Nonlinear benchmark case on n=1 TAE

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1. Initial equilibrium

The equilibrium considered here is the same as the linear benchmark case on the n=1 TAE constructed by N. N. Gorelenkov (http://w3.pppl.gov/~ngorelen/TAE_lin.html) with an exception for the energetic-particle distribution function. The magnetic surfaces are concentric circular with aspect ratio $R_0/\mathrm{a}=3$ where R_0 is the major radius of the magnetic axis and a is the plasma minor radius. The bulk plasma beta is zero and the plasma density is uniform.

The equilibrium magnetic field is given by

$$\begin{aligned} B_\theta &= -rB_0/q(r)R \\ B_\varphi &= R_0B_0/R \end{aligned} \tag{1}$$

where $q(r)$ is the safety factor. The poloidal magnetic flux $\psi(r)$ takes $\psi(0)=\psi_0$ and $\psi(\mathrm{a})=0$ with $\psi_0 \geq 0$. A normalized radial coordinate y is defined by $y=1-\psi/\psi_0$. The safety factor profile is given by

$$q(y) = q_0 + \Delta q y \tag{2}$$

with $q=q_0$ at $r=0$ and $q=q_0+\Delta q$ at $r=\mathrm{a}$. With this choice of the safety factor profile, the central value of the poloidal magnetic flux is given by $\psi_0 = B_0\mathrm{a}^2/(2q_0+\Delta q)$ and y is related to r by

$$y = \frac{q_0}{\Delta q}\left[\sqrt{1+\left(2\frac{\Delta q}{q_0}+\frac{\Delta q^2}{q_0^2}\right)\left(\frac{r}{\mathrm{a}}\right)^2}-1\right] \tag{3}$$

The parameters q_0 and Δq are chosen $q_0=1.1$ and $\Delta q=1.0$.

2. Energetic-ion distribution

The slowing-down distribution functions are assumed for the energetic ions. The parameters for the distributions are injection velocity $v_h=1.7v_A$, gyro-radius $m_hv_h/Z_heB_0=0.085\mathrm{a}$, and critical velocity $v_c=0.58v_h$. The energetic-ion beta profile is

$$\beta_h(y) = \beta_{h0}\exp(-y/0.37) \quad , \tag{4}$$

where β_{h0} is the energetic-ion beta value at the plasma center. As is described in the following sections, the initial energetic-ion distributions are kinetic equilibria and are not exactly the same as the beta profile mentioned above. The kinetic distributions are normalized so that the total energetic-ion energy is equal to that of the beta profile given by Eq. (4).

3. Linear benchmark on n=1 TAE

Linear properties of the n=1 TAE were investigated with the MEGA code and the results are compared with the NOVA analysis. The energetic-ion distribution function employed for the linear benchmark is given by

$$f_h(v,\mu,P_\varphi) = C\beta_{h0}\frac{1}{v^3+v_c^3}\frac{1}{2}\mathrm{erfc}\left(\frac{v-v_h}{\Delta v}\right)\exp(-z/0.37) \tag{5}$$

with $P_\varphi = Z_h e\psi + m_h R v_{//} B_\varphi / B$ and $v_{//} = \mathbf{v}\cdot\mathbf{B}/B$. The new variable z is related to y for passing particles by

$$\begin{aligned} z &= 1 - \frac{P_\varphi - m_h R_0\,\mathrm{sgn}(v_{//})\sqrt{v^2 - 2\mu B_{\min}/m_h}}{Z_h e\psi_0} \\ &= y - \frac{m_h}{Z_h e\psi_0}\left(R v_{//}\frac{B_\varphi}{B} - R_0\,\mathrm{sgn}(v_{//})\sqrt{v^2 - 2\mu B_{\min}/m_h}\right) \end{aligned} \tag{6}$$

with $B_{\min} = R_0 B_0/(R_0 + \mathrm{a})$. The second term in the bracket in Eq. (6) is introduced to minimize the imbalance between the co-passing and the counter-passing particles. For trapped particles, as they go both in co- and counter-directions, z is defined by

$$z = 1 - \frac{P_\varphi}{Z_h e\psi_0} = y - \frac{m_h R v_{//}\dfrac{B_\varphi}{B}}{Z_h e\psi_0} \tag{7}$$

The complementary error function erfc is introduced in Eq. (5) to model the cutoff at $v = v_h$ and the definition is

$$\mathrm{erfc}(x) \equiv 1 - \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt \tag{8}$$

The width of the cutoff is chosen $\Delta v = 5\times10^{-2}v_A$. The coefficient C is normalized so that the total energetic-ion energy with the distribution given by Eq. (5) is equal to that of the beta profile given by Eq. (4).

