

# Beam particle distribution modification by low amplitude Modes

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*DIID*

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## Motivation

- Recent observations on DIII-D show a strong modification of the high energy beam particle distribution from that predicted by TRANSP.
- TAE and RSAE modes are present, but with very low amplitude,  $dB/B = 2 \times 10^{-4}$ . The high energy beam profile is flattened significantly.
- This effect could result in TAE modification of the alpha profile in ITER

## We have examined the effects of

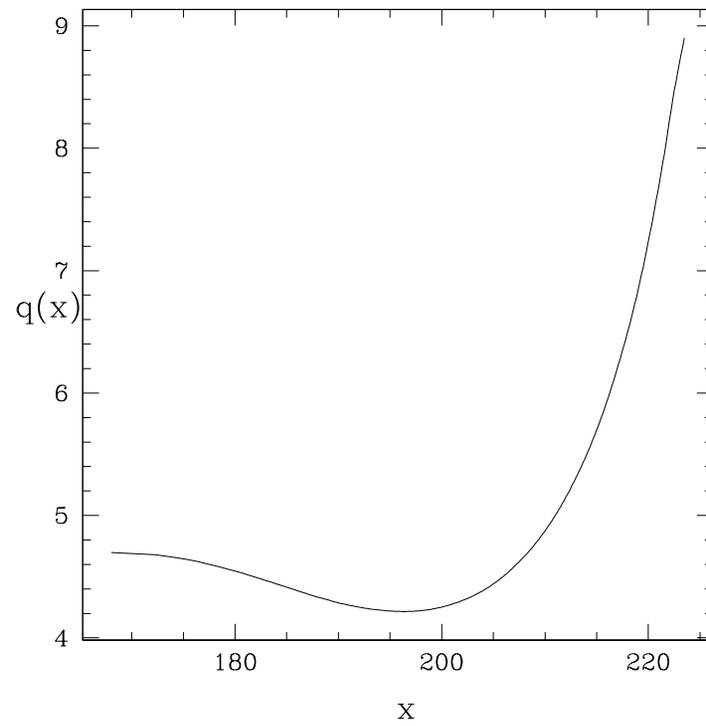
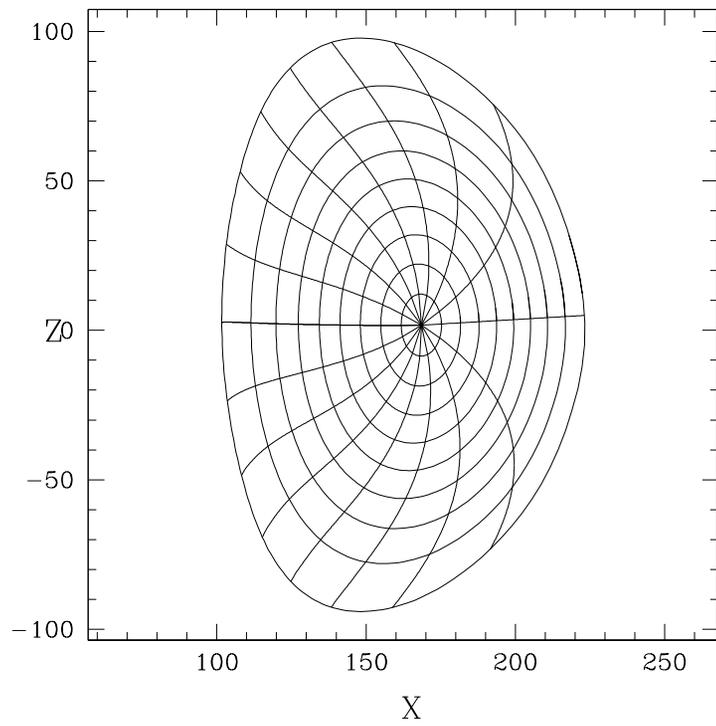
- Time dependence of the  $q$  profile
- Frequency chirping of modes
- Polarization potential
- Compression effects
- Mode spectrum and amplitude

## Tools for the study included

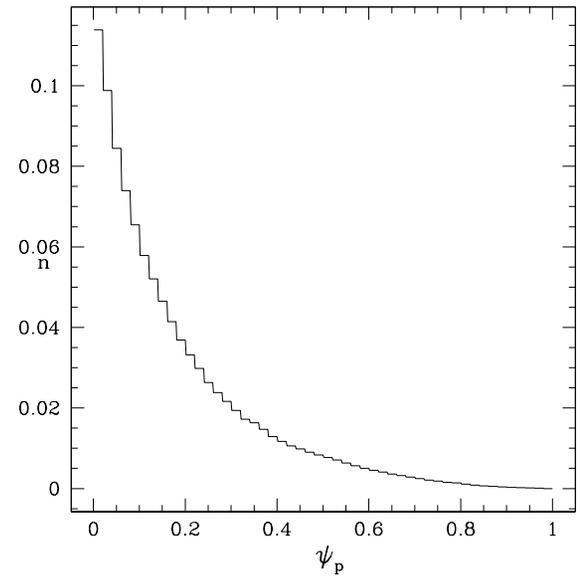
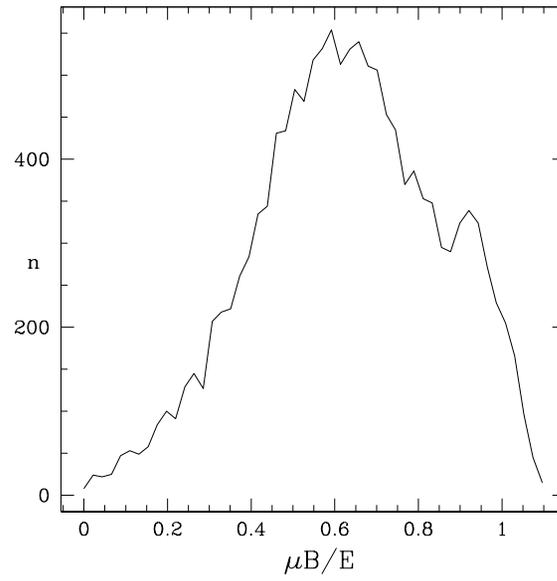
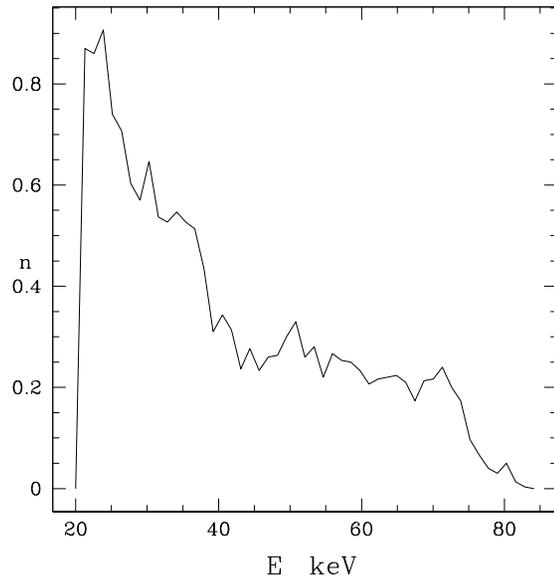
- Kinetic Poincaré plots showing resonances in  $\psi, \theta$
- Energy transfer plots in the plane of  $P_\zeta, \mu, E$ .
- Distribution modification calculations using full spectrum of modes

