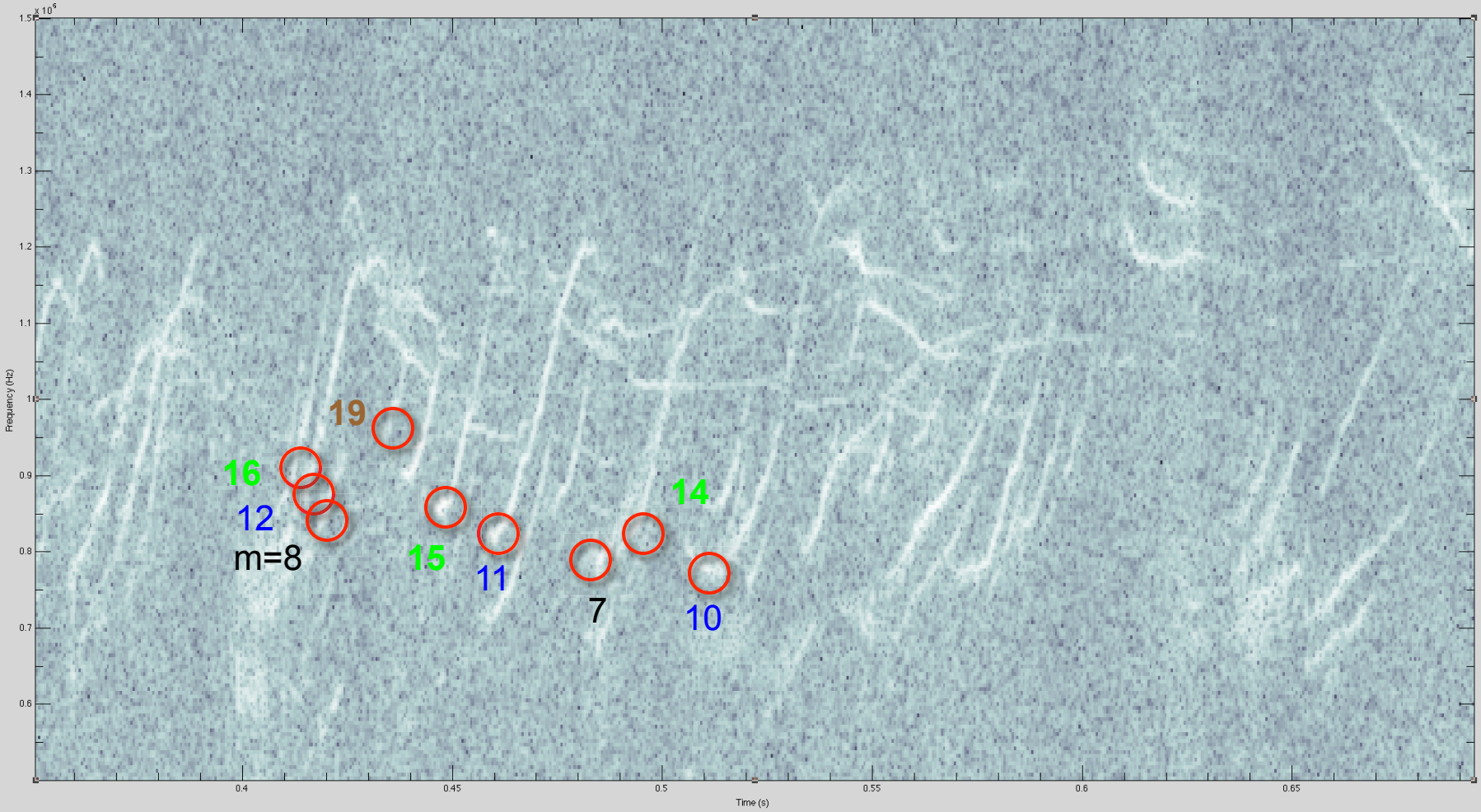
MHD spectroscopy using RSAEs in 144256



MHD spectroscopy using RSAEs in 144256



Modes that readily fit the pattern:

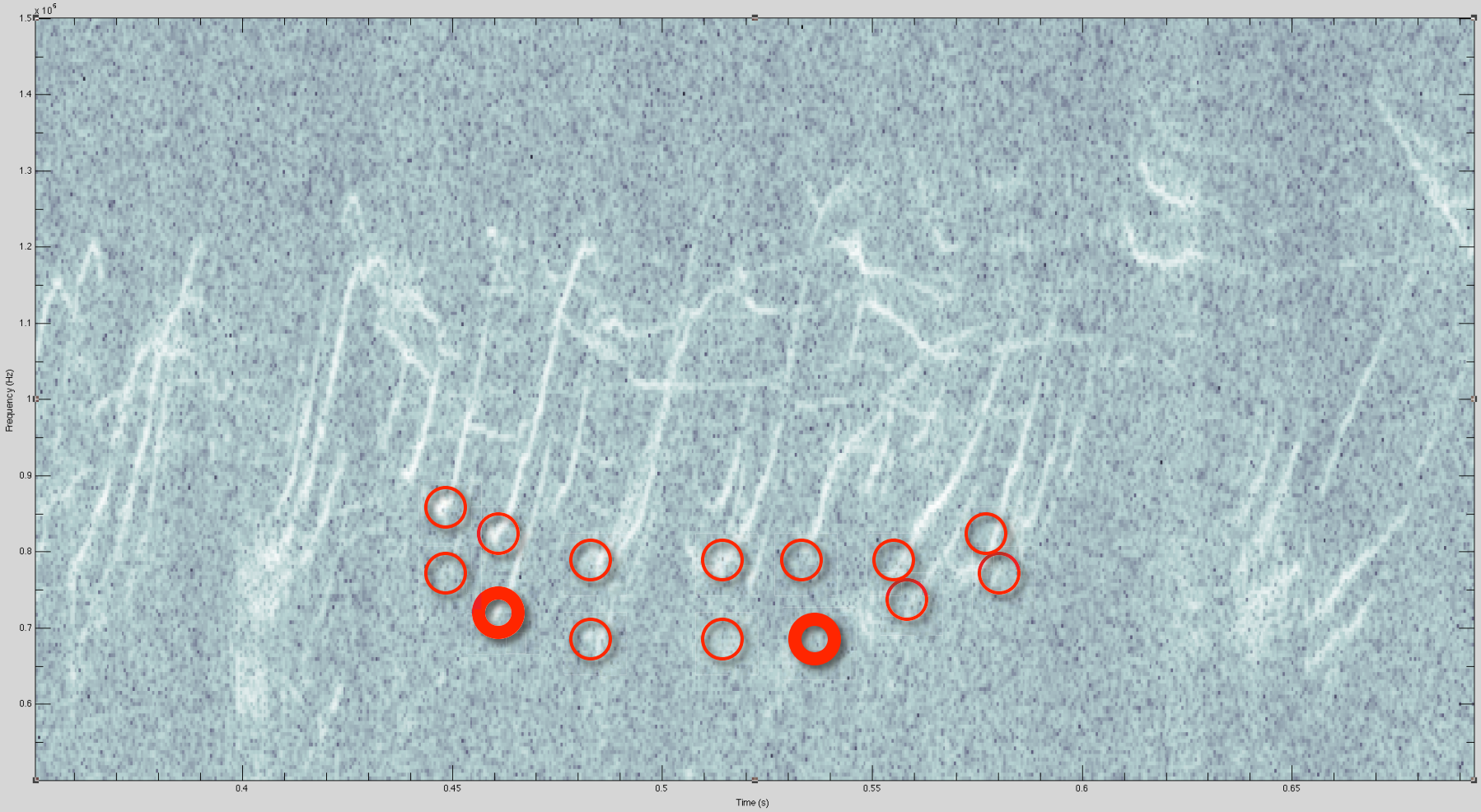


DIII-D NATIONAL FUSION FACILITY

PPPL

D.M.R.C Davis MM-Wave Research Center

Duplicate modes and radial harmonics:

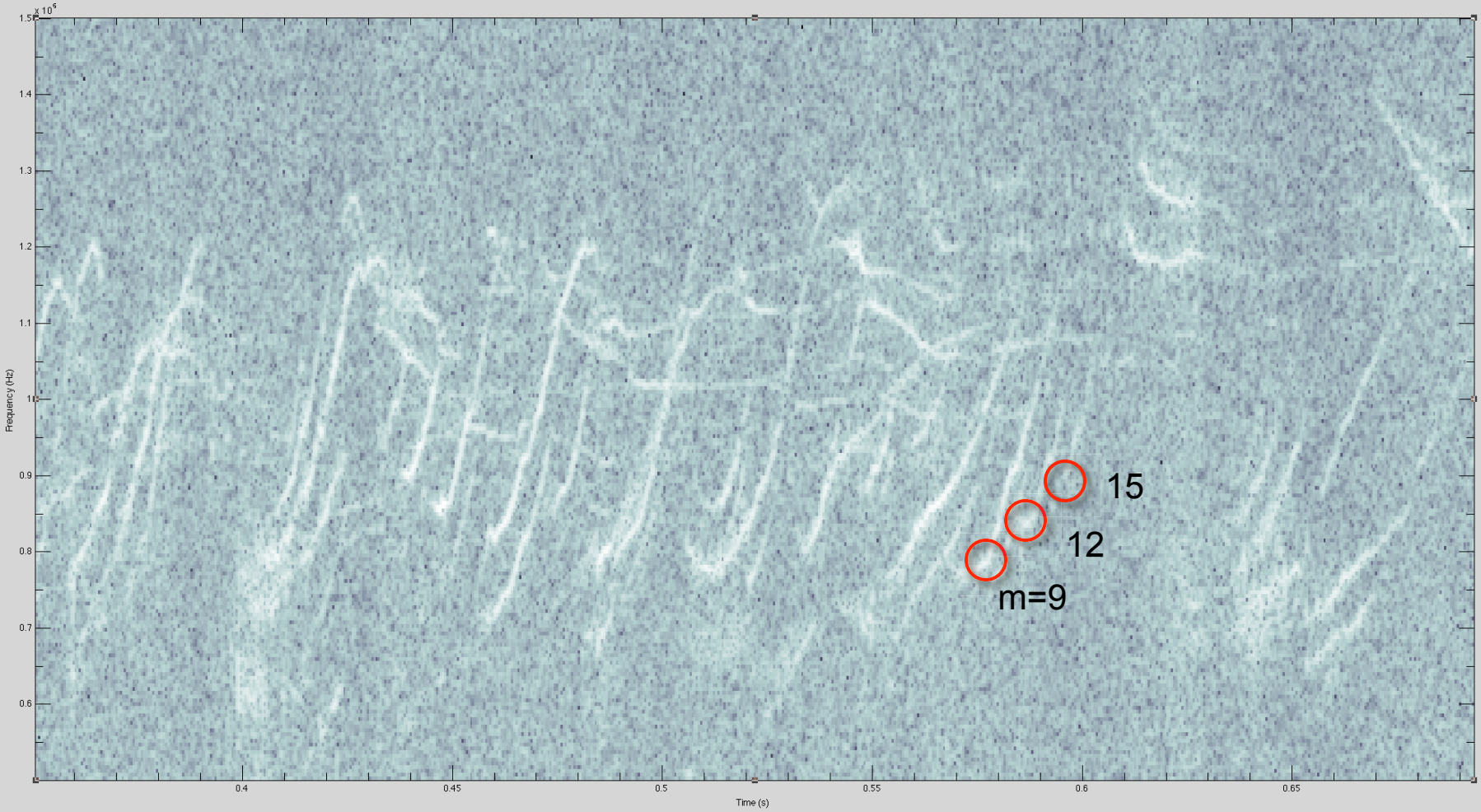


DIII-D NATIONAL FUSION FACILITY

PPPL

D.M.R.C Davis MM-Wave Research Center

Modes that come pretty close:

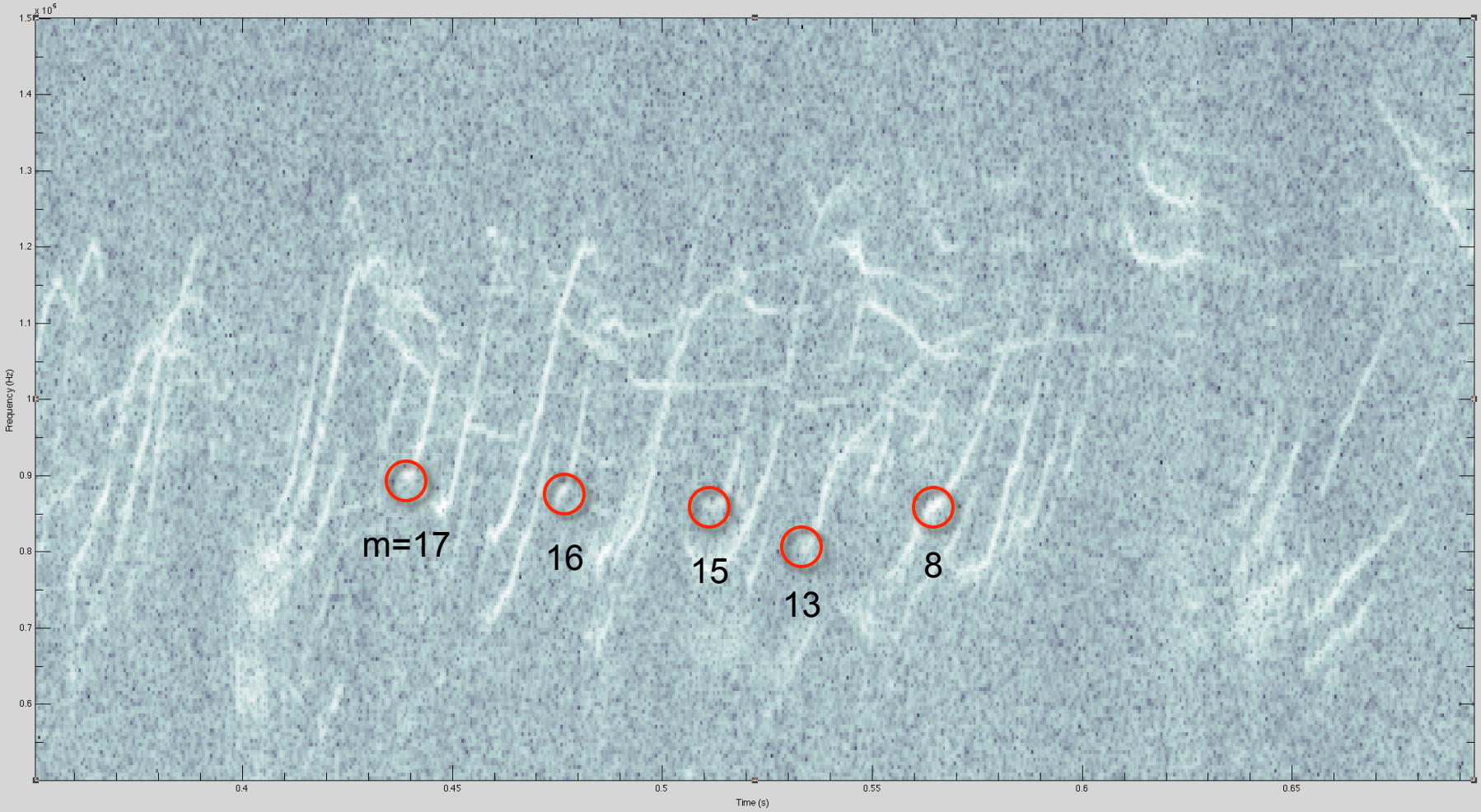


DIII-D NATIONAL FUSION FACILITY

PPPL

D.M.R.C Davis MM-Wave Research Center

Modes that are totally out of left field:

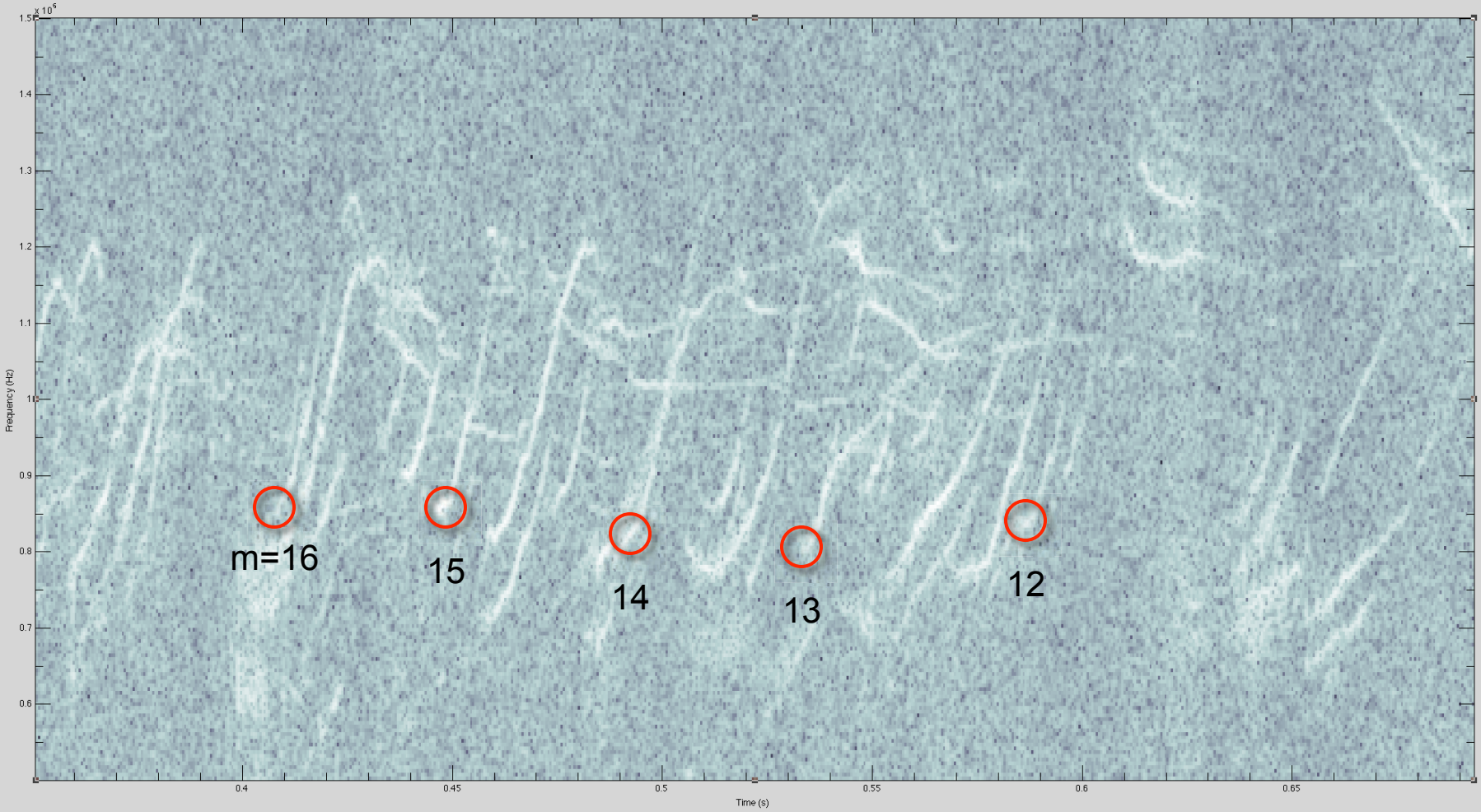


DIII-D NATIONAL FUSION FACILITY

PPPL

D.M.R.C Davis MM-Wave Research Center

A strange pattern that wants to fit so bad:



DIII-D NATIONAL FUSION FACILITY

PPPL

D.M.R.C Davis MM-Wave Research Center

Working on it still…

- **Looking carefully at the MSE data**
  - Uniform discrepancies (calibration), or a discontinuous evolution of q_{min}?
- **Does NOVA, for example, find the unexpected modes, too?**
- **Do I get consistent solutions working backwards from the next q_{min} crossing?**
- **What does this tell me that helps with understanding the BAAE?**