The spatial profile of an n=1 TAE simulated with the MEGA code for $\beta_{h0} = 3\times10^{-3}$ is shown in Fig. 1. We see good agreement in spatial profile between the MEGA simulation result and the NOVA analysis. The real frequency in the MEGA simulation is $\omega = 0.310\omega_A$ ($\omega_A = v_A / R_0$) while NOVA gives $\omega = 0.312\omega_A$. However, a discrepancy was found in the linear growth rate dependence on the energetic-ion beta. It is found with MEGA $\gamma/\omega\beta_{h0} \sim 1.7$, while NOVA predicts $\gamma/\omega\beta_{h0} \sim 1$.

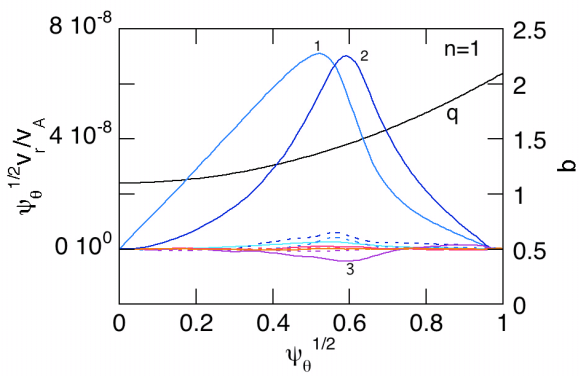

Fig. 1. Spatial profile of n=1 TAE in the MEGA simulation result for $\beta_{h0} = 3\times10^{-3}$. ($\psi_\theta = y$)

4. Nonlinear benchmark on n=1 TAE

The nonlinear evolution of the n=1 TAE was investigated with the MEGA code. The initial energetic-ion distribution is given by Eq. (5) with the variable z defined by Eq. (7) for all the particles. This is the only difference from the linear benchmark simulation in the previous section. In Fig. 2 the spatial profiles of the radial velocity and the radial magnetic fluctuations are shown for $\beta_{h0} = 10^{-2}$. The radial velocity profile is modified from Fig. 1 due to the energetic-ion effect. The linear growth rate dependence on energetic-ion beta value is shown in Fig. 3. The energetic-ion drive dependence on energetic-ion beta is $\gamma/\omega\beta_{h0} = 2.5$. The time evolution of the peak amplitude of the radial magnetic fluctuation is shown in Fig. 4. The saturation level of the radial magnetic fluctuation harmonic m/n=2/1 is plotted versus instability growth rate in Fig. 5. The saturation level can be fitted with a power law

$$\left(\frac{\delta B_{r\,2/1}}{B}\right)_{\text{sat}} = 2.2\times10^{-3}\left(\frac{\gamma}{10^{-2}\omega_A}\right)^{1.9} = 2.3\times10^{-4}\left(\frac{\gamma}{10^{-2}\omega}\right)^{1.9}.$$

The power of 1.9 is consistent with saturation caused by wave-particle trapping for which theory predicts the power of 2. Comparison among the simulation codes on the saturation level and the power law would be important.