# DIID Equilibrium

$$B_0 = 20.3Kg$$



# Beam Distribution from TRANSP in $E$ , $\mu B_0/E$ , $\psi_p$



## Flute like perturbations, no compression

$$\delta \vec{B} = \nabla \times \alpha(\psi_p, \theta, \phi) \vec{B}_0, \quad \alpha = \sum_{m,n} \alpha_{mn}(\psi_p, \theta) \sin(n\phi - m\theta - \omega t).$$

## Resonance using large aspect ratio circular approximation

$$[n - m'/ql]\omega_t = \omega, \quad q = \frac{m'/l}{n - \omega/\omega_t},$$

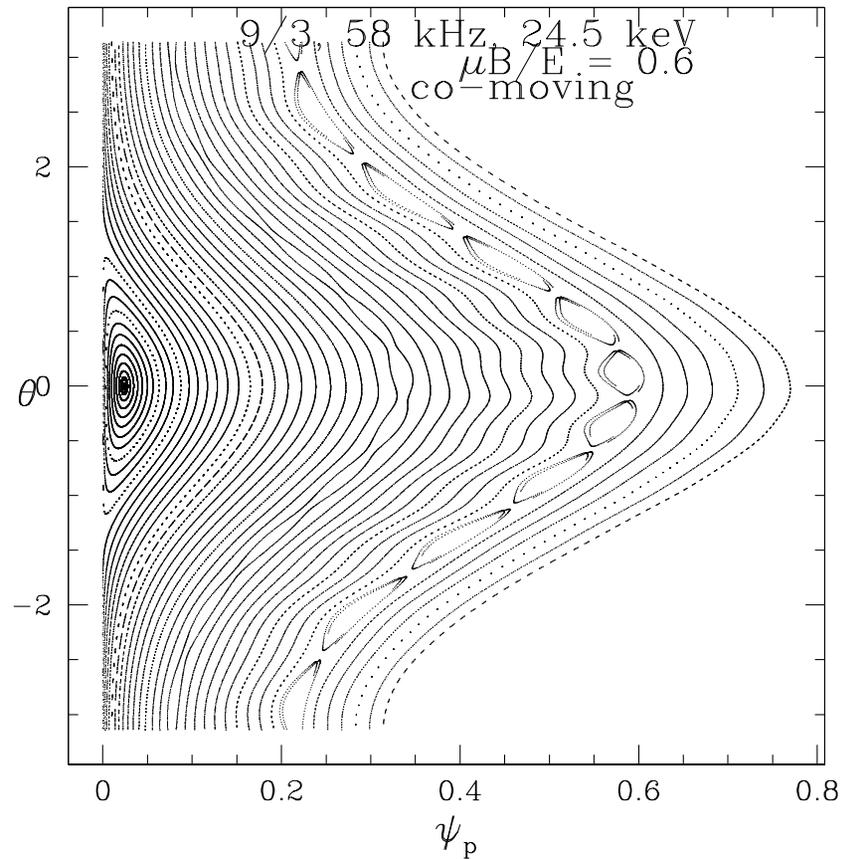
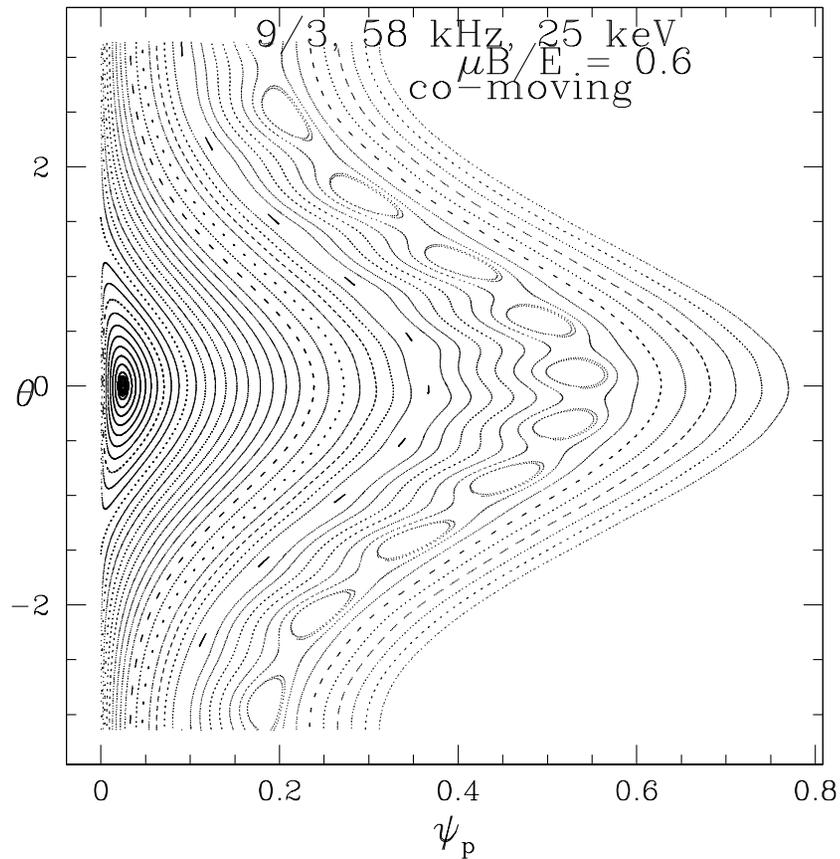
$\omega_t$  is the transit frequency.

Resonance appears when there exist integers  $m', l$  such that this relation is satisfied.

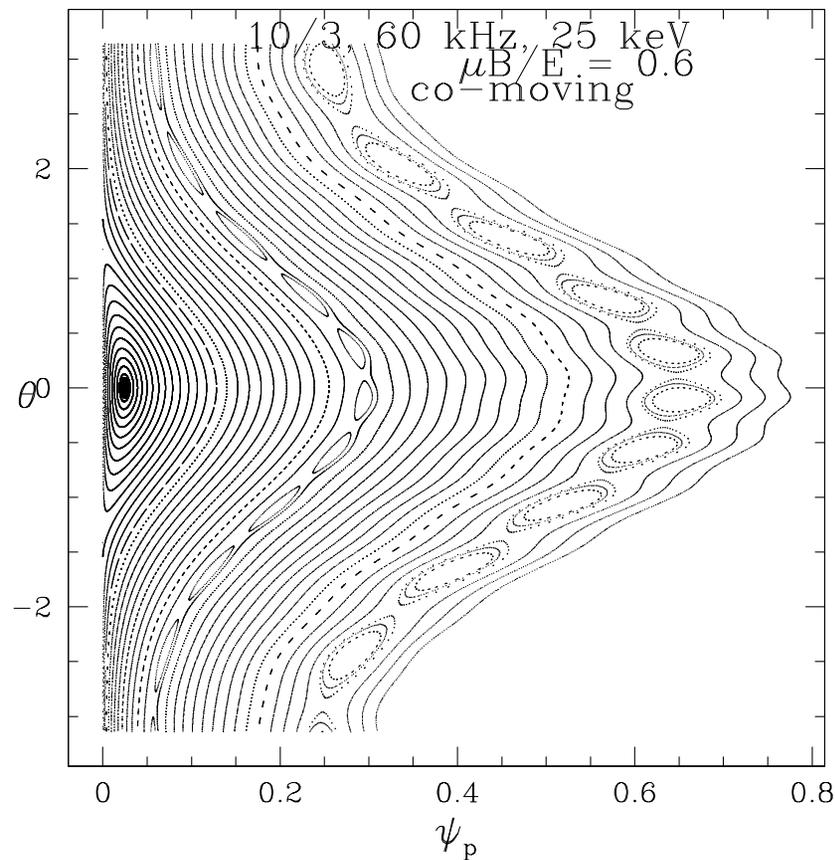
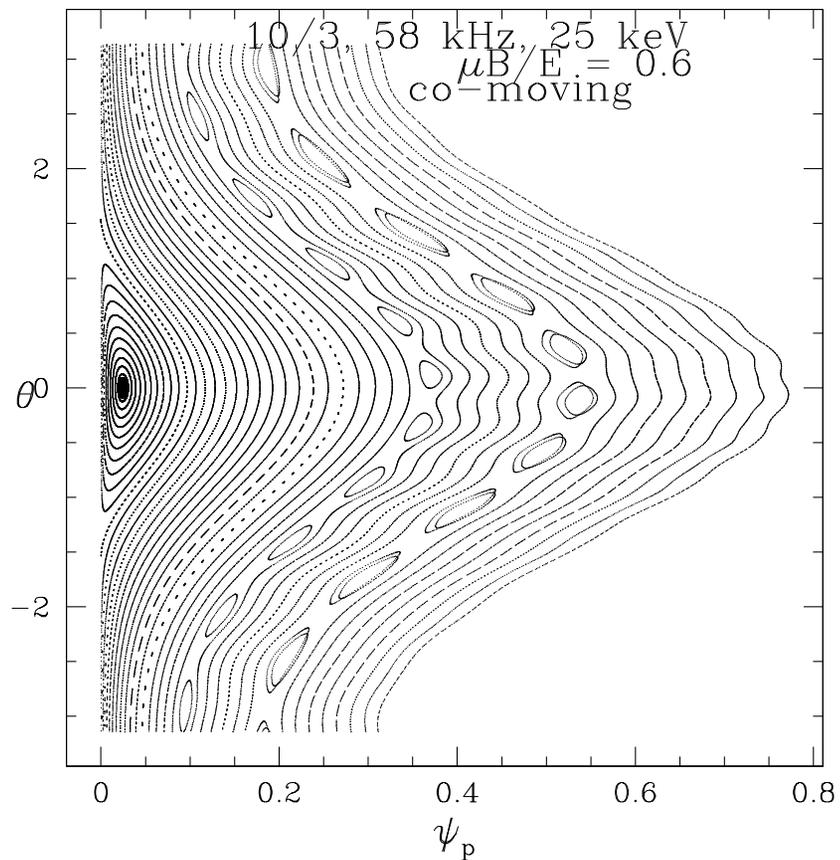
For co-moving ( $\omega_t > 0$ )

- increasing  $E$  gives resonance at smaller  $q$
- increasing  $\omega$  gives resonance at larger  $q$

Poincaré plot - distribution with a fixed value of  $\mu$  and  $\omega P_\zeta - nE = c$ .