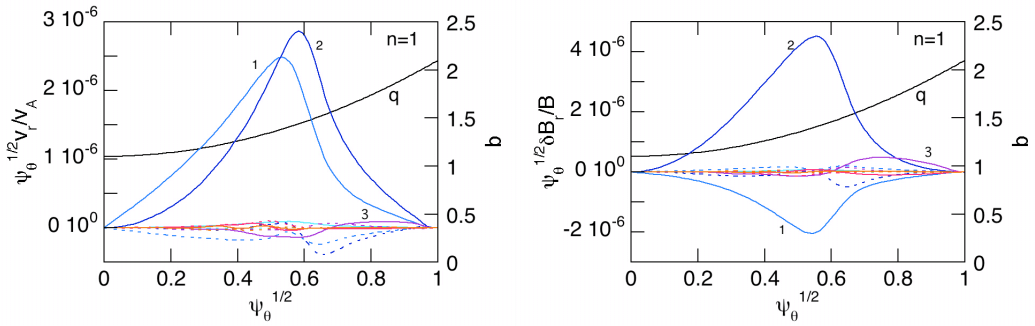

Fig. 2. Spatial profiles of the radial velocity and the radial magnetic fluctuations of the n=1 TAE for $\beta_{h0} = 10^{-2}$. ($\psi_\theta = y$)

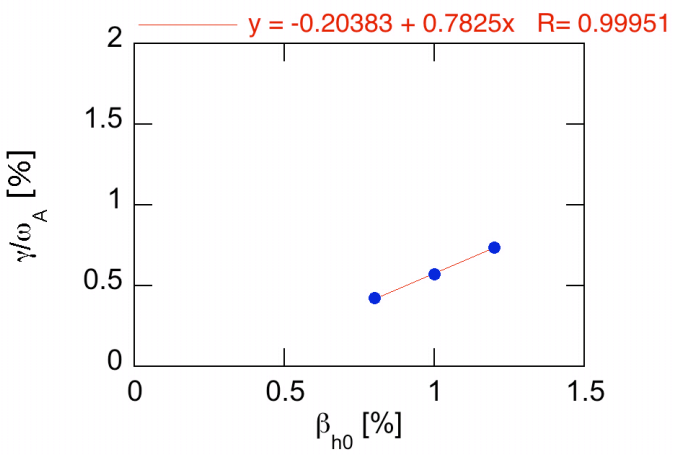

Fig. 3. Growth rate of the n=1 TAE vs. energetic-ion beta value.

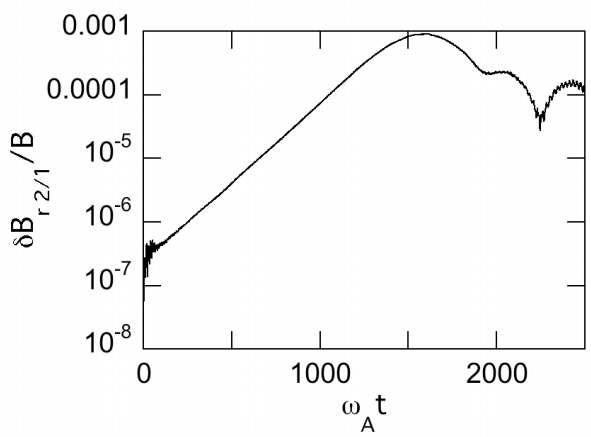

Fig. 4. Time evolution of the radial magnetic fluctuation harmonic m/n=2/1 at the peak location for $\beta_{h0} = 10^{-2}$.

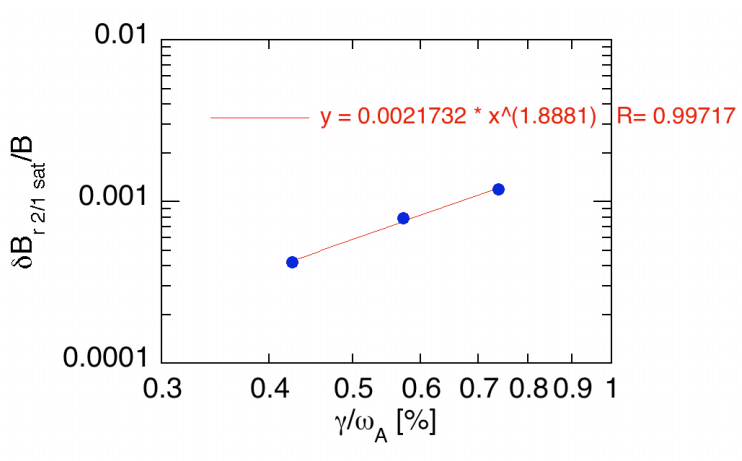

Fig. 5. Saturation level of the radial magnetic fluctuation harmonic m/n=2/1 vs. instability growth rate.