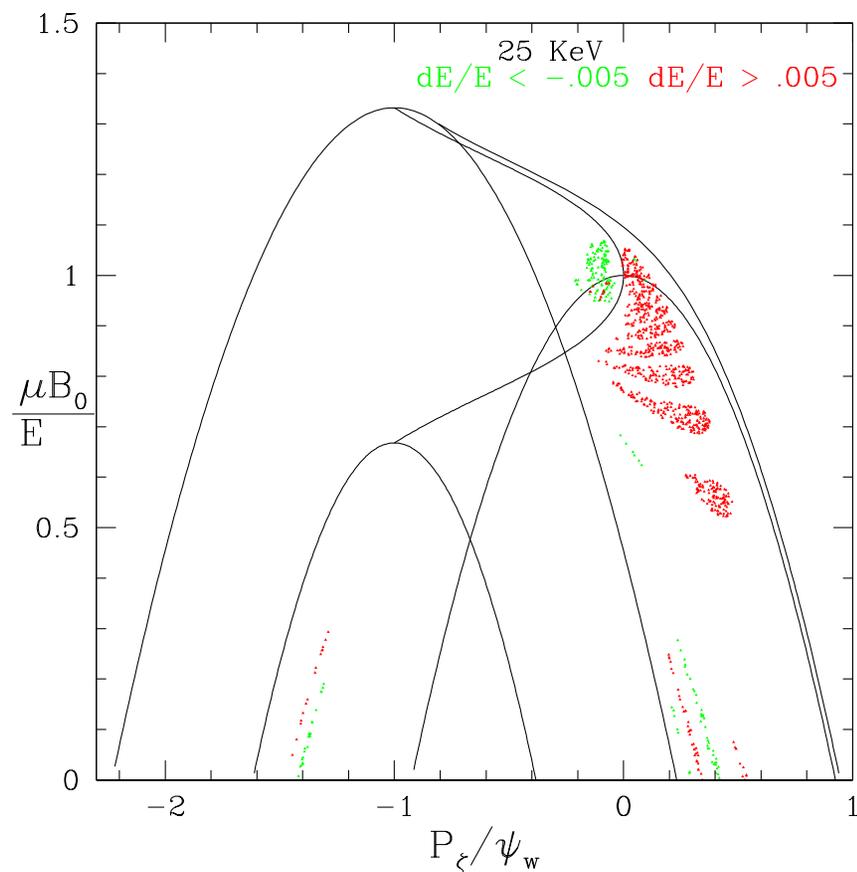
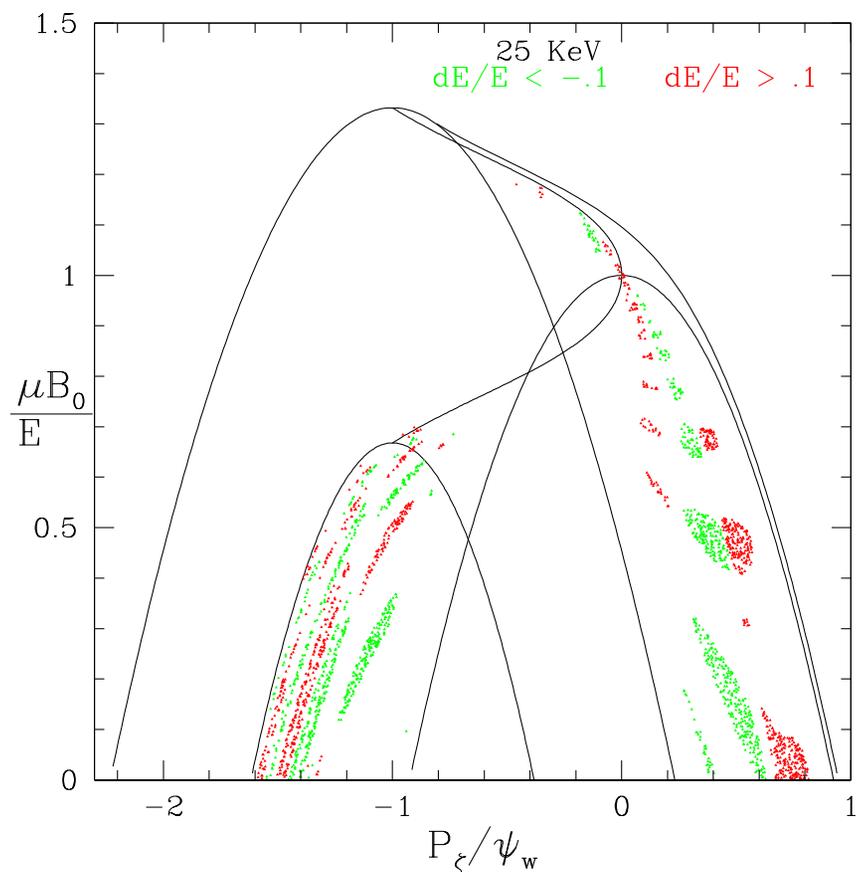


**Kinetic Poincaré plots for mode  $m/n = 9/3$ , showing energy dependence of the  $m' = 10$  resonances. Increasing  $E$  moves to smaller  $q$**



**Kinetic Poincaré plots for mode  $m/n = 10/3$ , showing frequency dependence of  $m' = 10$  resonances. Increasing  $\omega$  moves to larger  $q$**

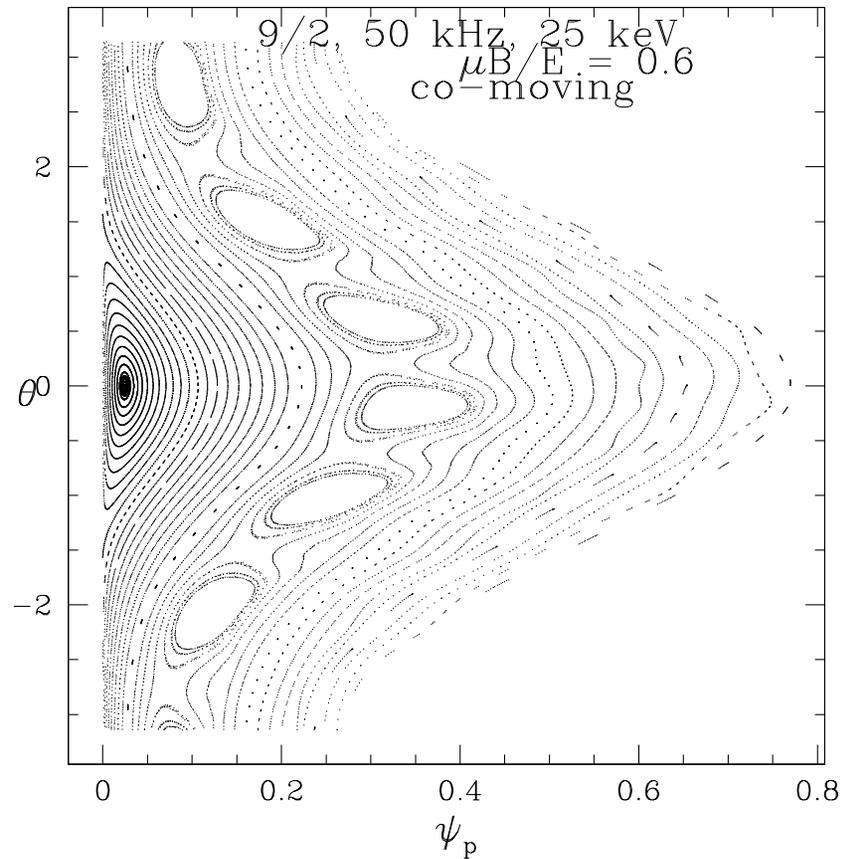
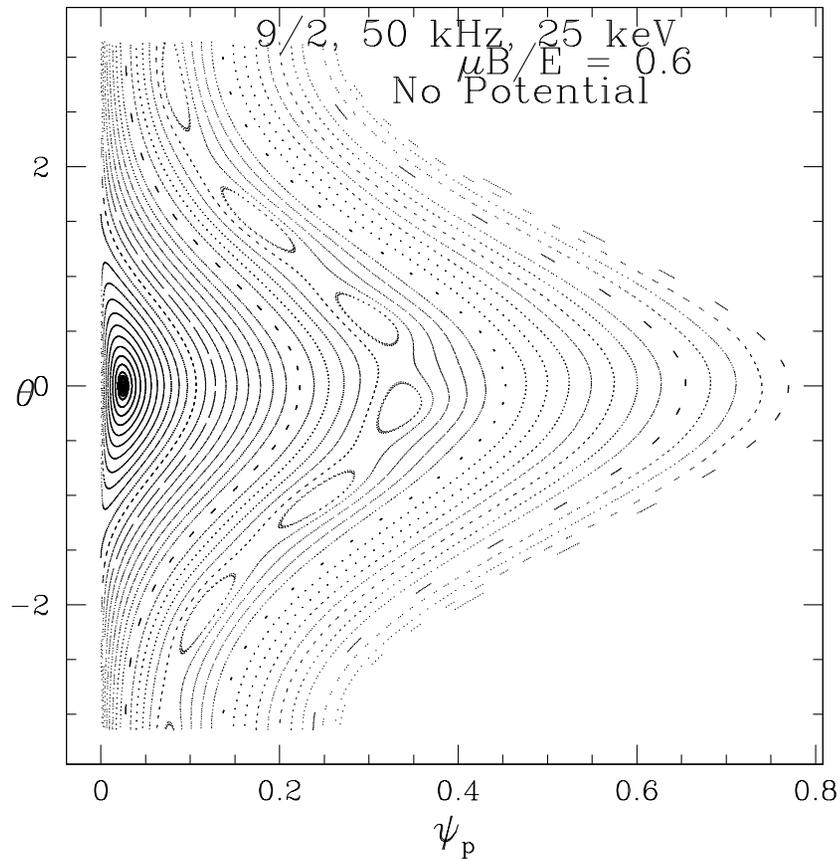
**Note resonant surface unchanged from the  $m = 9$  mode for 58 kHz.**



**A plot of the  $P_\zeta, \mu$  plane, showing only 25 keV particle orbits with energy loss or gain due to the mode, for a  $m/n = 9/2$ , mode at 90 kHz (left, showing 10 percent change) and a  $m/n = 10/3$  mode at 90 kHz (right, showing only 0.5 percent change). Both the co and counter moving parts of the distribution are shown.**

Electric field to cancel  $E_{\parallel}$ ,  $\Phi = \sum_{m,n} \Phi_{m,n} e^{i(n\zeta - m\theta - \omega t)}$

$$(gq + I)\omega\alpha_{m,n} = (nq - m)\Phi_{m,n},$$

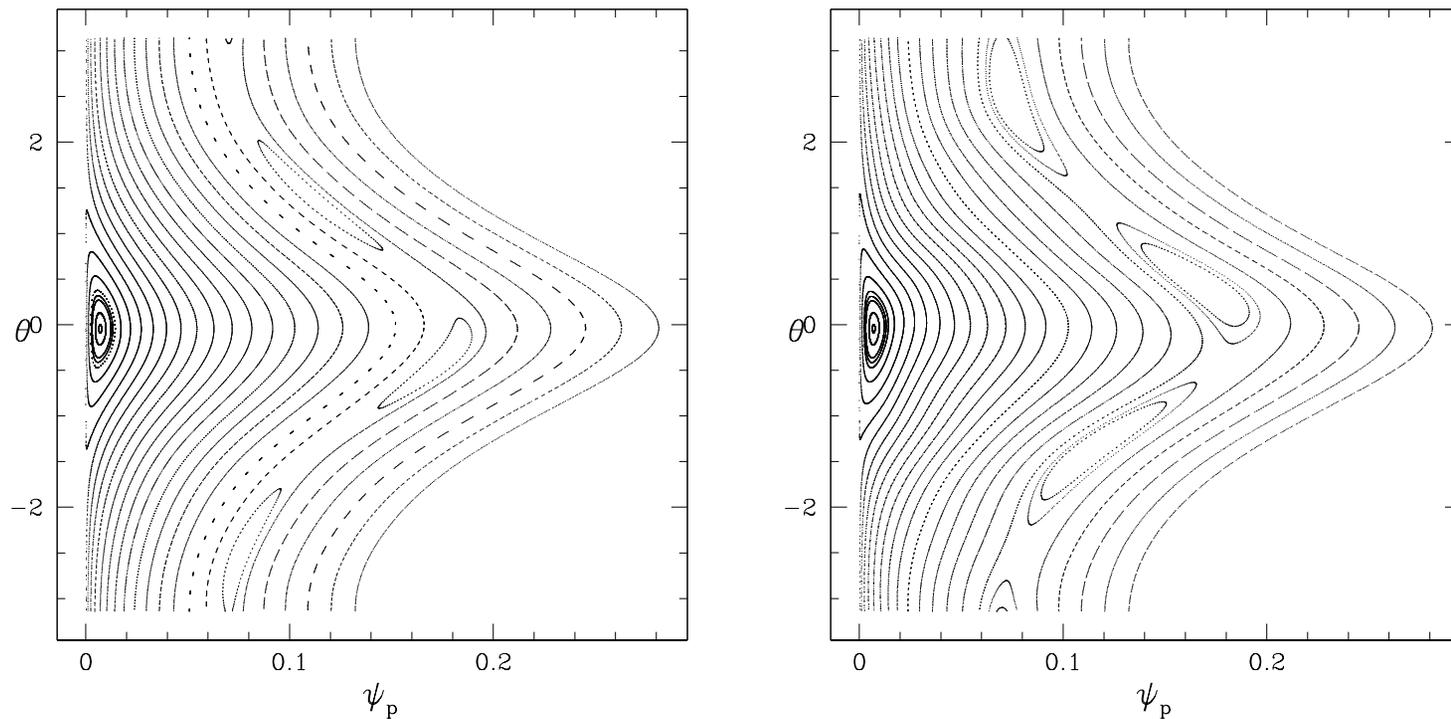


**Kinetic Poincaré plots for mode  $m/n = 9/2$ , showing the effect of the potential on 23 Kev beam particles for a 50 kHz mode.**

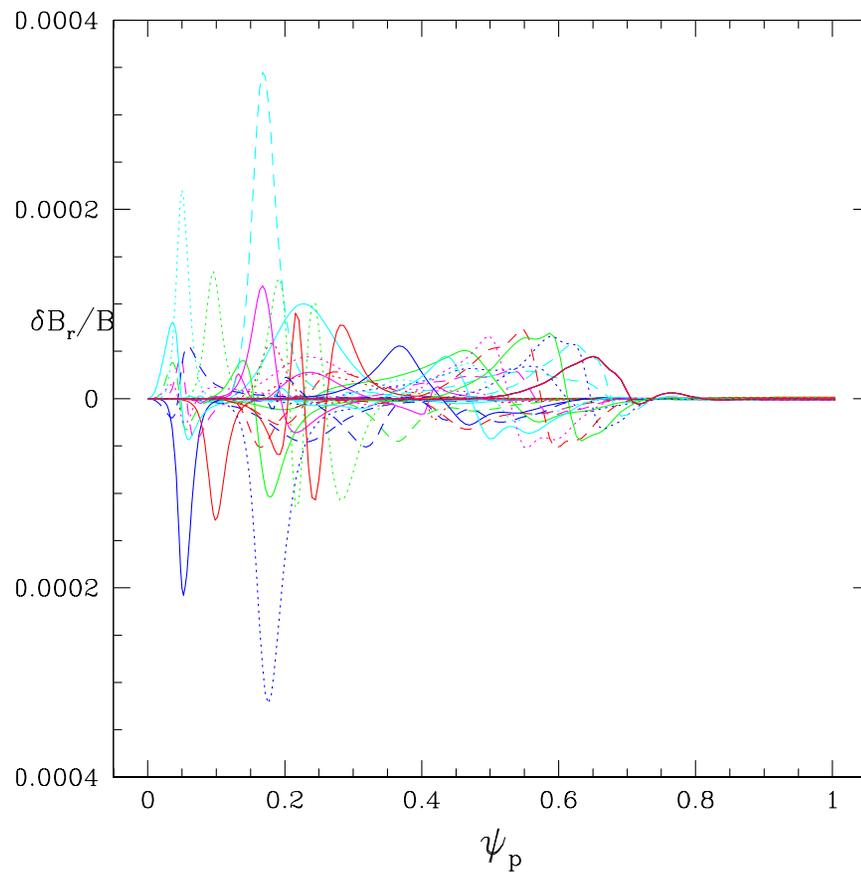
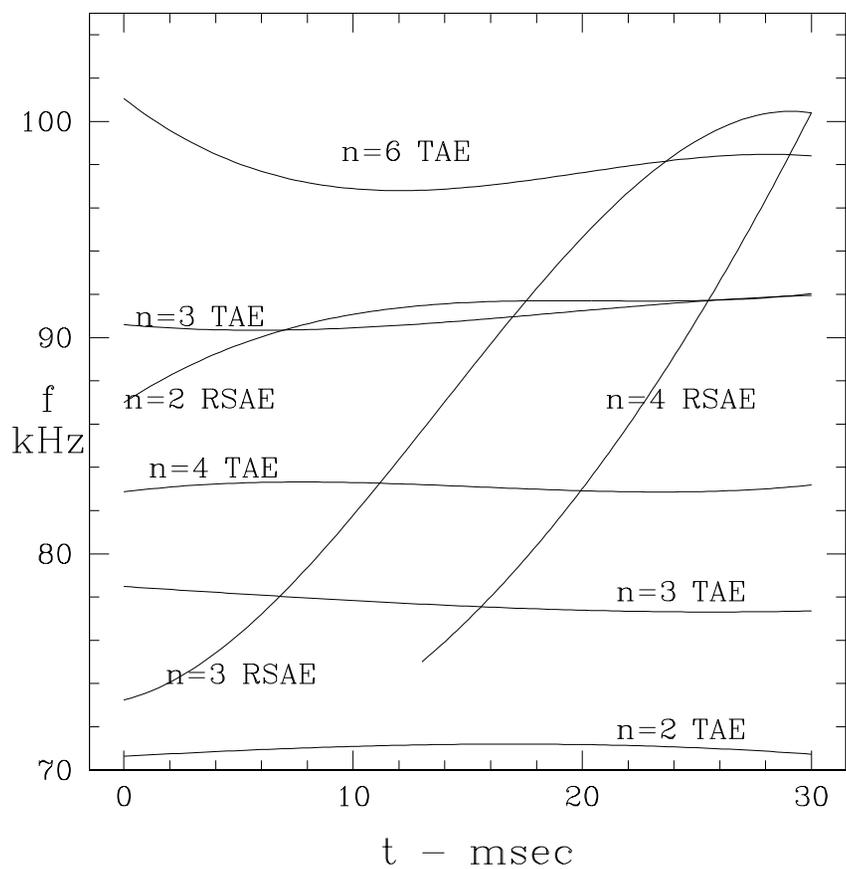
## Compressional Waves

Modified Guiding center equations derived

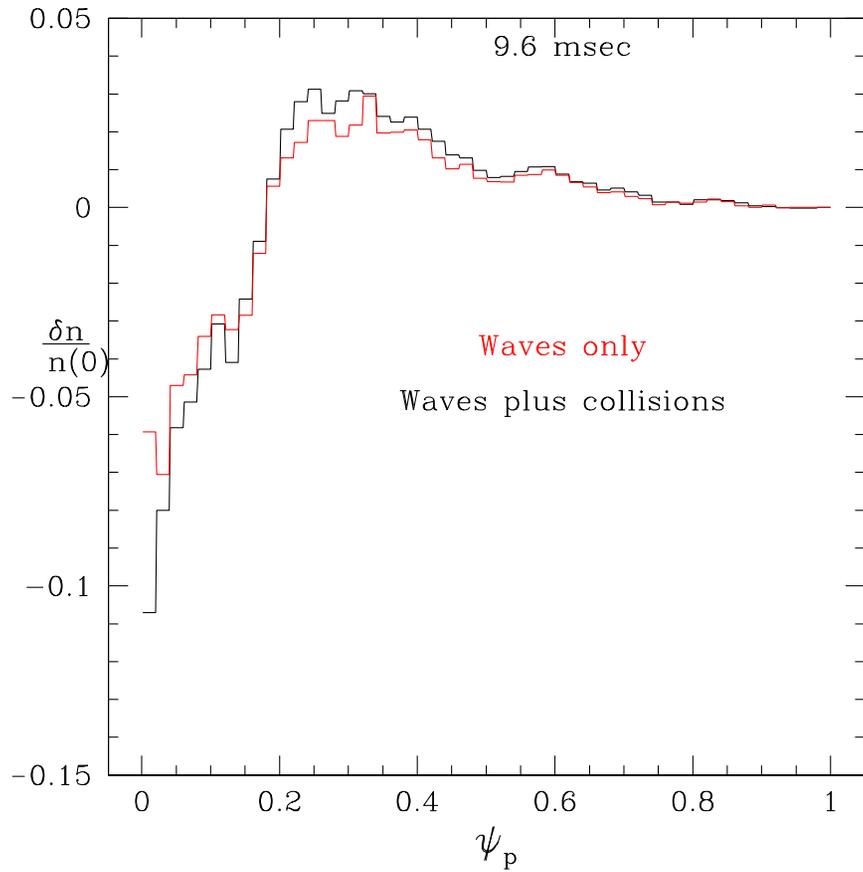
Numerically find that the only modification is due to the  $\delta B_{\parallel}$  contribution.



Kinetic Poincaré plots for mode  $m/n = 5/1$ , showing the effect of compression on 23 Kev beam particles for a 20 kHz mode.

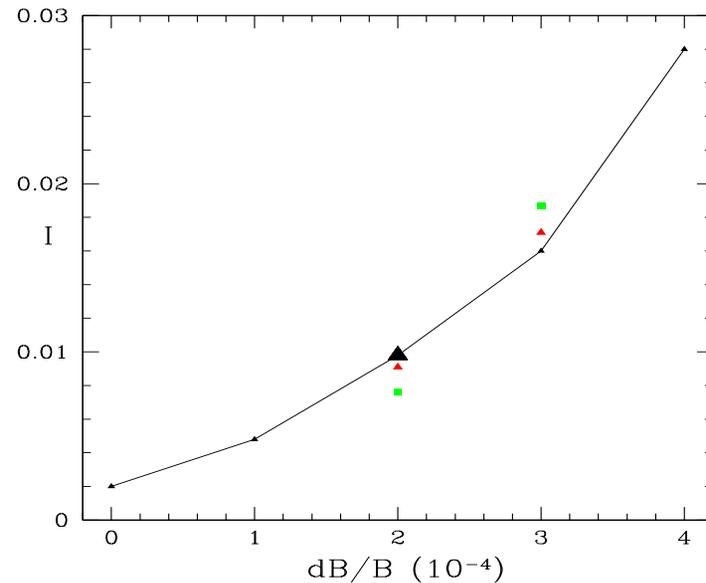
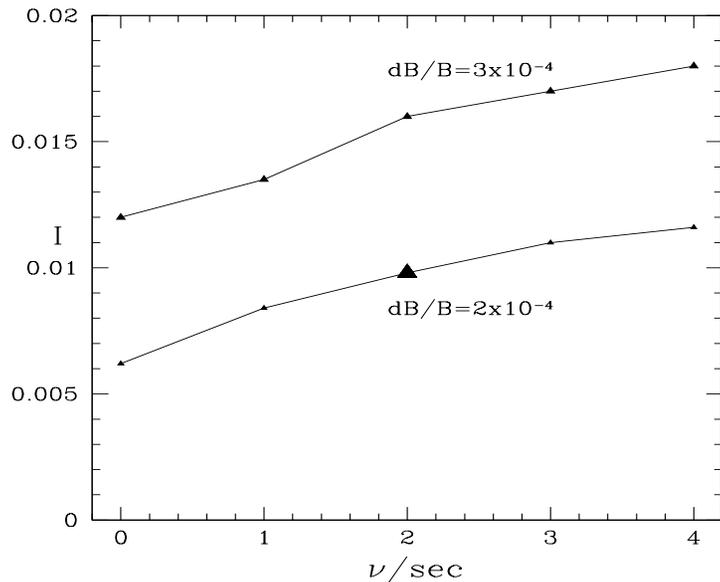


**Experimentally observed modes, showing time dependence of frequencies  
 133 harmonics used in the simulation with  $\delta B_r / B = 2 \times 10^{-4}$ .  
 Eigenfunctions produced by NOVAK**



### Effect of modes, with and without collisions.

For all values of collisionality and mode amplitude, the beam distribution modification is similar in shape, and changes are approximately linear in time, indicating that a diffusion formalism is appropriate.



## Scaling with mode amplitude and collisions

To compare the effect of perturbations with different collisionality and amplitude, introduce the mean total distribution shift, through

$$I = \frac{\int |n(\psi_p) - n_0(\psi_p)| d\psi_p}{n_0(0)}$$

Scaling with collisions and mode amplitude at a time of 4.8 msec

Note island width is  $w \sim \sqrt{\delta B}$  so should produce diffusion  $D \sim \delta B$  but it is stronger than linear. As islands begin to overlap  $D$  increases rapidly.

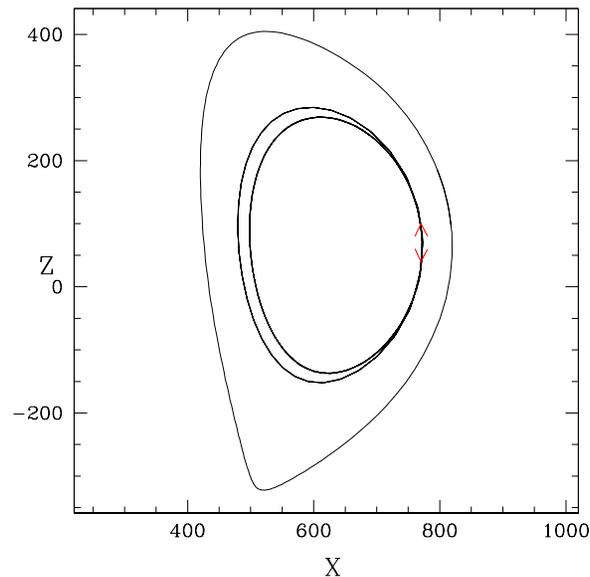
Colored points give the small effect of (two RSAE) mode sweeping.

## Diffusion Operator for TRANSP

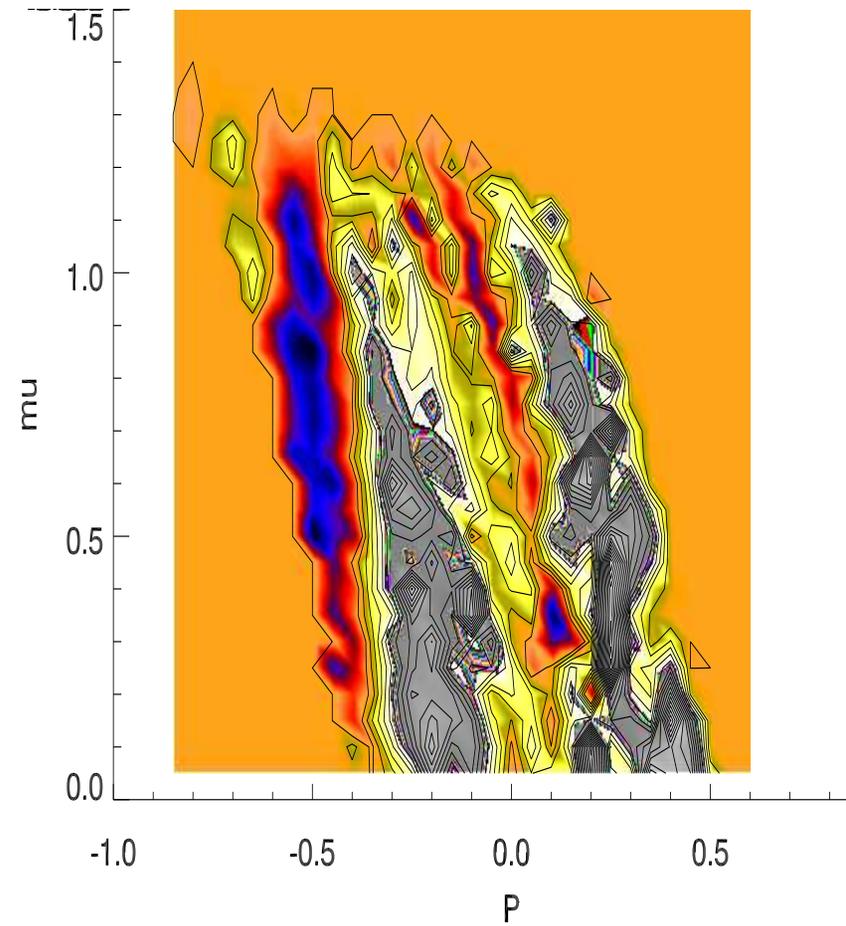
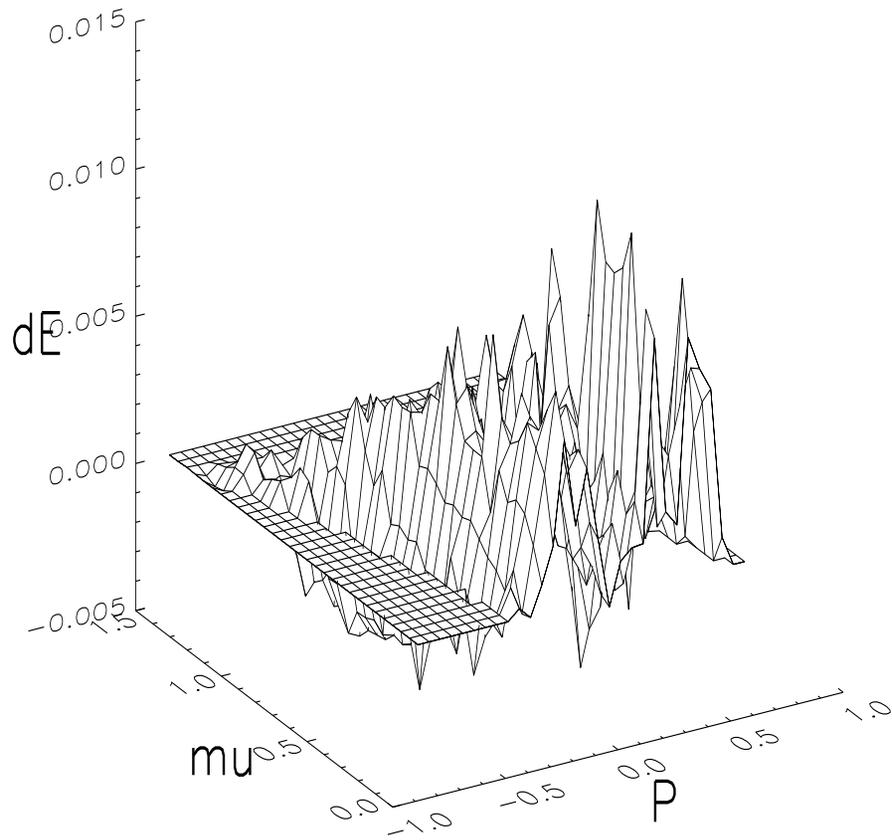
Assume an evolution equation  $df/dt = S + C + W$

$S$  is the source term,  $C$  Coulomb collisions,  $W$  wave-particle interactions.

Since all orbits determined by  $P_\zeta, \mu, E$  take  $f = f(P_\zeta, \mu, E)$  and formulate diffusion operator in this space. Flux surface is NOT a good quantity for energetic particles.



Co- and Counter alphas in ITER



**Energy change  $dE/E$ , 133 harmonics, 25 KeV, all co-moving orbits, 68  $\mu\text{sec}$  run**

## Wave Diffusion Operator

1. Produce beam distribution from TRANSP
2. Use NOVAK to find mode spectrum, amplitudes?
3. Run ORBIT  $\Delta t$  to find time averages of changes in  $P$ ,  $E$
4. Assume diffusion linear in time, step particles in  $P$ ,  $E$  for  $N \times \Delta t$
5. Reconstruct new beam distribution

## Conclusion

- A spectrum of TAE and RSAE modes can significantly modify the beam particle distribution, even with amplitudes of the level of  $dB/B \simeq 2 \times 10^{-4}$ . The simulated rate of profile modification is roughly linear in time and approximately agrees with the experimentally observed changes. This points to a diffusion formulation.
- The relevant factor is the presence of islands in the particle phase space, for particles at energies and pitches characteristic of the distribution.
- If two island chains are in close proximity, stochastic orbits result, but islands provide particle excursions from the initial drift surfaces, and collisions add to the transport.
- The time evolution of the mode frequencies assists a little by causing the resonance surfaces to move through the plasma volume but bucket transport (strong time dependent frequencies) is not significantly operative in the case studied.
- Modification of the  $q$  profile in time is not an important factor, except insofar as it determines the time dependence of the mode frequencies.
- Decreasing the spectrum by eliminating the smallest harmonics significantly decreases the effect of the modes. It appears that near overlap of islands is important.
- For a spectrum of TAE modes Orbit can find the diffusion functions  $D_E$  and  $D_P$   
$$\partial f / \partial t = D_P(P_\zeta, \mu, E) \partial f / \partial P_\zeta + D_E(P_\zeta, \mu, E) \partial f / \partial E.$$